Long Hedgers and Multiple Delivery Specifications on Futures Contracts

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Virtually all futures contracts provide sellers with various delivery options to ease their concerns of possible long squeezes as maturity dates near. With the exception of Livingston (1987), the literature seems to conclude delivery options have a nonzero value. (See Garbade and Silber, 1983; Gay and Manaster, 1984; Kane and Marcus, 1988; Boyle, 1989; Hedge, 1989; Boyle and Tse, 1990.) Moreover, it is argued that the price of a futures contract must be less than the price of a comparable forward contract, which usually does not provide any delivery options. By the same reasoning, a broad-based futures contract is priced below one that is narrow-based (i.e., a futures contract with the option of delivering multiple grades has a lower price compared to a contract that permits the delivery of a single grade).

This price difference stems from two sources. Contracts that offer delivery flexibility tend to be more attractive to sellers. Yet such contracts subject buyers to increased delivery risk; that is, the possibility of taking delivery of a grade not matching commercial needs. This makes them less attractive to buyers. In an efficient market, the difference between the value of the broad- and narrow-based contracts should reflect the expected value of the options. While Barnhill and Seale (1988) show that the T-bond futures market does not properly evaluate the delivery options, Kamara (1990) suggests the CBOT soybean futures market is efficient when the delivery options are explicitly accounted for.

Even though delivery options are costly to the short hedger (since the futures price is depressed when delivery options are included), it is commonly believed that they prefer the broad-based contract. Similarly, it is widely held that long hedgers prefer the narrow-based contract. Some real-world examples of these preferences are documented by Working (1954), Paul (1987), and Shaviro (1987).

However, an alternative view offered by Johnson (1957), and extended by Lien (1988), contends that the short hedger may prefer the narrow-based contract. More specifically, Lien considers a short hedger who possesses two grades, A and B, of a particular commodity. For a narrow-based contract that deals exclusively with A, the hedger establishes a perfect hedge on A and a cross hedge of B. In case of a broad-based contract, both grades are deliverable; hence, the hedger may hedge A and B directly, but imperfectly. That is, the broad-based contract improves the hedging effectiveness of B at the cost of the hedging effectiveness of A. The trade-

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off may go either way and, consequently, so may the hedger's contract preference. Lien (1988) provides conditions under which the short prefers a broad-based contract to a narrow-based contract.[1] Leaving aside the potential problem of a squeeze or a manipulation, a narrow-based contract may actually best serve the interest of short hedgers. The preferred contract specification, at least from the viewpoint of the short side, depends upon certain industry and hedger characteristics, e.g., storage costs, the correlation between the two spot prices, and risk preference.

This article attempts to illustrate a similar conclusion for long hedgers. A model with the long hedger purchasing futures contracts to hedge input demand is presented. Given sufficient flexibility in the production process, it is shown that the long hedger will choose a broad-based contract over a narrow-based contract. Conventional wisdom that the short prefers a broad-based contract and the long a narrow-based contract may not be applicable. When determining the contract specification, the industry characteristics of both long and short hedgers must be taken into account.

A SIMPLE EXAMPLE

The example is motivated by Ware and Winter (1988). In discussing the case of hedging risks in multiple foreign currencies, they show that an option may be a more effective instrument than a forward contract. The multiple delivery specification of those contracts embeds an implicit delivery option. Using the same logic, examples may be provided to demonstrate the possiblity that long hedgers choose a broad-based contract over a narrow-based contract.

More specifically, consider a producer employing two different grades of a particular commodity as inputs. The production technology is assumed to be linear and the two inputs are perfect substitutes. The derived demand functions for each input are

$$D_1 = Q^o/k \quad \text{if } P_1 < P_2; \quad \text{and} \quad 0 \text{ otherwise} \tag{1a}$$

$$D_2 = Q^o/k \quad \text{if } P_2 < P_1; \quad \text{and} \quad 0 \text{ otherwise} \tag{1b}$$

where P_i is the price of the ith grade (for the producer); D_i is the quantity demanded for the ith grade, $i = 1, 2$; k is the production coefficient such that one unit of input will produce k units of output. Finally, Q^o is the predetermined output level due, possibly, to the availability of other fixed inputs. Let P denote the unit output price, then unit profit is $P - (\min(P_1, P_2)/k)$. Suppose the producer is a price taker in all markets, and that production takes place in the future (holding k constant). A broad-based futures contract allowing delivery of either input is preferred to a narrow-based contract on either grade since it provides a perfect hedge.

