Tailing the Hedge: Why and How

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INTRODUCTION

Futures markets are cited as providing risk reduction for the agricultural and financial communities. It is not surprising, therefore, that both academics and practitioners devote considerable time and effort to devising the appropriate hedging rules for reducing risk. On a somewhat general level, academics focus on deriving optimal hedge ratios based on the risk/return characteristics of cash and futures markets (i.e., Stein (1961); Ederington (1979); and Cecchetti, Cumby, and Figlewski (1988). Practitioners, on the other hand, concentrate on the details of basis risk and financing costs that can complicate basic hedging principles (see Chicago Mercantile Exchange (1987); Kawaller (1986); and Kidder Peabody (1987).

As a general rule, everyone agrees that hedging in futures markets involves matching a specific cash market position with an offsetting position in futures contracts. The main problem is to determine the precise size of the futures position that minimizes overall risk exposure. One seemingly minor factor that turns out to be important for both academics and practitioners is called "tailing the hedge." This refers to the practice of reducing the size of the futures hedge to take account of interest cost. Unfortunately, the various discussions of tailing the hedge are confusing both from the vantage point of exactly why the hedge is tailed as well as from the decision rule perspective. An oversimplified example that illustrates the main points follows.

Suppose a producer buys 1 unit of a cash commodity and wants to reduce the risk exposure associated with changes in its price, P. Ignoring complications associated with risk aversion, the producer would like to combine the cash position with a futures position, h, such that the unanticipated change in net worth (NW) associated with random changes in P and in the futures price, F, is zero. Formally, the producer would like to find that position h which minimizes the variance of

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The Journal of Futures Markets, Vol. 11, No. 2, 201–212 (1991)
 CCC 0270-7314/91/020201-12$04.00

ΔNW, where

$$\Delta NW = \Delta P + h \cdot \Delta F \tag{1}$$

The well-known solution to this problem (i.e., Ederington (1979)) shows that $h = \rho\sigma_P/\sigma_F$, where the correlation coefficient measures "basis risk" or "tracking error." For simplicity, this study assumes that the cash instrument being hedged is the underlying asset for the futures. Thus if $\rho = 1$, then the minimum risk h will satisfy

$$h = -\frac{\Delta P}{\Delta F} \tag{2}$$

Eq. (2) indicates that the futures position, h, should be opposite the cash market position (short futures to offset long cash or vice versa), with the size of the futures position determined by the hedge ratio $\Delta P/\Delta F$. If $\Delta P/\Delta F < 1$ because futures prices move more than cash prices, then the futures position should be smaller than the cash position. The reverse is true if $\Delta P/\Delta F > 1$.

It is possible to derive values for $\Delta P/\Delta F$ in a number of ways. One of the simplest applies to storable commodities, such as gold, and is based on the cost of carry formula relating the equilibrium price of a futures contract to the price of the underlying commodity:

$$F = P(1 + i)^T. \tag{3}$$

Eq. (3) specifies that the equilibrium futures price equals the cash price plus the interest cost (i) associated with inventorying the commodity from the present until the expiration date of the futures contract (denoted as time period T). Eq. (3) can be solved for $\Delta P/\Delta F$ as follows:

$$\Delta P/\Delta F = \frac{1}{(1 + i)^T}. \tag{4}$$

Alternatively, using continuously compounded interest rates, eq. (4) becomes the familiar

$$\Delta P/\Delta F = e^{-iT} \tag{5}$$

Eq. (5) can be used in conjunction with eq. (2) to specify the appropriate futures position, h, to hedge one unit of the cash commodity. In particular, for the case of a storable commodity, the futures position should be smaller than the cash position by the present value factor e^{-iT}. Figlewski (1986, p.34) refers to this e^{-iT} adjustment as "tailing the hedge." In essence, it involves a reduction in the size of the futures position relative to cash because futures prices move more than cash prices and marking to market translates these price changes into mismatched cash flows. The reason futures prices move more than cash prices has to do with the interest cost relating equilibrium futures prices to cash prices.

