Margin Requirements and the Demand for Futures Contracts

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I. INTRODUCTION

Futures contracts are designed to allow hedgers to minimize or eliminate risk. Commodity futures contracts such as wheat futures contracts allow the hedger to hedge against uncertainty in the future price of the commodity. Currency futures contracts allow multinational corporations and investors in stocks of foreign companies to hedge foreign exchange risk. Interest rate futures contracts allow banks and other financial institutions to hedge against interest rate risk. Stock index futures contracts allow institutional investors such as pension funds to hedge the risk of their stock portfolios.

For instance, a pension fund that holds a particular stock portfolio and wishes to hedge uncertainty in the future value of the stock portfolio can sell futures contracts on the S & P 500 stock index. If the future value of the pension fund's stock portfolio and the price of the S & P 500 stock index are highly correlated, and if stock prices fall in the future, the loss suffered by the stock portfolio is offset by the gain made on the futures position. Depending upon the objective of the pension fund, the number of futures contracts sold could be chosen so as to be in line with the objective of either (1) eliminating all risk, (2) minimizing risk, or (3) minimizing risk while maximizing return.

When futures contracts are bought or sold, no initial investment changes hands. In this sense, a futures contract is merely an agreement to deliver the particular commodity in the future at a price agreed upon in the present. However, the exchanges in which futures contracts are traded specify that investors must pay an initial margin (or downpayment) for each futures contract traded. Brokers exact initial margin payments from their clients.

The authors are grateful for funding of this research through a grant from the Center for the Study of Futures Markets at Columbia University, New York, NY. The authors are also grateful for helpful comments from two anonymous referees.

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The Journal of Futures Markets, Vol. 11, No. 2, 213–237 (1991)
 CCC 0270-7314/91/020213-25$04.00

Previous research on margin requirements may be generally classified into (a) those that investigate the relationship between margin requirements and volatility in the stock market (e.g., Hardouvelis 1988, 1989); Salinger (1989); Schwert (1989); and Hsieh and Miller (1990)); (b) those that test the adequacy of margin requirements (Figlewski (1984) and Gay, Hunter, and Kolb (1986)); (c) those that theoretically address the cost of margin requirements and/or test the effects of changes in margin requirements upon some aspect of market activity (Hartzmark (1986) and Fishe and Goldberg (1986)).

Hardouvelis (1988) conducts empirical tests of the relationship between initial margin requirements and stock market volatility for the period 1934–present and concludes, from the results, that increasing margin requirements have the effect of reducing stock market volatility. Salinger (1989) criticizes this conclusion on the grounds that the relationship between margin requirements and volatility is not robust in the sense that it does not hold for all periods. Specifically, in the postwar period, 1946–present, no significant relationship is observed. The evidence of a relationship is due to the correlation between margin requirements and volatility observed in the 1930s. Schwert (1989) criticizes Hardouvelis' conclusions, based on the evidence of empirical tests he conducts, which show that high returns and low volatility in the stock market are followed by increases in margin requirements. He interprets this as evidence that the Federal Reserve responded to high levels of stock prices (associated with high returns and low volatility) by raising margin requirements. Schwert's results also indicate no significant effect of margin requirements upon subsequent stock returns and volatility. Hsieh and Miller (1990) claim that the relationship between margin requirements and volatility detected by Hardouvelis is due to serial correlation in the residuals of his regression (which they control for by using first differences) and due to a misspecification in his regression which uses both margin requirements and margin credit (correlated variables). When these are adjusted for, the relationship between margin requirements and volatility is not significant. Hardouvelis (1989) has responded to these criticisms by stating that these are criticisms of his tests on the relationship between margin requirements and *actual* volatility, whereas his research also includes tests of the relationship between *excess* volatility in the stock market and margin requirements, which is a more important issue.

Figlewski (1984) provides a theoretical model which determines the probability that a margin violation occurs at a particular time period. He uses the formula to analyze the degree of protection provided by initial margins in the stock market and the stock index futures market. Figlewski demonstrates that margin levels in futures markets provide adequate protection while those in stock markets are much higher. Gay, Hunter, and Kolb (1986) derive a theoretical formula similar to Figlewski's, which calculates the probability that the futures price moves by an amount equal to or greater than the margin during a particular time period. The formula is used to analyze margin-setting behavior in many commodity and financial futures markets. They find that the probability of futures price movements greater than or equal to the margin is the same across similar commodities such as, for example, wheat and oats. The margin-setting behavior is found to be consistent over time as well.

It is argued that initial margin requirements are costless because they can be satisfied by investing in T-bills which earn interest all the while that they are deposited with the broker (or exchange). The interest on the T-bill is available to the investor (or the broker). Thus, it is claimed that initial margin requirements have no opportunity cost attached to them.

However, Telser (1981) argues that initial margin requirements do have a cost attached, which is a liquidity cost. Once the margin is deposited with the broker, it is no longer available for other purposes. Thus, the investor is less liquid than before he/she bought or sold futures contracts.

Hartzmark (1986) argues that the liquidity cost is related to the probability that the trader (hedger or speculator) may be caught short of liquid assets. Therefore, the liquidity cost is different for every trader. Increasing margin increases the liquidity cost and reduces the demand for futures contracts. Hartzmark tests empirically if open interest and volume are significantly different in the two periods 15 days before and 15 days after changes in margin, for wheat, cattle, pork bellies, and bond futures contracts. He finds open interest and margin are significantly inversely related while there is no significant relationship between changes in margin and changes in volume.

Hartzmark also attempts to determine if there are changes in the composition of traders in the market. He tries to determine if an increase in margin requirements for speculators causes speculators to reduce their activity in the market. Hartzmark proxies speculator activity using an index he terms the noncommercial index. Unfortunately, as he himself remarks, it is difficult to identify target groups of speculators and hedgers. The index is constructed using the number of long contracts held by reporting commercial and noncommercial traders. Commercial traders are defined as those whose line of business is directly related to the underlying spot commodity. Since reports of traders are only required of traders exceeding reportable levels (set by the exchange), the index does not accurately measure speculative activity of nonreporting traders. Second, a commercial trader can carry hedging activity as well as speculative activity. Hartzmark's empirical tests also detect no significant effect of margin changes upon price volatility in the above futures contracts.

Fishe and Goldberg (1986) argue that the cost of the margin to a trader arises from the fact that the presence of the margin increases the costs associated with exercising the option of defaulting on the contract. They hypothesize that increasing margin should decrease the demand (long and short) for futures contracts. Their empirical tests involve initial margin changes for all contracts traded on the Chicago Board of Trade over the 1972–1978 period. A regression of changes in open interest and volume on changes in margin and other variables yield the predicted result that open interest is inversely related to margin changes but that volume is positively related to margin. They conjecture that the second result could arise due to increased activity if traders close out their futures position when margins are increased.

This article addresses the demand for futures contracts by *hedgers* and the consequences of increasing initial margin requirements upon this demand. If the cost of the margin is large enough to significantly lower the demand for futures contracts by hedgers, then the use of futures contracts by hedgers is threatened by any attempt to raise margin requirements.

This question merits attention. The stock market crash of October 19, 1987 has prompted critics to cite the lower margin requirements (approximately 15%) in stock index futures markets than in stock markets (50%) as making speculation in futures markets easier and thereby contributing to the crash. The chairman of the Securities and Exchange Commission (SEC), which oversees stock markets, has lobbied for veto control over the Commodities Futures Trading Commission (CFTC) which oversees futures markets. Reportedly, the chariman of the SEC and the Brady Commission which investigated the crash of October 19, are recommending that margin requirements in futures contracts be raised. This is not popular in the United States. However, in Canada, the Toronto Stock Exchange increased margin requirements on the TSE 35 futures contracts from 15% to 30% immediately after the October 19 crash.

This article theoretically examines the effect of initial margin requirements upon the demand for futures contracts by hedgers. The cost of the initial margin requirement to all traders is defined as the difference between the trader's borrowing and lending rates. Hartzmark reasons that once margin is deposited with the broker in the form of T-bills, it is unavailable to meet the trader's liquidity needs, if they arise. Therefore, the cost of the margin is a liquidity cost which depends on the probability of the emergence of world states in which a trader needs liquidity. It is argued here that even in those states of the world in which the trader does not require cash for liquidity purposes, the margin deposited with the broker is unavailable for investment in higher yielding assets. For the trader to retain the opportunity to invest in high-yielding assets he/she has to borrow. Therefore, the cost of the margin in these states is an opportunity cost. To be able to meet liquidity needs and to retain the opportunity to invest in higher yielding assets in the presence of the margin requirement, the hedger would have to borrow. However, the margin deposited with the broker earns the T-bill rate. Therefore, the cost of the margin in all states of the world is the difference between the trader's borrowing and lending (T-bill) rates.