The above example may be unrealistic; nonetheless, it illustrates the main ideas of this study. That is, if the long hedger's technology allows sufficient "production flexibility," then the long hedger's preference for the broad-based contract can readily be established. This ordering is ensured by the assumption that the two grades are perfect substitutes. Consider now a more realistic case of imperfect substitutes.

[1]If a hedger deals exclusively in a specific grade, then the most preferred contract specifies it as the only deliverable grade. In lieu of the ideal contract, a narrow-based contract that does not call for the delivery of the hedger's "specific grade" is less preferred compared to the broad-based contract that includes it as one of the deliverable grades. The conclusions derived in this article regarding deliverable grades also apply to the case of delivery points. Nonetheless, most hedgers have their business at one location. This may help explain the possible long list of delivery points in several commodity futures contracts, particularly when the delivery cost is high.

THE BASIC MODEL

Let $t = 0$ denote the current time. Production is anticipated to take place at $t = 1$. Again, let P_1, P_2, and P denote the input and output prices at $t = 1$, respectively. These prices are unknown at $t = 0$. The output level, y, is determined by $y = F(x_1, x_2)$. The function, $F(x_1, x_2)$ is the regular quasi-concave neoclassical production function, where x_i is the input level of ith grade, $i = 1,2$. Assume there exists a futures contract on the inputs with maturity at $t = 1$. Let f_0 and f_1 denote the futures price at $t = 0$ and $t = 1$, respectively. Due to uncertainty about prices, the producer may sell or purchase futures contracts for hedging purposes. Let V denote the hedger's futures position, with $V < 0$ indicating a short position, and $V > 0$ a long. The hedger's profits at $t = 1$ are given by

$$\pi = PF(x_1, x_2) - P_1x_1 - P_2x_2 + (f_1 - f_0)V \quad (2)$$

Profit maximization leads to the choice of (x_1^*, x_2^*) with the following properties

$$P \cdot \partial F(x_1, x_2)/\partial x_i = P_i; \qquad i = 1,2 \quad (3)$$

when evaluated at (x_1^*, x_2^*). One can alternatively write the derived demand functions as

$$x_i^* = h_i(P_1/P, P_2/P), \qquad i = 1,2 \quad (4)$$

The corresponding profit level is

$$\pi^* = PF(x_1^*, x_2^*) - P_1x_1^* - P_2x_2^* + (f_1 - f_0)V$$

While production takes place at $t = 1$—when prices P, f_1, P_1, and P_2 are known—the producer must take a futures position prior to actual production. That is, the optimal purchase of contracts is determined at $t = 0$, prior to the realization of the aforementioned (future) prices. It is assumed that prices (P, P_1, and P_2) at $t = 0$ are characterized by a joint density function. Arbitrage arguments imply that although f_1 is unknown at $t = 0$, its probability density function can be inferred from the joint density of (P, P_1, P_2). Moreover, since π^* is a function of these prices, its density function can be derived. In other words, at $t = 0$, when the producer chooses the futures position, π^* is a random variable with a known density function.

