Analyzing Portfolios with Derivative Assets: A Stochastic Dominance Approach Using Numerical Integration

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INTRODUCTION

Writing covered call options or buying protective put options has recently attracted the attention of both practitioners and academics, especially with the creation of index options for the purpose of manipulating portfolio returns (Heston, (1987), Fabozzi (1989)). Writing (or selling) covered call options against an existing portfolio can generate additional cash flows. Similarly, buying protective put options against an existing portfolio can reduce the client's downside risk.

Standard techniques, which rely on the mean-variance rule for evaluating portfolios, are not accurate for portfolios with options because, with options, the normality assumption is clearly violated. Bookstaber and Clarke (1985) identified this problem in evaluating the performance of portfolios on which option positions are taken on individual stocks. The tools best suited for solving this performance evaluation problem are stochastic dominance rules.

It is not a simple task to establish the preference between a portfolio with options vs. a portfolio without options. Several theoretical studies have investigated the conditions under which the underlying asset would be preferred over the call option (and vice versa) by risk averse investors. It turns out that the mathematical analyses are quite difficult (Perrakis and Ryan (1984), Levy (1985), Ritchen (1985)).

The purpose of this research is to examine, via numerical integration, the existence of stochastic dominance between various methods of employing derivative assets. For example, under what conditions will index-covered call writing be preferred to just owning the underlying stocks for all risk averse investors? Is it ever possible for just owning the stock to be preferable to covered call writing for all risk averse investors? This research is significant for several reasons. Using numerical integration to answer these questions has never been done before. This research provides a useful methodology for the portfolio manager in practice. The strength of the conclusions is superior to previous work using numerical simulation because the approach here does not suffer from bias due to the particular simulation.

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STOCHASTIC DOMINANCE RULES

Stochastic dominance criteria are developed in published articles by Hadar and Russell (1969), Hanoch and Levy (1969), Rothschild and Stiglitz (1970), and Whitmore (1970). Stochastic dominance (SD) rules allow inferences to be made with only partial information about investor preferences (such as risk aversion) and no assumption regarding the nature of the underlying return distribution. SD rules are based on three classes of utility functions $U(i)$, $(i = 1, 2, 3)$, where u belongs to $U(1)$ if $u' > 0$; u belongs to $U(2)$ if $u' > 0$ and $u'' < 0$; and u belongs to $U(3)$ if $u' > 0$, $u'' < 0$, and $u''' > 0$ (where $'$ denote derivatives of utility with respect to wealth).

The utility class $U(1)$ is the set of all utility functions which increases with a positive change in wealth. Economically, investors in this utility class are referred to as preferring more to less, or nonsatiated.

The utility class $U(2)$ is a subset of $U(1)$. Specifically, $U(2)$ contains only the utility functions in $U(1)$ which have a negative second derivative. Economically, investors in this utility class are referred to as exhibiting marginal diminishing utility. That is, an additional dollar at a wealth level of \$1000 means more to an investor (has greater utility) than an additional dollar at a wealth level of \$100,000. Investors who have marginal diminishing utility are also called risk averse. These investors are considered risk averse since they will not participate in a "fair" risky activity, fair meaning zero expected return.

The utility class $U(3)$ is a subset of $U(2)$, where the third derivative is positive. $U(3)$ investors have a preference for more positively skewed investments.

The decision rules are called first-, second-, and third-degree stochastic dominance (FSD, SSD, and TSD) based on the utility class $U(i)$, $(i = 1, 2, 3)$, respectively. The decision rules are as follows. Let F and G be the cumulative distribution of wealth of two distinct strategies (say, owning stock or owning stock and writing call options). Then, F dominates G (FDG) by FSD, SSD, and TSD, if and only if

$$\text{FSD: } F(x) \leq G(x) \text{ for all } x$$

$$\text{SSD: } \int_{-\infty}^{x} [G(t) - F(t)]dt \geq 0 \text{ for all } x$$

$$\text{TSD: } \int_{-\infty}^{x} \int_{-\infty}^{v} [G(t) - F(t)]\, dtdv \geq 0 \text{ for all } x$$

and

$$E_F(x) \geq E_G(x)$$

(Also, for all rules a strict inequality for at least one value must exist.)

Stochastic dominance rules can be illustrated graphically. Figure 1 presents the cumulative distribution function (CDF) for several alternative futures call strategies (to be explained later). The FSD rule above implies the CDFs do not intersect. Figure 1 clearly illustrates that these two distributions intersect, hence, there is no FSD.

The SSD rule implies the cumulative area must always be positive. Clearly, it is only possible that the 100% call-writing strategy dominates the 75% call-writing strategy, and so forth. Figure 1 shows that the positive area (lower part of graph) is less than the negative area (upper part of graph); hence, there is no SSD or TSD. Therefore, it is not possible to establish preferences between these strategies based solely on the partial information contained in $U(i)$, $i = 1, 2, 3$.

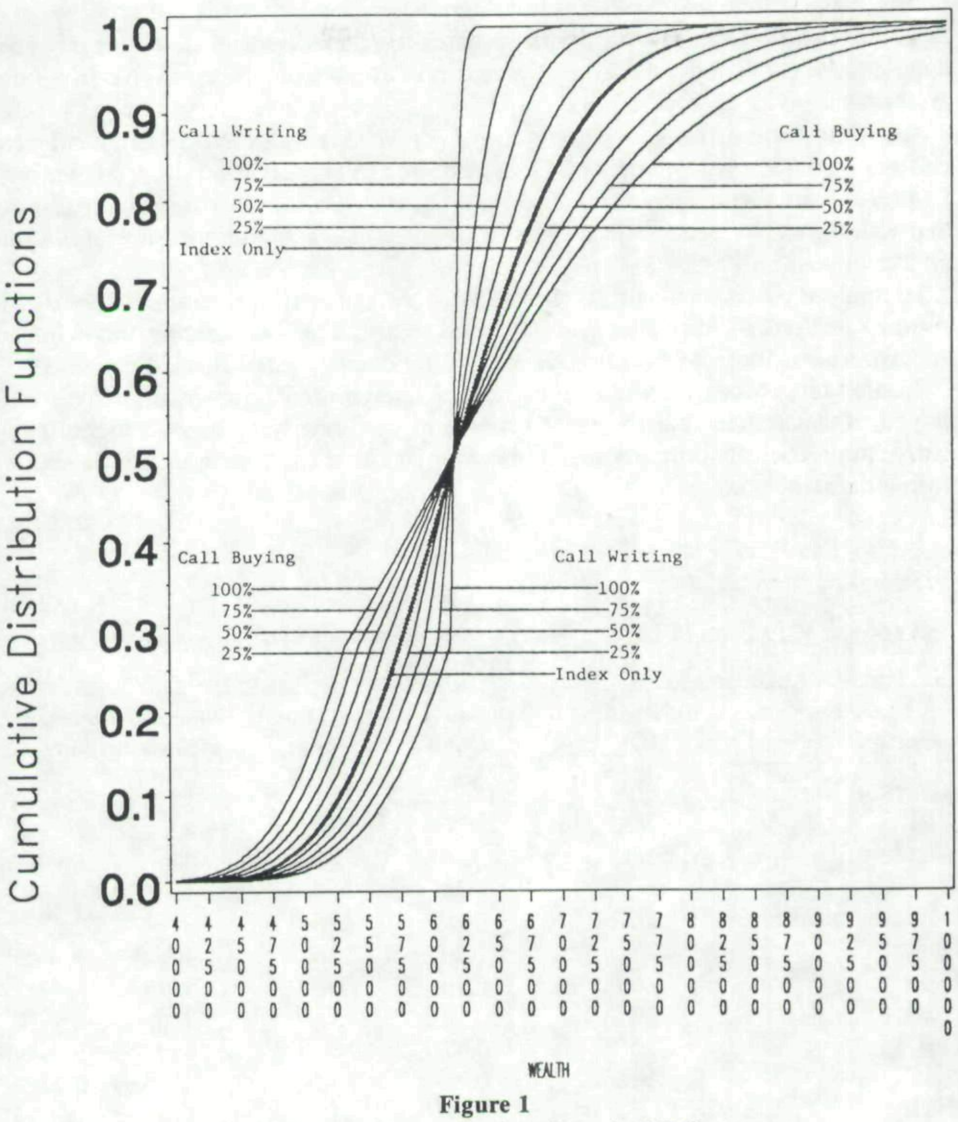

Figure 1
Futures call strategies based on the MMI.

END-OF-PERIOD WEALTH DISTRIBUTIONS

Index Only

Denote N_I as the number of "units" of the index owned. Let $\tilde{I}_1$ represent the value of the index at maturity and I_0 the initial index value. Thus, the end-of-period wealth from holding only stocks in the index is:

$$\tilde{W}_I^* = \tilde{I}_1 N_I + DYI_0 N_I \tag{1}$$

where DY denotes the dividend *yield* on the index (assumed to be certain).

There are basically two approaches to treating dividends: by assuming the dividends are paid continuously, or by assuming the dividends are paid at discrete points. The discrete assumption is used here because it better fits the problem. It is assumed also that the related dividend dates (e.g., ex-dividend date and payment date) can be predicted with perfect accuracy. Also, the appropriate risk-free rates are known.

As an illustration, Table I presents the Major Market Index (MMI) dividend yield calculation for the first quarter of 1990, assuming the observation date is January 1, 1990. Thus, in this example, $DY = 0.009047$ (or 0.9047%). It is important to note that although a dividend yield measure is used, reliance is on a measure of *discrete* dividends.

To analyze these various strategies, it is necessary to find the cumulative distribution functions (CDFs). The end of period wealth CDFs depend on the assumed underlying distribution of returns. Since the dividend adjusted Black–Scholes (1973) Option Pricing Model (Barone-Adesi and Whaley's (1987) approximation) is employed, it is assumed that the price relative of the underlying asset is lognormally distributed. The lognormality assumption implies that the return on the index, ignoring dividends, is

$$\tilde{R}_I = \ell n(\tilde{I}_1/I_0) \sim N(\mu, \sigma^2) \tag{2}$$

Table I
DIVIDEND YIELD CALCULATION FOR THE MMI FOR FIRST QUARTER OF 1990 (DIVISOR = 2.23510, MMI = 551.82)

Company	Ex-Dividend Date	Dividend Amount	Risk-Free Rate	Number of Days	Present Value[a]
American Express	3/29	$0.21	7.82%	88	$0.2062*
ATT	3/22	0.30	7.80	81	0.2950
Chevron	1/31	0.70	6.15	31	0.6965
Coca Cola	2/27	0.34	7.60	58	0.3361
Dow Chemical	3/22	0.65	7.80	81	0.6393
Dupont	2/9	1.20	7.18	40	1.1909
E.K.	2/27	0.50	7.56	58	0.4942
Exxon	2/7	0.80	7.18	38	0.5957
G.E.	3/5	0.47	7.74	64	0.4639
G.M.	2/10	0.75	7.18	41	0.7442
IBM	1/31	1.21	6.15	31	1.2039
Int. Paper	2/16	0.42	7.35	47	0.4162
INI	2/15	0.75	7.35	46	0.7433
MRK	3/5	0.45	7.74	64	0.4442
MMM	2/16	0.42	7.35	47	0.4162
MOB	1/31	0.65	6.15	31	0.6467
MO	3/11	0.74	7.74	70	0.3352
PE	1/16	0.45	6.15	16	0.4488
Sears	2/20	0.50	7.56	51	0.4949
USX	2/6	0.35	7.18	37	0.3475
Total					11.1589

DY = dividend yield = 11.1589/(551.82 * 2.23510) = 0.009047 or 0.9047%.
[a]Present value = dividend/$(1.0 + (\text{rate}/100))^{\text{days}/365}$.

