Reduction in Hedging Risk from Adjusting for Autocorrelation in the Residuals of a Price Level Regression

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This article addresses the question of whether correcting for autocorrelation in a price level regression can reduce hedging risk. When a hedging decision is being made, a hedger derives a target price to indicate the price he expects to achieve from hedging. Usually, the target price is developed from a linear (or price level) regression of local cash price on the nearby futures price. If the residuals in the regression are autocorrelated, the estimate of the autocorrelation coefficient from the regression residuals can be used to adjust the target price to obtain a more accurate indicator of the actual (net) price achieved by hedging. In theory, adjusting the target price for autocorrelation reduces hedging risk (defined as variance of net price minus target price).

Price level regressions are used extensively to estimate cross hedge ratios.[1] Hayenga and DiPietre (1982b) use a price level regression to estimate cross hedge ratios for wholesale pork products hedged in live hog futures. Miller and Luke (1982) and Hayenga and DePietre (1982a) estimate cross hedge ratios for wholesale beef products hedged in live cattle futures. Price level regression is used to analyze cross hedging of distiller's dried grain (Miller, 1982a), feeder pigs (Miller, 1982b), wheat mids (Miller, 1985), hay (Blake and Catlett, 1984), grain sorghum (Witt, Schroeder, and Hayenga, 1987; Schroeder, 1988), barley (Witt, Schroeder, and Hayenga, 1987; Anderson and Danthine, 1981), and oats (Anderson and Danthine, 1981). Price level regressions are used also to estimate hedge ratios for straight hedges, for example, wheat hedged in wheat futures (Brown, 1986), feeder cattle (Elam, 1988; Schroeder and Mintert, 1988), live cattle (Elam, 1986), and currencies (Dale, 1981).

Several studies using price level regression report autocorrelation in the regression residuals (Hill and Schneeweis (1981), Witt, Schroeder, and Hayenga (1987), Schroeder and Mintert (1988)). This is a concern because, usually, ordinary least squares (OLS) is used to estimate price level regressions. OLS estimates of the hedge ratio are inefficient when the regression residuals are autocorrelated. Standard statistical tests such as t and F tests are not valid when there is autocorrelation.

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[1]An alternative procedure for estimating the hedge ratio is price change regression, which involves regressing changes in cash prices on changes in futures prices (Ederington (1979), Bond, Thompson, and Lee (1987), Witt, Schroeder, and Hayenga (1987)). Price change regression is different from price level regression in that it uses *changes* in prices rather than *levels* of prices.

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Usually, generalized least squares (GLS) is used to estimate price level regressions when there is autocorrelation. By taking account of the autocorrelation, GLS improves the efficiency of the estimate of the hedge ratio, and makes it valid to use statistical tests on the hedge ratio estimate.

Witt, Schroeder, and Hayenga (1987) use GLS as well as OLS to estimate the hedge ratio for sorghum and barley and find only small differences in the GLS and OLS estimates. This should not temper the use of GLS because, theoretically, GLS estimates are superior to OLS estimates when autocorrelation is present. GLS estimates have smaller sampling variances than OLS estimates, and statistical tests are valid only if the standard deviation of an estimate is based on GLS (Johnston, 1972, p. 246).

An additional advantage of using GLS, other than estimation efficiency and statistical testing, is to improve prediction accuracy. In the context of this article, prediction accuracy refers to the accuracy of the target price as an indicator of the net price for a hedge. It is common practice in hedging discussions to refer to the actual price achieved from hedging as the net price, and the price the hedger expects to achieve from hedging as the target price (Hieronymus, 1977, p. 208, Miller (1985)). The target price is derived at the time a hedge is placed. If the residuals from a price level regression are autocorrelated, the estimate of the autocorrelation coefficient from the regression residuals can be used to adjust the target price, and potentially improve its accuracy in projecting the net price. Witt, Schroeder, and Hayenga (1987) mention this in their article, but they do not determine whether the autocorrelation coefficient can be used to improve the accuracy of the target price. The focus of the present article is on measuring the reduction in hedging risk that can be achieved by adjusting the target price using the autocorrelation coefficient calculated from the price level regression residuals.

Risk is involved in hedging because of "basis risk" which causes the net price to be different from the target price. To the extent that net and target prices are different, the hedger receives a price from hedging that is different from the price he expects to receive at the time the hedge is placed. A measure of the degree of risk involved in hedging is the variance (or standard deviation) of the difference between net and target prices. It is shown later in this article, that the residuals from a price level regression represent the errors made by the target price in projecting the net price. The variance (or standard deviation) of the regression residuals, which is equal to the variance of net minus target prices, provides a measure of hedging risk.

If the residuals from a price level regression are autocorrelated, then future residuals can be related to past residuals which means that future residuals can be predicted. And, if future residuals can be predicted, then the difference between net and target prices can be predicted (remembering that the regression residual is equal to the net price minus the target price). If the difference between net and target prices can be predicted, then an adjustment can be made to the target price such that it provides a more accurate estimate of the net price. The adjustment is based on the autocorrelation in the price level regression residuals much like autocorrelation adjustments are made to econometric predictions to improve their accuracy. In theory, the effect of adjusting the target price is to reduce hedging risk.

