Cointegration: Some Results on U.S. Cattle Prices

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The topic of cointegration and related topics of nonstationarity and unit root econometrics have been the center of considerable attention in the applied econometric literature over the last several years. A partial listing of articles includes: Engle and Granger (1987), Engle and Yoo (1987), Granger (1986), Hendry (1986), and Campbell and Shiller (1988). This article explores the application of cointegration techniques to the study of daily futures and cash prices on live cattle.

This article is presented in four sections. First, the basics of cointegration–econometrics are reviewed. A discussion follows on how cointegration reflects upon economic interrelationships in general and, more specifically, how it reflects on issues of interest to futures market researchers (e.g., informational efficiency, causality, forecasting, and basis relationships). Then, cointegration methods are applied to daily data for slaughter cattle cash and futures prices and the implications of the results with respect to the above issues are discussed.

SOME BASICS ON COINTEGRATION

A series of data indexed by time (a set of data in which order of observation is important) is said to be integrated of order d if it requires d first differences to reduce the resulting series to stationarity (e.g., $d = 2$ if $X(t) - X(t - 1) - X(t - 1) + X(t - 2) = Z(t)$ is stationary). Here, stationarity means that the characteristics of the times series are describable in terms of the time *separating* observations and not the particular time of the observations. Researchers find that many economic time series appear to require first differencing ($d = 1$) to achieve stationarity (Gould and Nelson (1974), Granger (1986)).

The standard approach to univariate time series analysis of data integrated of order d, is to model the dth differenced data as either an autoregression, a moving

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average, or some mixed autoregressive and moving average model (Box and Jenkins (1970)). That is, a model of these dth differenced data is sought.

The same mode of operation does not hold when modeling multiple time series. Here, analysts are not willing to difference each series when evidence of nonstationarity is present. Thus, Nerlove et al. (1979, p. 252) cancel stationary inducing transformations "to let the nonstationarity in ... (one) series explain the nonstationarity in the ... (other) series." Similar advice is offered by Tiao and Box (1981, p. 804): "... it should be noted here that for vector time series, linear combinations of the elements (of the vector) may often be stationary, and simultaneous differencing of all series can lead to unnecessary complications in model fitting."

The notion of cancellation of stationary-inducing transformations is given formal treatment in the cointegration literature introduced by Granger (1986). Two series, say $X(t)$ and $Y(t)$, are said to be cointegrated if, individually, they are integrated of orders d and b; but their linear combination, $Z(t) = X(t) - aY(t)$, is integrated of order $d - b$. The most prominent case studied in the literature is where $d = b = 1$. Where two series are shown to be cointegrated, an error correction model can be used to represent their dynamic joint process (Engle and Granger (1987)).

A discussion of the modeling of cointegrated systems follows. This discussion is presented in two major subsections: testing the order of integration of the original series along with their linear combinations, and modeling the cointegrated variables as an error-correction process.

Dickey and Fuller (1979) propose a simple test for nonstationarity. They suggest regressing the first differences of the series on lagged values of the levels of the series. Under the hypothesis that the underlying process is a random walk, the regression coefficient will be negative and significantly different from zero for a stationary series. As the distribution theory underlying such a test is nonstandard, Monte Carlo-generated critical values must be used (Dickey and Fuller (1979, pp. 134–136)). Engle and Granger (1987) suggest an additional test of nonstationarity which adds lags of the dependent variable, sufficient to produce white noise residuals in the above-described "Dickey–Fuller" regression. Termed the "augmented Dickey–Fuller" test, it too relies on Monte Carlo-generated critical values.

Given that one cannot reject the hypothesis that each of two series are integrated of order one, one can proceed to consider whether or not their joint process is cointegrated. Engle and Granger (1987) suggest that one begin by modeling the static relationship between the two series. Equation (1) is proposed as a starting point:

$$X(t) = aY(t) + Z(t). \tag{1}$$

A test for cointegratedness can be made from the observed residuals from the ordinary least squares regression of $X(t)$ on $Y(t)$. Consider that series $X(t)$ and $Y(t)$ are cointegrated and individually integrated of order one. An ordinary least squares regression of $X(t)$ on $Y(t)$ should yield residuals $\hat{Z}(t)$ which are stationary, by the definition of cointegratedness. If the estimated Durbin–Watson statistic exceeds the critical value for the particular sample size, then one should reject the null hypothesis that the two series are *not cointegrated*. The critical values for this statistic have been studied using Monte Carlo methods. Engle and Granger (1987, p. 269) provide 1%, 5%, and 10% critical values of .38, .36, and .32, respectively, for two commonly observed time series processes estimated with 100 data points.

An alternative test of cointegratedness is to apply the Dickey–Fuller test of unit roots to the observed residual series $\hat{Z}(t)$. If $X(t)$ and $Y(t)$ are cointegrated, one would expect the residuals from the cointegrating regression to be stationary. To test this, one would regress changes in the observed residuals on levels of the residuals lagged one period. One should reject the null hypothesis of noncointegratedness if the regression coefficient is negative and significantly different from zero. (An "augmented" Dickey–Fuller test, analogous to that described in footnote 1 might be considered also.) Again, because of the nonstandard distribution theory which underlies these tests, Monte Carlo critical values as given in, for example, Engle and Granger (1987) should be used.

