Risk-Return Hedging Effectiveness Measures for Stock Index Futures

Mary Lindahl

INTRODUCTION

Traditional measures of hedging effectiveness focus on risk reduction. One is the statistic known as the coefficient of determination, or R^2, which tells the percent of variance explained by a regression of spot price changes with futures price changes. The higher the R^2, the lower the minimum variance of the spot-futures or hedged portfolio, and the more effective the hedge. A related measure is σ_h/σ_u, the ratio of the standard deviation of the hedged portfolio to that of the unhedged portfolio; the closer this measure is to zero, the more effective the hedge. Though popular measures, a commonly recognized limitation is that these measures consider only the risk dimension, ignoring expected returns. During the last few years, measures that include the return dimension have become more common in the literature. For example, Graham and Jennings (1987) use what they term a "return retention" ratio as an additional hedging measure for stock index futures. Also, Howard and D'Antonio (1984, 1987) and Chang and Shanker (1987) introduce and refine what they call "a risk-return measure of hedging effectiveness" that is being used frequently in the literature. These newer measures have serious drawbacks, however, and should not be used to judge the effectiveness of stock index futures hedges.

HEDGING WITH STOCK INDEX FUTURES

Hedging with stock index futures allows portfolio managers to reduce exposure to market risk without altering the composition of their portfolio. For example, consider that the management of the Alaska Permanent Fund has a long-term goal of maintaining 10% of the money in a passively managed portfolio that mirrors the S&P 500 index. They believe, however, that stocks in general are currently overpriced, and they want to reduce exposure to market risk until the market corrects itself. They expect a market correction of 30% to occur in the near future. Two risk-reduction choices available include: (1) selling stocks and buying a risk-free asset such as T-bills; and (2) leaving the stock portfolio as is, and hedging with a short

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Mary Lindahl is an Associate Professor of Finance at the University of Alaska Fairbanks.

The Journal of Futures Markets, Vol. 11, No. 4, 399–409 (1991)

CCC 0270-7314/91/040399-11$04.00

position in S&P 500 stock index futures contracts (an opportunity available since 1982 when these contracts were first introduced. Under perfect equilibrium conditions and ignoring transaction costs, these two choices are virtually substitutable in terms of the resulting risk-return combinations. Consider eqs. (1) and (2).

The equilibrium relationship between spot (or cash) prices and futures prices is as follows:

$$f = s + s(i - d) \tag{1}$$

where: f = stock index futures price, s = spot or cash price, i = risk-free interest rate, and d = dividend yield on the spot index.

Equation (1) shows that the equilibrium futures price is equal to the spot price plus the cost of carrying the spot portfolio. The cost of carry is equal to the interest (that is not being earned on the money while it is tied up in stocks) less the dividends (that are being earned on the stocks). In eq. (1), i and d are both yields that are time related and, therefore, the differential becomes less important as the futures contract expiration date is approached. At expiration, the futures price is defined to be equal to the spot price.

The returns on a hedged spot-futures portfolio can be expressed as:

$$R_p = (s_1 - s_0 + D_t - h^*(f_1 - f_0))/s_0 \tag{2}$$

where: R_p = return on the hedged spot-futures portfolio during time period t; s_0, s_1 = spot price at the beginning and end of time period t; f_0, f_1 = futures price at the beginning and end of time period t; D_t = dividends paid during time period t; and h = hedge ratio.

When eq. (2) is restated, with futures prices being expressed as in equation (1), and when a fully hedged position is assumed ($h = 1$), the result is an R_p equal to the risk-free interest rate i.

Consider Figure 1a. The spot (or stock) portfolio is represented by point S and is graphed in risk-return space. Point S corresponds with a hedge ratio (h) of 0 and is the unhedged market return. The straight line between points S and i represent the possible outcomes from the hedged portfolio as the hedge ratio increases from 0 to 1, assuming that futures are always priced at their equilibrium value. Note that line i,S is identical to the Capital Market Line which combines the market portfolio with a risk-free asset. Thus, hedging at equilibrium futures prices produces the same set of risk-return choices as selling stocks and using the money to buy T-bills.

Figures 1b and 1c show typical risk-return profiles for stock index futures hedges compared to the equilibrium limit of line i,S. When basis risk exists, and spot and futures prices are not in perfect alignment, a hedge ratio less than 1 results in the minimum variance position. The curved profile plots under line i,S, as in Figure 1b, when futures are priced to the disadvantage of the short hedger; and the curved profile plots above line i,S, as in Figure 1c, when futures are priced to the advantage of the short hedger.