That is, the log of the price relative is normally distributed with some mean (μ) and variance (σ^2). Thus, $\tilde{I}_1/I_0$ has a lognormal distribution with mean (α) and variance (β^2) where

$$\alpha = \exp(\mu + \sigma^2/2) \tag{3}$$

and

$$\beta^2 = \exp(2\mu + \sigma^2) * (\exp(\sigma^2) - 1) \tag{4}$$

(See Johnson and Kotz (1970), p. 115.) To develop the CDFs of various option strategy distributions, the following theorem is employed.

Theorem 1: Let X be lognormal with mean $E(\ell nX) \equiv \mu$ and $\mathrm{Var}(\ell nX) \equiv \sigma^2$, and let a and b be constants, where $a > 0$. Then, $X' = aX + b$ is a "three-parameter lognormal distribution." The cumulative distribution function of the three parameter lognormal distribution is:

$$F(X') = \int_b^{x'} \frac{1}{(t-b)\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}\{\ell n(t-b) - (\ell n(a) + \mu)\}^2\right) dt \qquad \text{for } x' > b$$

and

$$F(X') = 0 \qquad \text{for } x' \leq b \tag{5}$$

See Aitchison and Brown (1966, p. 11) and Johnson and Kotz (1970, p. 112) for proof of Theorem 1. For an application of this theorem see Levy and Kroll (1976, p. 753).

From eq. (1) and by Theorem 1 above, $\tilde{W}_I^*$ is three-parameter lognormally distributed where $a = I_0N_I$ and $b = DYI_0N_I$. Applying eqs. (3) and (4) to the distribution of $\tilde{W}_I^*$, the expected end of period wealth and variance from holding the uncovered stock index are

$$E(\tilde{W}_I^*) = \exp(\ln(I_0N_I) + \mu + \sigma^2/2) + DYI_0N_I \tag{6}$$

$$\mathrm{Var}(\tilde{W}_I) = \exp(2(\ln(I_0N_I) + \mu) + \sigma^2)(\exp(\sigma^2) - 1) \tag{7}$$

Appendix A provides a method for calculating the moments of the various investment strategies. The formulas in Appendix A are employed in calculating the univariate statistics.

Index Options

Calls Let N_C be the number of index calls purchased (if negative) or written (if positive). If a position in calls on the index is taken (keeping the total investment at I_0N_I), the end of period wealth will be

$$\begin{aligned} \tilde{W}_{IC}^* = {} & (\tilde{I}_1/I_0)(I_0N_I + C_IN_C) - N_C\max(0, \tilde{I}_1 - E) \\ & + DY(I_0N_I + C_IN_C) \end{aligned} \tag{8}$$

where E is the exercise price of the option and C_I is the price of the index call option. This assumes that if calls are written, the proceeds are invested in the stock. If calls are bought there are fewer funds to be invested in the stock. Thus, the total equity put by the investor from his own resources is I_0N_I.

Equation (8) can be written as

$$\tilde{W}^*_{IC} = \{((\tilde{I}_1/I_0) + DY)(I_0N_I + C_IN_C)\} \qquad \text{for } \tilde{I}_1 \le E$$

and

$$\begin{aligned}\tilde{W}^*_{IC} = \{((\tilde{I}_1/I_0) &+ DY)(I_0N_I + C_IN_C)\} \\ &-N_C(\tilde{I}_1 - E)\} \qquad \text{for } \tilde{I}_1 > E\end{aligned} \tag{9}$$

Equation (9) provides the terminal wealth in two ranges, $\tilde{I}_1 \le E$ and $\tilde{I}_1 > E$. Since the cumulative distribution function is derived in terms of terminal wealth, the ranges in eq. (9) are converted to be based on terminal wealth levels, by noting that if $I_1 = E$, then

$$\tilde{W}^*_{IC} = ((E/I_0) + DY)(I_0N_I + C_IN_C) \equiv W_C \tag{10}$$

Where W_C is the terminal wealth level at which a move is made from being out of the money to being in the money. Therefore, rearranging eq. (9)

$$\tilde{W}^*_{IC} = \{(\tilde{I}_1/I_0)(I_0N_I + C_IN_C) + DY(I_0N_I + C_IN_C)\} \qquad \text{for } \tilde{W}^*_{IC} \le W_C$$

and

$$\tilde{W}^*_{IC} = \{(\tilde{I}_1/I_0)(I_0N_I + N_C(C_I - I_0)) + DY(I_0N_I + C_IN_C) + N_CE\} \qquad \text{for } \tilde{W}^*_{IC} > W_C \tag{11}$$

The terminal wealth is given as a combination of two random variables. Hence, the terminal wealth distribution from employing index call options is no longer a three-parameter lognormal distribution, but a combination of parts from two different truncated three-parameter lognormal distributions. However, since the two parts are functions of the same random variable $(\tilde{I}_1/I_0)$, the cumulative distribution function can be calculated. This distribution function can be defined for each range ($X' \le W_C$ and $X' > W_C$).

$$\begin{aligned}F_{\tilde{W}^*_{IC}}(X') = \Bigg\{\int_{b_1}^{x'} \frac{1}{(t - b_1)\sigma\sqrt{2\pi}} \exp(-\frac{1}{2\sigma^2}\{\ell\text{n}(t - b_1) \\ - \ell\text{n}(a) + \mu)\}^2)dt\Bigg\} \qquad \text{for } x' \le W_C\end{aligned}$$

and

$$\begin{aligned}F_{\tilde{W}^*_{IC}}(X') = &\Bigg\{\int_{b_1}^{W_C} \frac{1}{(t - b_1)\sigma\sqrt{2\pi}} \exp(-\frac{1}{2\sigma^2}\{\ell\text{n}(t - b_1) - (\ell\text{n}(a_1) + \mu)\}^2)dt\Bigg\} \\ &+ \Bigg\{\int_{W_C}^{x'} \frac{1}{(t - b_2)\sigma\sqrt{2\pi}} \exp(-\frac{1}{2\sigma^2}\{\ell\text{n}(t - b_2) - (\ell\text{n}(a_2) + \mu)\}^2)dt\Bigg\} \\ &\qquad \text{for } x' > W_C\end{aligned} \tag{12}$$

where:

$b_1 = DY(I_0N_1 + C_0N_C)$
$a_1 = I_0N_1 + C_0N_C$
$b_2 = b_1 + N_CE$
$a_2 = a_1 - I_0N_C$

Notice that as long as $N_C < N_I$, then $a_1 > 0$ (since $I_0 \ge C_0$). Also, $a_2 > 0$ (since N_C is negative when $-N_C$ is positive), and thus $\ell\text{n}(a_1)$ and $\ell\text{n}(a_2)$ are defined.

Puts If a position in puts is taken, then the end-of-period wealth will be:

$$\tilde{W}^*_{IP} = (\tilde{I}_1/I_0)(I_0N_I + P_IN_P) - N_P\max(0, E - \tilde{I}_1) + DY(I_0N_I + P_IN_P) \tag{13}$$

where P_I is the price of the index put option.

Recall, once again, that, by definition, N_P is negative if puts are purchased. Equation (13) can be written as

$$\tilde{W}^*_{IP} = ((\tilde{I}_1/I_0) + DY)(I_0N_I + P_IN_P) - N_P(E - \tilde{I}_1) \qquad \text{for } \tilde{I}_1 \leq E$$

and

$$\tilde{W}^*_{IP} = ((\tilde{I}_1/I_0) + DY)(I_0N_1 + P_1N_P) \qquad \text{for } \tilde{I}_1 > E \tag{14}$$

Rewriting, and notice if $\tilde{I}_1 = E$, then

$$\tilde{W}^*_{IP} = ((E/I_0) + DY)(I_0N_I + P_IN_P) \equiv W_P \tag{15}$$

where W_P is the terminal wealth level at which a move is made from being in the money to being out of the money. We have

$$\tilde{W}^*_{IP} = \{(\tilde{I}_1/I_0)(I_0N_I + N_P(P_I + I_0)) + DY(I_0N_I + P_IN_P) - N_PE\} \qquad \text{for } \tilde{W}^*_{IP} \leq W_P$$

and

$$\tilde{W}^*_{IP} = \{(\tilde{I}_1/I_0)(I_0N_I + P_IN_P) + DY(I_0N_I + P_IN_P)\} \qquad \text{for } \tilde{W}^*_{IP} > W_P \tag{16}$$

The cumulative distribution function for put options is similar in structure to the cumulative distribution function for call options (see eq. (12)) except in the case of put options

$$b_1 = DY(I_0N_I + P_IN_P) - N_PE$$
$$a_1 = I_0N_I + (P_I + I_0)N_P$$
$$b_2 = DY(I_0N_I + P_IN_P)$$
$$a_2 = I_0N_I + P_IN_P$$

Since $I_0 > P_0$, $a_2 > 0$ as long as $-N_P < N_I$ (when $N_P < 0$). In the case of a_1, however, $a_1 > 0$ only if $I_0N_I > (P_0 + I_0)N_P$ which could be violated even for $-N_P < N_I$ (when $N_P < 0$). Therefore, in this analysis of put buying (recall that with put buying $N_P < 0$), only partial insurance puts of the portfolio are examined.