If one is not able to reject the hypothesis of noncointegratedness, an error correction model of the joint process may be specified (Engle and Granger (1987)). An ordinary least squares regression of changes in $X(t)$ on past changes in $X(t)$ and $Y(t)$, and lags on residuals from the cointegrating regression in (eq. (1)) is proposed. An analogous specification is defined as the regression of changes of $Y(t)$ on past changes on $Y(t)$ and $X(t)$, and lags of the residuals from the cointegrating regression. That is, the following model (written without accompanying error terms) is suggested:

$$\begin{aligned}(1 - L)X(t) &= \sum_{k=1}^{K} A(k)(1 - L)X(t - k) + \sum_{k=1}^{K} B(k)(1 - L)Y(t - k) + C(1)\hat{Z}(t - 1)\\ (1 - L)Y(t) &= \sum_{k=1}^{K} D(k)(1 - L)X(t - k) + \sum_{k=1}^{K} E(k)(1 - L)Y(t - k)\\ &\quad + F(1)\hat{Z}(t - 1) \qquad (2)\end{aligned}$$

where $\hat{Z}(t - 1)$ is the observed residual from the cointegrating regression (eq. (1)), L is the lag operator ($LX(t) = X(t - 1)$), and $A(k)$, $B(k)$, and $C(1)$, $D(k)$, $E(k)$, and $F(1)$ are parameters to be estimated. Here, K is selected large enough to remove any autocorrelation in the residuals. The system is written for both $X(t)$ and $Y(t)$.

Forecasting cointegrated systems is not obvious from the specification of eq. (2), as the future value of $\hat{Z}(t - 1)$ is unknown for more than one step ahead forecast horizon. Following Campbell and Shiller (1988, p. 510), eq. (2) can be written in an equivalent form in terms of $(1 - L)X(t)$ and $\hat{Z}(t)$.

$$\begin{aligned}(1 - L)X(t) &= \sum_{k=1}^{K} G(k)(1 - L)X(t - k) + \sum_{k=1}^{K} H(k)\hat{Z}(t - k) + u(t)\\ \hat{Z}(t) &= \sum_{k=1}^{K} I(k)(1 - L)X(t - 1) + \sum_{k=1}^{K} J(k)\hat{Z}(t - k) + v(t) \qquad (3)\end{aligned}$$

where $u(t)$ and $v(t)$ are white noise residuals, and $G(k)$, $H(k)$, $I(k)$, and $J(k)$ are parameters defined on lags of $X(t)$ and $\hat{Z}(t)$. The system defined by eq. (3) is a $k + 1$th order vector autoregression, with two zero restrictions on coefficients of $X(t - k)$ (at lags k and $k + 1$ the parameters $F(k)$, $F(k + 1)$, $H(k)$, and $H(k + 1)$ equal zero). This allows one to forecast any h-step ahead horizon using standard "chain rule of forecasting" procedures (See Sargent (1979), p. 268).[1]

[1]Equation (3) is important as it imposes cointegration on multiple-step ahead forecasts. Engle and Yoo (1987, p. 146) show that long-run forecasts of X_t and Y_t will be tied together when the cointegration restrictions are imposed. For example, when cointegration exists between two series, X_t and Y_t, the long-run forecasts of X_t and Y_t from an error correction model will show lower mean squared forecast errors relative to forecasts from an unrestricted vector autoregression.

COINTEGRATION AND ECONOMIC INTERRELATIONSHIPS

Economic theory proposes forces that tend to keep a pair of economic series from drifting too far apart over time. Consider for example, arbitrage on similar commodities in different markets. Cointegration acts as evidence for this long-run equilibrium relationship, in which deviations would be due to some short-run shocks (Corbae and Ouliaris (1988)). Campbell and Shiller (1988) suggest that cointegration may arise when agents with rational expectations are forecasting. Hence, an error correction model should exist whenever there is forward-looking behavior of prices. This error correction representation also suggests a Granger-type causation between the two cointegrated variables. In the error correction model, Y may cause X either through the $Z(t - 1)$ term or the lagged terms, $(1 - L)Y(t - 1)$. Previous tests which fail to account for this cointegrating relationship may have incorrectly inferred a non-*prima facie* causal relationship (Granger (1988)).

Futures and cash market prices present an interesting case for application of cointegration-type relationships. One might expect, *a priori,* that a predictive relationship may exist between these two market prices. (See Leuthold (1974) or Gardner (1976) for formal discussion of the expectations or predictive role of futures markets.) If one considers the futures price at time t for delivery at time $t + k$ as the expectation held at time t of the cash price in period $t + k$, then the relationship between futures price and cash price is defined by the order of integration of cash price (the expectation of a series integrated of order i is itself integrated of order i).