When initial margin requirements are increased, the resulting increase in the cost of the margin causes the demand for futures contracts by hedgers to fall. This is modeled and tested empirically.

THE DEMAND FOR FUTURES CONTRACTS BY HEDGERS AND HEDGING EFFECTIVENESS OF FUTURES CONTRACTS

It is allowed that the hedger may either have a long position in the spot commodity or a short position in the spot commodity. If the hedger has a long position in the spot commodity, in general, the optimal futures position could either be a short position or a long position. (It is assumed that the hedger's objective is to maximize excess return and minimize risk.) If the hedger has a short position in the spot commodity, in general, the optimal futures position could either be a long position or a short position. Accordingly, four separate cases are analyzed: long spot–short futures, long spot–long futures, short spot–long futures, and short spot–short futures. The effect of an increase in the margin requirement upon the hedger's demand for futures contracts and upon hedging effectiveness is analyzed also.

Long Spot–Short Futures

Assume that a hedger holds C units of the spot commodity. This could be foreign currency or an agricultural commodity or shares of stock. The hedger is assumed to hedge the value of the spot commodity by selling N futures contracts in the commodity. If the spot position consists of foreign currency, the hedger sells futures contracts on the foreign currency. If the spot position consists of a stock portfolio, the hedger sells stock index futures contracts. It is assumed that the hedger is interested in maximizing the excess return per unit of risk of the hedged portfolio and accordingly chooses N, the number of futures contracts.

The return R_h from the hedged portfolio over the hedge period is given by:

$$R_h = \frac{C(P' - P + \text{div}) + NS\left(F - F' - \dfrac{M}{S} \cdot C_m\right)}{CP} \tag{1}$$

where R_h = return from the hedged portfolio over the hedge period; C = number of units of the spot commodity held (given); P' = price of the spot commodity at the end of the hedge period; P = price of the spot commodity at the beginning of the hedge period; div = any dividends received due to holding the spot commodity such as cash dividends, in the case of stock or interest, in the case of foreign currency; N = number of futures contracts in the hedged portfolio (constrained to be positive); S = size of each futures contract; F' = futures price of the futures contract at the end of the hedge period; F = futures price of the futures contract at the beginning of the hedge period; M = initial margin requirement per futures contract bought or sold (note that this is the value of the hedge margin for the contract); and C_m = cost per dollar of margin requirement. This could be a liquidity cost or an opportunity cost.

The cost per dollar of margin requirement is critical in determining the return from the hedged portfolio of the spot and futures instrument. It may be noted that once the margin (in whatever form) is deposited with the broker, it is no longer available for other purposes to meet liquidity needs or to retain the opportunity to invest in higher yielding assets even if the margin is deposited in the form of interest-earning T-bills. If the investor needs the funds deposited as margin to meet liquidity needs or to invest in other alternative investments, these funds would have to be borrowed. Therefore, the liquidity cost of the margin requirement is the investor's borrowing rate—the investor's lending rate.

Statman (1987) estimates the excess of the borrowing rate over the lending rate. The lending rate is proxied by the T-bill rate. The borrowing rate is proxied by the Call Money Rate which is the rate charged on loans to brokers on stock exchange collateral. Statman estimates that during 1985 the average difference between the Call Money Rate and the T-bill rate is approximately 2% on an annual basis. The margin cost C_m is defined, in this article, as the difference between the investor's borrowing rate (proxied by the average rate of interest on a broker's call loan) and the lending rate (proxied by the T-bill rate) over the period of the study.

It is shown that the magnitude of the margin cost C_m is critically important in determining if an increase in margin requirements will adversely affect the demand for futures contracts. The excess return per unit of risk θ_h of the hedged portfolio is given by

$$\theta_h = \frac{\bar{R}_h - i}{\sigma_h} \tag{2}$$

where $\bar{R}_h$ = expected return of the hedged portfolio, i = risk free rate, and σ_h = standard deviation of return of the hedged portfolio. Maximizing θ_h with respect to N, with the constraint that $N \geq 0$ (and C greater than 0) gives the optimum number of futures contracts N^* in the hedged portfolio. The first order conditions for a solution to the above problem (see Dano (1975), p. 13) is

$$\frac{\delta\theta_h}{\delta N} \le 0;\ N \ge 0;\ N \cdot \frac{\partial\theta_h}{\partial N} = 0 \tag{3}$$

The second-order condition for a solution to the above problem, which holds whether $N = 0$ or $N > 0$ is given by

$$\frac{\partial^2\theta_h}{\partial N^2} < 0 \tag{4}$$

A detailed derivation of the solution to the above using explicit Kuhn–Tucker analysis is given in Appendix 1.

Table I summarizes the possible solutions and their corresponding first- and second-order conditions. As can be seen, there is a range of parameter values at which $N = 0$. This study is more concerned with the case when $N > 0$, and the effect of raising margins upon this case. The analysis of this case follows.

It should be noted that in maximizing θ_h, the excess return per unit of risk or the Sharpe ratio, it is assumed implicitly that the hedger has a quadratic utility function or that return distributions are normal. A query may arise as to how the results may differ if the hedger has constant relative risk aversion with various possible values of relative risk aversion. An answer is provided in the work of Kroll, Levy, and Markowitz (1984). They compare the performance of optimal portfolios created by mean-variance maximization (with borrowing/lending at the risk-free rate allowed)

Table I
SOLUTIONS TO THE HEDGER'S PROBLEM AND IMPLIED FIRST- AND SECOND-ORDER CONDITIONS

$$r_s = \frac{P' - P + \text{div}}{P}$$

$$r_f = \left(F - F' - \frac{M}{S}C_m\right) \Big/ P$$

$\bar{r}_s$, $\bar{r}_f$ are expected values of r_s and r_f, σ_s and σ_f are standard deviations of r_s and r_f and ρ_{sf} is the correlation between r_s and r_f

Solution	Optimal Value of N N^*	Implied First-Order Condition	Implied Second-Order Condition
1. $N > 0; \frac{\partial\theta_h}{\partial N} = 0$	$\left[\frac{(\bar{r}_s - i)\,\sigma_{sf} - \bar{r}_f\,\sigma_s^2}{\bar{r}_f\,\sigma_{sf} - (\bar{r}_s - i)\,\sigma_f^2}\right]\frac{C}{S}$	$\rho_{sf}\frac{(\bar{r}_s - i)}{\sigma_s} < \frac{\bar{r}_f}{\sigma_f}$ and $\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{(\bar{r}_s - i)}{\sigma_s}$	$\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{(\bar{r}_s - i)}{\sigma_s}$
2. $N > 0; \frac{\partial\theta_h}{\partial N} = 0$	0	$\rho_{sf}\frac{(\bar{r}_s - i)}{\sigma_s} = \frac{\bar{r}_f}{\sigma_f}$	$\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{(\bar{r}_s - i)}{\sigma_s}$
3. $N > 0; \frac{\partial\theta_h}{\partial N} < 0$	0	$\rho_{sf}\frac{(\bar{r}_s - i)}{\sigma_s} > \frac{\bar{r}_f}{\sigma_f}$	$\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{(\bar{r}_s - i)}{\sigma_s}$

with that of optimal portfolios created by expected utility maximization (with borrowing/lending at the risk-free rate permitted) for many different utility functions, among them utility functions exhibiting constant relative risk aversion with various values of relative risk aversion. The approximation index

$$I = \frac{E^*U(\ ) - E_M U(\ )}{EU(\ ) - E_M U(\ )}$$

where $E^*\ U(\ )$ = expected utility of optimal portfolio from mean-variance maximization, $E\ U(\ )$ = expected utility of optimal portfolio from direct expected utility maximization, and $E_M\ U(\ )$ = expected utility of a "naive" portfolio with $1/M$ invested in each security where M represents the number of securities was used to measure the performance of mean-variance maximization relative to direct utility maximization. The approximation index is close to 1 (ranging from 0.973 to 0.998 for the utility functions with constant relative risk aversion). Since the returns data are skewed, Kroll, Levy, and Markowitz interpreted this as evidence that the results are not due to normality of data.