Index Futures Options

Denote N_{FC} and N_{FP} as the number of index futures calls and put options employed, respectively. If a position in calls on the index futures contract is taken, the end-of-period wealth will be:

$$\tilde{W}^*_{FC} = (\tilde{I}_1/I_0)(I_0N_I + C_FN_{FC}) - N_{FC}\max(0, \tilde{I}_1 - E) + DY(I_0N_I + C_FN_{FC}) \tag{17}$$

where C_F is the price of the futures call option on the index. To focus on the important differences between futures options and index options, it is assumed that the option expires on the same day as the futures contract. Thus, at expiration, the futures and index will have the same value. If a position in index futures put options is taken, then the end-of-period wealth will be

$$\tilde{W}^*_{FP} = (\tilde{I}_1/I_0)(I_0N_I + P_FN_{FP}) - N_{FP}\max(0, E - \tilde{I}_1) + DY(I_0N_I + P_FN_{FP}) \quad (18)$$

The CDFs for index futures options can be derived in exactly the same manner as for index options.

Index Futures

Let N_F denote the number of index futures employed. If a position in index futures is taken, the end of period wealth will be[1]

$$\begin{aligned}\tilde{W}^*_F = (\tilde{I}_1/I_0)(I_0N_I - m|N_F|) + m|N_F|\exp(rt) \\ + N_F(\tilde{I}_1 - F) + DY(I_0N_I - m|N_F|)\end{aligned} \quad (19)$$

where F is the futures price per contract, and m is the dollar amount of margin money required per index futures contract. Rearranging eq. (19):

$$\begin{aligned}\tilde{W}^*_F = (\tilde{I}_1/I_0)(I_0N_I - m|N_F| + I_0N_F) + m|N_F|\exp(rt) + FN_F \\ + DY(I_0N_I - m|N_F|)\end{aligned} \quad (20)$$

From eq. (20), and Theorem 1 above (eq. (5)), $\tilde{W}^*_F$ is three-parameter lognormally distributed where $a = I_0N_I - m|N_F| + I_0N_F$ and $b = m|N_F|\exp(rt) - F_{NF} + DY(I_0N_I - m|N_F|)$.

The next sections of this article provide an analysis of some specific distributions of $\tilde{I}_1/I_0$ and of some specific options and futures. These selected parameters allow portfolio insurance up to 75% of the index portfolio which assures $a_1 > 0$.

METHODOLOGY

Having the formula for cumulative distribution functions on options with the underlying asset (see eq. (12)), a numerical integration technique is used to calculate this cumulative distribution.[2] Obviously, the gain (or loss) on the investment strategies under consideration is a function of the observed value I_1 drawn out of the index distribution. Thus, it is assumed below that $\tilde{I}_1$ is lognormally distributed, and various sets of parameters characterizing this distribution are employed.

First, the univariate distributions on various investment strategies are analyzed (see Appendix A for the development of the first three central moments). Second, option strategies are evaluated by stochastic dominance criteria in an effort to establish preferences. The objective is to assess whether one strategy is preferable to another one by each of the stochastic dominance rules. (Recall that preference is based on numerical integration of estimated parameters, not based on historical data which, in this field of study, is particularly poor.)

It is particularly important to develop the appropriate pricing of these derivative assets. In Appendix B, the theoretical values for the derivative assets, based on Barone-Adesi and Whaley (1987), are developed.

[1]The absolute value of the number of futures contracts enters into eq. (19) because both sides of the contract are required to deposit margin. This approach differs from the implicit assumption that short option positions do not require margin.

[2]For a discussion of the numerical integration technique, see Piessens, et al. (1983).

RESULTS

The analysis is begun with a specific set of parameters based on the Major Market Index as of December 29, 1989. Thus, the results are relevant for a person who holds a portfolio similar to the MMI. Current values as well as estimated parameters from the U.S. market are used. Since some of the parameters are estimated, sensitivity analyses are carried out.

Assume the following underlying parameters:

- Current index value on December 29, 1989: $I_0 = 551.82$
- Standard deviation of index returns (based on implied volatility): $\sigma = 12.9\%$
- Risk-free rate on December 29, 1989 (based on one-year Treasury-bill rate from *Wall Street Journal,* January 2, 1990): $RF = 7.80\%$
- Expected return on the index (based on historical data): $ER = 11.4\%$
- Exercise price (assumed at the money): $E = 551.82$
- Dividend yield: $DY = 3.79\%$
- Margin requirement: $m = \$120.00$

where μ, σ, and RF are annualized continuously compounded values. The time to expiration is assumed to be one year and 100 units of the index are initially owned ($N_I = 100$). Thus, the initial wealth is $N_I I_0 = \$55,182.00$. By the dividend-adjusted American option pricing model and standard futures pricing techniques, Table II (part A) presents the option and futures prices (see Appendix B). The expected growth rate of the index is $ER - DY = 11.4 - 3.79 = 7.61\%$.

Before proceeding, it is necessary to convert the expected return (ER) into an estimate of μ, the mean of the normally distributed log of the price relative. (See eq. (1) and following.) From eq. (6), ER can be expressed as follows:

$$ER = \frac{E(\tilde{W}_T^*)}{W_0} - 1.0 \tag{21}$$

$$ER = \frac{\exp(\ell n(I_0 N_I) + \mu + \sigma^2/2) + DYI_0 N_I}{I_0 N_I} - 1.0 \tag{22}$$

Solving for μ:

$$\mu = \ell n(I_0 N_I(1 + ER - DY + 1.0)) - (\ell n(I_0 N_1) + \sigma^2/2) \tag{23}$$

which, in this case, equals 6.5%.

Table II (part B) presents a summary of the univariate statistics. The "No Options Strategy" means the manager holds only the index.

A few well-known results appear in Table II. First, moving from 100% covered call selling to 100% speculative call buying, the mean return, the standard deviation, and the skewness increase. Second, moving from 75% protective put buying to 100% speculative put writing, the mean return and the standard deviation increase while the skewness declines. These patterns are the same for index or futures options. As anticipated, the expected return changes in the same direction as the standard deviation.

The skewness is affected also by the selected investment strategy, a factor which has to be taken into account. With futures contracts, a move from a hedged posi-

Table II
SUMMARY STATISTICS

Part A: Prices and Critical Values

Index Options	**American Price**	**European Price**	**Critical Value**
Call	\$ 38.51	\$38.49	\$1245.39
Put	\$ 20.26	\$17.62	\$ 491.65
Futures Options:			
Call	\$ 39.63	\$38.50	\$ 673.37
Put	\$ 18.08	\$17.62	\$ 452.20
Futures:			
Index	\$574.40	N/A	N/A

Part B: Univariate Statistics

Position		**Long/Buying Mean**	**Long/Buying Standard Deviation**	**Long/Buying Skewness**	**Short/Selling/Writing Mean**	**Short/Selling/Writing Standard Deviation**	**Short/Selling/Writing Skewness**
Index only		12.60%	14.09%	0.391	N/A	N/A	N/A
Index call	25	13.35	16.63	0.565	11.84	11.61	−0.116
	50	14.11	19.20	0.683	11.09	9.22	−0.350
	75	14.87	21.78	0.766	10.33	7.02	−1.16
	100	15.62	24.38	0.83	9.58	5.24	−2.35
Index put	25	12.08	13.23	0.608	13.11	15.01	0.180
	50	11.57	12.42	0.821	13.63	15.97	−0.0181
	75	11.05	11.69	1.02	14.15	16.96	−0.202
	100	N/A	N/A	N/A	14.66	17.98	−0.370
Futures calls	25	13.30	16.62	0.565	11.90	11.62	0.116
	50	14.00	19.18	0.683	11.20	9.23	−0.348
	75	14.70	21.76	0.767	10.50	7.04	−1.16
	100	15.39	24.35	0.828	9.80	5.27	−2.34
Futures puts	25	12.19	13.24	0.607	13.00	15.00	0.180
	50	11.79	12.45	0.820	13.41	15.94	−0.0189
	75	11.39	11.73	1.015	13.81	16.92	−0.203
	100	N/A	N/A	N/A	14.22	17.93	−0.372
Futures	25	13.53	16.85	0.391	11.18	9.80	0.391
	50	14.47	19.61	0.391	9.75	5.52	0.391
	75	15.40	22.37	0.391	8.33	1.23	0.391
	100	16.34	25.12	0.390	N/A	N/A	N/A

Inputs: MMI = 551.82, expected growth rate of index = 7.61%, number of "units" owned = 100, strike price = 551.82, standard deviation = 12.9%, time to expiration = 365 days, risk-free rate = 7.8%, dividend yield = 3.79%, margin requirement = \$120/unit.

tion to a speculative position increases the mean and standard deviations. Skewness is unaffected by future positions.

Next, because of the important role that skewness plays, stochastic dominance tests are conducted. Recall that Barone-Adesi and Whaley's (1987) (BAW) option-pricing approximations are employed. Also, it is assumed that investors employ an option or futures strategy today, and then make no adjustments until the maturity of the contract. BAW's pricing values the opportunity for early exercise, hence, FSD is expected in certain cases. Specifically, index calls buying will dominate futures call buying because the end-of-period values for the options will be the same, and the purchase price for index calls is \$38.51 vs. \$39.63 for futures options. A similar analysis can be made for puts.

It is necessary to determine whether stochastic dominance exists outside of these obvious cases. The obvious cases include index call buying dominating futures call buying, futures call writing dominating index call writing, futures put buying dominating index put buying, and index put buying dominating futures put buying.

The critical value (the value of which early exercise will approximately occur) plays an important role in establishing preference between the various strategies. Consider the case of portfolio insurance with put options. Clearly one would prefer to pay as little as possible for the insurance; hence, one would invest in futures puts at \$18.08 (as opposed to \$20.26 for index puts) from Table II (part A). Early exercise of one's position would alter the cumulative distribution of wealth at expiration. When early exercise occurs, the time value of the option is essentially lost. The probability that the index put options will be exercised early is higher than for futures put options. Hence, it appears that futures index put options are clearly superior to index put options for portfolio insurance.

Now it is necessary to apply stochastic dominance rules to the various numerically generated cumulative distribution functions (CDFs). Figure 1 shows the CDFs of the futures call strategies from Table II. Clearly, because all the CDFs intersect, there is no FSD. By ordering the choices from the lower tail of the CDFs, one can examine the possibility of SSD. It is only possible for 100% covered call writing to dominate 75% covered call writing, and so forth. Because the distributions intersect only once, the means can be examined. Hence, there is no SSD or TSD because the means rise monotonically from 9.80% (100% writing) to 15.3% (100% buying).

Figure 2 shows the CDFs of the futures put strategies from Table II. Following the same procedure, it is found that there is no FSD, SSD, or TSD among these strategies.

Several interesting observations can be made by comparing Figures 1 and 2. First, by examining various amounts invested in puts or calls, it is apparent that the CDF can be rotated around one intersection point. The intersection point for calls is significantly higher than for puts. Second, because investments in calls require large amounts of wealth, the CDF is shifted by a larger magnitude. Third, depending on an investor's utility function, preferences can clearly be established. For example, if an investor has a severe disutility attached to receiving less than about \$52,500, then this investor will prefer 75% put buying (and even 100% put buying).