If, for example, cash prices follow a random walk

$$X(t) = X(t - 1) + u(t),$$

where $u(t)$ is a white noise process, the series $X(t)$ is integrated of order one. The expectation ($E\{.\}$) of cash price is thus itself integrated of order one, $E\{X(t) | X(t - 1)\} = X(t - 1)$. More generally, suppose that futures price ($Y(t)$) observed at t for delivery at $t + k$ is equal to current cash price ($X(t)$) plus other predictive information ($V(t)$). If $X(t)$ is integrated of order one, and $V(t)$ is integrated of order zero, the $Y(t)$ is integrated of order one (Granger (1986), p. 217). If cash market traders believe that futures prices summarize important information about future cash prices, beyond that contained in current cash price ($X(t)$); then $V(t)$ and, consequently, $Y(t)$ will Granger-cause $X(t + j), j \geq 1$. In such a case, a cointegration-type relationship exists between $X(t)$ and $Y(t)$. Further, the market represented by prices $X(t)$ does not pass usual market efficiency tests (Granger and Escribano (1988)).

Previous research on slaughter cattle markets make much of the issues of informational efficiency and causality. Many researchers conclude that the introduction of futures trading improves pricing efficiency in the cattle cash markets: Taylor and Leuthold (1974), Powers (1970), Cox (1976), and Brorson et al. (1989). Other research focuses on the price efficiency of the live cattle futures market with mixed results: Leuthold (1972), Leuthold and Hartmann (1981), Paul (1986), Hudson et al. (1987), and Garcia et al. (1988). Low degrees of informational inefficiency are found in four different regional cash markets (Bailey and Brorsen (1985)). Previous research (Purcell et al. (1979), Weaver and Banerjee (1982), Oellermann and Farris (1985)), using different within-sample tests, generally conclude that futures prices Granger-cause slaughter cattle cash prices with some evidence for feedback.

Futures research is concerned also with basis or intertemporal price relationships. For storable commodities (e.g., wheat or gold), spot and futures prices are related by storage costs. For the so-called nonstorables, such as cattle, no such relationship exists, and intertemporal prices are independent. The difference (basis) between two such independent price series would be a random walk. However, few commodities are purely storable or nonstorable. Cattle which have achieved the minimum weights necessary to be sold as slaughter cattle can be held on feed for several months depending on seller's price expectations relative to additional cost of feed (McCarty (1987), Naik and Leuthold (1988)). Naik and Leuthold (1988) showed that flexibility in the marketing decision strengthens the intertemporal price relationship for cattle which was previously attributed only to feed prices. Hence, applying the label "nonstorable" to a commodity such as cattle may be misleading. Discussion with market observers as reported by McCarty (1987), as well as previous research (Leuthold (1979), Naik and Leuthold (1988)), suggests slaughter cattle may be related to nearby but not more distant futures contracts. Thus, the terms "storable" and "nonstorable" might be better applied when referring to the relationship between the price of a commodity at a particular point in time and a specific futures contract.

APPLICATION TO FUTURES AND CASH MARKET PRICE DATA

The purpose of this study is to determine whether a statistical relationship (cointegration) exists between the futures market for live cattle and a major regional slaughter cattle cash market (Texas–Oklahoma). The results should yield further evidence regarding the economic interrelationships (price efficiency, Granger-causality, etc.) which may exist between the two markets.

Two price series are analyzed: the daily settlement price for the nearby live cattle futures contract for August 21, 1985 through August 20, 1986 (*The Wall Street Journal*) and the daily average cash price (per cwt.) for direct sale of choice 900–1300-lb. slaughter cattle steers for the Texas–Oklahoma (referred to as Amarillo) market over the same period (LS-214s, U.S. Department of Agriculture). Amarillo is a direct rather than auction sales market for slaughter cattle and, as such, conducts its sales throughout the entire five-day business week. According to market observers (McCarty (1987)), there exists no consistent intraday or intraweek pattern of cash market trade volume as occurs in the auction markets. Previous day's observations are used in place of missing observations, such as occasional holidays, for both price series since these represent the most recent information available to the marketplace participants.

Conducting tests for cointegration between spot slaughter cattle price and the nearby, as well as a distant live cattle futures price, allows inferences regarding basis relationships. Following the discussion above, one would expect the degree of cointegration to be a function of the strength of any existing relationship between two intertemporal prices. Following Leuthold (1979), one would expect to find evidence for cointegration for nearby contracts; but, perhaps, not for more distant contracts.

The transition from one futures contract to the next is made upon each contract's termination date (the 20th of each termination month). The delivery period is retained in the analysis because of the belief that the cash–futures price relationship continues up to the nearby contract's termination date. The approach is identical to Bessler and Kling's (1990) in their prequential analysis of cattle prices.