Note that Figures 1a–c assume that the expected return on the unhedged portfolio (R_s) is greater than the risk-free rate(i). Figures 2a–c show similar relationships when $R_s < i$. Again, a straight line i,S is obtained when futures are always priced at equilibrium (Fig. 2a); the risk-return profile plots below line i,S when futures are priced to the disadvantage of the short hedger (Fig. 2b); and the risk-return profile plots above line i,S when futures are priced to the advantage of the short hedger (Fig. 2c).

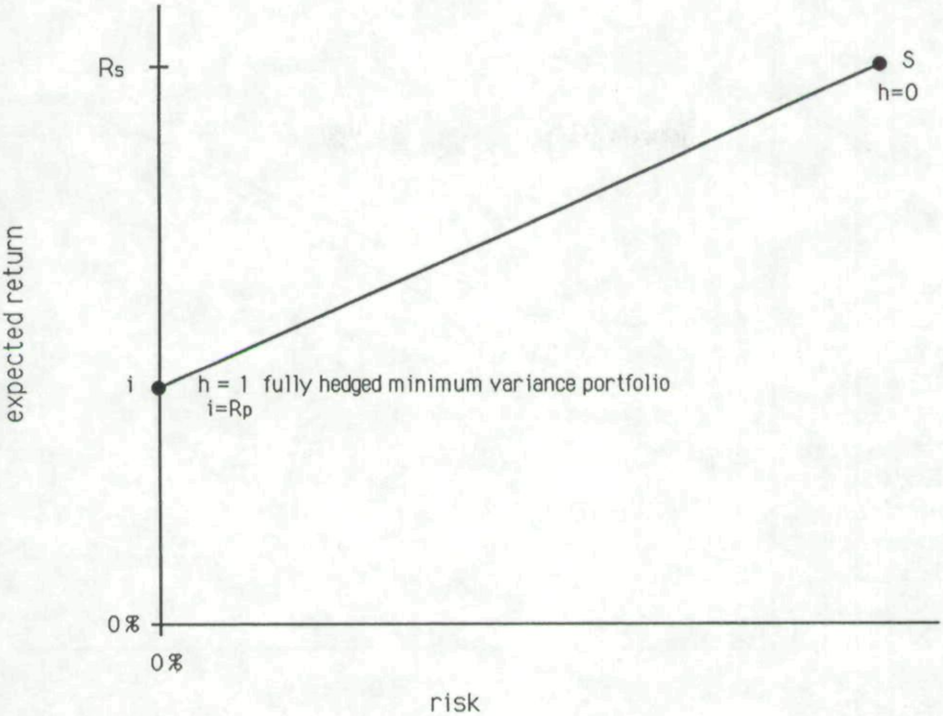

Figure 1a

Risk-return profile when futures are priced at equilibrium and $R_s > \mathrm{i}$.