Figure 3 shows the CDFs from various futures strategies. Again, due to the singular intersection point and the means from Table II, there is no FSD, SSD, or TSD. Comparing Figures 1, 2, and 3, it is clear that the futures strategies intersect at a point in between the lower put intersection point (Fig. 2) and the higher call intersection point (Fig. 1).

It is now possible to examine various hedging strategies. Figure 4 provides a small sample of these strategies. Clearly, there is no FSD due to the intersections of

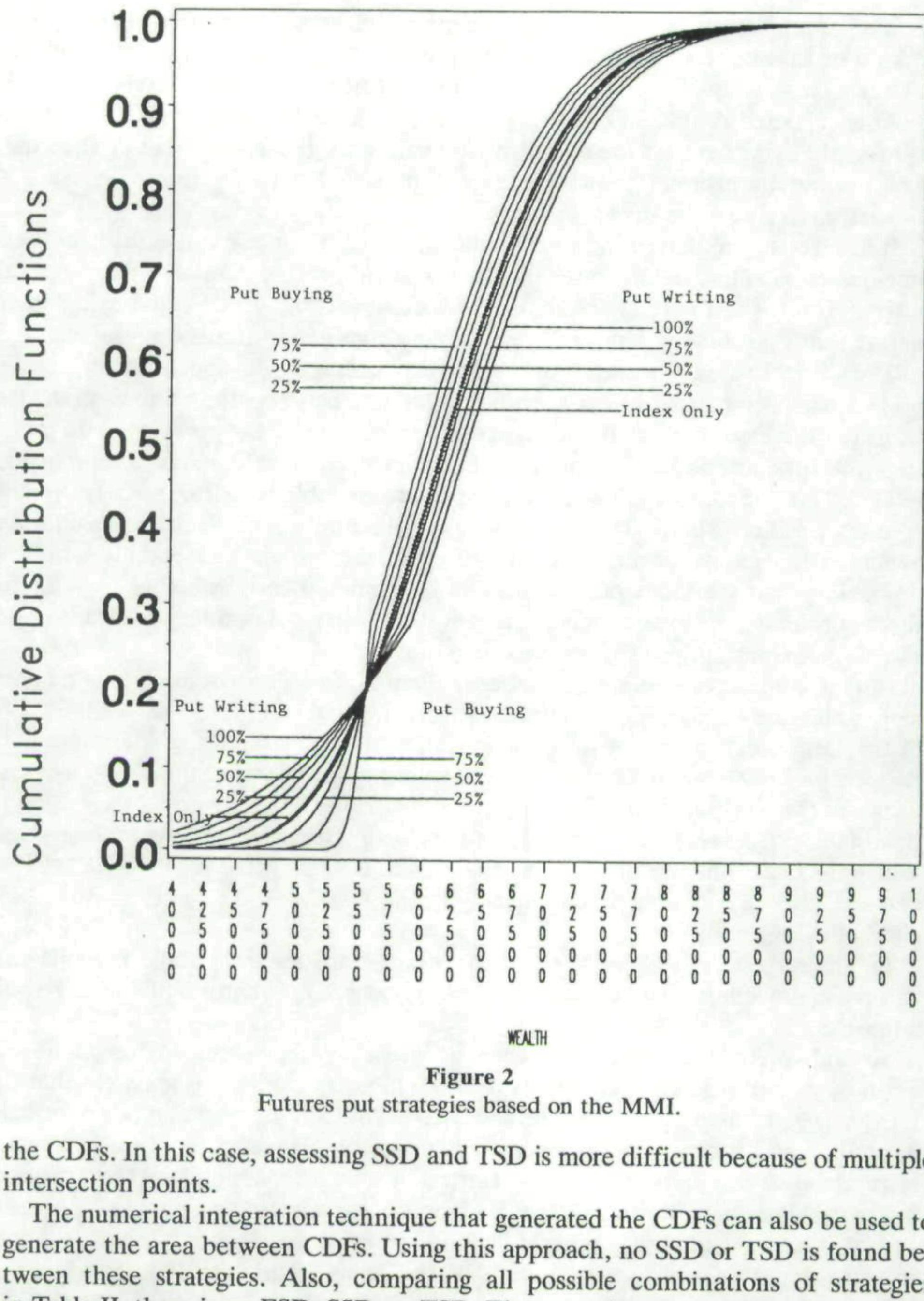

Figure 2
Futures put strategies based on the MMI.

the CDFs. In this case, assessing SSD and TSD is more difficult because of multiple intersection points.

The numerical integration technique that generated the CDFs can also be used to generate the area between CDFs. Using this approach, no SSD or TSD is found between these strategies. Also, comparing all possible combinations of strategies in Table II, there is no FSD, SSD, or TSD. These results are surprising because the underlying pricing of these derivative securities do *not* even depend on the existence of utility functions—much less risk aversion or a preference for positive skewness.

The results of sensitivity analysis are reported in Appendix C. Specifically, results are reported on lower and higher strike prices (Table C-1), lower and higher standard deviations (Table C-2), lower and higher risk-free rates (Table C-3), lower and higher index growth rates (Table C-4), lower and higher margin requirements

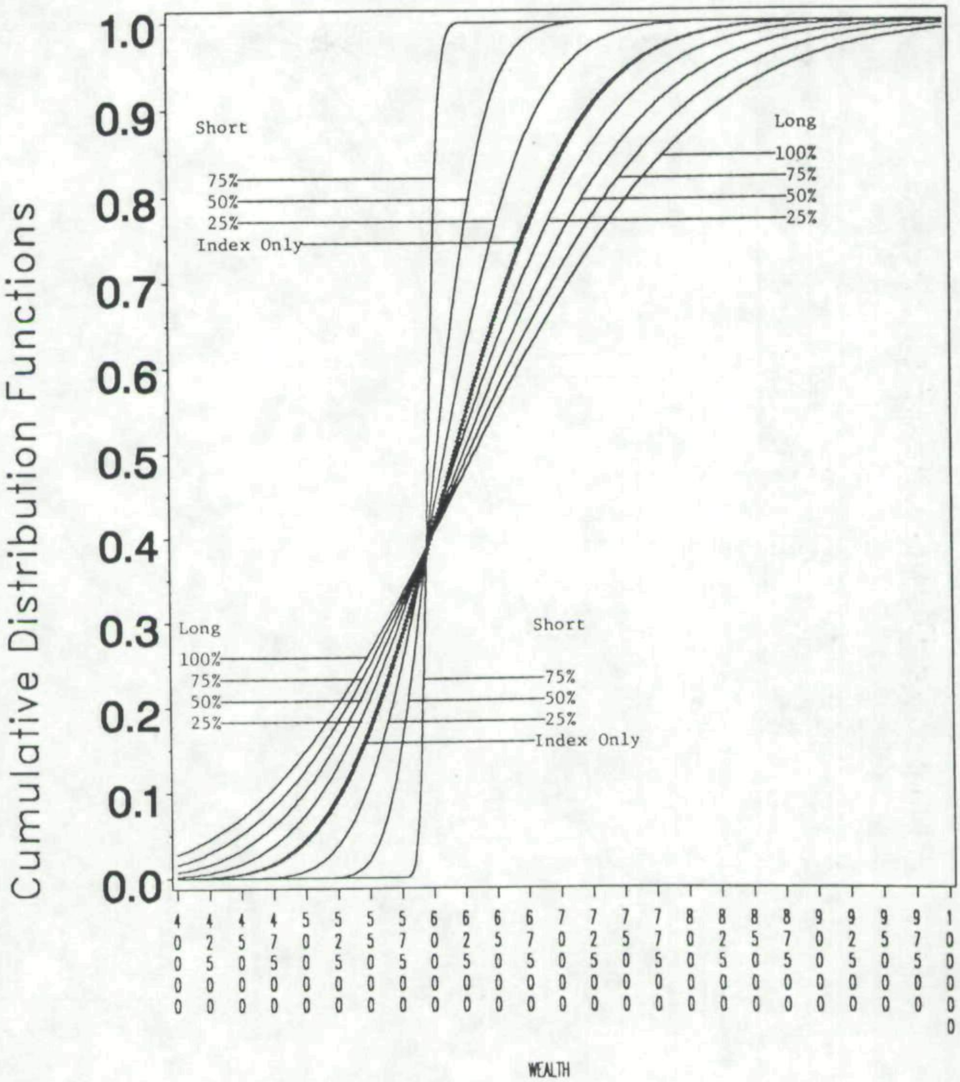

Figure 3
Futures strategies based on the MMI.

(Table C-5), and lower and higher dividend yield (Table C-6). The main conclusions are robust against the choice of the parameters.

CONCLUSIONS

This article develops a more powerful method, numerical integration for estimating portfolio distribution moments which is superior to Monte Carlo methods. Also, employing American option pricing approximation techniques, it is found that stochastic dominance exists. Specifically, portfolio insurance with futures index put options is cheaper and, hence, preferable to portfolio insurance with index put options. Also, covered call writing with futures index call options results in larger cash flows today and, hence, is preferable to covered call writing with index options.