To account for possible systematic relationships in the data associated with the construction of the nearby futures price random variable, several dummy variables, or time-trend specifications, are considered: a 0, 1 dummy variable set equal to 1 in the last two weeks of a contract (to account for the statistical effect of including the delivery period in the data set); a 0,1 dummy variable set equal to 1 at the first day of a new contract (to account for the statistical effect which may result in the transition from one futures contract to the next); and a time-to-expiration trend—43 days, 42 days, ..., 0 days (to determine whether the relationship between cash and futures changes systematically as each of the six contracts approaches maturity). All test statistics and estimated relationships show little sensitivity to these different specifications. In particular, Dickey–Fuller tests and cointegration tests are not affected (qualitatively) by these different specifications. Consequently, the results presented below do not include any of these dummy variables or time-trend specifications; although, the models used throughout the remainder of this report are estimated using the ROBUSTERRORS command in RATS (Doan and Litterman (1989)) which allows for heteroskedastic error processes in the models under study. (The reader may write the senior author for outputs on these results if particular specifications are of interest.)

Each price series is tested for order of integratedness using the Dickey–Fuller test and an augmented Dickey–Fuller test. Table I gives both tests for cash and futures prices and their first differences over the first 130 data points. The tests on levels are regressions of the first differences on lagged levels (Dickey–Fuller) and lagged levels and lagged first differences (augmented) of each series. The test of first differences are regressions of second differences on lagged first differences and second differences. Recall the null hypothesis on all regressions is that the series are random walks in their levels, so that the null hypothesis is rejected if the coefficient associated with the levels of the variable in each regression is negative and significantly different from zero. Recall too, usual critical values for both the Dickey–Fuller test and the augmented test are not standard. Monte Carlo critical values of Engle and Granger (1987) suggest a critical value (for 200 data points) of about −3.50.

From Table I, note that for levels of cash and nearby futures prices one is not able to reject the null hypothesis that each series is generated as a random walk

Table I
TESTS FOR ORDER OF INTEGRATION ON CASH PRICES ($C(t)$) AND FUTURES PRICES ($F(t)$)

Series	Dickey–Fuller[a]	Augmented Dickey–Fuller[b]
$C(t)$	−1.33	−1.03
$(1 - L)C(t)$	−8.34	−4.51
$F(t)$	−1.35	−1.00
$(1 - L)F(t)$	−12.00	−10.02

[a]Tests are defined as t-statistics on estimated coefficient $\hat{A}_1$ from the ordinary least squares regression fit to the first 130 data points.

$$(1 - L)X(t) = A_0 + A_1X(t - 1)$$

where $X(t)$ refers to series $C(t)$, $(1 - L)C(t)$, $F(t)$, and $(1 - L)F(t)$ in the body of the table.

[b]The augmented test is defined in a fashion analogous to that given in table footnote a, except lags of the dependent variable are included in each regression. Lags are determined by applying FPE (see Hsiao (1979) for details), and are given as follows: 1 for $C(t)$, 2 for $(1 - L)C(t)$, 1 for $F(t)$, and 1 for $(1 - L)F(t)$.

(t-statistics of approximately -1.4 for both cash and futures prices from the Dickey–Fuller tests). The null hypothesis, that each first difference is generated by a random walk, is rejected at very low levels of significance—t-statistics from the Dickey–Fuller tests of -8.3 for cash prices and -12.0 for futures prices. The augmented Dickey–Fuller tests on first differences yield qualitatively similar results—the first differences are not generated as random walks.

Tests for cointegration are carried out by checking the residuals from the cointegrating regression, which is estimated over the same first 130 observations as follows:

$$C(t) = \underset{(.12)}{-.15} + \underset{(.03)}{1.04}\, F(t); \qquad \text{DW} = .33 \tag{4}$$

The numbers in parentheses are estimated standard errors and $C(t)$ and $F(t)$ refer to cash and futures prices in period t. The residuals from this regression ($Z(t)$) should be stationary if cash and futures prices are cointegrated. The Durbin–Watson statistic (DW) associated with the residuals from eq. (4) is greater than the .32 value (for $N = 100$) suggested by Engle and Granger (1987) to reject the null hypothesis of no cointegration at the .10 level. That is, the Durbin–Watson statistic on the residuals from the cointegrating regression offers marginal support for the cointegrating hypothesis for cash and futures prices.

As a second test for cointegration, the Dickey–Fuller test is applied to the residuals from eq. (4). The Dickey–Fuller regression is given in eq. (5):

$$(1 - L)\hat{Z}(t) = \underset{(.00)}{.00} - \underset{(.05)}{.17}\, \hat{Z}(t-1); \qquad \text{DW} = 2.08 \tag{5}$$

Noting Monte Carlo-simulated critical values for this test (Engle and Yoo (1987)), the ratio of the estimated coefficient to its standard error (-3.59) is below the 0.5 critical value of -3.4. This test result suggests that the residuals from the cointegrating regression are stationary. On the other hand, the augmented Dickey–Fuller test on residuals from eq. (4) is not quite as conclusive. Equation (6) summarizes that test, where two lags of $(1 - L)\hat{Z}(t)$ are determined by FPE search.

$$(1 - L)\hat{Z}(t) = \underset{(.00)}{.00} - \underset{(.05)}{.12}\, \hat{Z}(t-1) - \underset{(.09)}{.11}\,(1 - L)\hat{Z}(t-1) - \underset{(.09)}{.23}\,(1 - L)\hat{Z}(t-2);$$

$$\text{DW} = 1.98 \tag{6}$$

Here, the ratio of the coefficient on $\hat{Z}(t-1)$ to its standard error (-2.34) is above the .05 critical level; this suggests that the residuals series may be nonstationary. These results offer mixed support for a cointegration-type relationship between cash and nearby futures prices.