The practitioners' literature on tailing the hedge also refers to a present value factor, but derives it in a very different way and focuses on a different interest rate. Kawaller (1986, p.34) introduces a "tailing factor" because futures contracts give rise to mark to market settlement, thereby requiring financing of cash outflows or reinvestment of cash inflows. Not surprisingly, he specifies that the present value factor needed to tail the futures position is based on the overnight financing rate. Moreover, Kawaller suggests that the relevant time period in the tailing factor should be based on "the number of days to the ultimate settlement of the variation margin financing." According to Kawaller, the time period covered by the tailing factor refers to the hedge horizon (the date the hedge is to be lifted), while in eq. (5) it refers to the settlement date of the futures contract.

Although tailing the hedge is properly considered a hedging refinement, one that can be important when "sharp-penciled" arbitrage trades are undertaken, there seems to be considerable confusion regarding both the purposes and procedures of the process. The objective of this study is to eliminate some of the mystery surrounding tailing the hedge.

Intuition suggests that financing costs are, in fact, at the root of tailing a futures hedge by considering the distinction between hedging a cash position with a futures contract compared with hedging via a forward contract. Although eq. (3) is the equilibrium pricing equation for both futures and forward contracts (assuming interest rates are not stochastic, see Cox, Ingersoll, and Ross (1981)), the proper hedge ratio when using forwards to hedge a cash position is 1.0 rather than e^{-iT}. In other words, a forward position used as a hedge is equal in size to the cash market position and is not reduced by any tailing factor whatsoever. This is true even though eq. (3) implies that forward prices move by e^{iT} more than cash prices in the same manner as futures prices. Although forward *prices* move by e^{iT} more than cash prices, a price change leads to a cash payment on a forward contract only at the end of time period T. To translate a price change into the change it causes in the current *value* of the forward contract it must, in turn, be discounted by e^{-iT} (producing a hedge ratio of 1.0). Futures contracts, on the other hand, are settled with daily cash flows, hence the futures price does not have to be discounted to derive its current value.

This discussion suggests that financing costs are at the root of tailing a futures hedge. Still, by dealing with stylized examples in which there is a single interest rate and in which the hedge horizon is the futures expiration date, the literature and the above intuitive discussions leave unresolved several important questions that arise in actual hedging decisions. For example, suppose a three-month futures contract is used in a hedge that is to be lifted one month from now: (a) Should you use three months or one month as T in the calculation? (b) Should you use the interest rate on three-month money, one-month money, or the overnight rate in the calculation? (c) Is there any adjustment that needs to be made if the security being hedged pays coupon interest or dividends during the hedge period?

INTEREST RATES AND TIME HORIZONS WITH COST OF CARRY FUTURES PRICES

As indicated, the need to tail a futures hedge is a result of the daily cash flows associated with mark to market settlement of a futures contract. In particular, the profits and losses associated with variation margin payments have financing returns or costs associated with them. These cash flows raise two related questions: First, what interest rate should be used as the financing cost: the overnight rate or a term rate to either the hedge horizon or the futures expiration date? Second, what is the proper time horizon to use: the planned hedging horizon or the futures expiration date?

To derive the optimal tail or hedge ratio, it is assumed that the goal of hedging is to lock in a certain payoff or certain return as of the planned hedging horizon. The analysis shows that the correct interest rate in the tailing factor depends on what the hedger believes the relationship between the cash and the futures prices will be at the hedge horizon. General results are derived and then illustrated with simple numerical examples.

Let RF_t be the compounded interest from time t to futures expiration, and similarly let RH_t be the compounded interest from time t to the hedge horizon. Assuming nonstochastic interest rates, which can be taken to imply a pure expectations term structure:

$$RF_t = (1 + r_t)(1 + r_{t+1})(1 + r_{t+2})\ldots(1 + r_{T-1}), \tag{6a}$$

$$RH_t = (1 + r_t)(1 + r_{t+1})(1 + r_{t+2})\ldots(1 + r_{H-1}), \tag{6b}$$

where r_t is the one-period rate, T is the futures expiration date, and H is the hedge horizon.

The payoff, Π, on a hedged position established at time period 0 and held until period H can be written as follows:

$$\Pi = h_0(F_1 - F_0)RH_1 + h_1(F_2 - F_1)RH_2 + \ldots + h_{H-1}(F_H - F_{H-1}) + P_H \tag{7}$$

where h_t is the hedge ratio for each period during the hedge horizon. Each period corresponds to the daily settlement of cash flows from variation margin payments. These flows are reinvested or financed to the hedge horizon at the rate RH_t.