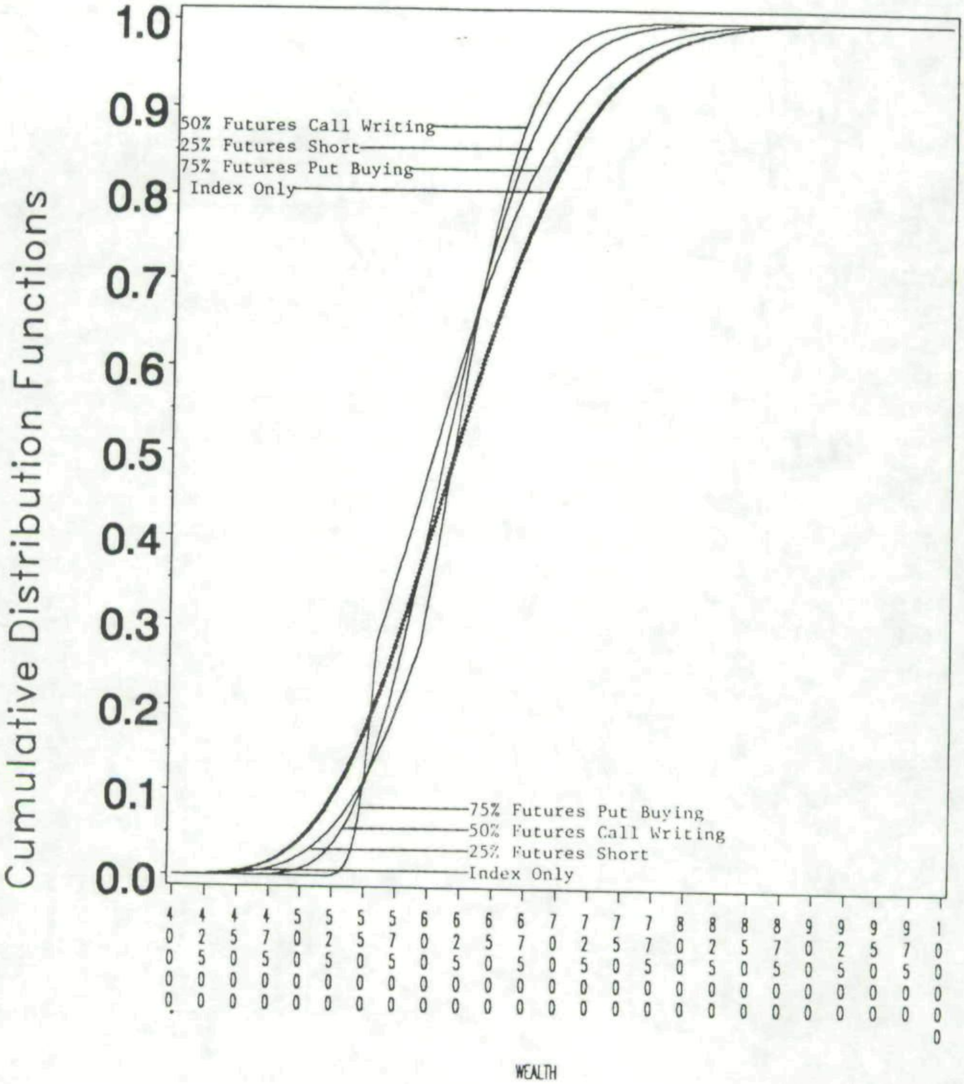

Figure 4
Call, put, and futures strategies based on the MMI.

Bibliography

Aitchison, J., and Brown, J. A. C. (1966): *The Lognormal Distribution*. Cambridge, UK: Cambridge University Press.

Arditti, F. R. (1971): "Another Look at Mutual Fund Performance," *Journal of Financial and Quantitative Analysis,* 6:909–912.

Barone-Adesi, G., and Whaley, R. E. (1987): "Efficient Analytic Approximation of American Option Values," *Journal of Finance,* 42:301–320.

Black, F. (1976): "The Pricing of Commodity Contracts," *Journal of Financial Economics,* 3:167–179

Black, F. and Scholes, M. (1973): "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy,* 81:637–654.

Bookstaber, R. and Clarke, R. (1985): "Problems in Evaluating the Performance of Portfolios with Options," *Financial Analysts Journal,* 41:48–62.

Bookstaber, R. and Clarke, R. (1984): "Option Portfolio Strategies: Measurement and Evaluation," *Journal of Business,* 57:469–492.

Bookstaber, R. and Clarke, R. (1980–81): "Options Can Alter Portfolio Return Distributions," *Journal of Portfolio Management,* 7:63–70.

Brooks, R., Levy, H., and Yoder, J. (1987): "Using Stochastic Dominance in Evaluating the Performance of Portfolios with Options," *Financial Analysts Journal,* 43:79–82.

Clarke, R. (1987): *Advances in Futures and Options Research,* Vol. 2. Greenwich, CT: JAI Press, pp. 1–18.

Cox, J. C., and Rubenstein, M. (1985): *Option Markets.* Englewood Cliffs, NJ: Prentice-Hall.

Fabozzi, F. J. (1989): *Portfolio and Investment Management.* Chicago, IL: Probus Publishing (Ch. 6).

Hadar, J., and Russell, W. R. (1969): "Rules for Ordering Uncertain Prospects," *American Economic Review* 59:25–34.

Hanoch, G. and Levy, H. (1969): "Efficiency Analysis of Choices Involving Risk," *Review of Economic Studies,* 36:335–346.

Heston, C. (1987): "Using Options and Futures to Tailor Portfolio Returns," *Futures,* 16:42–44.

Ibbotson Associates, Inc. (1989): *Stocks, Bonds, Bills, and Inflation 1989 Yearbook.* Chicago, IL: Ibbotson Associates.

Johnson, N. L., and Kotz, S. (1970): *Continuous Univariate Distributions—1.* New York, Wiley.

Kraus, A. and Litzenberger, R. H. (1976). "Skewness Preference and the Valuation of Risky Assets," *Journal of Finance,* 31:1085–1100.

Kroll, Y. and Levy, H. (1980): "Sampling Errors and Portfolio Efficiency Analysis," *Journal of Financial and Quantitative Analysis,* 15:655–688.

Kroll, Y. and Levy, H. (1980): "Stochastic Dominance: A Review and Some New Evidence," *Research in Finance,* 2:163–227.

Leland, H. E. (1980): "Who Should Buy Portfolio Insurance?" *Journal of Finance,* 35:581–594.

Levy, H. (1985): "Upper and Lower Bounds of Put and Call Option Value: Stochastic Dominance Approach," *Journal of Finance,* 40:1197–1217.

Levy, H. and Kroll, Y. (1976): "Stochastic Dominance with Riskless Assets," *Journal of Financial and Quantitative Analysis,* 11:743–773.

Levy, H. and Kroll, Y. (1978): "Ordering Uncertain Options with Borrowing and Lending," *Journal of Finance,* 33:553–573.

Levy, H. and Kroll, Y. (1979): "Efficiency Analysis with Borrowing and Lending: Criteria and Their Effectiveness," *Review of Economics and Statistics,* 61:125–130.

Levy, H. and Sarnat, M. (1984): *Portfolio and Investment Selection: Theory and Practice.* Englewood Cliffs, NJ: Prentice-Hall.

Markowitz, H. M. (1952): "Portfolio Selection," *Journal of Finance,* 6:77–91.

Merton, R. (1973): "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science,* 4:141–183.

Perrakis, S., and Ryan, P. J. "Option Pricing Bounds in Discrete Time," *Journal of Finance,* 39:519–525.

Piessens, R., de Doncker-Kapenga, E., Uberhuber, C.W., and Kahaner, D.K. (1983): *Quadpack: A Subroutine Package for Automatic Integration,* New York, Springer-Verlag.

Ritchken, P.H. (1985): "On Option Pricing Bounds," *Journal of Finance,* 40:1219–1233.

Rothschild, M., and Stiglitz, J.E. "Increasing Risk: I. A Definition." *Journal of Economic Theory*, 2:225–243.

Rubinstein, M. (1976): "The Valuation of Uncertain Income Streams and the Pricing of Options," *Bell Journal of Economics,* 7:407–425.

Rubinstein, M. and Leland, H.E. (1981): "Replicating Options with Positions in Stock and Cash," *Financial Analysts Journal,* 37:63–72.

Sharpe, W.F. (1966): "Mutual Fund Performance," *Journal of Business* 39:119–138.

Whitmore, G.A. (1970): "Third Degree Stochastic Dominance," *American Economic Review,* 60:457–459.

Appendix A

DERIVATION OF CENTRAL MOMENTS

The first three central moments of the end of period wealth distribution from owning only the index can be computed as follows:

Mean:
$$E(\tilde{W}_I^*) = \int_b^\infty x * INT_I dx \tag{A-1}$$

Variance:
$$E[(\tilde{W}_I^* - E(\tilde{W}_I^*))^2] = \int_b^\infty (x - E(\tilde{W}_I^*))^2 * INT_I dx \tag{A-2}$$

Skewness:
$$E[(\tilde{W}_I^* - E(\tilde{W}_I^*))^3] = \int_b^\infty (x - E(\tilde{W}_I^*))^3 * INT_I dx \tag{A-3}$$

where: INT_I is the integrand of eq. (5) where $a = I_0 N_1$ and $b = DYI_0 N_I$. (see eq. (1)). The analysis for employing futures is similar. The first three central moments of the end-of-period wealth distribution from owning the index and employing index (or futures) call options can be computed as follows:

Mean:
$$E(\tilde{W}_{IC}^*) = \int_{b_1}^{W_C} x * INT_{IC,OUT} dx + \int_{W_C}^\infty x * INT_{IC,IN} dx \tag{A-4}$$

Variance:
$$E[(\tilde{W}_{IC}^* - E(\tilde{W}_{IC}))^2] = \int_{b_1}^{W_C} (x - E(\tilde{W}_{IC}))^2 * INT_{IC,OUT} dx + \int_{W_C}^\infty (x - E(\tilde{W}_{IC}))^2 * INT_{IC,IN} dx \tag{A-5}$$

Skewness:
$$E[(\tilde{W}_{IC}^* - E(\tilde{W}_{IC}))^3] = \int_b^{W_C} (x - E(\tilde{W}_{IC}))^3 * INT_{IC,OUT} dx + \int_{W_C}^\infty (x - E(\tilde{W}_{IC}))^3 * INT_{IC,IN} dx \tag{A-6}$$

where: $INT_{IC,OUT}$ is the integrand of the first part of eq. (12), where $b_1 = DY(I_0 N_I + C_0 N_C)$ and $a_1 - I_0 N_I + C_0 N_C$ (this is the out-of-the-money portion),

$INT_{IC,IN}$ is the integrand of the second part of eq. (12), where $b_2 = b_1 + N_C E$ and $a_2 = a_1 - I_0 N_C$, and $W_C = ((E/I_0) + DY)(I_0 N_I + C_0 N_C)$.

The first three central moments of the end-of-period wealth distribution from owning the index and employing index (or futures) put options can be computed as follows:

Mean:

$$E(\tilde{W}^*_{IP}) = \int_{b_1}^{W_P} x * INT_{IP,IN} dx + \int_{W_P}^{\infty} x * INT_{IP,OUT} dx \tag{A-7}$$

Variance:

$$E[(\tilde{W}^*_{IP} - E(\tilde{W}_{IP}))^2] = \int_{b_1}^{W_P} (x - E(\tilde{W}^*_{IP}))^2 * INT_{IP,IN} dx + \int_{W_P}^{\infty} (x - E(\tilde{W}^*_{IP}))^2 * INT_{IP,OUT} dx \tag{A-8}$$

Skewness:

$$E[(\tilde{W}^*_{IP} - E(\tilde{W}_{IP}))^3] = \int_{b_1}^{W_P} (x - E(\tilde{W}^*_{IP}))^3 * INT_{IP,IN} dx + \int_{W_P}^{\infty} (x - E(\tilde{W}^*_{IP}))^3 * INT_{IP,OUT} dx \tag{A-9}$$

where: $INT_{IP,IN}$ is the integrand of the first part of eq. (12), where $b_1 = DY(I_0 N_I + P_0 N_P) - N_P E$ and $a_1 = I_0 N_I + (P_0 + I_0) N_P$ (see eq. (18)).