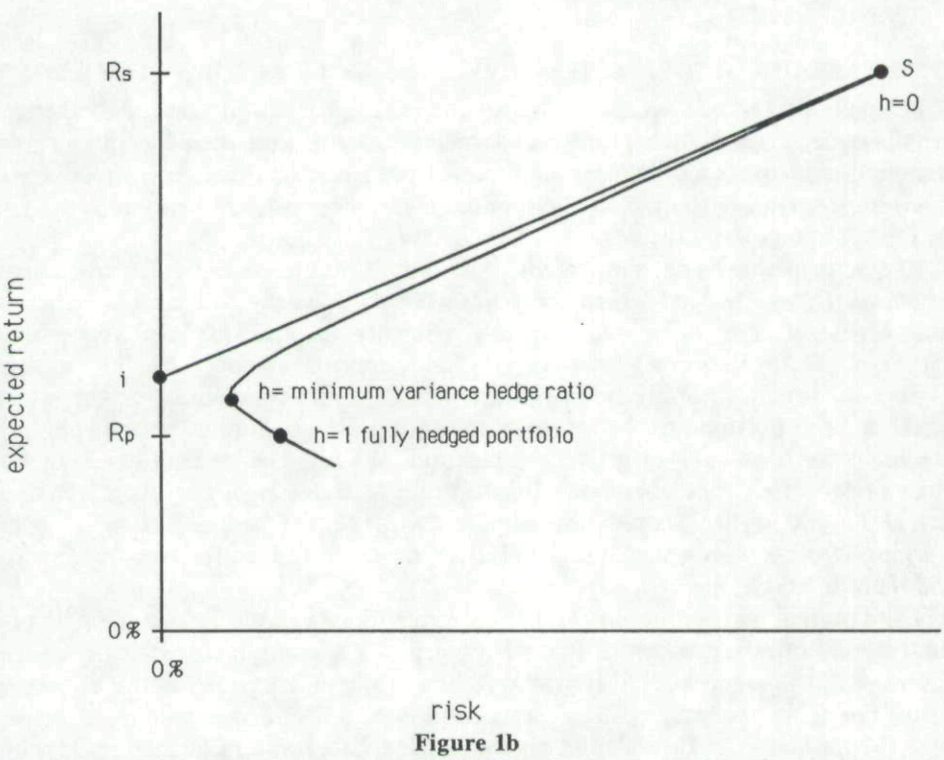

Figure 1b

Risk-return profile when futures are priced to the disadvantage of the short hedger and $R_s > \mathrm{i}$.

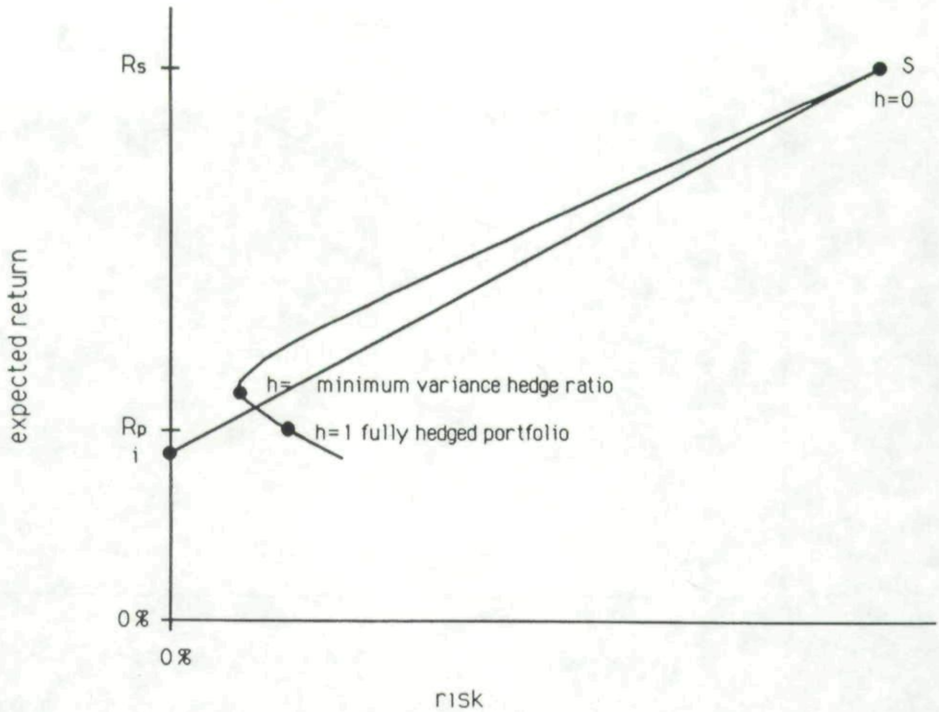

Figure 1c
Risk-return profile when futures are priced to the advantage of the short hedger and R_s > i.

ON MEASURING HEDGING EFFECTIVENESS—THE RETURN DIMENSION

Whenever basis risk exists, as in Figures 1b, 1c, 2b, and 2c, the risk of the hedged portfolio decreases to the point of minimum variance and then increases again. The minimum variance point is the focus of the traditional hedging effectiveness measure; comparing the risk of the minimum variance hedged portfolio to the risk of the spot portfolio indicates the effectiveness of the hedge.

The return dimension is different, however. Whether or not basis risk exists, whenever $R_s > i$, hedged return continually decreases as the hedge ratio increases; and whenever $R_s < i$, hedged return continually increases as the hedge ratio increases. Both the downward sloping and upward sloping straight lines of Figures 1a and 2a illustrate hedges that are 100% effective. Comparing the return of the hedged portfolio to the return of the spot portfolio is, thus, not an appropriate focus for a hedging effectiveness measure. Whether hedged return is greater than or less than spot does not indicate the effectiveness of the hedge. Rather, actual hedged return should be compared to expected hedged return in equilibrium. In other words, the closer the fully hedged return is to the risk-free rate (i), the more effective the hedge.