To test for cointegration for two price series where no dependence is expected, the analysis described above is repeated using a distant contract. At least five distant contracts exist for any point in time for slaughter cattle. The settle price for the second distant contract is used. For example, the spot price for September 6, 1985 is regressed on the settle price for the February 1986 contract. The distant contract series is constructed in a manner analogous to that used to construct the nearby futures series. Contracts are spliced together at the day when trading in a new (more distant) contract is initiated. That contract is used as the distant contract until trading commences in another, more distant, contract. A similar battery of dummy variables and time-trend variables, as described above, are studied to determine whether splicing has an affect on the qualitative patterns in the distant contract data. No such patterns are noted. Tests of stationarity (not reported here) for this distant contract series are consistent with those from the nearby and cash price series. That is, the distant contract appears to require one difference to achieve stationarity.

The cointegrating regression, analogous to eq. (4) for the distant contract, is given as eq. (7):

$$C(t) = \underset{(.46)}{-3.96} + \underset{(.11)}{1.97}F(t); \qquad \mathrm{DW} = .21 \tag{7}$$

Here $F(t)$ refers to the distant futures contract variable. Note the drop in the Durbin–Watson statistic relative to that from eq. (4). This suggests a weaker (or perhaps no) cointegration between cash prices and the distant contract relative to cash and the nearby futures. The Dickey–Fuller (DF) and augmented Dickey–Fuller (ADF) tests on the residuals from eq. (7) yield results ($t = -2.81$ for (DF), and -2.29 for (ADF)) that also do not support a cointegrating relationship between cash prices and the distant futures contract. Because no relationship is found between cash prices and the distant futures price, the analysis presented in the remainder of this study is confined to the nearby futures and cash-price relationship.

Additional evidence on cointegratedness is offered by out-of-sample forecasting. Using the VAR form of the cointegration (COVAR) model given above (eq. (3)), multiple step-ahead horizons are forecasted for cash prices over data points 131–261. The models used for forecasting are identified and fit using the following procedures over data points 1–130. The explicit error correction specification (lags) is determined by applying Hsiao's FPE search to changes in cash prices and residuals from the cointegrating regression. Table II summarizes this search. Note that changes in cash prices are generated by two lags of changes in cash prices and three lags of the residuals from the cointegrating regression. Further, residuals from the cointegrating regression are generated by two lags of changes in cash prices and three lags of residuals from the cointegrating regression.

Forecasts from this cointegrating VAR are contrasted with forecasts over the same horizons from a univariate time-series model and from a restricted vector autoregression. Both alternative models are specified using the Hsiao-search procedure. The patterns of search for these models are given in Table III. The cash price univariate representation is given as a first-order autoregression in first differences, while futures prices appear to be generated as a random walk. The bivariate representations are given as: one lag of differenced cash and four lags of differenced futures generate current cash price differences and, again, futures prices are generated as a random walk. Hence, Hsiao's procedure suggests futures prices Granger-cause cash prices without feedback. Any inefficiencies, therefore, are expected to be observable in the cattle cash market. If so, adding futures prices to the information set of a cash-price model should result in improvement in out-of-sample predictions (Granger (1980)).

Table IV presents mean squared error (MSE) measures on forecasts at various horizons for all three specifications. The models used to generate the out-of-sample forecasts are given in footnotes 2, 3, and 4 of Table IV. Note that, at short horizons, both the restricted and the cointegration vector autoregression outperform the univariate model. That is, futures prices Granger-cause cash prices (Granger (1980, 1988)). The cointegration model is outperformed by the restricted vector autoregres-

Table II
FPE-STATISTICS ON CAMPBELL–SHILLER VAR SPECIFICATION OF CASH-FUTURES ERROR CORRECTION MODEL[a]

	ΔCash $= W(t)$					$\hat{Z}(t)$					
FPE[b]	Constant	−1	−2	−3	−4	Constant	−1	−2	−3	−4	−5
$(W(t))$:											
.0000895	X										
.0000584	X	X					X	X			
.0000550	X	X	X				X	X	X		
.0000563	X	X	X	X			X	X	X	X	
.0000579	X	X	X	X	X		X	X	X	X	X
$(\hat{Z}(t))$:											
.0008981						X					
.0002919		X				X	X	X			
.0002805		X	X			X	X	X	X		
.0002897		X	X	X		X	X	X	X	X	
.0002980		X	X	X	X	X	X	X	X	X	X

[a]The model studied is of the general form:

$$W(t) = A_0 + \sum_{i=1}^{k} A_i W(t-i) + \sum_{k=1}^{k+1} B_k \hat{Z}(t-k)$$

$$\hat{Z}(t) = C_0 + \sum_{i=1}^{k} D_i W(t-i) + \sum_{k=1}^{k+1} E_k \hat{Z}(t-k)$$

Each line of the table gives the FPE statistic associated with an ordinary least squares regression of the variable listed in parentheses on lags of $W(t)$ and $Z(t)$. The X's indicate the particular lags in each equation.