$INT_{IP,OUT}$ is the integrand of the second part of eq. (12), where $b_2 = DY(I_0 N_I + P_0 N_P)$ and $a_2 = I_0 N_I + P_0 N_P$, and $W_P = ((E/I_0) + DY)(I_0 N_I + P_0 N_P)$. The analysis for futures options is identical.

Appendix B

The Barone-Adesi and Whaley (1987) quadratic approximation values American options written on underlying securities that exhibit a constant continuous leakage, δ. For call options, the model is:

$$C^A = C^E + A_2 (S/\bar{S})^{q_2}$$

where $S < \bar{S}$, and

$$C^A = S - X$$

where

$$S > \bar{S}, \qquad A_2 = (\bar{S}/q_2)\{1 - e^{-\delta T} N[d_2(\bar{S})]\},$$

$$d_2(\bar{S}) = \{[\ell n(\bar{S}/X) + (r - \delta + (\sigma^2/2))T]/\sigma\sqrt{T}\}, \qquad q_2 = (1 + \sqrt{1 + 4k})/2,$$

and

$$k = 2r/[\sigma^2(1 - e^{rt})].$$

The notation used is:

C^A : the value of the American option,

C^E : the value of the corresponding European option,

S : the underlying asset value,
X : the option's exercise price,
$N(\cdot)$: the cumulative standard normal probability distribution function,
σ^2 : the instantaneous variance of the returns on the underlying asset,
T : the option's maturity expressed as a fraction of a year, and
$\overline{S}$: the critical underlying asset price above which it is optimal to prematurely exercise.

Barone-Adesi and Whaley's model states that if the underlying asset price is below $\overline{S}$, then the American call option value is equal to its corresponding European value plus the early exercise premium, as approximated by $A_2(S/\overline{S})^{q_2}$. Above $\overline{S}$, the value of C^A is its exercisable proceeds, $S - X$.

The critical asset price, $\overline{S}$, can be determined iteratively by solving the following equation:

$$\overline{S} - X = C^E(\overline{S}, t) + \{1 - e^{-\delta T}N(d_2(\overline{S}))\}\overline{S}/q_2.$$

Barone-Adesi and Whaley provide a rapidly converging algorithm (used here) to solve $\overline{S}$. A similar model exists for American put options.

For index call options, the above model is applicable where S is the underlying index, σ is the instantaneous standard deviation of returns on holding the index, and δ is the dividend yield.

Appendix C

SENSITIVITY ANALYSIS

Table C-1a
LOWER STRIKE PRICE: SUMMARY STATISTICS

Part A: Prices and Critical Values

Index Options	American Price	European Price	Critical Value
Call	$ 76.06	$76.04	$1120.85
Put	$ 4.72	$ 4.13	$ 442.49
Futures Options:			
Call	$ 79.52	$76.06	$ 606.03
Put	$ 4.27	$ 4.13	$ 406.98
Futures:			
Index	$574.40	N/A	N/A

Part B: Univariate Statistics

Position		Long/Buying Mean	Long/Buying Standard Deviation	Long/Buying Skewness	Short/Selling/Writing Mean	Short/Selling/Writing Standard Deviation	Short/Selling/Writing Skewness
Index only		12.60%	14.09%	0.391	N/A	N/A	N/A
Index call	25	13.51	16.94	0.448	11.68	11.26	0.286
	50	14.39	19.74	0.475	10.76	8.45	0.0770
	75	15.14	22.45	0.481	9.83	5.69	−0.468
	100	15.67	24.99	0.479	8.91	3.17	−2.63
Index put	25	12.46	13.89	0.464	12.74	14.32	0.310
	50	12.32	13.67	0.0530	12.88	14.57	0.223
	75	12.18	13.48	0.0587	13.02	14.82	0.130
	100	N/A	N/A	N/A	13.16	15.08	0.0315
Futures calls	25	13.34	16.92	0.452	11.85	11.28	0.290
	50	14.09	19.76	0.491	11.11	8.49	0.0800
	75	14.83	22.60	0.519	10.36	5.76	−0.455
	100	15.58	25.44	0.538	9.62	3.24	−2.52
Futures puts	25	12.48	13.88	0.464	12.72	14.32	0.310
	50	12.36	13.68	0.530	12.83	14.56	0.223
	75	12.24	13.49	0.590	12.95	14.81	0.129
	100	N/A	N/A	N/A	13.07	15.07	0.0311
Futures	25	13.53	16.85	0.391	11.18	9.80	0.391
	50	14.47	19.61	0.391	9.75	5.51	0.391
	75	15.40	22.37	0.391	8.33	1.22	0.391
	100	16.34	25.12	0.390	N/A	N/A	N/A

Inputs: MMI = 551.82, expected growth rate of index = 7.61%, number of "units" owned = 100, strike price = 496.64, standard deviation = 12.9%, time to expiration = 365 days, risk-free rate = 7.8%, dividend yield = 3.79%, margin requirement = $120/unit.

Table C-1b
HIGHER STRIKE PRICE: SUMMARY STATISTICS

Part A: Prices and Critical Values

Index Options	American Price	European Price	Critical Value
Call	$ 15.55	$15.54	$1369.93
Put	$ 55.95	$45.71	$ 540.82
Futures Options:			
Call	$ 15.96	$15.54	$ 740.71
Put	$ 47.10	$45.70	$ 497.42
Futures:			
Index	$574.40	N/A	N/A

Part B: Univariate Statistsics

Position		Long/Buying			Short/Selling/Writing		
		Mean	Standard Deviation	Skewness	Mean	Standard Deviation	Skewness
Index only		12.60%	14.09%	0.391	N/A	N/A	N/A
Index call	25	13.06	15.85	0.658	12.13	12.42	0.0476
	50	13.49	17.61	0.856	11.66	10.85	−0.370
	75	13.81	19.31	0.996	11.19	9.47	−0.830
	100	13.97	20.88	1.096	10.72	8.36	−1.22
Index put	25	11.30	12.09	0.748	13.89	16.19	0.127
	50	10.01	10.21	1.22	15.19	18.33	−0.0709
	75	8.72	8.54	1.78	16.48	20.51	−0.221
	100	N/A	N/A	N/A	17.77	22.72	−0.338
Futures calls	25	13.05	15.85	0.663	12.15	12.42	0.0477
	50	13.50	17.67	0.877	11.70	10.86	−0.370
	75	13.95	19.53	1.047	11.25	9.48	−0.829
	100	14.39	21.41	1.18	10.80	8.36	−1.22
Futures puts	25	11.76	12.14	0.746	13.44	16.13	0.126
	50	10.91	10.32	1.21	14.28	18.22	−0.0737
	75	10.07	8.70	1.76	15.12	20.35	−0.262
	100	N/A	N/A	0.391	15.97	22.50	−0.345
Futures	25	13.53	16.85	N/A	11.18	9.80	0.391
	50	14.47	19.61	0.391	9.75	5.51	0.391
	75	15.40	22.37	0.391	8.33	1.22	0.391
	100	16.34	25.12	0.391	N/A	N/A	N/A

Inputs: MMI = 551.82, expected growth rate of index = 7.61%, number of "units" owned = 100, strike price = 607.00, standard deviation = 12.9%, time to expiration = 365 days, risk-free rate = 7.8%, dividend yield = 3.79%, margin requirement = $120/unit.

Table C-2a
LOWER STANDARD DEVIATION: SUMMARY STATISTICS

Part A: Prices and Critical Values

Index Options	American Price	European Price	Critical Value
Call	$ 31.02	$31.00	$1213.20
Put	$ 12.60	$10.13	$ 515.53
Futures Options:			
Call	$ 31.99	$31.01	$ 634.35
Put	$ 10.40	$10.13	$ 480.02
Futures:			
Index	$574.40	N/A	N/A

Part B: Univariate Statistics

Position		Mean	Long/Buying Standard Deviation	Skewness	Mean	Short/Selling/Writing Standard Deviation	Skewness
Index only		12.14%	9.80%	0.272	N/A	N/A	N/A
Index call	25	12.89	11.73	0.415	11.39	7.91	0.0335
	50	13.64	13.66	0.508	10.63	6.07	−0.411
	75	14.39	15.61	0.571	9.88	4.35	−1.34
	100	15.13	17.55	0.614	9.13	2.98	−3.07
Index put	25	11.74	9.36	0.449	12.54	10.27	0.0913
	50	11.34	8.95	0.614	12.94	10.77	−0.0888
	75	10.94	8.57	0.761	13.33	11.28	−0.264
	100	N/A	N/A	N/A	13.73	11.82	−0.433
Futures calls	25	12.84	11.72	0.415	11.43	7.91	0.0338
	50	13.55	13.66	0.508	10.73	6.08	−0.410
	75	14.25	15.60	0.573	10.03	4.36	−1.34
	100	14.95	17.55	0.620	9.32	2.99	−3.06
Futures puts	25	11.85	9.37	0.449	12.42	10.26	0.0911
	50	11.56	8.97	0.614	12.71	10.75	−0.0895
	75	11.28	8.60	0.759	13.00	11.25	−0.266
	100	N/A	N/A	N/A	13.29	11.78	−0.435
Futures	25	12.98	11.72	0.272	10.85	6.82	0.272
	50	13.83	13.64	0.272	9.57	3.84	0.272
	75	14.67	15.56	0.272	8.29	0.852	0.273
	100	15.52	17.48	0.272	N/A	N/A	N/A

Inputs: MMI = 551.82, expected growth rate of index = 7.61%, number of "units" owned = 100, strike price = 551.82, standard deviation = 9.03%, time to expiration = 365 days, risk-free rate = 7.8%, dividend yield = 3.79%, margin requirement = $120/unit.