The Graham and Jennings (1987) "return retention" ratio is a clear example of a hedging effectiveness measure that suffers from the wrong focus. It is defined as the ratio of the mean hedged portfolio return to the mean return of the unhedged (spot) portfolio. As just explained, this ratio does not indicate hedging effectiveness. Mean hedged return should be compared to equilibrium hedged return, not

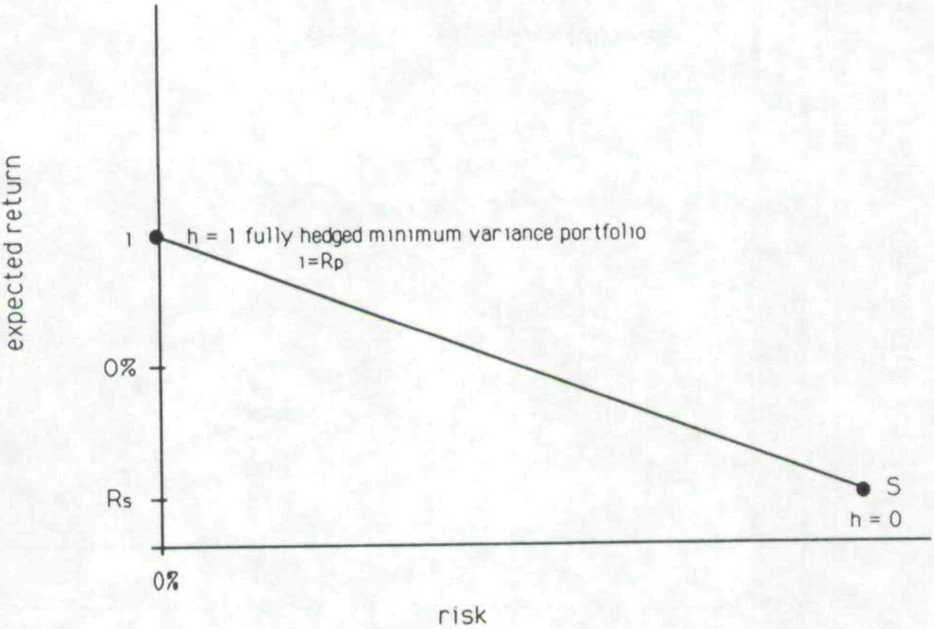

Figure 2a
Risk-return profile when futures are priced at equilibrium and $R_s < 1$.

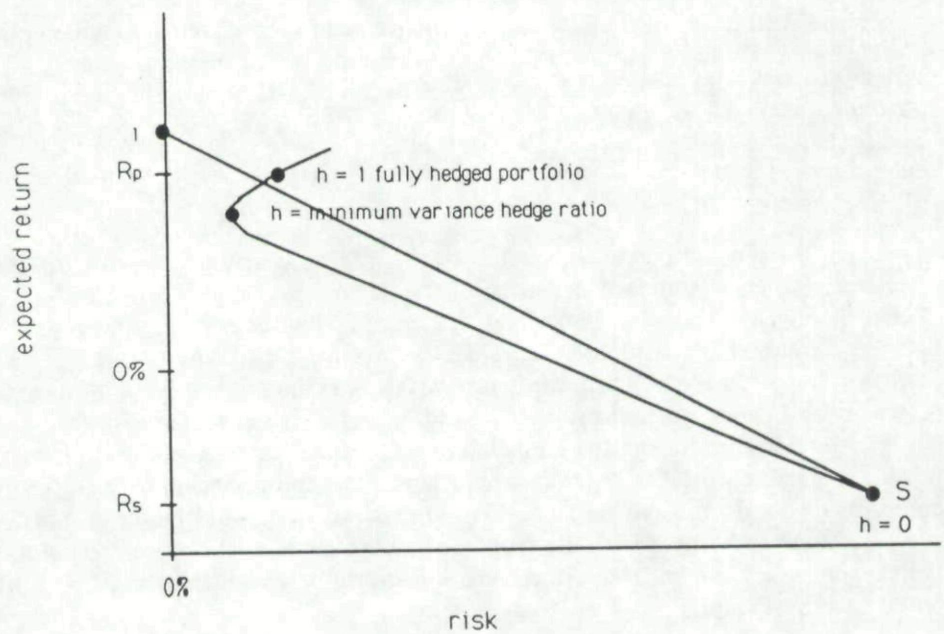

Figure 2b
Risk-return profile when futures are priced to the disadvantage of the short hedger and $R_s <$ i.