It is assumed that the producer is an expected utility maximizer. Thus, the futures position, V, is selected to maximize expected utility, $Eu(\pi^*)$, conditional on information available at $t = 0$, where $u(\cdot)$ is the utility function and $E(\cdot)$ is the expectation operator. It is assumed also that the mean-variance method is a reasonable framework for analyzing the producer's problem of expected utility maximization. Consequently, the producer chooses V to maximize $E(\pi^*) - \lambda \text{Var}(\pi^*)/2$, where $\text{Var}(\cdot)$ is the variance operator and λ is the risk aversion coefficient such that an increasing λ implies the producer is more risk averse. Further, it is assumed that in an unbiased futures market $E(f_1) = f_0$. Hence, $E(\pi^*)$ is independent of V, and the producer's problem is simplified, making it comparable to the problem faced by a bona fide hedger. In this case, the hedger chooses an optimal futures position V^* to minimize the risk of future profit. Decomposing π^* into $\pi_p^* + (f_1 - f_0)V$, and solving for the optimal futures position we obtain

$$V^* = -\text{Cov}(\pi_p^*, f_1)/\text{Var}(f_1) \quad (5)$$

where π_p^* is the profit from the production process and $\text{Cov}(\cdot,\cdot)$ is the covariance operator. The corresponding risk level is

$$W^* = \text{Var}(\pi_p^*) - \text{Cov}^2(\pi_p^*, f_1)/\text{Var}(f_1)$$

NARROW-BASED VERSUS BROAD-BASED CONTRACTS

The above result applies to any futures contract specification. Consider now two hypothetical futures contracts: a narrow-based contract with grade 1 being the only deliverable grade, and a broad-based contract allowing both grades to be delivered at par. Let f_{b1} and f_{n1}, respectively, denote the futures prices for the broad-based and narrow-based contracts at their common maturity date $t = 1$. Given sufficient arbitrage activity, convergence of cash to futures prices occurs at contract maturity, thus $f_{b1} = \min(P_1, P_2)$ and $f_{n1} = P_1$. Since π_p^* decreases as either P_1 or P_2 increase, it is clear that both $\mathrm{Cov}(\pi_p^*, f_{b1})$ and $\mathrm{Cov}(\pi_p^*, f_{n1})$ are less than zero. Then, by eq. (5), the optimal futures position is always positive when either the narrow- or broad-based contract is traded. Therefore, the producer in this model is always a long hedger. To determine which of the contract designs is preferable, the producer compares the risk levels, denoted W_n^* and W_b^*, associated with each

$$W_i^* = \mathrm{Var}(\pi_p^*) - \mathrm{Cov}^2(\pi_p^*, f_{i1})/\mathrm{Var}(f_{i1}), \qquad i = b, n \tag{6}$$

This expression gives the minimum risk achieved with optimal transactions in the two contracts, respectively. If W_n^* is greater than W_b^* or, alternatively, $\mathrm{Cov}^2(\pi_p^*, f_{n1})/\mathrm{Var}(f_{n1})$ is smaller than $\mathrm{Cov}^2(\pi_p^*, f_{b1})/\mathrm{Var}(f_{b1})$, the broad-based contract is preferred; and vice versa for $W_n^* < W_b^*$. The conclusion clearly rests on the functional form of $F(.,.)$, which determines π_p^* and the joint probability distribution of input and output prices.

An alternative explanation for the choice between the two contracts is as follows. Let h_i denote the hedging effectiveness of the ith contract. Following Ederington (1979) we have

$$h_i = 1 - W_i^*/\mathrm{Var}(\pi_p^*), \qquad i = b, n$$

If $h_n/h_b > 1$, then the long hedger prefers the narrow-based contract; if $h_n/h_b < 1$, the broad-based contract is favored. This criterion is similar to the one utilized by Black (1986) regarding the success or failure of a new futures contract competing against existing contracts.