Therefore, building upon the results of Kroll, Levy, and Markowitz, the results here are expected to be robust, in spite of the fact that mean-variance maximization is used instead of direct utility optimization with various utility functions.

The solution to the case when $N > 0$ is

$$N^* = \left[\frac{(\bar{r}_s - i)\sigma_{sf} - \bar{r}_f \sigma_s^{\,2}}{\bar{r}_f \sigma_{sf} - (\bar{r}_s - i)\sigma_f^2}\right]\frac{C}{S} \tag{5}$$

where

$$r_s = (P' - P + \text{div})/P = \text{effective return from the spot position} \tag{6}$$

and

$$r_f = \left(F - F' - \frac{M}{S}C_m\right)\Big/ P = \text{effective return from the futures position,} \tag{7}$$

$\bar{r}_s$ and $\bar{r}_f$ are the expected values of r_s and r_f, σ_s, and σ_f are the standard deviations of r_s and r_f, ρ_{sf} is the correlation between r_s and r_f and σ_{sf} is the covariance between r_s and r_f.

To determine the effect of an increase in margin requirements upon the demand for futures contracts, eq. (5) is differentiated with respect to the margin requirement M.

$$\frac{\partial N^*}{\partial \mathrm{M}} = \left[\frac{\sigma_s^2\sigma_f^2(\bar{r}_s - i)(\rho_{sf}^2 - 1)}{(\bar{r}_f\sigma_{sf} - (\bar{r}_s - i)\sigma_f^2\,)^2}\right]\cdot\left(\frac{C_m}{P}\right)\cdot\frac{C}{S^2} \tag{8}$$

A detailed derivation of the above is provided in Appendix 2. The right hand side of eq. (8) is negative as long as the expected return from the spot position $\bar{r}_s$ is greater than the risk-free rate and $|\rho_{sf}| < 1$. The conclusion is that as margin requirements are increased, the demand for futures contracts by hedgers decreases. It is evident from eq. 8 that the cost per dollar of margin deposited, C_m, is critical in determining the reduction in the demand for futures contracts.

The hedging effectiveness of futures contracts may be proxied by the excess return per unit of risk of the portfolio of the spot position and the futures contract to that of the original spot position. The effect of increasing margin requirements upon the excess return per unit of risk of the optimal portfolio of the spot position and futures contracts may be analyzed. Figure 1 shows this graphically. The expected return of different portfolio positions are graphed against their standard deviation of return. S with coordinates $\bar{r}_s$ and σ_s denotes the spot position, with r_s given by eq. (6). F_1 with coordinates $\bar{r}_f$ and σ_f denotes the futures position with r_f given by eq. (7). O_1 represents the optimal portfolio of the spot, futures, and risk-free asset. When margin requirements are increased, the mean return from the futures position is reduced while its risk remains the same. F_2 represents the futures position under increased margin requirements. O_2 now represents the optimal portfolio of the spot, futures, and the risk-free asset under increased margin requirements. The difference between the slopes of iO_1 and iO_2 is a measure of the reduction in the excess return per unit of risk due to increased margin requirements.

This may be analyzed mathematically. The optimal value of θ_h, $\theta_h{}^*$ is determined by substituting N^* from eq. (5) for N in eq. (2). Differentiating $\theta_h{}^*$ with respect to M gives:

$$\frac{\partial \theta_h{}^*}{\partial (M)} = \frac{-N^* C_m}{\sigma_h^* CP} \tag{9}$$

where σ_h^* is the standard deviation of the optimal portfolio of the spot and futures. The value of eq. (9) will be negative if N^* is constrained to be positive.

Suming up, as the margin requirement is increased, the demand for futures contracts is reduced. The excess return per unit of risk of the optimal portfolio of spot and futures is reduced as well.

Long Spot–Long Futures Position

Since the hedger's objective is to maximize excess return per unit of risk, a situation could arise where a long futures position is optimal (see Howard and D'Antonio (1984). This solution is not the generally expected intuitive solution.

The return R_h from the hedged portfolio over the hedge period is given by:

$$R_h = \frac{C(P' - P + \text{div}) + NS(F' - F - \frac{M}{S} \cdot C_m)}{CP} \tag{10}$$

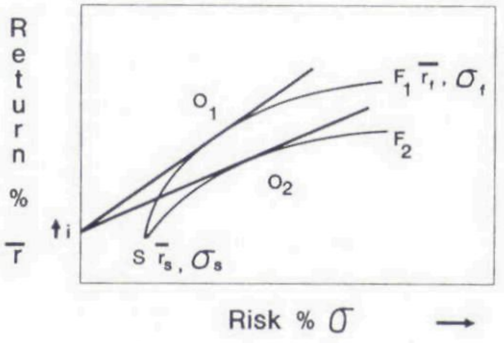

Figure 1
Margin Requirements and Hedging Effectiveness.

where all variables are defined as in the case of a long spot–short futures. The hedger's problem is to maximize

$$\theta_h = \frac{\bar{R}_h - i}{\sigma_h}$$

subject to the constraint that $N \geq 0$ (and $C > 0$).

The solutions, first-order and second-order conditions of Table I, also apply to this problem with one difference, namely, that r_f is defined as:

$$r_f = (F' - F - \frac{M}{S} \cdot C_m)/P \tag{11}$$

When $N > 0$, the effect of increase in margin upon the demand for futures contracts and the hedging effectiveness of futures contracts is given by eqs. (8) and (9), with r_f defined by eq. (11). As before, it may be deduced that the effect of increasing margin is to reduce the demand for futures contracts (as long as the expected return from the spot position $\bar{r}_s$ is greater than the risk-free rate i and $|\rho_{sf}| < 1$) and reduce hedging effectiveness (unconditionally).

Short Spot–Long Futures Position

The return R_h from the hedged portfolio is given by

$$R_h = \frac{C(P - P' - \text{div}) + NS(F' - F - \frac{M}{S} \cdot C_m)}{CP} \tag{12}$$

where all variables are as defined in the case of long spot–short futures. The hedger's problem is to maximize

$$\theta_h = \frac{\bar{R}_h - i}{\sigma_h}$$

subject to the constraint that $N \geq 0$ (and $C > 0$ (note that even though the hedger holds a *short* spot position, because of the way of defining the return from the short spot position which allows the use of the results of the case of long spot–short futures in a general way to fit all positions, C is effectively constrained to be > 0)). The solutions, first- and second-order conditions of Table I apply to this case also with the substitution that:

$$r_s = \frac{P - P' - \text{div}}{P} \tag{13}$$

$$r_f = \left(F' - F - \frac{M}{S} \cdot C_m\right) \Big/ P \tag{14}$$

substitute for previously defined values of r_s and r_f. When $N > 0$, the effect of increasing margin upon the demand for and hedging effectiveness of futures contracts is given by eqs. (8) and (9), with r_s and r_f defined by eqs. (13) and (14). As before, it may be deduced that an increase in margin requirements will reduce the demand for futures contracts, as long as the expected return from the spot position

$\bar{r}_s$ is greater than the risk-free rate i and $|\rho_{sf}| < 1$ and decrease hedging effectiveness unconditionally.

Short Spot–Short Futures Position

Again, since the hedger's objective is to maximize excess return per unit of risk, it could arise that a short futures position is optimal, although this is not the generally expected intuitive solution.

The return R_h from the hedged portfolio is given by:

$$R_h = \frac{C(P - P' - \text{div}) + NS\left(F - F' - \frac{M}{S} \cdot C_m\right)}{CP} \tag{15}$$

where all variables are as defined in the case of long spot–short futures. The hedger's problem is to maximize

$$\theta_h = \frac{\bar{R}_h - i}{\sigma_h}$$

subject to the constraint that $N \geq 0$ (and $C > 0$). The solutions, first- and second-order conditions of Table I apply to this case also with the substitution that r_s, as defined in eq. (13) and r_f as defined in eq. (7) substitute for previously defined values of r_s and r_f. When $N > 0$, the effect of increasing margin upon the demand for and hedging effectiveness of futures contracts is given by eqs. (8) and (9), with r_s and r_f defined by eqs. (13) and (7) respectively. As before, it may be deduced that an increase in margin requirements will reduce the demand for futures contracts as long as the expected return from the spot position $\bar{r}_s$ is greater than the risk-free rate, and $|\rho_{sf}| < 1$ and decrease hedging effectiveness unconditionally.