Table 2-b
HIGHER STANDARD DEVIATION: SUMMARY STATISTICS

Part A: Prices and Critical Values

Index Options	**American Price**	**European Price**	**Critical Value**
Call	$ 46.24	$46.22	$1278.98
Put	$ 28.11	$25.35	$ 467.08
Futures Options:			
Call	$ 47.55	$46.23	$ 714.80
Put	$ 26.01	$25.34	$ 426.00
Futures:			
Index	$574.40	N/A	N/A

Part B: Univariate Statistics

Position		**Long/Buying** Mean	**Long/Buying** Standard Deviation	**Long/Buying** Skewness	**Short/Selling/Writing** Mean	**Short/Selling/Writing** Standard Deviation	**Short/Selling/Writing** Skewness
Index only		13.23%	18.48%	0.511	N/A	N/A	N/A
Index call	25	4.02	21.38	0.700	12.42	15.40	0.223
	50	14.82	24.82	0.828	11.62	12.45	−0.239
	75	15.61	28.02	0.913	10.82	9.74	−0.985
	100	16.35	31.17	0.967	10.01	7.54	−1.98
Index put	25	12.60	17.17	0.747	13.85	19.86	0.290
	50	11.97	15.95	0.987	14.48	21.30	0.0870
	75	11.35	14.84	1.21	15.10	22.78	−0.0956
	100	N/A	N/A	N/A	15.73	24.30	−0.259
Futures calls	25	13.96	21.62	0.700	12.49	15.41	0.223
	50	14.69	24.80	0.828	11.75	12.47	−0.237
	75	15.41	27.99	0.914	11.02	9.77	−1.981
	100	16.09	31.13	0.968	10.28	7.57	−1.97
Futures puts	25	12.71	17.19	0.747	13.74	19.84	0.290
	50	12.19	15.99	0.986	14.26	21.26	0.0862
	75	11.67	14.90	1.21	14.78	22.73	−0.0971
	100	N/A	N/A	N/A	15.30	24.23	−0.261
Futures	25	14.28	22.10	0.511	11.61	12.86	0.511
	50	15.34	25.71	0.510	10.00	7.23	0.511
	75	16.39	29.31	0.505	8.38	1.61	0.511
	100	17.40	32.86	0.494	N/A	N/A	0.512

Inputs: MMI = 551.82, expected growth rate of index = 7.61%, number of "units" owned = 100, strike price = 551.82, standard deviation = 16.77%, time to expiration = 365 days, risk-free rate = 7.8%, dividend yield = 3.79%, margin requirement = $120/unit.

Table-3a
LOWER RISK-FREE RATE SUMMARY STATISTICS

Prices and Critical Values

Index Options	**American Price**	**European Price**	**Critical Value**
Call	$ 31.75	$31.71	$870.60
Put	$ 24.11	$22.92	$471.30
Futures Options:			
Call	$ 32.28	$31.71	$683.51
Put	$ 23.31	$22.92	$445.51
Futures:			
Index	$561.11	N/A	N/A

Univariate Statistics

Position		**Mean**	**Long/Buying Standard Deviation**	**Skewness**	**Mean**	**Short/Selling/ Writing Standard Deviation**	**Skewness**
Index only		12.60%	14.09%	0.390	N/A	N/A	N/A
Index call	25	13.69	16.66	0.560	11.50	11.57	0.114
	50	14.73	19.21	0.661	10.40	9.13	−0.356
	75	15.62	21.64	0.719	9.30	6.90	−1.19
	100	16.24	23.87	0.755	8.20	5.11	−2.40
Index put	25	11.89	13.20	0.608	13.31	15.03	0.181
	50	11.18	12.37	0.823	14.02	16.02	−0.0168
	75	10.46	11.62	1.02	14.73	17.03	−0.199
	100	N/A	N/A	N/A	15.45	18.08	−0.365
Futures calls	25	13.67	16.67	0.565	11.52	11.57	0.115
	50	14.75	19.27	0.682	10.45	9.14	−0.356
	75	15.82	21.90	0.764	9.38	6.91	−1.19
	100	16.89	24.53	0.824	8.30	5.12	−2.39
Futures puts	25	11.93	13.21	0.608	13.27	15.03	0.181
	50	11.26	12.38	0.822	13.94	16.01	−0.0171
	75	10.59	11.63	1.02	14.61	17.02	−0.200
	100	N/A	N/A	N/A	15.28	18.06	−0.366
Futures	25	14.00	16.85	0.391	10.44	9.80	0.391
	50	15.40	19.61	0.391	8.28	5.51	0.391
	75	16.80	22.37	0.391	6.12	1.22	0.391
	100	18.20	25.12	0.390	N/A	N/A	N/A

Inputs: MMI = 551.82, expected growth rate of index = 7.61%, number of "units" owned = 100, strike price = 551.82, standard deviation = 12.9%, time to expiration = 365 days, risk-free rate = 5.46%, dividend yield = 3.79%, margin requirement = $120/unit.

Table C-3b
HIGHER RISK-FREE RATE: SUMMARY STATISTICS

Prices and Critical Values

Index Options	American Price	European Price	Critical Value
Call	$ 45.97	$45.95	$1623.98
Put	$ 17.19	$13.28	$ 503.68
Futures Options:			
Call	$ 47.99	$45.96	$ 666.00
Put	$ 13.76	$13.27	$ 457.21
Futures:			
Index	$588.00	N/A	N/A

Univariate Statistics

Position		Long/Buying Mean	Long/Buying Standard Deviation	Long/Buying Skewness	Short/Selling/Writing Mean	Short/Selling/Writing Standard Deviation	Short/Selling/Writing Skewness
Index only		12.60%	14.09%	0.390	N/A	N/A	N/A
Index call	25	12.97	16.57	0.561	12.22	11.66	0.117
	50	13.30	19.04	0.667	11.85	9.31	−0.341
	75	13.52	21.43	0.729	11.47	7.15	−1.13
	100	13.55	23.66	0.766	11.10	5.39	−2.29
Index put	25	12.24	13.25	0.607	12.96	14.99	0.180
	50	11.88	12.46	0.820	13.32	15.93	−0.0192
	75	11.52	11.74	1.015	13.67	16.90	−0.204
	100	N/A	N/A	N/A	14.03	17.90	−0.373
Futures calls	25	12.87	16.57	0.566	12.33	11.67	0.117
	50	13.14	19.08	0.685	12.01	9.34	−0.339
	75	13.42	21.60	0.769	11.78	7.19	−1.12
	100	13.69	24.14	0.831	11.51	5.43	−2.27
Futures puts	25	12.41	13.27	0.607	12.78	14.97	0.180
	50	12.23	12.50	0.819	12.97	15.88	−0.0204
	75	12.05	11.81	1.012	13.15	16.84	−0.206
	100	N/A	N/A	N/A	13.33	17.82	−0.377
Futures	25	13.06	16.85	0.391	11.93	9.80	0.391
	50	13.52	19.61	0.391	11.26	5.51	0.391
	75	13.97	22.37	0.391	10.59	1.22	0.391
	100	14.43	25.12	0.390	N/A	N/A	N/A

Inputs: MMI = 551.82, expected growth rate of index = 7.61%, number of "units" owned = 100, strike price = 551.82, standard deviation = 12.9%, time to expiration = 365 days, risk-free rate = 10.14%, dividend yield = 3.79%, margin requirement = $120/unit.

Table C-4a
LOWER GROWTH RATE: SUMMARY STATISTICS

Prices and Critical Values

Index Options	American Price	European Price	Critical Value
Call	$ 38.51	$38.49	$1245.39
Put	$ 20.26	$17.62	$ 491.65
Futures Options:			
Call	$ 39.63	$38.50	$ 673.37
Put	$ 18.08	$17.62	$ 452.20
Futures:			
Index	$574.40	N/A	N/A

Univariate Statistics

Position		Long/Buying Mean	Long/Buying Standard Deviation	Long/Buying Skewness	Short/Selling/Writing Mean	Short/Selling/Writing Standard Deviation	Short/Selling/Writing Skewness
Index only		8.94%	13.62%	0.391	N/A	N/A	N/A
Index call	25	9.10	15.78	0.606	8.78	11.52	0.0735
	50	9.24	17.96	0.752	8.62	9.52	−0.400
	75	9.32	20.11	0.846	8.46	7.72	−1.08
	100	9.30	22.17	0.906	8.30	6.27	−1.86
Index put	25	8.71	12.50	0.663	9.17	14.80	0.145
	50	8.49	11.46	0.950	9.39	16.04	−0.0703
	75	8.26	10.53	1.23	9.62	17.31	−0.258
	100	N/A	N/A	N/A	9.85	18.61	−0.419
Futures calls	25	9.05	15.78	0.608	8.84	11.53	0.0737
	50	9.15	17.78	0.762	8.73	9.54	−0.399
	75	9.26	20.21	0.875	8.63	7.74	−1.08
	100	9.36	22.45	0.960	8.52	6.29	−1.85
Futures puts	25	8.82	12.51	0.662	9.06	14.79	0.145
	50	8.70	11.49	0.949	9.18	16.00	−0.0711
	75	8.58	10.57	1.22	9.30	17.27	−0.259
	100	N/A	N/A	N/A	9.42	18.55	−0.422
Futures	25	9.16	16.29	0.391	8.63	9.48	0.391
	50	9.38	18.95	0.391	8.32	5.33	0.391
	75	9.60	21.62	0.391	8.01	1.18	0.391
	100	9.82	24.28	0.391	N/A	N/A	N/A

Inputs: MMI = 551.82, expected growth rate of index = 4.19%, number of "units" owned = 100, strike price = 551.82, standard deviation = 12.9%, time to expiration = 365 days, risk-free rate = 7.8%, dividend yield = 3.79%, margin requirement = $120/unit.

Table C-4b
HIGHER GROWTH RATE: SUMMARY STATISTICS

Prices and Critical Values

Index Options	American Price	European Price	Critical Value
Call	$ 38.51	$38.49	$1245.39
Put	$ 20.26	$17.62	$ 491.65
Futures Options:			
Call	$ 39.63	$38.50	$ 673.37
Put	$ 18.08	$17.62	$ 452.20
Futures:			
Index	$574.40	N/A	N/A

Univariate Statistics

Position		Long/Buying Mean	Long/Buying Standard Deviation	Long/Buying Skewness	Short/Selling/Writing Mean	Short/Selling/Writing Standard Deviation	Short/Selling/Writing Skewness
Index only		16.38%	14.58	0.390	N/A	N/A	N/A
Index call	25	17.81	17.43	0.513	14.94	11.75	0.170
	50	19.12	20.19	0.578	13.49	8.99	−0.243
	75	20.16	22.77	0.612	12.04	6.40	−1.13
	100	20.75	25.07	0.642	10.59	4.28	−2.91
Index put	25	15.65	13.93	0.554	17.12	15.27	0.224
	50	14.91	13.32	0.707	17.86	15.99	0.0572
	75	14.17	12.76	0.843	18.60	16.74	−0.106
	100	N/A	N/A	N/A	19.33	17.51	−0.263
Futures calls	25	17.77	17.45	0.523	14.99	11.76	0.171
	50	19.16	20.33	0.609	13.60	9.01	−0.242
	75	20.55	23.23	0.668	12.21	6.42	−1.12
	100	21.94	26.13	0.709	10.82	4.30	−2.90
Futures puts	25	15.76	13.95	0.554	17.01	15.26	0.224
	50	15.14	13.95	0.706	17.63	15.96	0.0565
	75	14.52	12.80	0.842	18.25	16.70	−0.107
	100	N/A	N/A	N/A	18.87	17.45	−0.265
Futures	25	18.06	17.44	0.391	13.81	10.15	0.391
	50	19.74	20.29	0.391	11.23	5.71	0.391
	75	21.41	23.15	0.391	8.66	1.27	0.391
	100	23.09	26.00	0.390	N/A	N/A	N/A

Inputs: MMI = 551.82, expected growth rate of index = 11.03%, number of "units" owned = 100, strike price = 551.82, standard deviation = 12.9%, time to expiration = 365 days, risk-free rate = 7.8%, dividend yield = 3.79%, margin requirement = $120/unit.