A MORE SPECIFIC EXAMPLE

The following production function is assumed[2]

$$F(x_1, x_2) = \alpha \log x_1 + \beta \log x_2; \qquad \alpha, \beta > 0 \tag{7}$$

Then, $x_1^* = \alpha P/P_1$ and $x_2^* = \beta P/P_2$; also

$$\pi_p^* = \alpha P \log(\alpha P/P_1) + \beta P \log(P/P_2) - \alpha P - \beta P \tag{8}$$

Suppose the producer locks in the output price through forward sales so that P is a known number at $t = 0$. To simplify, P is normalized by setting $P = 1$. Consequently

$$\mathrm{Cov}^2(\pi_p^*, f_{i1}) = -\mathrm{Cov}(\alpha \log P_1 + \beta \log P_2, f_{i1}), \qquad i = b, n \tag{9}$$

[2]A referee suggested that an alternative production function may reduce the complexity of the analysis while maintaining the same conclusions. Unfortunately, such efforts failed. The most popular Cobb–Douglas production function incurs more sophisticated algebra than the log–linear production function adopted in this study. More specifically, one must compute the expectation of the fractional power of a random variable for the former case. The computation is generally difficult and, most likely, some approximation needs to be applied.

Since both grades are deliverable at par for the broad-based contract, it seems reasonable to assume the two prices, P_1 and P_2, are symmetric, and thus have identical density functions. As a consequence,

$$\mathrm{Cov}^2(\pi_p^*, f_{n1})/\mathrm{Var}(f_{n1}) = (\alpha + \beta\theta)^2\, \mathrm{Cov}^2(\log P_1, P_1)/\mathrm{Var}(P_1) \tag{10a}$$

$$\mathrm{Cov}^2(\pi_p^*, f_{b1})/\mathrm{Var}(f_{b1}) = (\alpha + \beta)^2\, \mathrm{Cov}^2(\log P_1, \min(P_1, P_2))/\mathrm{Var}(\min(P_1, P_2)) \tag{10b}$$

where $\theta = \mathrm{Cov}(\log P_2, P_1)/\mathrm{Cov}(\log P_1, P_1)$. Further, is is assumed that the future spot prices, P_1 and P_2, are bivariately lognormally distributed with $E[\log P_i] = \mu$, $\mathrm{Var}[\log P_i] = \sigma^2$; $i = 1, 2$, and $\mathrm{Cov}(\log P_1, \log P_2) = \rho\sigma^2$. As a result

$$\mathrm{Cov}(\log P_1, P_1) = \sigma^2 \exp(\mu + \sigma^2/2) \tag{11a}$$

$$\mathrm{Cov}(\log P_1, \min(p_1, p_2)) = (1 + \rho)\sigma^2\Phi(-\sigma\sqrt{(1 - \rho)/2}) \cdot \exp(\mu + \sigma^2/2) \tag{11b}$$

$$\mathrm{Var}(P_1) = \exp(2\mu + 2\sigma^2) - \exp(2\mu + \sigma^2) \tag{11c}$$

$$\mathrm{Var}(\min(P_1, P_2)) = \{2 \exp(2\sigma^2)\Phi(-\sigma\sqrt{2(1 - \rho)}) - 4 \exp(\sigma^2) \cdot \Phi^2(-\sigma\sqrt{(1 - \rho)/2})\}\exp(2\mu) \tag{11d}$$

where $\Phi(.)$ is the cumulative density function for a standard normal random variable. The derivations of eqs. (10a)–(11d) are described in the Appendix.

Upon applying eqs. (11a)–(11d) to the general results, it may be determined which contract is preferred by the long hedger. In general, the choice depends on the values of σ, ρ, and (β/α). Some numerical results are provided for the case $\sigma = 1$. When $\beta/\alpha = 0$, the narrow-based contract is always preferred, regardless of the value of ρ. In this case the hedger has absolutely no production flexibility, since grade 2 cannot be utilized in the production process. On the other hand, if β/α is rather large, then the broad-based contract is always chosen. The hedger, in this case, is a cross-hedger for the narrow-based contract, since the large spot position on grade 2 is hedged on a contract dealing exclusively with grade 1. Thus, incorporating grade 2 into the contract specification certainly increases the contracts hedging effectiveness. Nonetheless, this case is not of interest because a large β/α tends to create a narrow-based contract with grade 2 being the only deliverable grade (not grade 1 as assumed above).[3]