An article by Herbst, Kare, and Caples (1989) (HKC) addresses some of the same issues. HKC propose that time series analysis be used (rather than OLS) to estimate the hedge ratio because of autocorrelation in the residuals of an OLS regression.

The GLS procedure used in this research is an alternative procedure for obtaining improved hedge ratio estimates based on correcting for autocorrelation (Granger and Newbold, 1977, ch. 6). HKC relate hedging risk to the variance of the regression residuals. In this article, hedging risk is related to the degree of autocorrelation in the regression residuals, as well as the residual variance. A formula is derived which measures the reduction in hedging risk based on the time period a hedge is held and the degree of autocorrelation in the regression residuals. By contrast, HKC claim that the reduction in hedging risk is equal to the relative reduction in the residual variance of the time series model over the OLS model. Their estimates for reduction in hedging risk apply to all hedges, regardless of the time period a hedge is held; whereas, the results in this article show that the reduction in hedging risk depends on the hedging period.

HEDGING RISK AND THE MINIMUM VARIANCE HEDGE RATIO

The price a hedger expects to achieve by hedging is usually not equal to the actual price achieved by hedging. To the extent that the net (actual) price differs from the target (expected) price, risk is involved in hedging.[2] The remainder of this section defines hedging risk in terms of the variance of net minus target prices.

The net price for a hedge is the sum of the cash price at the time the hedge is lifted plus the return on the futures position:

$$N_t = C_t + b_1(F^t_{t-j} - F^t_t) \tag{1}$$

where N_t is the per unit net price for a j-period hedge that is lifted at time t, C_t is the actual per unit price of the cash commodity at time t, b_1 is the estimated hedge ratio (discussed below), and F^t_{t-j} and F^t_t are the per unit futures prices at times $t - j$ and t, respectively, for the futures contract maturing nearest to, but not before, time t.

The hedge ratio can be estimated from a price level regression:

$$C_t = b_0 + b_1 F^t_t + v_t \tag{2}$$

where b_0 and b_1 are least squares estimates, and v_t is the regression residual with expected mean of zero and variance of $E[(v_t)^2] = \sigma^2_v$. The estimated slope coefficient (b_1) represents the number of units of futures required to hedge one unit of a given (or anticipated) cash position. For example, a short hedge against cattle in a feedlot would require the sale of b_1 pounds of live cattle futures for each pound of anticipated production of cattle; and a long hedge by a meat packer against a future need for cattle would require the purchase of b_1 pounds of live cattle futures for each pound of cattle that will be needed.

An estimate of the target price is needed at the time a decision is being made concerning whether or not to hedge. The target price is the expected net price, where the expectation is based on what is known at the time the hedging decision is being made. The target price can be derived by substituting C_t from eq. (2) into (1), and then taking expectations of both sides of the resulting equation to obtain:

$$T^t_{t-j} = b_0 + b_1 F^t_{t-j} \tag{3}$$

where T^t_{t-j} is the per unit target price as calculated at time $t - j$ for a hedge to be

[2]This definition of hedging risk is consistent with Peck's (1975) concept of risk as "exposure to unpredictable price variation."

lifted at time t.[3] In deriving eq. (3), it is assumed that the expected value of v_t equals zero, which holds if there is zero autocorrelation in the regression residuals from eq. (2). (The situation where v_t is autocorrelated is considered in the next section.)

As discussed above, risk is involved in hedging because the net and target prices can differ. A measure of the risk associated with a hedge is the variance of net minus target prices:

$$\mathrm{var}(N_t - T^t_{t-j}) = \frac{1}{n}\sum_{t=1}^{n}(C_t - b_0 - b_1 F^t_t)^2 \tag{4}$$

where n is the number of sample observations for (C_t, F^t_t).[4,5] Hedging risk can be minimized by choosing b_0 and b_1 so as to make the variance from eq. (4) as small as possible.[6] A necessary condition is that the partial derivatives of eq. (4), with respect to b_0 and b_1, should both be zero:

$$\frac{\partial\ \mathrm{var}(N_t - T^t_{t-j})}{b_0} = -\frac{2}{n}\sum_{t=1}^{n}(C_t - b_0 - b_1 F^t_t) = 0 \tag{5}$$

$$\frac{\partial\ \mathrm{var}(N_t - T^t_{t-j})}{b_1} = -\frac{2}{n}\sum_{t=1}^{n}F^t_t(C_t - b_0 - b_1 F^t_t) = 0. \tag{6}$$

Simplifying these two equations gives the standard normal equations for a linear regression of cash on nearby futures prices (eq. (2)):

$$\sum_{t=1}^{n} C_t = nb_0 + b_1 \sum_{t=1}^{n} F^t_t \tag{7}$$

$$\sum_{t=1}^{n} C_t F^t_t = b_0 \sum_{t=1}^{n} F^t_t + b_1 \sum_{t=1}^{n} (F^t_t)^2. \tag{8}$$

[3]Price level regression has its roots in practical applications of hedging. Textbook examples of hedging show a hedger adjusting the futures price with the basis to obtain a target price (Hieronymus, 1977, pp. 207–208):

$$T^t_{t-j} = \text{avg. basis} + F^t_{t-j}$$

The above equation is similar to eq. (3) in the text, except b_0 is replaced with the average basis and the hedge ratio (b_1) is assumed to equal 1.0.