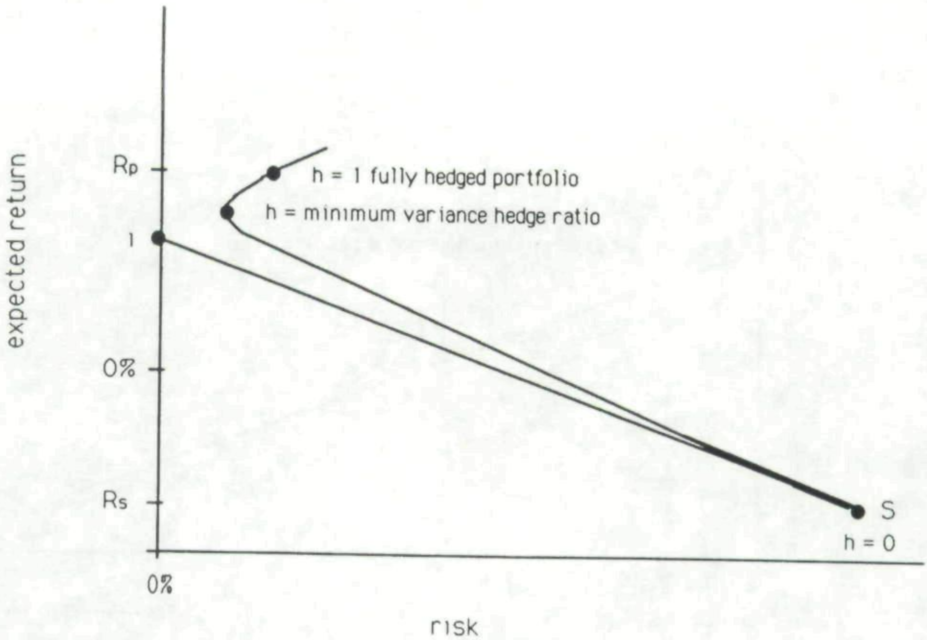

Figure 2c
Risk-return profile when futures are priced to the advantage of the short hedger and $R_s < \text{i}$.

spot return. The Howard and D'Antonio (hereafter HD) and Chang and Shanker (hereafter CS) risk-return hedging effectiveness measures have the same drawback, though the definitions of the HD and CS measures are more complex. Following is a brief review of their measures and the model upon which they are based.

The 1984 HD paper proposed a model where the investors' optimization problem is to choose the appropriate hedge ratio so as to maximize θ_H, the expected excess return per degree of risk of the hedged portfolio:

$$\theta_H = (R_p - i)/\sigma_p \tag{3}$$

where: R_p = expected percentage return of the hedged portfolio, i = risk-free rate of return, and σ_p = standard deviation of the hedged portfolio returns.

Graphically, the model is illustrated in Figure 3. Points S, i, R_p, and R_s are as defined earlier. The curve is a possible risk-return profile as the hedge ratio increases from 0 to the point of minimum variance. Point T is the point of tangency between the hedged portfolio possibility curve and a line from the risk-free asset (i); it is the point where θ_H, the expected excess return per degree of risk, is maximized. According to HD, investors would hold the appropriate number of futures contracts to obtain tangent portfolio T (and this position could be long, short, or zero) and then combine T with the risk-free asset to move to the preferred point on the tangent line. The hedging effectiveness measure based on this model and proposed in HD (1984) is:

$$HE = \theta_H/\theta_s \tag{4}$$

where: $\theta_s = (\text{R}_s - i)/\sigma_s$ and σ_s = standard deviation of the spot portfolio returns.

