# Determining the Relevant Fair Value(s) of S&P 500 Futures: A Case Study Approach

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A fundamental consideration for potential users of stock index futures is the determination of the futures' break-even price or fair value. Conceptually, being able to sell futures at prices above the break-even, or to buy futures at prices below the break-even, offers opportunity for incremental gain. This article points out an important, though widely unappreciated, caveat. That is, no single break-even price is universally appropriate. Put another way, the break-even price for a given institution depends on the motivation of that firm as well as on its marginal funding and investing yield alternatives.

In this article, five differentiated objects are identified, and the calculations of the respective break-even futures prices are provided. The various objectives are: (a) to generate profits from arbitrage activities, (b) to create synthetic money market instruments, (c) to reduce exposure to equities, (d) to increase equity exposure, and (e) to maintain equity exposure using the most cost-effective instrument via stock/futures substitution. All of these objectives have the same conceptual starting point, which relates to the fact that a combined long stock/short futures position generates a money market return composed of the dividends on the stock position as well as the basis[1] adjustment of the futures contract. Under the simplified assumptions of zero transactions costs and equal marginal borrowing and lending rates, the underlying spot/features relationship can be expressed as follows:

$$F = S\left(1 + (i - d)\frac{t}{360}\right) \tag{1}$$

where: $F$ = break-even futures price, $S$ = spot index price, $i$ = interest rate (expressed as a money market yield), $d$ = projected dividend rate (expressed as a money market yield), and $t$ = number of days from today's spot value date to the value date of the futures contract.

In equilibrium, the actual futures price equals the break-even futures price, and thus the market participant would either have no incentive to undertake the transactions or be indifferent between competing tactics for an equivalent goal.

The author appreciates helpful comments from Dan Siegel and two anonymous reviewers.

[1]"Basis," in this article, is defined as the futures price minus the spot index value. Elsewhere, the calculation might be made with the two prices reversed.

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The Journal of Futures Markets, Vol. 11, No. 4, 453–460 (1991)

CCC 0270-7314/91/040453-08$04.00

Moving from the conceptual to the practical simply requires the selection of the appropriate marginal interest rate for the participant in question, as well as precise accounting for transactions costs. This article demonstrates that these considerations foster differences between the break-even prices among the alternative goals considered. Each goal is explained more fully, and the respective theoretical futures prices are presented.

## GENERATING PROFITS FROM ARBITRAGE ACTIVITIES

Generally, arbitrage is explained as a process whereby one identifies two distinct marketplaces where something is traded and then waits for opportunities to buy in one market at one price and sell in the other market at a higher price. This same process is at work for stock/futures arbitrage, but these market participants tend to view their activities with a slightly different slant. They enter an arbitrage trade whenever (a) buying stock and selling futures generates a return that exceeds financing costs, or (b) selling stocks and buying futures results in an effective yield (cost of borrowing) that falls below marginal lending rates. Completed arbitrages require a reversal of the starting positions, and the cost for both buying and selling stocks and futures must be included in the calculations.[2] Thus, the total costs of an arbitrage trade reflects the bid/ask spreads on all of the stocks involved in the arbitrage, the bid/ask spread for all futures positions, and all commission charges on both stocks and futures.[3] Table I calculates these arbitrage costs under three different scenarios. In all cases, the current starting value of the stock portfolio, based on last-sale prices, is \$100 million, and the S&P 500 index is valued at 335.00. The hedge ratio is calculated in the traditional manner.[4]

$$H = \frac{V \times \text{Beta}}{\text{S\&P} \times 500}$$

where: $H$ = hedge ratio (number of futures contracts required), $V$ = value of the portfolio, Beta = portfolio beta, and S&P = spot S&P 500 index price. The average price per share is estimated to be the S&P 500 index divided by five.