[4]The variance of net minus target prices is:

$$\mathrm{var}(N_t - T^t_{t-j}) = \frac{1}{n}\sum_{t=1}^{n}[(N_t - T^t_{t-j}) - E(N_t - T^t_{t-j})]^2. \tag{*}$$

The second term in the brackets is equal to v_t, the least squares regression residual (refer to equation (11) in text). An assumption of least squares regression is that $E(v_t) = 0$. Therefore, eq. (*) simplifies to:

$$\mathrm{var}(N_t - T^t_{t-j}) = \frac{1}{n}\sum_{t=1}^{n}(N_t - T^t_{t-j})^2$$

which is the basis of eq. (4) in the text.

[5]In the case of a textbook hedge where the hedge ratio is 1.0, eq. (4) can be written as:

$$\mathrm{var}(N_t - T^t_{t-j}) = \frac{1}{n}\sum_{t=1}^{n}[(C_t - F^t_t) - \bar{B}]^2 = \frac{1}{n}\sum_{t=1}^{n}(B_t - \bar{B})^2, \tag{+}$$

where $\bar{B}$ is the average basis. Note that eq. (+) was derived using the net price from eq. (1) in the text and the target price from footnote 3. Equation (+) shows that the variance of net minus target prices is equal to the variance of the basis. This demonstrates the connection between eq. (4) and the textbook concept of hedging risk as basis variation.

[6]The reader should note that the hedge ratio estimate, b_1, is the minimum variance hedge ratio, which is not necessarily the optimal hedge ratio derived from maximizing expected utility. Minimum variance hedge ratios have been extensively studied by academic researchers (refer to studies by Brown (1986) and Ederington (1979)).

Equations (7) and (8) can be solved for b_1 and b_0:

$$b_1 = \frac{\sum_{t=1}^{n}(C_t - \bar{C})(F_t^t - \bar{F})}{\sum_{t=1}^{n}(F_t^t - \bar{F})^2} \tag{9}$$

$$b_0 = \bar{C} - b_1\bar{F} \tag{10}$$

where $\bar{C}$ and $\bar{F}$ are the means of cash and nearby futures prices, respectively. The coefficient b_1 is the estimated hedge ratio which minimizes hedging risk, defined as the variance of net minus target prices.

REDUCTION IN HEDGING RISK FROM ADJUSTING FOR AUTOCORRELATION

The residuals from a price level regression represent the errors made by the target price in projecting the net price for a hedge. This can be seen by subtracting eq. (3) from (1) and substituting for C_t using (2):

$$N_t - T_{t-j}^t = v_t. \tag{11}$$

When v_t is autocorrelated, the error made by the target price in projecting the net price in one period will be correlated with the error(s) in some previous period(s). This correlation provides a means for improving the accuracy of the target price as an indicator of the net price for a hedge, and thereby reducing hedging risk.

For example, assume that the residuals from eq. (2) follow a first-order autoregressive process:

$$v_t = rv_{t-1} + e_t \tag{12}$$

where r is the autoregressive coefficient and e_t is a random error with expected mean of zero and variance of $E[(e_t)^2] = \sigma_e^2$. A more efficient estimate of the hedge ratio can be obtained by re-estimating eq. (2) using a Cochrane–Orcutt transformation to take account of the autocorrelation in v_t:

$$C_t - rC_{t-1} = b_0^* + b_1^*(F_t^t - rF_{t-1}^{t-1}) + e_t \tag{13}$$

where b_0^* and b_1^* are least squares estimates from the transformed price level regression.[7] The slope coefficient estimate b_1^* provides a more efficient estimate of the hedge ratio than b_1 because the information in the autocorrelated error process (eq. (12)) is used in the Cochrane–Orcutt estimation procedure (Johnston, 1972, pp. 208–11).

By taking account of the autocorrelation in v_t, an adjusted target price can be derived which provides a more accurate estimate of the net price than is provided by the target price, T, from eq. (3). The steps involved in deriving the adjusted target price are the following. First, equations for $C_{t-1}, C_{t-2}, \ldots, C_{t-j+1}$ are developed using eq. (13); and then these equations are successively substituted into (13) to obtain:

$$\begin{aligned} C_t = r^jC_{t-j} + b_0^*(1 + r + r^2 + \ldots + r^{j-1}) + b_1^*(F_t^t - r^jF_{t-j}^{t-j}) \\ + e_t + re_{t-1} + \ldots + r^{j-1}e_{t-j+1}. \end{aligned} \tag{14}$$

[7]The reader should note that correcting for first-order autocorrelation as in eq. (13), which involves first differencing the data, is not generally equivalent to using a price change regression. The two procedures are equivalent only in the situation where the time period a hedge is held is equal to the time interval of an analyst's data (e.g., for a hedge held one month and monthly data) (Witt, Schroeder, and Hayenga, 1987).