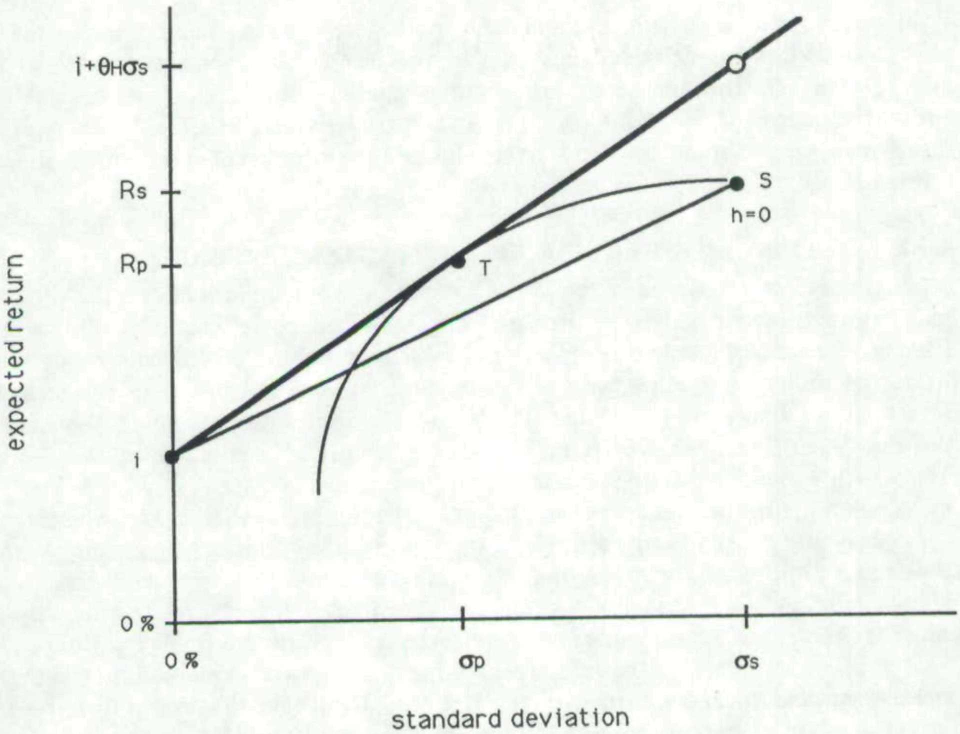

Figure 3
The HD model to maximize expected excess return per degree of risk.

The above *HE* measure is refined by CS (1987), who show that it is inconsistent when the excess return on the spot portfolio $(R_s - i)$ is negative. CS suggest the following measure as an alternative:

$$HE_1 = (\theta_H - \theta_s)/|\theta_s| \tag{5}$$

Their measure is positive when θ_H is greater than θ_s, and negative when θ_H is less than θ_s. In their response to CS, HD (1987) agree that their first HE measure is ambiguous, but they criticize HE_1 because it (like *HE*) still has the undesirable property that if the difference between the mean spot return and the risk-free rate is very small or zero, the measure becomes very large. HD derive the following measure, called HBS for hedging benefit per unit of risk, to solve this problem:

$$HBS = (i + \theta_H\sigma_s - R_s)/\sigma_s \tag{6}$$

Note that $i + \theta_H\sigma_s$ is indicated on Figure 3, and represents the amount of return on the line of tangency that corresponds with the σ_s level of risk. HBS is the extra return on the combined hedged-risk-free asset portfolio (compared to the return of the spot portfolio) that can be earned per unit of risk.

Since both HD and CS agree that problems exist with the first HE measure, the following discussion is limited to the HE_1 and HBS measures. A major problem, as in the Graham and Jennings measure, is that the spot portfolio return rather than equilibrium hedged return is the benchmark for measuring effectiveness on the return dimension. Remember, that as basis risk decreases, the hedged risk-return profile approaches a straight line between the risk-free asset (i) and the spot portfolio (S). Also, as basis risk decreases, θ_H approaches θ_s. So, given the definitions of HE_1 and HBS, the measures are getting smaller and smaller as basis risk approaches zero; at the limit, $\theta_H = \theta_s$ and both HE_1 and $HBS = 0$. Therefore, these measures cannot be used to evaluate the effectiveness of stock index futures hedges.

A NEW RETURN-RISK HEDGING EFFECTIVENESS MEASURE

A hedging effectiveness measure for stock index futures should focus on the difference between actual hedged returns and expected hedged returns in equilibrium. This difference approaches zero as basis risk approaches zero. The sign of the difference indicates whether basis risk is advantageous or not, but does not reflect on the effectiveness of the hedge. For a fully hedged stock portfolio, the equilibrium return is the risk-free rate. This rate is usually approximated by the T-bill rate, which varies for different durations and over time.