It should be noted that though based on *actual* historical data, the expected return from the spot position $\bar{r}_s$ may not *simultaneously* be greater than the risk-free rate i for a short spot position *and* a long spot position, in terms of the *expectations* of a hedger with a short spot position, the expected return $\bar{r}_s$ may be $> i$ while simultaneously in terms of the expectations of another hedger with a long spot position, the expected return $\bar{r}_s$ may be $> i$.

METHODOLOGY AND RESULTS OF EMPIRICAL TESTS

Data

Weekly data on the interest paid on brokers' call loans (high and low) for the years 1982–88 are from I. P. Sharpe Associates. Weekly data on the three-month T-bill rate for the years 1983–88 are also from I. P. Sharpe Associates. Daily data on the returns on the S&P 500 stock index (includes dividends and capital gains) are from the Center for Research in Security Prices daily index data tape. Daily data on the settlement price, trading volume, and open interest of the S&P 500 futures contract for the period May 3, 1982–June 30, 1988 are from the Centre for the Study of Futures Markets at Columbia University which originate from the International Monetary Market of the Chicago Mercantile Exchange. Data on margin requirements history on the S&P 500 futures contract are from the CME and presented in Table II. Note that over the period of the study, hedging margins are changed only once in 1982 and six times in 1987. Over a major portion of the period of the study (10/25/82 to 01/30/87) there is no change in hedging margin.

Table II
S & P 500 MARGIN HISTORY 1982–88

Date	Futures Margin $		Firm
	Speculators	Hedgers/Members	
04/21/82	6000 I	2500 I	1500 I
	2500 M	1500 M	1500 M
10/25/82	6000 I	3000 I	2500 I
	2500 M	2500 M	2500 M
01/30/87	10,000 I	5000 I	
	5000 M	5000 M	
10/19/87	10,000 I	7500 I	
	7500 M	7500 M	
10/22/87	15,000 I	10,000 I	
	10,000 M	10,000 M	
10/28/87	20,000 I	12,500 I	
	12,500 M	12,500 M	
10/29/87	20,000 I	15,000 I	
	15,000 M	15,000 M	
12/18/87	15,000 I	10,000 I	
	10,000 M	10,000 M	
03/08/88	18,000 I	10,000 I	
	10,000 M	10,000 M	
04/12/88	19,000 I	10,000 M	
	10,000 M	10,000 M	
07/12/88	20,000 I	10,000 I	
	10,000 M	10,000 M	
09/21/88	20,000 I	4000 I	
	4000 M	4000 M	

Note: I = initial margin requirements; M = maintenance margin requirements.
Source: Chicago Mercantile Exchange.

Methodology and Results

Estimates of Actual Values of Parameters. The average interest rate on brokers' call loans is calculated as the average of the high and low interest rates on brokers' call loans. The mean of the average interest rate on brokers' call loans and the three-month T-bill rate is calculated for each of the years 1983–88. The margin cost C_m per year is calculated as the difference between the mean average interest rate on brokers' call loans and the mean T-bill rate for the year. Table III shows the results. The margin cost varies from a low of 1.44% in 1983 to a high of 2.07% in 1987. The average margin cost for the years 1983–88 is 1.75%. The margin cost per day is obtained as $C_m = (1.0175)^{1/365} - 1$. This value of $C_m = 0.00475\%$ is used in subsequent calculations to determine the effect of an increase in margin upon the demand for futures contracts and hedging effectiveness. As shown in Table III, the mean T-bill rate is calculated to be 7.5% per year. This is converted into a daily rate to calculate the risk-free rate i as $i = ((1.075)^{1/365} - 1)\ .100$.

A time series of daily futures contract settlement prices is constructed for the period January 1982 through June 1988 by rolling over each maturing futures contract

Table III
CALCULATION OF THE MARGIN COST C_m/YEAR

Year	Mean of Average Interest Rate on Brokers' Call Loans %	Mean of the 3-month T-bill Interest Rate %	Margin Cost C_m/year %
1982	13.38	Not available	Not calculated
1983	10.06	8.62	1.44
1984	11.23	9.53	1.70
1985	9.03	7.47	1.56
1986	7.76	5.99	1.77
1987	7.85	5.78	2.07
1988	8.36	6.37	1.99

Mean margin cost C_m/year = 1.75%.
Mean T-bill rate i/year = 7.5%.

to the futures contract with the next closest maturity on the last day of trading of the maturing contract. Daily returns from the spot position and the futures position are calculated using the eqs. (6) and (7) for the long spot–short futures position, eqs. (6) and (11) for the long spot–long futures position, eqs. (13) and (14) for the short spot–long futures position, and eqs. (13) and (7) for the short spot–short futures position. Note that in eqs. (7), (11), and (14) the contract size S = \$500. The margin M is the appropriate hedging margin for the date for which the return is calculated. This information is provided in Table II.

Actual values of mean returns ($\bar{r}_s$ and $\bar{r}_f$), standard deviation of returns (σ_s and σ_f), and correlation between the returns (ρ_{sf}) on the spot and futures position are calculated for the years 1982–88 for each of the four cases: long spot–short futures, long spot–long futures, short spot–long futures, and short spot–short futures. The mean daily trading volume ($\bar{N}$) and open interest ($\bar{O}$) in the S & P 500 futures contract and the standard deviations of the daily trading volume (σ_N) and open interest (σ_O) in the S & P 500 futures contract are calculated for each of the years 1982–88. These results are shown in Table IV. All returns are on a daily basis.

Note from Table IV that the mean returns from the short spot position is the negative of the mean return from the long spot position and the standard deviations of returns are the same under the long spot and short spot position. This is consistent with eq. (6) (return under the long spot position) and eq. (13) (return under the short spot position). Note that the mean return under the long futures position is positive, and that under the short futures position it is negative. The absolute values of the mean return under the long futures and short futures position have different values. This is consistent with eq. (7) (return under the short futures position) and eq. (11) (return under the long futures position). The difference in the values, however, is very small. This is due to the small magnitude of the margin cost per dollar of margin on a daily basis (0.00475%). Note also that the absolute values of the correlation between the returns on the spot and futures positions are the same for the long spot–short futures, long spot–long futures, short spot–short futures, and short spot–long futures positions. This is consistent with the definition of the spot return and futures returns given by the pairs of eqs. (6) and (7), (6) and (11), (13) and (14), and 13 and 7 for the above four cases, respectively. As expected, the correlations are positive for the long spot–long futures and short spot–short futures cases and negative for the other two cases.

The general conclusions, illustrated in Table IV, are that the magnitude of the absolute value of the excess return from the spot position $\bar{r}_s - i$ is slightly higher than that from the futures position, the standard deviation of the return from the futures position is higher than that from the spot position, and the absolute values of the correlations between the spot and futures position is close to 1.

As far as the figures on trading volume and open interest are concerned, mean trading volume is higher than mean open interest for the years 1982–84 and vice versa in the years 1985–88. The standard deviation of trading volume is higher than that of open interest for the years 1982–85 and vice versa in the years 1986–88. These figures are but an imperfect proxy for hedger activity in the S & P 500 stock index futures markets. Direct accurate measures of hedger activity in the stock index futures markets are impossible to obtain at this time because reports of trades are filed with the Market Surveillance Department of the Chicago Mercantile Exchange only if trades exceed defined reportable levels set by the exchange or if a hedging trader wishes to obtain permission to exceed speculative position limits set by the exchange. (See *Chicago Mercantile Exchange Clearing House Operations Manual,* January 1989 and *Rules of the Chicago Mercantile Exchange,* March 1989.) In addition, a hedging trader may simultaneously take part in speculative activity.