Second, C_t from eq. (14) is substituted into (1) and then expectations are taken of both sides of the equation based on what is known at time $t - j$ when the hedge is placed. The resulting equation for the target price adjusted for autocorrelation (*TA*) is:

$$TA_{t-j}^{t} = r^{j}C_{t-j} + b_0^*(1 + r + r^2 + \dots + r^{j-1}) + b_1^*(F_{t-j}^{t} - r^{j}F_{t-j}^{t-j}). \quad (15)$$

In deriving eq. (15), it is assumed that the expected values of the future error terms $(e_t, e_{t-1}, \dots, e_{t-j+1})$ equal zero because they are independent of past errors by virtue of the assumption that the error term e_t in eq. (13) is nonautocorrelated.

The difference between the net price—i.e., NA_t derived from eq. (1) with b_1 replaced with the estimated hedge ratio (b_1^*) from eq. (13)—and *TA* is[8]:

$$NA_t - TA_{t-j}^{t} = e_t + re_{t-1} + \dots + r^{j-1}e_{t-j+1}; \quad (16)$$

and the variance of the difference is:

$$\mathrm{var}(NA_t - TA_{t-j}^{t}) = (1 + r^2 + r^4 + \dots + r^{2(j-1)})\sigma_e^2. \quad (17)$$

By contrast, if the target price is taken from eq. (3), the variance of net minus target prices from (11) is $\sigma_v^2 = (1 + r^2 + r^4 + \dots)\sigma_e^2$. The difference between σ_v^2 and the variance from eq. (17) is:

$$\mathrm{var}(N_t - T_{t-j}^{t}) - \mathrm{var}(NA_t - TA_{t-j}^{t}) = \frac{r^{2j}}{(1 - r^2)}, \quad (18)$$

which is positive for $r \neq 1$.[9] The fact that the difference in the variances is positive indicates that hedging risk is less with the target price adjusted for autocorrelation than with the target price from eq. (3).

The percentage reduction in hedging risk with *TA* compared to *T* is obtained by multiplying 100 times the autocorrelation coefficient raised to the power of two times the number of periods that the hedge will be held:

$$1 - \frac{\mathrm{var}(NA_t - TA_{t-j}^{t})}{\mathrm{var}(N_t - T_{t-j}^{t})} = r^{2j} \times 100 \quad (19)$$

The percentage reductions in hedging risk are shown in Table I for different hedging periods (j) and different degrees of autocorrelation (r) in the residuals. Note that the reduction in hedging risk increases as r increases. For example, for one-week hedges with $r = 0.25$, hedging risk is expected to be reduced 6.2% when using *TA* compared to *T* as the target price. For one-week hedges as r increases to 0.50, 0.63, 0.75, and 0.85, hedging risk is expected to be reduced 25.0%, 39.7%, 56.2%, and 72.2%, respectively, when the target price used is *TA* rather than *T*. Hedging risk is reduced, because as r increases, the error made by *TA* in projecting the net price becomes more predictable. That is, *TA* provides a more accurate indication of

[8]The difference between *NA* and *TA* depends on the errors from eq. (13) for the period a hedge is held. For example, a 22-week hedge placed in the first week of May 1985 is lifted during the first week in October 1985. The error from eq. (13) for the first week in October (for October cash versus October futures prices) is e_t in eq. (16); whereas the error from (13) for the first week in May (for May cash versus June futures prices) is e_{t-21} in (16). The difference between *NA* and *TA* is equal to the weighted sum of the errors e_t to e_{t-21} (where r-values are used as weights).

[9]When $r = 1$, the difference in the variances in (18) is infinite because the $\mathrm{var}(N - T) = \sigma_v{}^2$ is infinite. Also note that when $r = 1$, first differences of the data are used to estimate the transformed price level regression (eq. (13)).

Table I
PERCENT REDUCTION IN HEDGING RISK WITH *TA* COMPARED TO *T* FOR DIFFERENT HEDGING PERIODS AND DIFFERENT DEGREES OF AUTOCORRELATION IN THE RESIDUALS FROM A PRICE LEVEL REGRESSION

	Number of Periods (j) that Hedge is Held					
First-order Autocorrelation (r)	1	2	4	9	13	22
0.25	6.2	0.4	0.0	0.0	0.0	0.0
0.50	25.0	6.2	0.4	0.0	0.0	0.0
0.63	39.7	15.8	2.5	0.0	0.0	0.0
0.75	56.2	31.6	10.0	0.6	0.0	0.0
0.85	72.2	52.2	27.2	5.4	1.5	0.1

Note: The percentage reduction in hedging risk with *TA* compared to *T* is $r^{2j} \times 100$ from eq. (19).

NA because the information in the autocorrelated error series is used in deriving the target price, *TA*. Since hedging risk is defined as the variance of net minus target prices, as the accuracy of *TA* improves (i.e., as the prediction, *TA*, is closer to the net price), hedging risk decreases.