[b]The FPE-metric is defined in Hsiao (1979) and is given as:

$$\text{FPE}(k) = \frac{T + k_1 + k_2 + 1}{T - k_1 - k_2 - 1} \sum_{t=1}^{T} (W(t) - \hat{W}(t|k_1, k_2))^2/_T,$$

where k_1 is the number of lags on Δcash price, k_2 is the number of lags on the innovations from the cointegrating regression, T is the number of observations used to fit the model, and $\hat{W}(t|k_1, k_2)$ is the OLS fit value of W given a model of k_1 and k_2 lags on Δcash and $\hat{Z}(t)$, respectively.

sion at most horizons, with the reductions in MSE appearing to be more dramatic at longer lags. This result is not consistent with results found for cointegration systems in Engle and Yoo (1987) (although their results are obtained with Monte Carlo data). These out-of-sample forecasts do not offer evidence supporting a cointegrating relationship between cash and the nearby futures prices over the out-of-sample period.

Previous literature indicates that a superiority in long-run forecasts from the error-correction specification over the restricted VAR would be found. The discrepancy between this study's tests of fit and out-of-sample forecasts is, perhaps, consistent with that found in Granger and Escribano (1987) for gold and silver prices. Cointegration-type relationships may appear in subsets or runs of data sets, while not being consistently present over time. This is consistent with the rather marginal significance levels required to reject the null hypothesis of noncointegration on the residuals from the cointegrating regression (eq. (5)).

Table III
FPE-STATISTICS ON UNIVARIATE AND BIVARIATE MODELS ON CHANGES IN CASH AND FUTURES PRICES[a]

FPE	ΔCash (*t*) Constant	−1	−2	−3	−4	−5	ΔFutures (*t*) Constant	−1	−2	−3	−4	−5
ΔCash (*t*):												
.0000895	X											
.0000839	X	X										
.0000853	X	X	X									
.0000847	X	X	X	X								
.0000861	X	X	X	X	X							
.0000872	X	X	X	X	X	X						
.0000717	X	X						X				
.0000606	X	X						X	X			
.0000605	X	X						X	X	X		
.0000603	X	X						X	X	X	X	
.0000610	X	X						X	X	X	X	X
ΔFutures (*t*):												
.0002052							X					
.0002085							X	X				
.0002103							X	X	X			
.0002108							X	X	X	X		
.0002137							X	X	X	X	X	
.0002172							X	X	X	X	X	X
.0002073		X					X					
.0002105		X	X				X					
.0002137		X	X	X			X					
.0002157		X	X	X	X		X					
.0002190		X	X	X	X	X	X					

[a]The FPE-metric is defined in a manner analogous to that given in Table II.

CONCLUSION

Cointegration is studied through the example of futures and cash prices for slaughter cattle. Tests for cointegration and its VAR representation, that have appeared heretofore in the literature, are reviewed. This literature suggests that models of the cointegration relationship should show improved long-range forecasts relative to models which do not impose the cointegration restrictions. These models are applied to 261 data points on daily live cattle prices. Results are mixed. First, within-sample fits (conducted on the first 130 data points) indicate that both cash and futures prices (both nearby and distant contract series) are generated by processes not statistically distinguishable from a random walk. Tests for cointegration, based on residuals from a static regression (based on the same 130 data points), show marginal support for the cointegration hypothesis between cash prices and the nearby futures contract. No cointegration is discovered between cash prices and the distant contract. Out-of-sample forecasts of cash prices (forecasts of the subse-

Table IV
ROOT MEAN SQUARED ERRORS ON OUT-OF-SAMPLE FORECASTS OF CASH PRICE[a]

Horizon	UNIV[b]	RVAR[c]	COVAR[d]	# Obs.
1	.0102	.0082	.0082	131
2	.0178	.0138	.0140	130
3	.0238	.0199	.0205	129
4	.0288	.0248	.0257	128
5	.0335	.0295	.0307	127
10	.0449	.0429	.0471	122

[a]UNIV, RVAR, and COVAR refer to univariate autoregression, restricted vector autoregression, and cointegration vector autoregression, respectively. Orders of lags on UNIV, COVAR, and RVAR are determined using FPE (see Tables II and III).

[b]The univariate model identified by FPE (Table III) and used to generate the UNIV forecasts is given as:

$$w(t) = \underset{(.01)}{.00} + \underset{(.09)}{.27}\, w(t-1)$$

where $w(t) = (1 - L)C(t)$ is the first difference of cash price in period t, and the numbers in parentheses are standard errors.