The returns on the futures position r_f used to calculate the mean return on the long futures and short futures position presented in Table IV are determined using actual values of the margin in use in the appropriate period as given in Table II. In addition, mean values of the return on the futures position $\bar{r}_f$ are calculated assuming that the margin is 50% of the contract value. This value of margin is chosen to coincide with the stock market's margin requirements of 50%. The mean return $\bar{r}_f$ is lower by 0.002%, on a daily basis, for all years for the short futures position and for five years (1982, 1983, 1986, 1987, and 1988) for the long futures position. It is lower by 0.003% for two years (1984, 1985) for the long futures position.

The parameters estimated in Table IV could have been used to determine the sensitivity of the demand for futures contracts to increase in initial margin requirement to 50% of the contract value. This is not done because of the instability in the parameter estimates and because the in-sample parameter estimates of the mean return on the short spot position $\bar{r}_s$ are negative for all years. It may be argued that a hedger may not hold a short spot position if the expectation is that the return from the short spot position will be negative. It is reasonable that in terms of expectations at least, the mean return from the spot position should be positive. Therefore, some reasonable values of the parameters are assumed, the decrease in the demand for futures contracts by hedgers on an increase in initial margin requirement to 50% of the contract value are determined, and, the sensitivity of the fall in demand to a change in the different parameters is calculated.

Effect of the Change in Margin Requirement Upon the Demand for Futures Contracts by Hedgers. Some annual values of the relevant parameters are assigned as follows: $\bar{r}_s = 16.5\%$, $\sigma_s = 10\%$, $\bar{r}_f = -8\%$, $\sigma_f = 15\%$, $\rho_{sf} = -0.9$, and $i = 7.5\%$. These values are referred to hereinafter as the base case values. The above values have the characteristics observed in the actual values computed in Table IV, i.e., the absolute value of $\bar{r}_s - i$ is approximately equal to the absolute value of $\bar{r}_f$; $\sigma_s < \sigma_f$ and $|\rho_{sf}|$ close to 1. The above values have the desirable property that the

Table IV
ACTUAL VALUES OF PARAMETERS FROM THE S & P 500 STOCK INDEX SPOT AND FUTURES MARKETS ON A DAILY BASIS

	Spot Position				Futures Position								All S & P 500 Index Futures			
	Long		Short		Long		Short		Correlation Between Spot and Futures ρ_{sf}				Trading Volume (No. of Contracts)		Open Interest (No. of Contracts)	
Year/ Number of days of data	Mean $\bar{r}_s$ %	Standard Deviation σ_s %	Mean $\bar{r}_f$ %	Standard Deviation σ_f %	Mean $\bar{r}_f$ %	Standard Deviation σ_f %	Mean $\bar{r}_f$ %	Standard Deviation σ_f %	Long Spot–Short Futures	Long Spot–Long Futures	Short Spot–Long Futures	Short Spot–Short Futures	Mean $\bar{N}$	Standard Deviation σ_N	Mean $\bar{O}$	Standard Deviation σ_O
1982/169	0.117	1.238	−0.117	1.238	0.1029	1.6571	−0.1033	1.6571	−0.8761	0.8761	−0.8761	0.8761	17,386	7965	12,076	3478
1983/252	0.066	0.841	−0.066	0.841	0.0704	1.0094	−0.0708	1.0094	−0.9053	0.9053	−0.9053	0.9053	31,130	9413	24,071	6548
1984/249	0.007	0.807	−0.007	0.807	0.0307	1.0624	−0.0310	1.0624	−0.8324	0.8324	−0.8324	0.8324	47,459	15,554	35,078	10,841
1985/252	0.095	0.640	−0.095	0.640	0.0716	0.8168	−0.0719	0.8168	−0.8526	0.8526	−0.8526	0.8526	57,540	18,847	60,944	14,585
1986/250	0.055	0.928	−0.055	0.928	0.0579	1.0718	−0.0582	1.0718	−0.9421	0.9421	−0.9421	0.9421	73,815	24,641	98,643	33,511
1987/253	0.030	2.025	−0.030	2.025	0.0385	2.6872	−0.0389	2.6872	−0.9621	0.9619	−0.9619	0.9619	72,937	27,346	120,764	26,310
1988/126	0.089	1.298	−0.089	1.298	0.0972	1.4657	−0.0979	1.4657	−0.9608	0.9608	−0.9608	0.9608	47,207	14,480	14,733	23,188

second-order condition for a maximum

$$\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{\bar{r}_s - i}{\sigma_s}$$

is satisfied and the first-order conditions for a non-negative solution to N, given by:

$$\rho_{sf}\frac{(\bar{r}_s - i)}{\sigma_s} < \frac{\bar{r}_f}{\sigma_f} \quad \textit{and}$$

$$\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{\bar{r}_s - i}{\sigma_s}$$

are satisfied. The optimal values of the number of futures contracts N^* and hedging effectiveness θ_h is then calculated using eq. (5) and eq. (2). This gives a result of $N^* = 0.4392\,\frac{C}{S}$ and $\theta_h^* = 1.10$.

The optimal values of the number of futures contracts $N^{*\prime}$ and the hedging effectiveness $\theta_h^{*\prime}$ when margin requirements are increased to 50% of the contract value is then determined. This is determined by assuming that the parameters $\bar{r}_s$, σ_s, σ_f, ρ_{sf}, and i have the same values as in the base case. The mean return on the futures position $\bar{r}_f$ is assumed to be reduced by 1% per year to a value $\bar{r}_f' = -9\%$. This is based on the previously observed effect of increasing margin requirements to 50% of the contract value upon the actual values of $\bar{r}_f$. $\bar{r}_f$ reduced by 0.002% to 0.003% on a daily basis or by 0.73% $((1.00002^{365} - 1) \times 100)$ to 1.10% $((1.00003^{365} - 1) \times 100)$. This gives a value of $N^{*\prime} = 0.3889\,\frac{C}{S}$ and $\theta_h^{*\prime} = 1.02$.

Therefore, if margins increase to 50% of contract value, the *decrease* in the demand for futures contracts by hedgers would be $(N^{*\prime} - N^*)/N^* = 11.45\%$ and the *decrease* in hedging effectiveness would be $(\theta_h^{*\prime} - \theta_h^*)/\theta_h^* = 7.27\%$. The effect of increasing margin is calculated as an explicit discrete change in the demand for futures contracts and in hedging effectiveness, rather than the continuous change given by eqs. (8) and (9). This is because of the requirement that to apply eqs. (8) and (9) to calculate the effect of a change in margin requirements, $\frac{\partial^2 N^*}{\partial M^2}$ and $\frac{\partial^2 \theta_h^*}{\partial M^2}$ should be equal to 0 at the status quo which may not hold.

Sensitivity of the Decrease in Demand for Futures Contracts by Hedgers Upon the Parameters of the Spot and Futures Markets and T-Bill Rates. Each of the parameters $\bar{r}_s$, σ_s, σ_f, ρ_{sf}, and i are increased or decreased by 1%, 5%, and 10% from the base case values while keeping all other parameter values the same as under the base case. The optimal values of the number of futures contracts N^* and the hedging effectiveness θ_h^* under current margin levels are computed using these values and $\bar{r}_f$ = expected return from the futures position = -8%. The optimal values of the number of futures contracts $N^{*\prime}$ and the hedging effectiveness $\theta_h^{*\prime}$ under margin levels equal to 50% of the contract value are calculated by using the expected return under the futures position $\bar{r}_f' = -9\%$. The percentage changes in the optimal number of futures contracts and hedging effectiveness is calculated. The results are shown in Table V.