The reduction in hedging risk with *TA* compared to *T* decreases as the length of time a hedge is held increases. For example, if $r = 0.63$, hedging risk is expected to be reduced 39.7%, 15.8%, and 2.5%, respectively, for 1-, 2-, and 4-week hedges, when the target price used is *TA* rather than *T*. For hedges held for 9 weeks or longer, hedging risk is expected to be the same (0% reduction as shown in Table I) for *TA* and *T*.[10]

The decline in the accuracy of *TA* compared to *T* (as the length of time a hedge is held increases) is due to the fact that the information in the autocorrelated residuals has less predictive value the further the net price is projected in the future (i.e., the longer a hedge is held). An analogous situation occurs in time series analysis when forecasting with an autoregressive process (Nelson, 1973). As the forecast horizon moves further into the future, the autoregressive forecast has relatively less predictive accuracy, compared to the mean, as a forecast. For distant horizons, the autoregressive forecast approaches the mean (for a stationary autoregressive process) and the accuracy of the autoregressive forecast approaches that of the mean. Similarly, in the hedging context, *TA* approaches *T* as the time a hedge is held increases; and from this it follows that hedging risk for *TA* approaches that of *T*. This is reflected in Table I by the zeros which indicate that hedging risk with *TA* is the same as hedging risk with *T*. The positive numbers in Table I reflect reductions in hedging risk with *TA* compared to *T*, and the reductions go to zero (for a given r) as the length of time a hedge is held increases.

[10]The figures in Table I show that the variance of the regression residuals cannot be used as an indicator of hedging risk (as suggested by Herbst, Kare, and Caples (1989, p. 191)). The process of correcting for autocorrelation guarantees that the residual variance of the nonautocorrelated regression (σ_ϵ^2) will be lower than that of the autocorrelated (OLS) regression (σ_v^2) (see eq. (18)). Because the residual variance is equal to hedging risk for 1-week hedges (eq. (17) with $j = 1$), hedging risk should be lower for 1-week hedges after correcting for autocorrelation.

The results for 1-week hedges (based on residual variance) do not apply for longer-period hedges. Reductions in hedging risk from correcting for autocorrelation depend on the time period a hedge is held and the degree of autocorrelation in the regression residuals (Table I). For example, correcting for autocorrelation with $r = 0.63$ is expected to reduce hedging risk 40% for 1-week hedges and 0% for 9-week hedges.

HEDGING SIMULATION

A simulation is conducted to compare the accuracy of T from eq. (3) and TA from (15) as indicators of the net price for a hedge. Beef cattle is chosen as the commodity. The simulation covers the years 1981–85. The data include Wednesday cash prices for "choice" steers at Omaha, obtained from the *Chicago Mercantile Exchange Yearbook* (CME) and the *Wall Street Journal*. Closing prices for live beef cattle futures are from the Commodity Futures Trading Commission and the *Wall Street Journal*.

The hedge ratios are obtained by estimating eqs. (2) and (13) with monthly intercept and slope shifters.

$$C_t = b_0 + \sum_{i=2}^{12} c_i D_{ti} + b_1 F_t^l + \sum_{i=2}^{12} d_i D_{ti} F_t^l \tag{20}$$

$$C_t - rC_{t-1} = b_0^* + \sum_{i=2}^{12} g_i D_{ti} + b_1^*(F_t^l - rF_{t-1}^l) + \sum_{i=2}^{12} h_i D_{ti}(F_t^l - rF_{t-1}^l) \tag{21}$$

where b_0 and b_1 (b_0^* and b_1^*) are intercept and slope coefficients for January for the autocorrelated (nonautocorrelated) regression; c_i and d_i (g_i and h_i) for $i = 2$ to $i = 12$ are fixed coefficients that measure changes in the intercept and slope coefficients from the January base for the autocorrelated (nonautocorrelated) regression; and D_{ti} is a 0–1 dummy variable with $D_{t2} = 1$ if the month is February and 0 otherwise, $D_{t3} = 1$ if the month is March and 0 otherwise, . . . , and $D_{t12} = 1$ if the month is December and 0 otherwise. Slope and intercept shifters are included as regressors to account for seasonal changes in the relationship between cash and futures prices, and to account for the fact that the relationship between cash and futures prices might be different for months when futures contracts mature (February and every other month for live beef cattle) compared to months when futures do not mature (January, March, etc.). The slope coefficients from eqs. (20) and (21) provide estimates of the hedge ratios. For example, the estimated monthly hedge ratios from eq. (20) are b_1 for January, $(b_1 + d_2)$ for February, . . . , and $(b_1 + d_{12})$ for December. Hedge ratios from eq. (21) are determined in a similar manner. Equations (20) and (21) were first estimated using data for 1975–80. The residuals from eq. (20) are autocorrelated with $r = 0.63$ and a Durbin–Watson statistic of 0.73. This level of autocorrelation in v_t implies that hedging risk for TA compared to T can be reduced 40%, 16%, and 2%, respectively, for hedges held for 1, 2, and 4 weeks (Table I).