[c]The restricted vector autoregression (RVAR), identified by FPE (Table III), and used to generate the RVAR forecasts is given as:

$$\begin{bmatrix} w(t) \\ h(t) \end{bmatrix} = \begin{bmatrix} .00 \\ (.01) \\ .00 \\ (.00) \end{bmatrix} + \begin{bmatrix} .03 & .26 \\ (.09) & (.05) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w(t-1) \\ h(t-1) \end{bmatrix} + \begin{bmatrix} 0 & .29 \\ & (.05) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w(t-2) \\ h(t-2) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & .09 \\ & (.06) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w(t-3) \\ h(t-3) \end{bmatrix} + \begin{bmatrix} 0 & .07 \\ & (.05) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w(t-4) \\ h(t-4) \end{bmatrix};$$

where $w(t) = (1 - L)C(t)$ is the first difference of cash price in period t, $h(t) = (1 - L)F(t)$ is the first difference of futures price in period t, the numbers in parenthesis are standard errors.

[d]The cointegration VAR (COVAR), used to generate the forecasts, is given as:

$$\begin{bmatrix} w(t) \\ Z(t) \end{bmatrix} = \begin{bmatrix} .00 \\ (.00) \\ -.00 \\ (.00) \end{bmatrix} \begin{bmatrix} .20 & -.25 \\ (.10) & (.05) \\ .09 & .73 \\ .22 & .08 \end{bmatrix} \begin{bmatrix} w(t-1) \\ \hat{Z}(t-1) \end{bmatrix} + \begin{bmatrix} .13 & -.04 \\ (.09) & (.08) \\ .31 & -.15 \\ (.22) & (.14) \end{bmatrix} \begin{bmatrix} w(t-2) \\ \hat{Z}(t-2) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & .17 \\ & (.06) \\ 0 & .34 \\ & (.12) \end{bmatrix} \begin{bmatrix} w(t-3) \\ \hat{Z}(t-3) \end{bmatrix};$$

where $w(t) = (1 - L)C(t)$ is the first difference of cash price in period t, $\hat{Z}(t)$ is the residual from cointegrating regression (eq. (4) of the text) observed in period t. Equation (4) and the associated $\hat{Z}(t)$ value is reestimated recursively one step at a time through the entire out-of-sample period. Standard errors are in parentheses.

quent 131 data points) from an error-correction model, using the nearby futures contract series, which is consistent with the cointegration hypothesis, outperform forecasts of cash prices from a univariate autoregression. The error-correction model's forecasts do not outperform forecasts from a restricted VAR in first differences of cash and nearby futures prices.

The evidence for a weak cointegrating relationship between cash and nearby futures suggests some dependency between the two price series, which may arise when cash traders use the nearby futures price as a means of predicting short-run price movements in the cash markets. The possibility for continuing slaughter cattle on feed for at least part of the interval between contracts and delivering on the nearby contract would also explain the existence of the statistical relationship. These results, plus the clear absence of any cointegration relationship between cash prices and the distant futures contract, confirms prior work of Leuthold (1979) and Naik and Leuthold (1988), which suggest the greater the distance over time the greater the degree of independence.

The informational inefficiency in the cattle cash market, suggested by the within-sample tests, is confirmed by the improvement in out-of-sample forecasting performance when nearby futures prices are added to an information set of a cash price forecasting model. Such an improvement in predictive performance suggests a Granger mean-causal relation running from today's settle price for the nearby cattle futures contract to tomorrow's average spot price for Texas–Oklahoma slaughter cattle. This relationship continues for up to 10 trading days. Information from futures may be added to that already present in cash market price to improve the slaughter cattle price predictions over this short-run marketing horizon (10 days). This should provide additional evidence when judging the marginal benefits to cash market participants resulting from information generated in the futures market. While the strength of the cointegrating relationship between cash and the nearby futures price is not strong, it does yield forecasts of cash prices which outperform a univariate representation of cash prices.

Further research on the cointegrating relationship between cash and futures prices is recommended. For example, one would expect to find stronger evidence for cointegration between cash and futures prices for the so-called storable commodities. Feeder cattle would be expected to be cointegrated with six-month distant live cattle futures contracts, but perhaps not with the nearby contracts. Additional research with longer and shorter data sets may prove valuable because many of the tests applied here are based on Monte Carlo-generated critical values. Here one would place strong emphasis on the ability of any uncovered relationship to forecast out-of-sample data.

Bibliography

Bailey, D., and Brorsen, B.W. (1985): "Dynamics of Regional Fed Cattle Prices," *Western Journal of Agricultural Economics,* 10:126–33.

Bessler, D.A., and Kling, J.L. (1990): "Prequential Analysis of Cattle Prices," *Applied Statistics: Journal of the Royal Statistical Society,* Series c, 39(1):95–106.

Box, G., and Jenkins, G. (1970): *Time Series Analysis: Forecasting and Control,* San Francisco: Holden Day.

Brorsen, B.W., Oellermann, C.M., and Farris, P.L. (1989): "The Live Cattle Futures Market and Daily Cash Price Movements," *Journal of Futures Markets,* 9:273–282.

Campbell, J., and Shiller, R. (1988): "Interpreting Cointegration Models," *Journal of Economic Dynamics and Control,* 12:505–522.