Table C-5a
LOWER MARGIN REQUIREMENTS: SUMMARY STATISTICS

Prices and Critical Values

Index Options	American Price	European Price	Critical Value
Call	$ 38.51	$38.49	$1245.39
Put	$ 20.26	$17.62	$ 491.65
Futures Options:			
Call	$ 39.63	$38.50	$ 673.37
Put	$ 18.08	$17.62	$ 452.20
Futures:			
Index	$574.40	N/A	N/A

Univariate Statistics

Position		Long/Buying Mean	Long/Buying Standard Deviation	Long/Buying Skewness	Short/Selling/Writing Mean	Short/Selling/Writing Standard Deviation	Short/Selling/Writing Skewness
Index only		12.60%	14.09%	0.391	N/A	N/A	N/A
Index call	25	13.35	16.62	0.561	11.84	11.61	0.116
	50	14.05	19.13	0.664	11.09	9.22	−0.349
	75	14.62	21.54	0.723	10.33	7.02	−1.16
	100	14.96	23.77	0.760	9.58	5.24	−2.35
Index put	25	12.08	13.23	0.608	13.11	15.011	0.180
	50	11.57	12.42	0.821	13.63	15.97	−0.0181
	75	11.05	11.69	1.017	14.14	16.96	−0.202
	100	N/A	N/A	N/A	14.66	17.98	−0.370
Futures calls	25	13.30	16.62	0.565	11.90	11.62	0.116
	50	14.00	19.18	0.683	11.20	9.23	−0.350
	75	14.69	21.76	0.770	10.50	7.04	−1.16
	100	15.39	24.35	0.828	9.80	5.27	−2.34
Futures puts	25	12.19	13.24	0.607	13.00	15.00	0.180
	50	11.79	12.45	0.820	13.41	15.94	−0.0189
	75	11.39	11.73	1.015	13.81	16.92	−0.203
	100	N/A	N/A	N/A	14.22	17.93	−0.372
Futures	25	13.78	17.62	0.391	11.42	10.57	0.391
	50	14.96	21.14	0.391	10.24	7.05	0.391
	75	16.14	24.67	0.391	9.06	3.52	0.391
	100	17.31	28.28	0.389	N/A	N/A	N/A

Inputs: MMI = 551.82, expected growth rate of index = 7.61%, number of "units" owned = 100, strike price = 551.82, standard deviation = 12.9%, time to expiration = 365 days, risk-free rate = 7.8%, dividend yield = 3.79%, margin requirement = $0/unit.

Table C-5b
HIGHER MARGIN REQUIREMENT: SUMMARY STATISTICS

Prices and Critical Values

Index Options	**American Price**	**European Price**	**Critical Value**
Call	$ 38.51	$38.49	$1245.39
Put	$ 20.26	$17.62	$ 491.65
Futures Options:			
Call	$ 39.63	$38.50	$ 673.37
Put	$ 18.08	$17.62	$ 452.20
Futures:			
Index	$574.40	N/A	N/A

Univariate Statistics

Position		**Long/Buying Mean**	**Long/Buying Standard Deviation**	**Long/Buying Skewness**	**Short/Selling/Writing Mean**	**Short/Selling/Writing Standard Deviation**	**Short/Selling/Writing Skewness**
Index only		12.60%	14.09%	0.391	N/A	N/A	N/A
Index call	25	13.35	16.63	0.565	11.84	11.61	0.116
	50	14.11	19.20	0.683	11.09	9.22	−0.349
	75	14.87	21.78	0.766	10.33	7.02	−1.16
	100	15.62	24.38	0.827	9.58	5.24	−2.35
Index put	25	12.08	13.23	0.608	13.11	15.01	0.180
	50	11.57	12.42	0.821	13.63	15.97	−0.018
	75	11.05	11.69	1.02	14.14	16.96	−0.202
	100	N/A	N/A	N/A	14.66	17.98	−0.370
Futures calls	25	13.30	16.62	0.565	11.90	11.62	0.116
	50	14.00	19.18	0.683	11.20	9.23	−0.348
	75	14.69	21.76	0.767	10.50	7.04	−1.16
	100	15.39	24.35	0.828	9.80	5.27	−2.34
Futures puts	25	12.19	13.24	0.607	13.00	15.00	0.180
	50	11.79	12.45	0.820	13.41	15.94	−0.018
	75	11.39	11.73	1.02	13.81	16.92	−0.203
	100	N/A	N/A	N/A	14.22	17.93	−0.372
Futures	25	13.29	16.09	0.391	10.93	9.04	0.391
	50	13.98	18.08	0.391	9.26	3.98	0.391
	75	14.67	20.07	0.391	N/A	N/A	N/A
	100	15.36	22.06	0.391	N/A	N/A	N/A

Inputs: MMI = 551.82, expected growth rate of index = 7.61%, number of "units" owned = 100, strike price = 551.82, standard deviation = 12.9%, time to expiration = 365 days, risk-free rate = 7.8%, dividend yield = 3.79%, margin requirement = $240/unit.

Table C-6a
LOWER DIVIDEND YIELD: SUMMARY STATISTICS

Prices and Critical Values

Index Options	**American Price**	**European Price**	**Critical Value**
Call	\$ 52.85	\$52.83	\$ 0.00
Put	\$ 15.73	\$11.45	\$508.28
Futures Options:			
Call	\$ 54.60	\$52.85	\$673.37
Put	\$ 11.75	\$11.44	\$452.20
Futures:			
Index	\$596.59	N/A	N/A

Univariate Statistics

		Long/Buying			Short/Selling/ Writing		
Position		**Mean**	**Standard Deviation**	**Skewness**	**Mean**	**Standard Deviation**	**Skewness**
Index only		13.01%	14.64%	0.391	N/A	N/A	N/A
Index call	25	13.87	17.45	0.519	12.15	11.87	0.179
	50	14.73	20.77	0.603	11.29	9.16	−0.214
	75	15.60	23.12	0.661	10.43	6.61	−1.04
	100	16.45	25.96	0.702	9.57	4.47	−2.77
Index put	25	12.52	14.04	0.548	13.50	15.28	0.228
	50	12.03	13.47	0.694	13.99	15.94	0.0648
	75	11.54	12.95	0.824	14.48	16.63	−0.0966
	100	N/A	N/A	N/A	14.98	17.35	−0.253
Futures calls	25	13.78	17.44	0.519	12.24	11.88	0.179
	50	14.56	20.25	0.603	11.47	9.18	−0.213
	75	15.33	23.08	0.661	10.70	6.64	−1.03
	100	16.10	25.92	0.703	9.92	4.51	−2.74
Futures puts	25	12.72	14.06	0.548	13.30	15.25	0.228
	50	12.44	13.53	0.693	13.59	15.89	−0.0636
	75	12.15	13.03	0.822	13.87	16.56	−0.0992
	100	N/A	N/A	N/A	14.16	17.25	−0.258
Futures	25	13.97	17.50	0.391	11.52	10.18	0.391
	50	14.93	20.37	0.391	10.03	5.73	0.391
	75	15.89	23.23	0.391	8.54	1.27	0.391
	100	16.84	26.09	0.390	N/A	N/A	N/A

Inputs: MMI = 551.82, expected growth rate of index = 7.61%, number of "units" owned = 100, strike price = 551.82, standard deviation = 12.9%, time to expiration = 365 days, risk-free rate = 7.8%, dividend yield = 0%, margin requirement = \$120/unit.

Table C-6b
HIGHER DIVIDEND YIELD: SUMMARY STATISTICS

Prices and Critical Values

Index Options	**American Price**	**European Price**	**Critical Value**
Call	$ 27.44	$26.84	$680.31
Put	$ 26.51	$25.71	$455.82
Futures Options:			
Call	$ 27.57	$26.84	$673.37
Put	$ 26.41	$25.71	$452.20
Futures:			
Index	$553.04	N/A	N/A

Univariate Statistics

		Long/Buying			Short/Selling/ Writing		
Position		**Mean**	**Standard Deviation**	**Skewness**	**Mean**	**Standard Deviation**	**Skewness**
Index only		12.34%	13.57%	0.391	N/A	N/A	N/A
Index call	25	12.94	15.76	0.608	11.74	11.44	0.0681
	50	13.50	17.95	0.753	11.14	9.43	−0.413
	75	13.98	20.08	0.844	10.54	7.61	−1.105
	100	14.28	22.08	0.903	9.94	6.16	−1.86
Index put	25	11.80	12.38	0.669	12.89	14.82	0.143
	50	11.25	11.28	0.968	13.43	16.12	−0.0718
	75	10.71	10.29	1.26	13.97	17.46	−0.256
	100	N/A	N/A	N/A	14.52	18.83	−0.414
Futures calls	25	12.94	15.76	0.612	11.75	11.44	0.0681
	50	13.53	17.99	0.768	11.16	9.43	−0.413
	75	14.12	20.25	0.882	10.56	7.61	−1.104
	100	14.71	22.53	0.968	9.97	6.16	−1.86
Futures puts	25	11.80	12.38	0.669	12.88	14.82	0.143
	50	11.27	11.28	0.968	13.42	16.12	−0.0719
	75	0.730	10.29	1.26	13.96	17.46	−0.257
	100	N/A	N/A	N/A	14.50	18.83	−0.415
Futures	25	13.25	16.23	0.391	10.98	9.44	0.391
	50	14.15	18.88	0.391	9.61	5.31	0.391
	75	15.06	21.53	0.391	8.25	1.18	0.390
	100	15.96	24.19	0.391	N/A	N/A	N/A

Inputs: MMI = 551.82, expected growth rate of index = 3.82%, number of "units" owned = 100, strike price = 551.82, standard deviation = 12.9%, time to expiration = 365 days, risk-free rate = 7.8%, dividend yield = 7.58%, margin requirement = $120/unit.