In column A, transactions are assumed to be costless, reflected by zero values for bid/ask spreads as well as zero commissions. In column B, more typical conditions are shown. Commissions on stock are assumed to be \$.02 per share; bid/ask spreads on stocks are assumed to be ⅛ (\$.125 per share); commissions on futures are assumed to be \$12 on a round-turn basis (i.e., for both buy and sell transactions); and bid/ask spreads on futures are assumed to be 1 tick or 0.05, worth \$25.[5] Column C

[2]If any fees or charges apply to the borrowing or lending mechanisms, these too would have to be incorporated in the calculations. Put another way, for the calculations that are presented in this article, the marginal borrowing and lending rates are effective rates, inclusive of all such fees.

[3]Brennan and Schwaltz (1990) note that the cost of closing an arbitrage position may differ if the action is taken at expiration versus prior to expiration. Thus, the appropriate arbitrage bound should reflect whether or not the arbitrageur is expecting (or hoping) to exercise "early close-out option."

[4]See Kawaller (1985) for a discussion of the justification for this hedge ratio.

[5]In practice, it may be appropriate to assume two different cost structures for the upper- and lower-bound break-even calculations because costs differ depending on whether the trade starts with long stock/short futures or vice versa. The difference arises because initiating the short stock/long futures arbitrage requires the sales of stock on an uptick. The "cost" of this requirement is uncertain because the transactions price is not known at the time the decision is made to enter the arbitrage. No analogous uncertainty exists when initiating the arbitrage in the opposite direction.

**Table I**
**ARBITRAGE BREAK-EVENS**

| **Arbitrage Costs** | **A** | **B** | **C** |
|---|---|---|---|
| Index value | 335 | 335 | 335 |
| Size of portfolio | 1,000,000,000 | 1,000,000,000 | 1,000,000,000 |
| Average price per share | 67 | 67 | 67 |
| Number of shares | 1,492,537 | 1,492,537 | 1,492,537 |
| Commission per share of stock | 0 | 0 | 0 |
| Stock commissions per side | 0 | 29,851 | 29,851 |
| Stock commissions (Rt) | 0 | 59,701 | 59,701 |
| Bid/ask per unit of stock | 0 | 0.125 | 0.5 |
| Bid/ask stock | 0 | 186,567 | 746,269 |
| Number of futures contracts | 597 | 597 | 597 |
| Commissions per round turn | 0 | 12 | 12 |
| Futures commissions | 0 | 7164 | 7164 |
| Bid/ask per futures contract | 0.00 | 0.05 | 0.05 |
| Bid/ask futures | 0 | 14,925 | 149,250 |
| Dollar costs | 0 | 268,358 | 962,384 |
| Index point cost | 0.00 | 0.90 | 3.22 |
| Marginal borrowing rate | 9.00% | 9.00% | 9.00% |
| Marginal lending rate | 8.00% | 8.00% | 8.00% |
| Dividend rate | 3.50% | 3.50% | 3.50% |
| *Shorter horizon (case a):* | | | |
| Days to expiration | 30 | 30 | 30 |
| Upper bound | 336.54 | 337.43 | 339.76 |
| Lower bound | 336.26 | 335.36 | 333.03 |
| No-arbitrage range | 0.28 | 2.08 | 6.73 |
| *Longer horizon (case b):* | | | |
| Days to expiration | 60 | 60 | 60 |
| Upper bound | 338.07 | 338.97 | 341.29 |
| Lower bound | 337.51 | 336.61 | 334.29 |
| No-arbitrage range | 0.56 | 2.36 | 7.01 |

assumes the same commission structure as that of column B; but bid/ask spreads are somewhat higher, reflecting a decline in liquidity relative to the former case. This scenario might also be viewed as representing the case where impact costs of trying to execute a stock portfolio are expected to move initial bids or offers for a complete execution. The index point costs in all cases reflects the respective dollar costs on a per contract basis.

The arbitrageur would evaluate two independent arbitrage bounds: an upper bound and a lower bound. During those times when futures prices exceed the upper arbitrage boundary, profit could be made by financing the purchase of stocks at the marginal borrowing rate and selling futures. When the futures prices are below the lower bound, profits could be made by selling stocks and buying futures, thus creating a synthetic borrowing, and investing at the marginal lending rate. In both cases, the completed arbitrages would require an unwinding of all the original trades.