Consider again, the scenario described at the beginning of this article, where the manager of the Alaska Permanent Fund stock portfolio is considering hedging with stock index futures as a way to eliminate market risk and lock in current risk-free interest rates. Assume that this hedge is implemented, is lifted three months later, and that the manager must report on the effectiveness of the hedge to the Board of Trustees. The straightforward effectiveness measure, as just explained, is to compare the *ex-post* hedged return with the risk-free T-bill rate that could have been earned during the same time period. The smaller the difference, whether positive or negative, the more effective the hedge. Given a T-bill yield of 7%, for example, a hedged return of 6% would be judged as more effective than a hedged return of 10%, even though, of course, 10% is more desirable. Similarly, whether the spot return was −15% or +15% does not influence the effectiveness of the hedge, though it certainly affects the satisfaction quotient.

Imagining the application of a hedging effectiveness measure to an individual hedge helps put the emphasis in the right place—on return rather than risk. The effectiveness of one hedge cannot be measured by traditional risk measures because more than one observation is necessary to compute a standard deviation. When more than one observation exists, however, the standard deviation can certainly be a helpful measure. Imagine now, that the Permanent Fund manager has hedged the stock portfolio a total of 10 times during the past two years, and a report is being made to summarize the effectiveness of the hedges. Effectiveness can then be measured in terms of the mean standard deviation of the difference between portfolio return and the corresponding T-bill rate. The manager might find, for example, that the average hedge earned a return of 1% above the relevant T-bill rate with a standard deviation of 5%.

The hedging effectiveness measure proposed here is thus a joint return-risk measure and is defined as the mean and standard deviation of the fully hedged portfolio return less the corresponding T-bill rate for the same time period and duration:

$$M_r = \text{Mean of } (R_{pt} - R_{ft}) \tag{7}$$

and

$$\sigma_r = \sigma \text{ of } (R_{pt} - R_{ft}) \tag{8}$$

where: R_{pt} = return of the fully hedged portfolio in time period t, R_{ft} = risk-free T-bill rate for time period t, and $r = (R_{pt} - R_{ft})$.

The mean of the difference in returns, M_r, indicates the percent that fully hedged portfolios earn above or below the risk-free rate, on average. The standard deviation of the difference in returns, σ_r, indicates the dispersion around the mean. The closer M_r is to zero and the smaller the σ_r, the more effective the hedge.

Fortunately, portfolio managers do not need to generate their own hedge data to gather expectations about effectiveness. This section uses the M_r and σ_r measures to analyze the hedging effectiveness of S&P 500 stock index futures. The spot portfolio is represented by a long position in the S&P 500 cash index, and hedging durations of 1, 2, 4, and 13 weeks are examined. Futures data on the S&P 500 index are from the Chicago Mercantile Exchange while cash S&P 500 data and T-bill yields are from the *Wall Street Journal*. Weeks are defined using Friday to Friday closing prices and the futures price is represented by the nearest futures contract in the March–June–September–December series. Equation (2) is used to calculate fully hedged portfolio returns[1] for nonoverlapping periods from 1983–1988. Each hedged portfolio return is matched with the corresponding T-bill yield for the same time period and duration. Over the six years, T-bill yields range from 2.02% to 11.44%. Average T-bill yields for the different durations range from 6.42% to 7.58% and are given in Table I, along with the M_r and σ_r results.

The information in Table I shows that the shorter duration hedges are less effective: the σ_r's are 22.79%, 12.51%, 6.19%, and 2.36% for 1-, 2-, 4-, and 13-week hedges, respectively; and the corresponding M_r's are 1.89%, .59%, .62%, and .79% (all returns are annualized). For each of the durations, the frequency distributions of $(R_{pt} - R_{ft})$ are tested, and the chi square statistics indicate that they approxi-

Table I
RETURN-RISK HEDGING EFFECTIVENESS MEASURES FOR S&P 500 STOCK INDEX FUTURES

Hedge Duration (Weeks)	M_r[a] Mean of R_{pt}-R_{ft} (%)	σ_r[a] σ of R_{pt}-R_{ft} (%)	Average T-Bill Yield (%)	Number of Observations
1	1.89	22.79	6.42	306
2	.59	12.51	6.74	144
4	.62	6.19	6.72	72
13	.79	2.36	7.58	24

[a]R_{pt} = fully hedged portfolio return, R_{ft} = risk-free rate of return; all returns are annualized.