For any intermediate value of β/α, the choice between the two contracts depends on ρ, the correlation coefficient of the two prices (in logarithms). Alternatively, for each ρ, one can characterize a critical level, Ω, such that the narrow-based (broad-based) contract is chosen if β/α is smaller (greater) than Ω. Note that β/α measures the ratio of productivities of the two grades when the same amount of each grade is applied to production. Ω may be called the critical relative productivity, or critical flexibility, because it affords the processor more freedom to choose either grade as inputs as Ω rises. Table I shows various critical flexibility levels corresponding to different correlation coefficients. In general, Ω decreases as ρ increases. For example, suppose the two prices are more highly correlated, then less flexibility is

[3]It is assumed here that long hedgers are dominant players in the futures market. Notice that a large β/α implies there is a much greater demand in grade 2 than in grade 1. The concern of long hedging interests leads exchanges to choose a narrow-based contract on grade 2 if the possibility of a broad-based contract is excluded. A referee argued that short hedgers also affect the contract specification. This is certainly true; however, this article considers the problem from the viewpoint of long hedgers and complements the analysis of Lien (1988), who discusses the same problem from the viewpoint of short hedgers.

Table I
CRITICAL FLEXIBILITY ($\sigma = 1$)

Correlation Coefficient	Critical Flexibility
0.1	0.394847
0.2	0.377367
0.3	0.361965
0.4	0.348460
0.5	0.336752
0.6	0.326838
0.7	0.318877
0.8	0.313326
0.9	0.311618

required for the hedger to choose the broad-based contract over the narrow-based contract. The most important conclusion is that, whenever the productivity of the second grade is no less than 40% of the productivity of the first grade, the hedger always prefers the broad-based contract. A small amount of flexibility is sufficient to overcome the additional uncertainty created from incorporating grade 2 into the contract.

CONCLUSIONS

This article demonstrates the possibility that long hedgers may prefer a broad-based contract to a narrow-based contract. Several assumptions are imposed. It is assumed that both grades are deliverable at par in the broad-based contract. This assumption can be relaxed; however, a premium or discount must then be added to the structure. A similar analysis could then be conducted. Martingale equilibrium conditions are imposed on both futures markets. Certainly, a futures market may be established at a backwardation or contango equlibrium. Nonetheless, more assumptions must be imposed to derive the wedge between the current futures price and the expected futures price in the future (i.e., $E(f_1) - f_0$). A rational expectation equilibrium framework may be called upon to investigate these possibilities.

The choice of the production function is a difficult task. It is argued that flexibility in the production process may generate the conclusion that a broad-based contract is preferred. Linear production functions maintain complete flexibility, but they are rather restrictive. To increase generality, the basic analysis is repeated using log-linear production functions. Perhaps other production functions can lead to the same conclusions—this is left for future research. Finally, the simulation study with $\sigma = 1$ only provides an illustration. Similar analyses can be performed with different σ values; however, these conclusions are retained qualitatively.

In sum, the conventional wisdom regarding the choice of contract types by long hedgers may not apply universally. Consideration must be given to a particular industry's needs. If the producer operates with a flexible production process, then a broad-based contract may be preferred to a narrow-based contract for the producer's hedging purposes.

Appendix

To derive eqs. (10a) and (10b), note that $f_{n1} = P_1$ and $f_{b1} = \min(P_1, P_2)$. Consequently, eq. (9) implies

$$\begin{aligned}\mathrm{Cov}^2(\pi_p^*, f_{n1}) &= [\alpha \,\mathrm{Cov}(\log P_1, P_1) + \beta \,\mathrm{Cov}(\log P_2, P_1)]^2\\ &= \mathrm{Cov}^2(\log P_1, P_1)[\alpha + \beta \,\mathrm{Cov}(\log P_2, P_1)/\mathrm{Cov}(\log P_1, P_1)]^2\\ &= \mathrm{Cov}^2(\log P_1, P_1)(\alpha + \beta\theta)^2\end{aligned}$$

where $\theta = \mathrm{Cov}(\log P_2, P_1)/\mathrm{Cov}(\log P_1, P_1)$. Since $\mathrm{Var}(f_{n1}) = \mathrm{Var}(P_1)$, eq. (10a) follows. For the case of the broad-based futures contract

$$\mathrm{Cov}^2(\pi_p^*, f_{b1}) = [\alpha \,\mathrm{Cov}(\log P_1, \min(P_1, P_2)) + \beta \,\mathrm{Cov}(\log P_2, \min(P_1, P_2))]^2$$