The upper bound is found by substituting the arbitrage firm's marginal borrowing rate in eq. (1), and adding the arbitrage costs (in basis points) to this calculated value. In the case of the lower arbitrage boundary, the marginal lending rate is used for the variable $i$ in eq. (1), and the arbitrage costs are subtracted. The calculations in Table I assume marginal borrowing and lending rates of 9% and 8%, respectively, and a dividend rate of 3.5%. The upper and lower arbitrage boundaries are given for the three alternative cost structures. For comparative purposes, two sets of arbitrage boundaries are generated for two different terms.

Most obvious is the conclusion that an arbitrageur with a higher (lower) cost structure or a wider (narrower) differential between marginal borrowing and lending costs would face wider (narrower) no-arbitrage boundaries. In addition, Table I also demonstrates the time-sensitive nature of the difference between the two bounds, or the no-arbitrage range. As time to expiration expands, this range increases, monotonically, and all other considerations hold constant.

## CREATING SYNTHETIC MONEY MARKET SECURITIES

The case of the firm seeking to construct a synthetic money market income security by buying stocks and selling futures is a slight variant of the arbitrage case described in the prior section.[6] In this situation, too, the firm seeks to realize a rate of return for the combined long stock/short futures positions, however, the relevant interest rate that underlies the determination of the break-even futures price is different. While the arbitrageur who buys stock and sells futures does so whenever the resulting gain betters his marginal borrowing rate, the synthetic fixed-income trader endeavors to outperform the marginal lending rate. For both, however, the imposition of transaction costs necessitates the sale of the futures at a higher price than would be dictated by the costless case.

Not surprisingly, the break-even price for this firm is directly related to both transaction costs and time to expiration. What may not be quite as readily apparent is the fact that, at least theoretically, situations may arise that provide no motivation for arbitrageurs to be sellers of futures, while at the same time offering a motivation for a potentially much larger audience of money managers to be futures sellers. Put another way, large-scale implementation of the synthetic money market strategy by many market investors could certainly enhance these firms' returns, but also have the more universally beneficial effect of reducing the range of futures price fluctuation that *do not* induce relative-price-based trading strategies.

Yet another seemingly perverse condition that is highlighted by these calculations is that firms that operate less aggressively in the cash market, and thereby tend to have lower marginal lending rates, likely have a greater incremental benefit from arranging synthetic securities than firms that seek out higher cash market returns. For example, assume firm A has access to Euro Deposit markets while firm B deals only with lower-yielding U.S. domestic banks; and assume further that the difference in marginal lending rates is .25%. Firm B's break-even futures price necessarily falls below that of firm A. At any point in time, however, the current futures bid is relevant for both firms. Assuming the two firms face the same transaction cost structures, this futures price would generate the same effective yield for the two firms. Invariably, firm B will find a greater number of yield enhancement opportunities than will firm A; and any time both firms are attracted to this strategy simultaneously, B's incremental gain will be greater.

[6]The case where the firm already holds the stock is considered later.

## DECREASING EQUITY EXPOSURES

The case of the portfolio manager who owns equities and is looking to eliminate that exposure requires a further determination before the break-even calculation can be made. That is, two different break-evens would result depending on whether the desired reduction in equity exposure is expected to be permanent or temporary.

First, consider the case where the shift out of equities is expected to be permanent. Hedging with futures simply defers the actual stock transaction. At the same time, it introduces futures transactions costs that would otherwise be saved if the immediate sale of stock were chosen. The determination of this break-even, therefore, requires an evaluation of the return that one could realize by liquidating stock today and investing the resulting funds in some money market security maturing at the futures value date, versus the return of hedging the stock portfolio today and subsequently liquidating it, on the futures value date.

In calculating the returns from the traditional sell stocks/buy money market securities tactic, one should recognize that the liquidation cost effectively "haircuts" the portfolio. For example, the liquidation of a $100 million portfolio involves an immediate expense such that some amount *less than* the original $100 million becomes available for reinvestment. Thus, the portfolio manager realizes a lower fixed-income return than the nominal yield on the proposed money market security. The break-even futures price would be that price which, when including all transactions costs, generates the same realized yield as the net money market return available from the shift into money market instruments.