[1]The empirical analysis in this article assumes a fully hedged portfolio, which is interpreted as using a hedge ratio equal to the beta of the cash portfolio (and beta = 1 in the case of an index portfolio). A debate exists in the literature over whether the cash beta or the minimum variance hedge ratio is ideal. See, for example, Figlewski (1985) and Kawaller (1985). If the minimum variance hedge ratio is preferred, the M_r and σ_r return-risk measures defined in this article can still be used , and the h in eq. (2) would be set equal to the minimum variance hedge ratio. The equilibrium return would also be adjusted to reflect the minimum variance hedge ratio. The minimum variance hedge ratio can be considered a special case of a partially hedged portfolio. And if a portfolio is only partially hedged, the appropriate benchmark return for an equilibrium portfolio can be expressed as $h*R_f + (1 - h)*R_s$.

mate that of a normal curve. Therefore, 68% of the observations are estimated to lie within ±1 SD of the mean, M_r, and about 95% of the observations lie within ±2 SD of M_r. If a risk-free rate of 7% is expected, this translates into average R_p's of 8.89%, 7.59%, 7.62%, and 7.79% for 1-, 2-, 4-, and 13-week hedges, respectively. And the corresponding 95% confidence intervals are: −36.69% to 54.47% for 1-week hedges; −17.43% to 32.61% for 2-week hedges; −4.76% to 20.00% for 4-week hedges; and 3.07% to 12.51% for 13-week hedges. These confidence ranges show that considerable risk exists in the shorter duration hedges. The 13-week hedges are judged to be quite effective, however, with 95% of the returns being within about 5% of the corresponding risk-free rate.

Hedges are subdivided also according to the week prior to contract expiration, and effectiveness measures are calculated for each subset. Table II gives the results. While the 2- and 4-week hedges exhibit no significant patterns, the 1-week hedges show a definite trend toward increased effectiveness as expiration is approached. Hedges that are lifted between 0 and 4 weeks before expiration have an average σ_r of 17.47%, while the hedges 5 to 12 weeks from expiration have an average σ_r of 25.08%. However, all 1-, 2-, and 4-week hedges are judged to be poor substitutes for a risk-free asset. Only the 13-week hedges have returns that are consistently positive.

CONCLUSIONS

This article shows that when stock index futures are priced in equilibrium, fully hedged portfolios earn the risk-free rate of return. The most important indication of hedging effectiveness for stock index futures thus focuses on the difference between *ex-post* hedged portfolio return and the corresponding risk-free rate ($R_{pt} - R_{ft}$). The Graham and Jennings "return retention" measure and the HD and CS "risk-return" measures all suffer from the drawback that they focus on hedged return vs. spot return rather than hedged return vs. equilibrium return. The study

Table II
HEDGING EFFECTIVENESS MEASURES WHEN SUBDIVIDED BY WEEK TO EXPIRATION

Weeks Before Expiration	1-Week Hedges		2-Week Hedges		4-Week Hedges	
	M_r	σ_r	M_r	σ_r	M_r	σ_r
0	−2.6	19.21	−2.16	9.55	0.75	6.02
1	1.25	12.9				
2	4.6	19.41	3.47	14.51		
3	3.16	21.31				
4	0.39	14.53	1.58	12.62	−0.32	6.34
5	3.57	23.44				
6	−7.14	21.4	−2.56	12.67		
7	2.38	24.05				
8	−4.89	27.62	2.83	11.49	1.47	6.36
9	10.35	26.46				
10	−3.62	25.3	0.35	13.69		
11	3.87	25.14				
12	15.54	27.25				

defines a new two-part return-risk hedging effectiveness measure as: (1) M_r, the mean of $(R_{pt} - R_{ft})$; and (2) σ_r, the standard deviation of $(R_{pt} - R_{ft})$. The closer both of these measures are to zero, the more effective the hedge.

The M_r and σ_r return-risk measures are applied to six years of S&P 500 stock index cash and futures data to analyze the hedging effectiveness of 1-, 2-, 4-, and 13-week hedges. While the M_r's are less than 2% for all durations, the σ_r's are 22.79%, 12.51%, 6.19%, and 2.36% for 1-, 2-, 4-, and 13-week hedges, respectively. When the hedges are subdivided by week prior to contract expiration; the 1-week hedges show a trend toward increased effectiveness as the contract expiration date approaches. However, all 1-, 2-, and 4-week duration hedges are concluded to be unsatisfactory substitutes for a risk-free asset. Only the 13-week hedges have returns that are consistently positive, with 95% of the returns being within about 5% of the corresponding risk-free rate of return.

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