Since P_1 and P_2 are assumed to be symmetric, we have

$$\mathrm{Cov}(\log P_1, \min(P_1, P_2)) = \mathrm{Cov}(\log P_2, \min(P_1, P_2))$$

and consequently

$$\mathrm{Cov}^2(\pi_p^*, f_{b1}) = (\alpha + \beta)^2 \,\mathrm{Cov}^2(\log P_1, \min(P_1, P_2))$$

Eq. (10b) is then immediate upon substituting $\mathrm{Var}(\min(P_1, P_2))$ for $\mathrm{Var}(f_{b1})$.

Turn now to eqs. (11a)–(11d). First, let $x_i = \log P_i$, $i = 1, 2$. Then, (x_1, x_2) is a bivariate normal vector with $E(x_1) = E(x_2) = \mu$ and $\mathrm{Var}(x_1) = \mathrm{Var}(x_2) = \sigma^2$; $\mathrm{Cov}(x_1, x_2) = \rho\sigma^2$. To derive eq. (11a), note that $\mathrm{Cov}(\log P_1, P_2) = \mathrm{Cov}(x_1, \exp(x_1))$. Straight algebraic manipulation leads to the result. Alternatively, the moment generating function of a normal random variable may be applied

$$E(\exp(tx_1)) = \exp(t\mu + t^2\sigma^2/2) \tag{A1}$$

where t is a real number. If one takes the partial derivative of eq. (A1) with respect to t, this leads to

$$E(x_1 \exp(tx_1)) = (\mu + t\sigma^2)\exp(t\mu + t^2\sigma^2/2) \tag{A2}$$

Upon substituting $t = 1$ into eq. (A1) and (A2)

$$\begin{aligned}\mathrm{Cov}(x_1, \exp(x_1)) &= E(x_1 \exp(x_1)) - E(x_1)E(\exp(x_1))\\ &= (\mu + \sigma^2)\exp(\mu + \sigma^2/2) - \mu \exp(\mu + \sigma^2/2)\\ &= \sigma^2 \exp(\mu + \sigma^2/2)\end{aligned}$$

which is eq. (11a). Similarily, upon substituting $t = 1$ and $t = 2$ into eq. (A1), we have

$$\begin{aligned}\mathrm{Var}(P_1) &= E(P_1^2) - (E(P_1))^2 = E(\exp(2x_1)) - (E(\exp(x_1)))^2\\ &= \exp(2\mu + 2\sigma^2) - \exp(2\mu + \sigma^2)\end{aligned}$$

Eq. (11c) is established.

The two other eqs., (11b) and (11d), are more complicated. Both of them involve an order statistic of a bivariate lognormal vector. Let $Z = \min(P_1, P_2)$. From Lien (1986), one can derive the following

$$\begin{aligned}E(Z) &= 2\exp(\mu + \sigma^2/2)\Phi(-\sigma\sqrt{(1 - \rho)/2})\\ E(Z^2) &= 2\exp(2\mu + 2\sigma^2)\Phi(-\sigma\sqrt{2(1 - \rho)})\end{aligned}$$