Hedges are placed each week during 1981 based on the estimated hedge ratios from eqs. (20) and (21). Hedges are placed in the futures contract nearest to maturity at the time the hedge is to be lifted. The hedges are held for 1, 2, 4, 9, 13, and 22 weeks. T, TA, and the respective net prices are calculated each week during 1981 for each of the 6 hedges. At the end of 1981, eqs. (20) and (21) are re-estimated using data for 1975–81. Hedges are placed each week during 1982 based on the re-estimated hedge ratios, and target and net prices are calculated. This process of re-estimating eqs. (20) and (21) at the end of each year, placing hedges each week of the following year based on the re-estimated hedge ratios and calculating target and net prices, is continued through the end of 1985. The number of simulated hedges is 260 for each of the 6 hedging periods.

Mean differences (MDs) between net and target prices and mean square differences (MSDs) are calculated using the target price (T) from the autocorrelated price level regression and the adjusted target price (TA) from the nonautocorrelated price level regression. The MDs can be used to determine whether target prices are unbiased estimates of subsequent net prices. The MSDs provide a measure of the risk associated with the divergence of net and target prices. If MD = 0, then MSD is equal to the variance of net minus target prices. Since it is not known for certain whether MD = 0, MSD is used rather than the variance, as an empirical measure of hedging risk.

The simulation results for cattle hedges are shown in Table II. The average hedge ratio for the autocorrelated price level regression (0.98) is slightly larger than the average hedge ratio for the nonautocorrelated regression (0.94).[11] The small difference in the hedge ratios is consistent with the results of Witt, Schroeder, and Hayenga which show small differences in hedge ratios for grain sorghum and barley estimated with OLS and GLS. In this article, OLS is used to estimate the autocorrelated price level regression and the Cochrane–Orcutt procedure, a form of GLS, is used to estimate the nonautocorrelated regression.[12] Herbst, Kare, and Caples (1989) report that hedge ratios estimated with time series analysis (a form of GLS—Granger and Newbold (1977, ch. 6)) are 1% to 8% smaller than those estimated with OLS. This is in line with the 4% smaller hedge ratio in Table II for GLS (0.94) over OLS (0.98).

Even though the difference in the hedge ratios is small, the amount of futures required to hedge a pen of cattle can be quite different. For example, assume that a cattle feeder would like to hedge a pen of 125 head of fed cattle with an anticipated sale weight of 1100 lbs. per head. If the target price is based on the autocorrelated price level regression, a hedge for a pen of 125 head of cattle would require the sale of 134,750 lbs. of cattle futures (0.98 × 125 × 1100). The live cattle futures contract on the CME is for 40,000 lbs. while the Mid-America Commodity Exchange (MACE) contract is for 20,000 lbs. The closest to 134,750 lbs. is three CME contracts plus one MACE contract for 140,000 lbs.[13] By contrast, if the target price is based on the nonautocorrelated price level regression, a hedge for 125 head of 1100 lb. cattle would require selling 129,250 lbs. of futures (0.94 × 125 × 1100). The closest to 129,250 lbs. is three CME contracts for 120,000 lbs. which is 20,000 lbs. less than required to hedge the pen of 125 head of cattle based on the autocorrelated price level regression.

The average net prices for hedges based on the autocorrelated and nonautocorrelated price level regressions are approximately the same (Table II). This means that a cattle feeder using a continuous hedge will achieve approximately the same price with either procedure. The average net prices for 1- and 2-week hedges for both the

[11]The hedge ratios represent averages of the monthly slope coefficients from eqs. (20) and (21). Each equation has 12 monthly slope coefficients, and each equation is estimated 5 times in the simulation analysis (using data for 1975–80, 1975–81, 1975–82, 1975–83, and 1975–84). A total of 60 (12 × 5) slope coefficients are estimated for each equation. A t test is used to test the hypothesis that the estimated slope coefficients are equal to 1.0 (the hedge ratio for a traditional (lb.-for-lb.) hedge). Of the 60 coefficients, 16 are significantly different from 1.0 at the 0.05 level of significance in eq. (20), and 30 are significant in eq. (21). The simulation analysis is performed using the estimated hedge ratios (rather than the traditional hedge ratio of 1.0) because the coefficients from eqs. (20) and (21) provide the most accurate estimates of the true (but unknown) hedge ratios.

[12]There is no evidence of heteroscedasticity in the regression residuals.

[13]It is assumed that prices of the CME and MACE live cattle futures contracts are equal. While the author is not aware of empirical evidence to support this claim, there is reason to believe that the two prices are approximately equal due to the action of MACE "changers" who arbitrage prices on the MACE and CME exchanges (MidAmerica Commodity Exchange, 1990).