Corbae, D., and Ouliaris, S. (1988): "Cointegration and Tests of Purchasing Power Parity," *The Review of Economics and Statistics,* 70:508–511.

Cox, C. C. (1976): "Futures Trading and Market Information," *Journal of Political Economy,* 84:1215–1237.

Dickey, D. A., and Fuller, W. (1979): "Distribution of the Estimators for Autoregression Time Series with a Unit Root," *Journal of the American Statistical Association,* 74: 427–431.

Doan, T., and Litterman, R. (1989): *"User Manual: RATS,"* Minneapolis: VAR Econometrics.

Engle, R. F., and Granger, C. (1987): "Cointegration and Error Correction: Representation, Estimation and Testing," *Econometrica,* 55:251–276.

Engle, R., and Yoo, S. (1987): "Forecasting and Testing in Cointegrated Systems," *Journal of Econometrics,* 35:143–159.

Garcia, P., Leuthold, R. M., Fortenbery, T. R., and Sarassoro, G. F. (1988): "Pricing Efficiency in the Live Cattle Futures Market: Further Interpretation and Measurement," *American Journal of Agricultural Economics,* 70:162–169.

Gardner, B. (1976): "Futures Prices in Supply Analysis," *American Journal of Agricultural Economics,* 58:81–84.

Granger, C. (1986): "Developments in the Study of Cointegrated Random Variables," *Oxford Bulletin of Economics and Statistics.* 68:213–228.

Granger, C. (1980): "Testing for Causality: A Personal Viewpoint," *Journal of Economic Dynamics and Control,* 2:329–352.

Granger, C. (1988): "Some Developments in a Concept of Causality," *Journal of Econometrics,* 39:199–211.

Granger, C., and Escribano, A. (1988): "The Long Run Relationship Between Prices from an Efficient Market: The Case of Gold and Silver," Unpublished Paper, Department of Economics, University of California, San Diego.

Gould, J., and Nelson, C. (1974): "The Stochastic Behavior of the Velocity of Money," *American Economic Review,* 64:405–417.

Hendry, D. (1986): "Econometric Modelling with Cointegrated Variables: An Overview," *Oxford Bulletin of Economics and Statistics,* 48:201–211.

Hsiao, C. (1979): "Autoregressive Modeling of Canadian Income and Money Data," *Journal of the American Statistical Association,* 74:533–560.

Hudson, M., Leuthold, R. M., and Sarassoro, G. F. (1987): "Commodity Futures Price Changes: Recent Evidence for Wheat, Soybeans and Live Cattle," *Journal of Futures Markets,* 7:287–301.

Leuthold, R. M. (1979): "An Analysis of the Futures—Cash Price Basis for Live Beef Cattle," *North Central Journal of Agricultural Economics,* 1:47–52.

Leuthold, R. M. (1974): "The Price Performance of the Futures Market of a Nonstorable Commodity," *American Journal of Agricultural Economics,* 56:271–279.

Leuthold, R. M. (1972): "Random Walk and Price Trends: The Live Cattle Futures Market," *The Journal of Finance,* 27:879–889.

Leuthold, R. M., and Hartmann, P. A. (1981): "An Evaluation of the Forward-Pricing Efficiency of Livestock Futures Markets," *North Central Journal of Agricultural Economics,* 3:71–80.

McCarty, J. (1987): Personal Communication, U.S. Department of Agriculture, Agricultural Marketing Service, Livestock Division, Amarillo, TX.

Naik, G., and Leuthold, R. M. (1988): "Cash and Futures Price Relationships for Nonstorable Commodities: An Empirical Analysis Using a General Theory," *Western Journal of Agricultural Economics,* 13:327–338.

Nerlove, M., Grether, D., and Carvalho, J. (1979): *Analysis of Economic Time Series,* New York: Academic Press.

Oellermann, C. M., and Farris, P. L. (1985): "Futures or Cash: Which Market Leads Live Beef Cattle Prices?" *Journal of Futures Markets,* 5:529–538.

Paul, A. B. (1986): "Liquidation Bias in Futures Price Spreads," *American Journal of Agricultural Economics,* 68:312–321.

Powers, M. J. (1970): "Does Futures Trading Reduce Price Fluctuations in the Cash Markets?" *The American Economic Review,* 60:460–464.

Purcell, W. D., Flood, D., and Plaxico, J. S. (1979). "Cash-Futures Interrelationships in Live Cattle: Causality, Variability, and Pricing Processes," *Livestock Futures Research Symposium.* pp. 135–156.

Sargent, T. (1979): *Macroeconomic Theory,* New York: Academic Press.

Taylor, G. S., and Leuthold, R. M. (1974): "The Influence of Futures Trading on Cash Cattle Price Variations," *Food Research Institute Studies,* 13:323–329.

Tiao, G., and Box, G. (1981): "Modeling Multiple Time Series with Applications," *Journal of the American Statistical Association,* 75:802–816.

Weaver, R. D., and Banerjee, A. (1982): "Cash Price Variation in the Live Beef Cattle Market: The Causal Role of Futures Trade," *Journal of Futures Markets,* 2:367–389.