As expected, the greater the magnitude of the change from the base case, the greater the difference in the decrease in demand for futures contracts and hedging effectiveness from those under the base case. However, decrease in hedging effectiveness is not as sensitive to the shifts in the parameters as the decrease in the demand for futures contracts. Hedgers, according to this model, could therefore

Table V

SENSITIVITY OF DECREASE IN OPTIMAL NUMBER OF FUTURES CONTRACTS AND HEDGING EFFECTIVENESS TO ESTIMATES OF THE PARAMETERS IN THE S & P 500 STOCK INDEX SPOT AND FUTURES MARKETS AND T-BILL RATES

				Optimal Number of Futures Contracts*			Optimal Value of Hedging Effectiveness		
Magnitude of Change from Base Case	Parameter Changed	Increase I or Decrease D in Absolute Value	Value of Parameter	Under Current Margin N^*	Under Margin of 50% of Contract Value $N^{*\prime}$	Decrease %	Under Current Margin θ_R^*	Under Margin of 50% of Contract Value $\theta_R^{*\prime}$	Decrease %
1%	$\bar{r}_s$	I	16.665	0.4453	0.3981	10.60	1.13	1.05	7.08
		D	16.335	0.4326	0.3787	12.46	1.07	0.99	7.48
	σ_s	I	10.1	0.4401	0.3874	11.97	1.08	1.00	7.41
		D	9.9	0.4465	0.3988	10.68	1.12	1.04	7.14
	σ_f	I	15.15	0.4382	0.3902	10.95	1.11	1.03	7.21
		D	14.85	0.4400	0.3874	11.95	1.09	1.01	7.34
	ρ_{sf}	I	−0.909	0.4573	0.4100	10.34	1.13	1.04	7.96
		D	−0.891	0.4215	0.3684	12.60	1.08	1.00	7.41
	i	I	7.575	0.4363	0.3844	11.89	1.09	1.01	7.34
		D	7.425	0.4420	0.3932	11.04	1.12	1.03	8.04
5%	$\bar{r}_s$	I	17.325	0.4656	0.4282	8.03	1.27	1.18	7.09
		D	15.675	0.3999	0.3261	18.45	0.94	0.86	8.51
	σ_s	I	10.5	0.4420	0.3784	14.39	1.02	0.94	7.84
		D	9.5	0.4327	0.3927	9.24	1.20	1.11	7.50

	σ_f	I	15.75	0.4329	0.3928	9.26	1.14	1.05	7.89
		D	14.25	0.4419	0.3775	14.57	1.06	0.99	6.60
	ρ_{sf}	I	−0.945	0.5340	0.5015	6.09	1.32	1.18	10.61
		D	−0.855	0.5320	0.4380	17.67	0.91	0.91	0
	i	I	7.875	0.4242	0.3655	13.84	1.03	0.95	7.78
		D	7.125	0.4524	0.4088	9.64	1.18	1.09	7.63
10%	$\bar{r}_s$	I	18.15	0.4846	0.4552	6.06	1.44	1.35	6.25
		D	14.85	0.3352	0.2013	37.26	0.79	0.75	5.06
	σ_s	I	11	0.4404	0.3595	18.37	0.94	0.88	6.38
		D	9	0.4231	0.3913	7.52	1.31	1.21	7.63
	σ_f	I	16.5	0.4251	0.3920	7.78	1.17	1.08	7.69
		D	13.5	0.4391	0.3538	19.43	1.03	0.96	6.80
	ρ_{sf}	I	−0.99	0.6410	0.6339	1.11	2.69	2.25	16.36
		D	−0.81	0.2788	0.2077	25.50	0.96	0.93	3.12
	i	I	8.25	0.4042	0.3333	17.54	0.95	0.89	6.32
		D	6.75	0.4636	0.4253	8.26	1.25	1.16	7.20

Base case values: $\bar{r}_s$ = expected return from the spot position, 16.5%; σ_s = standard deviation of return from the spot position, 10%; $\bar{r}_f$ = expected return from the futures position under current margin levels, −8%; $\bar{r}_f'$ = expected return from the futures position under margin level equal to 50% of the contract value, −9%; σ_f = standard deviation of return from the futures position, 15%; ρ_{sf} = correlation between the return on the spot and futures position, −0.9; i = risk-free rate, 7.5%.

Decrease in demand for futures contracts under base case = 11.45%; decrease in hedging effectiveness under base case = 7.27%.

*Optimal number of futures contracts N^* (or $N^{*\prime}$) = figure in the column * (C/S) where C = number of units of the spot position and S = size of each futures contract.

obtain not very different measures of hedging effectiveness by suitably reducing the number of futures contracts in their portfolios.

The decrease in demand for futures contracts and hedging effectiveness are not symmetric, i.e., an increase in a parameter by 1% does not give rise to the same decrease in demand for futures contracts and hedging effectiveness upon increase in margin levels to 50% of the contract value as does the decrease in the parameter. This is more clearly evident at 5% and 10% shifts in the parameters from the base case. Table V shows that when the expected return on the spot position $\bar{r}_s$ is low, when its volatility σ_s is high, when the volatility of the futures position is low, the absolute value of the correlation between the spot and futures position $|\rho_{sf}|$ is low, when the risk-free rate i is high, the decrease in the demand for futures contracts with an increase in margin to 50% of contract levels is higher than in the other cases. In addition, the decrease in demand for futures contracts to 50% of contract levels is most sensitive to decreases in the expected return on the spot position $\bar{r}_s$ and to decreases in the correlation between the spot and the futures position. The magnitudes of the sensitivity of the decrease in demand for futures contracts to the increase in the volatility of the spot market, the decrease in volatility of the futures market and the increase in the risk-free rate are approximately equal.

The above analysis of the effect of increasing margin requirements upon the demand for futures contracts and hedging effectiveness are applicable to a hedger wishing to hedge a long spot position or to a hedger wishing to hedge a short spot position whose expectations of the expected return on the spot position is described by the base case value $\bar{r}_s = 16.5\%$ (positive value). It is assumed that the hedger wishes to hedge a long spot position with a short futures position ($\bar{r}_f$ and ρ_{sf} are negative) or that the hedger wishes to hedge a short spot position with a long futures position ($\bar{r}_f$ and ρ_{sf} are negative). (The long spot–long futures and short spot–short futures are omitted from the empirical analysis of sensitivities since a preliminary analysis with the actual values of the parameters of Table IV indicates that these would result in the second-order condition for a maximum being unsatisfied or in values of $N^* = 0$.)

CONCLUSIONS

Futures contracts are important instruments which allow hedgers to manage risk. The initial margin paid by an investor in futures contracts poses a cost to the investor. The theoretical concept of the cost and how it may be measured are explained and empirically measured. The results indicate that the cost of the margin averages 1.75%/year. The effect of an increase in initial margin requirement upon the demand for futures contracts by hedgers and the hedging effectiveness of futures contracts are examined theoretically. The impact of an increase in margin requirements is shown to lower the demand for futures contracts by hedgers and the hedging effectiveness of futures contracts.

The effect of an increase in margin requirements upon the demand for the S & P 500 spot index futures contract by hedgers is simulated. The results indicate that the demand for the S & P 500 spot index futures contracts drop substantially when margin requirements are increased to be compatible with the stock market. The drop in the demand for futures contracts is also higher, the lower the expected return on the spot market and the lower the absolute value of the correlation between the spot and futures markets. The drop in the demand has approximately the same sensitivity to increases in the volatility of the spot market, decreases in the volatility of the futures market, and increases in the T-bill rate. The results are significant, since this reduction in demand would cause a decrease in the liquidity of these markets, which could further increase costs to market participants and endanger the continuing use of these markets by hedgers.

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Appendix 1

Following is the solution to the hedger's problem of maximizing expected excess return per unit of risk.

LONG SPOT–SHORT FUTURES POSITION

The return from the hedged portfolio R_h is given by:

$$R_h = \frac{C(P' - P + \text{div}) + NS\left(F - F' - \dfrac{M}{S} \cdot C_m\right)}{CP} \tag{1.1}$$

where all terms are as defined earlier. The hedger's problem is to maximize θ_h with respect to N, subject to the constraint that $N \geq 0$, (and $C > 0$) where

$$\theta_h = \frac{\overline{R}_h - i}{\sigma_h}$$

where R_h = expected return from the hedged portfolio, i = risk-free rate, σ_h = standard deviation of return of the hedged portfolio. $\overline{R}_h$ and σ_h may be written as

$$\overline{R}_h = \bar{r}_s + \frac{NS}{C} \cdot \bar{r}_f \tag{1.2}$$

$$\sigma_h^2 = \sigma_s^2 + \left(\frac{NS}{C}\right)^2 \sigma_f^2 + 2\frac{NS}{C} \rho_{sf} \sigma_s \sigma_f \tag{1.3}$$

where

$$r_s = \frac{P' - P + \text{div}}{P} = \text{effective return from the long spot position} \tag{1.4}$$

$$r_f = \frac{F - F' - \dfrac{M}{S} \cdot C_m}{P} = \text{effective return from the short futures position} \tag{1.5}$$

and $\bar{r}_s$ and $\bar{r}_f$ are the expected values of r_s and r_f, σ_s, and σ_f are the standard deviations of r_s and r_f and ρ_{sf} is the correlation between r_s and r_f. Note that in the maximization problem, N is constrained to be ≥ 0, because of the way in which r_f is defined. r_f is the return on a short futures position which effectively takes into account the cost of the initial margin deposited to fulfill the margin requirement.