Table II

SUMMARY OF SIMULATED HEDGING RESULTS FOR AUTOCORRELATED (EQ. (2)) AND NONAUTOCORRELATED (EQ. (13)) PRICE LEVEL REGRESSIONS FOR CATTLE, 1981–85

		Autocorrelated Price Level Reg.				Nonautocorrelated Price Level Reg.				
Number of Weeks Hedge is Held	Average Cash Price \$/cwt.	Average Hedge Ratio[a]	Average Net Price \$/cwt.	MD[b] \$/cwt.	MSD[c] (\$/cwt.)^2	Average Hedge Ratio[d]	Average Net Price \$/cwt.	MD[b] \$/cwt.	MSD[d] (\$/cwt.)^2	Change in Hedging Risk[e] (%)
1	63.31	0.98	63.22	−0.02	3.31[g]	0.94	63.22	0.11	1.62[g]	−51.1
2	63.31	0.98	63.18	0.01	3.37[g]	0.94	63.18	0.20[f]	2.53[g]	−24.9
4	63.28	0.98	63.11	0.04	3.44	0.94	63.09	0.30[f]	3.53	2.6
9	63.20	0.98	62.86	0.05	3.44	0.94	62.91	0.36[f]	3.99	16.0
13	63.13	0.98	62.66	0.01	3.46	0.94	62.68	0.33[f]	4.04	16.8
22	62.76	0.98	62.59	0.05	3.46	0.94	62.57	0.34[f]	4.04	16.8

[a] Average of the estimated slope coefficients for the months January through December (from eq. (20)).

[b] MD is the average difference between net and target prices.

[c] MSD is the average squared difference between net and target prices. Hedging risk is the same for different length hedging periods because it depends on the uncertainty in the relationship between cash and futures prices at the time a hedge is lifted (v_t from eq. (2) and (11)) and this does not vary with the length of time a hedge is held. The MSDs differ slightly because of differences in the simulation periods. For example, for 1-week hedges, the first and last hedges are lifted during the first and last weeks of 1981 and 1985, respectively; whereas for 22-week hedges, the first and last hedges are lifted during weeks 21 of 1981 and 1986, respectively.

[d] Average of the estimated slope coefficients for the months January through December (from eq. (21)).

[e] One minus column 10 divided by column 6 multiplied by 100.

[f] Significantly different from zero at the 0.05 level based on a two-tailed t test.

[g] MSD for the nonautocorrelated price level regression is significantly different from MSD for the autocorrelated price level regression at the 0.05 level based on a test developed by Ashley, Granger, and Schmalensee (1980).

autocorrelated and nonautocorrelated price level regressions are approximately equal to the cash price. But as the number of weeks a hedge is held increases, the average net price declines relative to the average cash price.

The expected MDs, determined by taking expectations of eqs. (11) and (16), are equal to zero and, therefore, both T and TA should provide unbiased estimates of the respective net prices. The small MDs in Table II for the autocorrelated price level regression indicate that T from eq. (3) is an unbiased estimate of the net price from hedging. By contrast, the positive values for the MDs for the nonautocorrelated price level regression indicate that TA from eq. (15) underestimates, on average, the net price from hedging.

The MSDs in Table II provide a measure of the risk due to the divergence of net and target prices. Comparing the MSDs, hedging risk is lower for TA compared to T for hedges held for 1 and 2 weeks and approximately the same for hedges held for 4 weeks. For 1- and 2-week hedges, the reduction in hedging risk is 51% and 25%, respectively, which is 11 and 9 percentage points greater than the expected reductions from Table I. By contrast, hedging risk for 9-, 13-, and 22-week hedges is 16% to 17% higher for TA compared to T. For hedges held for 9 weeks or longer and for $r = 0.63$, hedging risk was expected to be approximately the same for TA and T (Table I).

The simulation results in Table II indicate that hedging risk can be reduced substantially for hedges held for short periods of time by using TA, rather than T, as the target price. The accuracy of TA as an indicator of the net price from a hedge is improved because TA takes account of the information in the autocorrelated regression residuals, whereas T ignores this information. An analogous situation occurs in econometric prediction where there is autocorrelation in the prediction equation residuals (Johnston, 1972, pp. 212–213). If the information in the autocorrelated residuals is used when deriving a prediction, the accuracy of the prediction is improved compared to the situation where the autocorrelation is ignored.

When hedges are held for longer periods of time, reductions in hedging risk when using TA compared to T are expected to be small or zero (Table I). This is because the information in the autocorrelated residuals has less predictive power the longer the time interval between when the target price is derived—the target price being the prediction—and when the net price is realized. A similar situation occurs in econometric prediction with autocorrelated regression errors where the predictive value of the autocorrelation diminishes as the prediction moves further into the future. In the beef hedging simulation with $r = 0.63$, hedging risk is expected to be the same for TA and T for hedges held for 9 weeks or longer (Table I). The fact that hedging risk is expected to be the same says that the autocorrelation in the price level regression residuals has no value in improving the accuracy of the target price, T, from eq. (3), as an indicator of the net price for a hedge. The actual results in the beef simulation are somewhat different from the expected results in that hedging risk actually increases for TA (which uses the information in autocorrelated residuals) compared to T (which ignores the autocorrelation information) (Table II). The increase in hedging risk for TA compared to T for hedges held for 9 weeks or longer is not significant, and is likely due to sampling error.