In Table II, the haircut is estimated to reflect half of the bid/ask spread as well as the stock commissions. The same commission and bid/ask structure is assumed as that which faces the firms analyzed in the prior section; and, similarly, the same marginal investment rate (8%) is incorporated. Under these conditions, the manager who chooses the liquidation of the stock portfolio, and the investment of the proceeds at 8% (rather than hedging), realizes an effective net money market return of 6.51% for 30 days or 7.25% for 60 days. Respective break-even futures prices are 336.33 and 337.58.

The case where the decision to reduce exposure is more likely to be temporary involves a minor modification to the above calculations. That is, operating exclusively in the arena of stocks would add the cost of repurchasing a portfolio, thereby lowering the net money market return even further. In contrast, the hedging alternative generates no stock charges. As a consequence, the break-evens in this case are substantially below the break-evens required for the former example.

## INCREASING EQUITY EXPOSURE

Perhaps the easiest situation to explain is the choice between buying today at the spot price versus buying in the future at the futures price. This determination simply requires calculating the forward value of the index, which, in turn, reflects the opportunity costs of foregoing interest income of a fixed-income investment alternative as well as an adjustment for transactions costs of futures, alone.[7] For the case

[7]Stock costs would be roughly comparable whether one were to buy now or later, so they do not enter into the calculation. This treatment, admittedly, is not precise. For example, with a significant market move, the number of shares required may vary, as may the average bid-ask spreads; therefore, some differences may arise. Moreover, the statement ignores the fact that although absolute magnitudes may be identical in both the buy-now or buy-later cases, the present values of these charges may differ. This consideration, if taken into account more rigorously, would bias decision toward a later purchase. For the purposes of this analysis, however, these differences are ignored.

**Table II**
**REDUCING EQUITY EXPOSURES:**
**PERMANENT ADJUSTMENT**

| | |
|---|---|
| *Short hedging considerations:* | |
| Index value | 335 |
| Size of portfolio | 1,000,000,000 |
| Portfolio beta | 1.0 |
| Average price per share | 67 |
| Number of shares | 1,492,537 |
| Commission per share | 0.02 |
| Commissions per side | 29,850.74 |
| Bid/ask per stock | 0.125 |
| 1/2 bid/ask stock | 93,283.56 |
| Total stock costs | 123,134.30 |
| Investable funds | 99,876,865.70 |
| Money market return | 8.00% |
| Hedge calculation | 597.0 |
| Number of futures contracts | 597 |
| Commissions (rnd trn) | 12.00 |
| Futures commissions | 7164.00 |
| Bid/ask per contract | 0.05 |
| Bid/ask futures | 14,925.00 |
| Total futures costs | 22,089.00 |
| Total stock costs | 123,134.30 |
| Dollar costs/contract | 243.26 |
| Index point cost | 0.49 |
| Dividend rate | 3.50% |
| *Shorter horizon (case a):* | |
| Days to end point | 30 |
| Ending value | 100,542,711.47 |
| Net money market return | 6.51% |
| Break-even futures price | 336.33 |
| *Longer horizon (case b):* | |
| Days to end point | 60 |
| Ending value | 101,208,557.24 |
| Net money market return | 7.25% |
| Break-even futures price | 337.58 |

of the same prototype firm discussed in the earlier sections, and given the same portfolio, the opportunity cost is generated using the marginal lending rate of 8.0%. Thus, the portfolio manager would be indifferent between buying stocks now and hedging for a future purchase if the futures are cheaper than the price calculated from eq. (1), inputting 8.0% for $i$. In this case, with the spot S&P 500 index at 335.00 and 30 days to the futures value date, the break-even price is 336.18. For a 60-day horizon, the break-even becomes 337.44.