Using Kuhn–Tucker analysis, the first-order conditions for a maximum are:

$$N \geq 0; \quad \frac{\partial \theta_h}{\partial N} \leq 0; \quad N \cdot \frac{\partial \theta_h}{\partial N} = 0 \tag{1.6}$$

(See (Dan (1975), p. 13).) The second-order conditions for a maximum are:

$$\frac{\partial^2 \theta_h}{\partial N^2} < 0 \tag{1.7}$$

$$\theta_h = \frac{\bar{r}_s + \dfrac{NS}{C} \cdot \bar{r}_f - i}{\left[\sigma_s^2 + \left(\dfrac{NS}{C}\right)^2 \sigma_f^2 + 2\dfrac{NS}{C} \rho_{sf} \sigma_s \sigma_f\right]^{1/2}} \tag{1.8}$$

$$\frac{\partial \theta_h}{\partial N} = \frac{\bar{r}_f \dfrac{S}{C}}{V^{1/2}} - \frac{1}{2} \cdot \frac{\left(\bar{r}_s + \dfrac{NS}{C} \bar{r}_f - i\right)}{V^{3/2}} \cdot \frac{\partial V}{\partial N} \tag{1.9}$$

where

$$V = \sigma_s^2 + \left(\frac{NS}{C}\right)^2 \sigma_f^2 + 2\frac{NS}{C}\rho_{sf}\sigma_s\sigma_f$$

$$\begin{aligned}\frac{\partial\theta_h}{\partial N} = \Bigg[&\bar{r}_f\frac{S}{C}\left(\sigma_s^2 + \left(\frac{NS}{C}\right)^2\sigma_f^2 + 2\frac{NS}{C}\rho_{sf}\sigma_s\sigma_f\right) \\ &- \left(\bar{r}_s + \frac{NS}{C}\bar{r}_f - i\right)\left(\frac{NS^2}{C^2}\sigma_f^2 + \frac{S}{C}\rho_{sf}\sigma_s\sigma_f\right)\Bigg]\Bigg/ V^{3/2}\end{aligned} \tag{1.10}$$

$$\frac{\partial\theta_h}{\partial N} = \left[\frac{\frac{\bar{r}_f}{C}S\sigma_s^2 - \frac{(\bar{r}_s - i)\,\rho_{sf}\sigma_s\sigma_f S}{C} + \frac{NS^2}{C^2}(\bar{r}_f\rho_{sf}\sigma_s\sigma_f - (\bar{r}_s - i)\,\sigma_f^2)}{V^{3/2}}\right] \tag{1.11}$$

$$\begin{aligned}\frac{\partial^2\theta_h}{\partial N^2} = &\frac{\frac{S^2}{C^2}(\bar{r}_f\rho_{sf}\sigma_s\sigma_f - (\bar{r}_s - i)\sigma_f^2)}{V^{3/2}} \\ &- \frac{3}{2}\cdot\left[\frac{\bar{r}_f}{C}S\sigma_s^2 - \frac{(\bar{r}_s - i)\rho_{sf}\sigma_s\sigma_f S}{C} + \frac{NS^2}{C^2}(\bar{r}_f\rho_{sf}\sigma_s\sigma_f - (\bar{r}_s - i)\sigma_f^2)\right] \\ &\cdot\frac{1}{V^{5/2}}\cdot\frac{\partial V}{\partial N}\end{aligned} \tag{1.12}$$

$$\begin{aligned}\frac{\partial^2\theta_h}{\partial N^2} = \Bigg[&\frac{S^2}{C^2}(\bar{r}_f\rho_{sf}\sigma_s\sigma_f - (\bar{r}_s - i)\sigma_f^2)\left(\sigma_s^2 + \left(\frac{NS^2}{C}\right)\sigma_f^2 + 2\,\frac{NS}{C}\cdot\rho_{sf}\sigma_s\sigma_f\right) \\ &- \frac{3}{2}\left[\frac{\bar{r}_f\sigma_s^2 S}{C} - \frac{(\bar{r}_s - i)\rho_{sf}\sigma_s\sigma_f S}{C} + \frac{NS^2}{C^2}(\bar{r}_f\rho_{sf}\sigma_s\sigma_f - (\bar{r}_s - i)\sigma_f^2)\right] \\ &\cdot\left[2\,\frac{NS^2}{C^2}\,\sigma_f^2 + 2\,\frac{S}{C}\cdot\rho_{sf}\sigma_s\,\sigma_f\right]\Bigg]\Bigg/ V^{5/2}\end{aligned} \tag{1.13}$$

Three solutions to eqs. (1.6) and (1.7) are possible.
(1) *Solution 1.* The first-order conditions are $N > 0$, $\partial\theta_h/\partial N = 0$. The second-order condition is

$$\frac{\partial^2\theta_h}{\partial N^2} < 0.$$

(2) *Solution 2.* The first-order conditions are $N = 0$, $\partial\theta_h/\partial N = 0$. The second-order condition is

$$\frac{\partial^2\theta_h}{\partial N^2} < 0.$$

(3) *Solution 3.* The first-order conditions are $N = 0$, $\partial\theta_h/\partial N < 0$. The second-order condition is

$$\frac{\partial^2\theta_h}{\partial N^2} < 0.$$

Solution 1

Substituting the first-order conditions of solution 1 in eq. (1.11), we obtain:

$$\frac{\partial \theta_h}{\partial N} = 0$$

$$N = \left[\frac{(\bar{r}_s - i)\rho_{sf}\sigma_s\sigma_f - \bar{r}_f\sigma_s^2}{\bar{r}_f\rho_{sf}\sigma_s\sigma_f - (\bar{r}_s - i)\sigma_f^2}\right]\frac{C}{S} \tag{1.14}$$

Substituting eq. (1.14) into eq. (1.13) we obtain (note that this causes the second term in square brackets in eq. (1.13) to equal 0)

$$\frac{\partial^2 \theta_h}{\partial N^2} = \frac{S^2}{C^2}\left(\bar{r}_f\rho_{sf}\sigma_s\sigma_f - (\bar{r}_s - i)\ \sigma_f^2\right)/V^{3/2} \tag{1.15}$$

The second-order condition $\partial^2\theta_h/\partial N^2 < 0$ implies the condition

$$\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{(\bar{r}_s - i)}{\sigma_s} \tag{1.16}$$

The condition $N > 0$ can be satisfied if both the numerator *and* denominator of the right hand side of eq. (1.14) are greater than 0 or both the numerator *and* denominator of the right hand side of eq. (1.14) are less than 0. However, the condition that the denominator of eq. (1.14) is greater than 0 conflicts with the second-order condition for a maximum 1.16. Therefore, the only acceptable first-order condition for solution 1 is that both the numerator and demoninator of eq. (1.14) be less than 0. This implies

$$\rho_{sf}\frac{(\bar{r}_s - i)}{\sigma_s} < \frac{\bar{r}_f}{\sigma_f}$$

and

$$\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{\bar{r}_s - i}{\sigma_s} \tag{1.17}$$

Solution 2

Substituting the first-order conditions of solution 2 into eq. (1.11), we obtain

$$\frac{\partial \theta_h}{\partial N} = \frac{\dfrac{\bar{r}_f S\sigma_s^2}{C} - \dfrac{(\bar{r}_s - i)\rho_{sf}\sigma_s\sigma_f S}{C}}{V^{3/2}} = 0 \tag{1.18}$$

This implies

$$\rho_{sf}\frac{(\bar{r}_s - i)}{\sigma_s} = \frac{\bar{r}_f}{\sigma_f} \tag{1.19}$$