From a practical standpoint, the recommendation from the theoretical and simulation results in Tables I and II, is to use the autocorrelation information to adjust the target price for hedges held for short periods of time (where "short," in the beef simulation, is less than 4 weeks). An example of a situation where a hedge would be held for a short period of time is the case of packer who has purchased cattle from feeders and is holding the cattle until they can be moved to the slaughterhouse. A short hedge could be used to protect the packer from a drop in prices over the 1- to 2-week period that the cattle are to be held. Another example is a packer who has sold wholesale beef on a cash forward contract that calls for delivery over the next month. The packer could buy cattle futures to cover the cost of the live cattle needed to fill the contract. The reduction in hedging risk achieved by adjusting the target price depends on the degree of autocorrelation and the length of time a hedge is held. The higher the autocorrelation in the price level regression residuals, and the shorter the length of time a hedge is held, the greater the reduction in hedging risk.

When hedges are held for longer periods of time, a small (to zero) reduction in hedging risk is expected from utilizing the information in the autocorrelated residuals. Even with a relatively high r of 0.85, the expected reduction in hedging risk for a 9-week hedge is only 5.6% (Table I). When expected reductions in hedging risk are this small, it is probably not worth the effort to calculate TA compared to T.

SUMMARY AND CONCLUSIONS

This research addresses the question of whether the autocorrelation in the residuals from a price level regression can be used to adjust the target price to make it a more accurate indicator of the net price for a hedge. Hedging risk is defined as the variance of net minus target prices, where the net price is the actual price achieved by hedging, and the target price is the price the hedger expects to achieve by hedging. Assuming that the residuals from a price level regression are first-order autocorrelated, it is shown that the percentage reduction in hedging risk from using the information in the autocorrelated residuals is equal to $r^{2j} \times 100$. The expected reduction in hedging risk increases as the degree of autocorrelation (r) increases, and as length of time a hedge is held (j) decreases.

A simulation is used to determine if the expected reductions in hedging risk can be achieved in practice. The simulation uses cattle prices for 1981–85. For hedges held for 1 and 2 weeks, hedging risk is reduced by 51% and 25%, respectively, when using a target price adjusted for autocorrelation (TA). The actual reductions in hedging risk from correcting for autocorrelation are 11 and 9 percentage points greater, respectively, than the expected reductions. For hedges held for 4 weeks or longer, hedging risk increases 3% to 17% by adjusting the target price for autocorrelation even though hedging risk is expected to be approximately the same regardless of whether the information in the autocorrelated residuals is used to adjust the target price.

The simulation results indicate that unless a substantial reduction in hedging risk is expected, as in the case of 1- and 2-week hedges in the beef hedging simulation, the autocorrelation in the price level regression residuals should not be used to derive the target price. This is consistent with the theoretical derivations in Table I, which show small to zero expected reductions in hedging risk from using TA (rather than T) for hedges held for 4 weeks or longer. Therefore, the recommendation from both the theoretical derivations and simulation results is to ignore the autocorrelation in the regression residuals and use T as the target price when a hedge is held for longer periods of time. This recommendation is good news for practicing hedgers because it provides support for the commonly followed practice of calculating a target price from a linear regression of cash on nearby futures prices as in eq. (3). Moreover, it justifies the simpler calculations involved in deriving the traditional target price, T.

Another reason for correcting for autocorrelation when estimating a price level regression is to improve the efficiency of the hedge ratio estimate. When the regression residuals are autocorrelated, correcting for autocorrelation with GLS reduces the sampling variance of the hedge ratio estimate compared to OLS. In addition, it is valid to use statistical tests such as t and F tests after correcting for autocorrelation with GLS. If a researcher is interested in using statistical tests (e.g., testing whether the hedge ratio is different from 1.0), then a GLS procedure which corrects for autocorrelation should be used. However, one should not expect any great change in the size of the hedge ratio estimate when using GLS compared to OLS (based on the results for cattle hedges in this article, and for sorghum and barley hedges (Witt, Schroeder, and Hayenga, 1987 and foreign currency hedges (Herbst, Kare, and Caples, 1989).

The results in this study of beef cattle hedges show that obtaining more efficient estimates of the hedge ratio can cause an increase in hedging risk. The increase can be avoided by using OLS (i.e., not correcting for autocorrelation); however, the problem with using OLS is that hedge ratio estimates are less efficient and hypothesis tests are not valid (because of the autocorrelated error term). A researcher interested in obtaining efficient estimates of a hedge ratio should use GLS. By comparison, a practicing hedger is not concerned with estimation efficiency, but with reducing hedging risk. The simulation results for beef cattle show that if a hedge is to be held for less than one month, then correcting for autocorrelation will reduce hedging risk as well as provide a more efficient hedge ratio estimate. However, if a hedge is to be held for one month or more, OLS will provide the lowest hedging risk, but not the most efficient hedge ratio estimate. The tradeoff between estimation efficiency (and the validity of hypothesis tests) and practical hedging risk needs to be more carefully considered in the hedging literature.

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