## MAINTAINING EQUITY EXPOSURE IN THE MOST COST EFFECTIVE INSTRUMENT

Consider the case of the portfolio manager who currently holds equities with the existing degree of exposure at the desired level. Even this investor may find using

**Table III**
**ALTERNATIVE BREAK-EVEN PRICES**

| | Days to Expiration | |
|---|---|---|
| | **30-Days** | **60-Days** |
| Lower arbitrage boundary[a]/futures substitution break-even | 335.36 | 336.61 |
| Temporary equity adjustment (short hedge) break-even | 335.50 | 336.76 |
| Long hedge break-even | 336.18 | 337.44 |
| Permanent equity adjustment (short hedge) break-even | 336.33 | 337.58 |
| Synthetic fixed income break-even | 337.16 | 338.41 |
| Upper arbitrage boundary | 337.43 | 338.97 |

[a]Not reflective of costs associated with the uptick rule.

futures to be attractive if they are sufficiently cheap. At some futures price it becomes attractive to sell the stocks and buy the futures, thereby maintaining the same equity exposure. The break-even price for this trader, then, would be the trigger price. That is, any lower futures price than this break-even would induce the substitution of futures for stocks and generate incremental benefits.

Like the prior case, this strategy rests on the comparison of present versus futures values; and again, the firms' marginal lending rate is the appropriate discounting factor. Regarding trading costs, commissions and bid/ask spreads for both stocks and futures must be taken into account, as the move from stocks to futures would be temporary. Thus, the break-even price would be lower than the zero-cost theoretical futures price by the basis point costs of the combined commissions and bid/ask spreads.

For the prototype firm with the marginal lending rate of 8.00%, under the same normal market assumptions used throughout, the break-even price for 30- and 60-day horizons becomes 335.36 and 336.61, respectively.[8]

## CONSOLIDATION AND SUMMARY

Table III shows the respective break-even prices that are relevant to the various applications discussed in the article, in ascending order. All calculations relate to a firm with a marginal borrowing rate of 9.00% and a marginal lending rate of 8.00%. Break-even prices are given for two different time spans for the hedging period: 30 days and 60 days. Further, these calculations reflect the additional assumption of "normal" transactions cost and bid/ask spreads.

The highest price for which it becomes advantageous to take a long futures position is the long hedger's break-even price; and if prices decline sufficiently from this value, such that they fall below the lower arbitrage boundary, additional market participants—namely arbitrageurs—will be induced to buy futures as well. The lowest price for which it becomes advantageous to sell futures would be the break-even for the temporary short hedger; and, in a similar fashion, if prices rise sufficiently above this level, additional short sellers will be attracted to these markets.

Note that regardless of the time horizon, the lowest price for which buying futures is justified (336.18 or 337.44) is higher than the highest price for which selling

[8]This result happens to be identical to that shown for the lower arbitrage bound of the firm operating with the same cost structure. As explained in footnote 5, however, the arbitrage firm that sells stock short has additional costs that do not apply to the stock/futures substituter. Thus, in practice, the break-even for the substituter is likely to be a higher price than the lower bound for the equivalent firm involved with arbitrage.

futures are justified (335.50 or 336.76). Thus, at every futures price, there is at least one market participant who "should" be using this market. Moreover, it is also interesting that if the futures price enables the arbitrageur to operate profitably, at least one other market participant would find the futures to be attractively priced as well. For example, if the futures are below the lower arbitrage bound, aside from the arbitrageur, the long-hedger would certainly be predisposed to buying futures rather than buying stocks; and, if the futures price is above the upper arbitrage bound, willing sellers would include arbitrageurs, both temporary and permanent short hedgers, and those constructing fixed-income securities.

The overall conclusion, then, is that it pays (literally) to evaluate the relevant break-even prices for any firm interested in any of the above strategies—a population that includes all firms that manage money market of equity portfolios. At every point in time, at least one strategy will dictate the use of futures as the preferred transactions vehicle because use of futures in the given situation will add incremental value. Failure to make this evaluation would undoubtedly result in either using futures at inopportune moments or, more likely, failing to use futures when it would be desirable to do so. In either case, neglecting to compare the currently available futures price to the correct break-even price ultimately results in suboptimal performance.

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