Substituting $N = 0$ in eq. (1.13) implies

$$\frac{\partial^2 \theta_h}{\partial N^2} = \left[\frac{S^2}{C^2}\left(\bar{r}_f\rho_{sf}\sigma_s\sigma_f - (\bar{r}_s - i)\sigma_f^2\right)\sigma_s^2 \right.$$
$$\left. -3\left[\frac{\bar{r}_f\sigma_s^2 S}{C} - \frac{(\bar{r}_s - i)\rho_{sf}\sigma_s\sigma_f S}{C}\right]\frac{S}{C}\rho_{sf}\sigma_s\sigma_f\right] \Big/ V^{5/2}$$

$$\frac{\partial^2\theta_h}{\partial N^2} = \left[3(\bar{r}_s - i)\rho_{sf}^2\sigma_s^2\sigma_f^2\frac{S^2}{C^2} - (\bar{r}_s - i)\sigma_s^2\sigma_f^2\frac{S^2}{C^2} - 2\bar{r}_f\rho_{sf}\sigma_s^3\sigma_f\right] \Big/ V^{5/2}$$

$$\frac{\partial^2\theta_h}{\partial N^2} = \frac{S^2}{C^2}\sigma_s^3\sigma_f^2\left[3\rho_{sf}^2\frac{(\bar{r}_s - i)}{\sigma_s} - \frac{(\bar{r}_s - i)}{\sigma_s} - 2\frac{\bar{r}_f}{\sigma_f}\rho_{sf}\right] \Big/ V^{5/2}$$

Noting eq. (1.19), this implies

$$\frac{\partial^2\theta_h}{\partial N^2} = \frac{S^2}{C^2}\frac{(\bar{r}_f\rho_{sf}\sigma_s\sigma_f - (\bar{r}_s - i)\sigma_f^2)}{V^{3/2}}$$

The second-order condition $(\partial^2\theta_h/\partial N^2) < 0$ implies

$$\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{(\bar{r}_s - i)}{\sigma_s} \tag{1.20}$$

Solution 3

Substituting the first-order conditions of solution 3 into eq. (1.11):

$$\frac{\partial\theta_h}{\partial N} = \frac{\dfrac{\bar{r}_f S\sigma_s^2}{C} - \dfrac{(\bar{r}_s - i)\rho_{sf}\sigma_s\sigma_f S}{C}}{V^{3/2}} < 0 \tag{1.21}$$

This implies

$$\rho_{sf}\frac{(\bar{r}_s - i)}{\sigma_s} > \frac{\bar{r}_f}{\sigma_f} \tag{1.22}$$

Substituting $N = 0$ into eq. (1.13), this implies

$$\frac{\partial^2\theta_h}{\partial N^2} = \left[\frac{S^2}{C^2}(\bar{r}_f\rho_{sf}\sigma_s\sigma_f - (\bar{r}_s - i)\sigma_f^2)\sigma_s^2 - 3\left[\frac{\bar{r}_f\sigma_s^2 S}{C} - \frac{(r_s - i)\rho_{sf}\sigma_s\sigma_f S}{C}\right]\frac{S}{C}\rho_{sf}\sigma_s\sigma_f\right] \Big/ V^{5/2}$$

$$\frac{\partial^2\theta_h}{\partial N^2} = \left[3(\bar{r}_s - i)\rho_{sf}^2\sigma_s^2\sigma_f^2\frac{S^2}{C^2} - (r_s - i)\sigma_s^2\sigma_f^2\frac{S^2}{C^2} - 2\bar{r}_f\rho_{sf}\sigma_s^3\sigma_f\right] \Big/ V^{5/2}$$

$$\frac{\partial^2\theta_h}{\partial N^2} = \frac{S^2}{C^2}\sigma_s^3\sigma_f^2\left[3\rho_{sf}^2\frac{(\bar{r}_s - i)}{\sigma_s} - \frac{(\bar{r}_s - i)}{\sigma_s} - 2\frac{\bar{r}_f}{\sigma_f}\rho_{sf}\right] \Big/ V^{5/2}$$

Substituting $\bar{r}_f/\sigma_f$ in place of $\rho_{sf}(\bar{r}_s - i)/\sigma_s$ and noting the implication of eq. (1.22) the second-order condition $\partial^2\theta_h/\partial N^2 < 0$, this implies

$$3\rho_{sf}\frac{\bar{r}_f}{\sigma_f} - \frac{(\bar{r}_s - i)}{\sigma_s} - 2\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < 0$$

$$\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{(\bar{r}_s - i)}{\sigma_s} \tag{1.23}$$

Summing up the solutions and their first- and second-order conditions:
Solution 1:

$$N = \left[\frac{(\bar{r}_s - i)\rho_{sf}\sigma_s\sigma_f - \bar{r}_f\sigma_s^2}{\bar{r}_f\rho_{sf}\sigma_s\sigma_f - (\bar{r}_s - i)\sigma_f^2}\right]\frac{C}{S}$$

The first-order conditions are

$$\rho_{sf}\frac{(\bar{r}_s - i)}{\sigma_s} < \frac{\bar{r}_f}{\sigma_f} \quad \text{and} \quad \rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{(\bar{r}_s - i)}{\sigma_s}$$

The second-order condition is

$$\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{(\bar{r}_s - i)}{\sigma_s}$$

Solution 2:

$$N = 0$$

The first-order condition is

$$\rho_{sf}\frac{(\bar{r}_s - i)}{\sigma_s} = \frac{\bar{r}_f}{\sigma_f}$$

The second-order condition is

$$\rho_{sf}\frac{\bar{r}_f}{\sigma_f} < \frac{(\bar{r}_s - i)}{\sigma_s}$$

Solution 3:

$$N = 0$$

The first-order condition is

$$\rho_{sf}\frac{(\bar{r}_s - i)}{\sigma_s} > \frac{\bar{r}_f}{\sigma_f}$$

The second-order condition is

$$\rho_{sf}\frac{r_f}{\sigma_f} < \frac{(\bar{r}_s - i)}{\sigma_s}$$

Appendix 2

The derivation of the effect of an increase in margin requirements M upon the optimal demand for futures contracts by the hedger N^* are detailed below.

From eq. (5), the optimal number of futures contracts demanded N^* is given by

$$N^* = \left[\frac{(\bar{r}_s - i)\sigma_{sf} - \bar{r}_f\sigma_s^2}{\bar{r}_f\sigma_{sf} - (\bar{r}_s - i)\sigma_f^2}\right]\frac{C}{S} \tag{2.1}$$

Since the margin requirements affect the expected effective return from the futures position $\bar{r}_f$

$$\begin{aligned}\frac{\partial N^*}{\partial M} &= \frac{\partial N^*}{\bar{r}_f}\cdot\frac{\partial \bar{r}_f}{\partial M}\\ &= \left[\frac{-\sigma_s^2}{[\bar{r}_f\sigma_{sf} - (\bar{r}_s - i)\sigma_f^2]} - \frac{((\bar{r}_s - i)\sigma_{sf} - \bar{r}_f\sigma_s^2)\sigma_{sf}}{[\bar{r}_f\sigma_{sf} - (\bar{r}_s - i)\sigma_f^2]^2}\right]\frac{C}{S}\cdot\left(-\frac{C_m}{SP}\right)\end{aligned}$$

Implicit in the above derivation is the assumption that the futures price does not respond to the change in margin requirement. The basis for this is the Schwert

(1989) results that subsequent stock market returns and volatility do not respond to changes in margin requirements, and the Hartzmark (1986) result that price volatility in different futures contracts are not significantly affected by changes in margin requirements.

$$\frac{\partial N^*}{\partial M} = \left[\frac{\sigma_s^2 \bar{r}_f \sigma_{sf} - \sigma_s^2 (\bar{r}_s - i)\sigma_f^2 + (\bar{r}_s - i)(\sigma_{sf})^2 - \sigma_s^2 \bar{r}_f \sigma_{sf}}{[\bar{r}_f \sigma_{sf} - (\bar{r}_s - i)\sigma_f^2]^2}\right] \frac{CC_m}{S^2 P}$$

$$\frac{\partial N^*}{\partial M} = \frac{\sigma_s^2 \sigma_f^2 (\bar{r}_s - i)\,(\rho_{sf}^2 - 1)}{[\bar{r}_f \sigma_{sf} - (\bar{r}_s - i)\sigma_f^2]^2} \frac{C}{S^2} \frac{C_m}{P} \tag{2.2}$$

As long as the expected effective return from the spot position $\bar{r}_s$ is greater than the risk free rate i and $|\rho_{sf}|$, the absolute value of the correlation between the effective return on the spot and futures position is < 1; $\partial N^*/\partial M$ is always negative.