Note that there is a typographical error in eq. (5) of Lien (1986): instead of $\exp(\mu_i + 2\sigma_i^2)$, it should be $\exp(2\mu_i + 2\sigma_i^2)$. Eq. (11b) follows immediately, once one notes that $\mathrm{Var}(Z) = E(Z^2) - (E(Z))^2$. Finally, to show eq. (11d), define $w_i = x_i - \mu$, $i = 1,2$. Hence, (w_1,w_2) is a bivariate normal vector such that $E(w_i) = 0$, $\mathrm{Var}(w_i) = \sigma^2$; $i = 1,2$. Also, $\mathrm{Cov}(w_1,w_2) = \rho\sigma^2$. As a consequence

$$\begin{aligned}\mathrm{Cov}(\log P_1, \min(P_1,P_2)) &= \mathrm{Cov}(x_1, \min(\exp(x_1), \exp(x_2)))\\ &= \mathrm{Cov}(w_1 + \mu, \min(\exp(w_1 + \mu), \exp(w_2 + \mu)))\\ &= \exp(\mu)\,\mathrm{Cov}(w_1, \min(\exp(w_1), \exp(w_2)))\end{aligned}$$

Eq. (11d) is derived after algebraic manipulation on the above covariance term. In particular, the following equation is applied

$$E[\Phi(\alpha y + \beta)] = \Phi(\beta/\sqrt{\alpha^2 + 1})$$

with y being a standard normal random variable (Lien, 1986). Details are available upon request from the author.

Bibliography

Barnhill, T. M., and Seale, W. E. (1988): "Optimal Exercise of the Switching Option in Treasury Bond Arbitrages," *Journal of Futures Markets,* 5:517–532.

Black, D. (1986): Success and Failure of Futures Contracts: Theory and Empirical Evidence," NYU Business School Monograph Series in Finance and Economics.

Boyle, P. P. (1989): "The Quality Option and Timing Option in Futures Contracts," *Journal of Finance,* 44:101–114.

Boyle, P. P., and Tse, Y. K. (1990): "An Algorithm for Computing Values of Options on the Maximum or Minimum of Several Assets," *Journal of Financial and Quantitative Analysis,* 25:215–227.

Ederington, L. H. (1979): "The Hedging Performance of the New Futures Markets," *Journal of Finance,* 34:157–170.

Garbade, K. D., and Silber, W. L. (1983): "Futures Contracts in Commodities with Multiple Varieties: An Analysis of Premiums and Discounts," *Journal of Business,* 56:249–272.

Gay, G. D., and Manaster, S. (1984): "The Quality Option Implicit in Futures Contracts," *Journal of Financial Economics,* 13:353–370.

Hedge, S. P. (1989): "On the Value of the Implicit Delivery Options," *Journal of Futures Markets,* 9:421–437.

Johnson, L. J. (1957): "Price Instability, Hedging, and Trade Volume in the Coffee Futures Markets," *Journal of Political Economy,* 65:306–321.

Kamara, A. (1990): "Delivery Uncertainty and the Efficiency of Futures Markets," *Journal of Financial and Quantitative Analysis,* 25:45–64.

Kane, A., and Marcus, A. J. (1988): "The Delivery Option on Forward Contract: A Note," *Journal of Financial and Quantitative Analysis,* 23:337–341.

Lien, D. (1986): "Moments of Ordered Bivariate Log–Normal Distributions," *Economic Letters,* 20:45–47.

Lien, D. (1988): "Hedger Response to Multiple Grades of Delivery on Futures Markets," *Journal of Futures Markets,* 8:687–702.

Livingston, M. (1987): "The Delivery Option on Forward Contracts," *Journal of Financial and Quantitative Analysis,* 22:79–87.

Paul, A. B. (1987): "Pricing Implications of Alternative Delivery Mechanisms for Futures Contracts," in Purcell, W., and Rowsell, J., Eds., *Key Issues in Livestock Pricing: A Perspective for the 1990s,* Blackburg, VA: Research Institute on Livestock Pricing.

Shaviro, F.W. (1987): "An Analysis of Cash and Futures Prices in the Delivery Period of Maturing Contracts in the Coffee C Market, 1972–1981," *Journal of Futures Markets,* 4:413–441.

Ware, R., and Winter, R. (1988): "Forward Markets, Currency Options and the Hedging of Foreign Exchange Risk," *Journal of International Economics,* 25:291–302.

Working, H. (1954): "Whose Markets? Evidence on Some Aspects of Futures Trading," *Journal of Marketing,* 19:1–11.