Tailing the hedge involves a present value factor. The key practical problem is which interest rate should be used in the calculation. First examine the case where the hedge ratio is set up using compound interest to the futures expiration date, i.e., a rate corresponding to that used in the cost of carry relationship expressed in eq. (3). In the current framework,

$$h_t = -\frac{1}{RF_{t+1}}, \tag{8}$$

Note that according to eq. (8) the hedge ratio is set up at the beginning of period t using the financing cost from the next day because that is when the first cash flow is generated from mark to market settlement. This realistic specification can also be seen in the payoff function (eq. (7)), where the cash flow $(F_t - F_{t-1})$ for period t involves the hedge ratio established in the previous period, h_{t-1}, but with the reinvestment rate RH_t as of t.

Substituting for the hedge ratios from eq. (8) into eq. (7) yields:

$$\Pi = \frac{1}{RF_H}(F_0 - F_H) + P_H \tag{9}$$

This relatively simple expression follows from the fact that for any $t < H$, $RH_t/RF_t = 1/RF_H$, where RF_H is the cumulative interest from the hedge horizon to futures expiration. In this way one can combine all of the cash flows under one coefficient.

In eq. (9) there are two uncertain terms, F_H and P_H, and therefore Π is also uncertain. But now assume that the cost of carry relation expressed in eq. (3) above holds between futures and cash. This can be rewritten as:

$$F_t = P_t RF_t \tag{10}$$

Recall that eq. (10) is an arbitrage/equilibrium relationship based on the inventory cost of holding the cash commodity to the futures expiration date. In the case where the cost of carry holds both at time 0 and at the hedge horizon, by substituting (10) into (9), the following is obtained:

$$\Pi = \frac{P_0 RF_0}{RF_H} = P_0 RH_0 \tag{11}$$

Eq. (11) shows that with $h_t = -1/RF_{t+1}$ in each period, the uncertainty associated with F_H and P_H disappears, and a riskless return of RH_0 is locked in on the initial cash position, P_0.

Example 1

Example 1 (Table I) illustrates a simple three-period example. It covers four points in time: $t = 0$, when the hedge is initiated; $t = 1$, the first date at which the futures position is marked to market; $t = 2$, the hedge horizon (H); and $t = 3$, the futures expiration date (T). One period is one month.

This example is sufficient to capture the essential elements of a realistic hedging situation in which a hedge is initiated, it is kept in place for a time during which the futures must be marked to market, and it is then lifted prior to the expiration date of the futures. The hedge horizon and futures expiration dates are clearly separated. In practice, of course, there are a number of periods like $t = 1$, during which the hedged position simply remains open and marking to market leads to variation margin payments. The time from $t = 0$ to $t = 1$ is normally one trading day or less.

The first section of the table shows the prevailing interest rates. At $t = 0$, the hedger is facing a slightly rising yield curve, with the one-month rate at 1.00% (12% annually, at simple interest), the two-month rate at 1.05% and the three-month rate at 1.10%. Interest rate uncertainty is not considered so the forward rates embodied in this term structure are assumed to be the one-period rates that actually occur in the future periods, i.e., $r_1 = 1.1000\%$ and $r_2 = 1.2001\%$. The RF_t and RH_t variables that represent the compounded carrying costs to futures expiration and the hedge horizon, are as shown, respectively.

It is assumed arbitrarily that the cash price rises from 100 to 105 over the hedge period, and then drops back to 100 at expiration. The futures price at each point is determined by the cost of carry relation, eq. (10).

The last part of the example shows the performance of the hedged position. A tailed hedge ratio equal to $h_t = -1/RF_{t+1}$ is used. One multiplies the untailed hedge ratio (−1.0) by the tailing factor of one divided by the cumulative interest rate from the first mark to market period until the futures expiration date. In other words, the interest rate for tailing the hedge is the forward rate for the period beginning at the time of the next variation margin cash flow, and the compounding interval is the time from that point to futures expiration.

In the first period, the cash price increases by 2 points. If the cash price did not move, the futures price would drop from $P_0RF_0 = 103.34$ to $P_0RF_1 = 102.31$. With the cash 2 points higher, the futures are higher also, by an amount equal to $(2.00)(RF_1) = 2.05$, so $F_1 = 104.36$. Since the hedge is tailed by the factor $1/RF_1$, the variation margin cash flow from the futures is exactly −1.00, and the total value of the hedged position goes from 100 to 101. If the hedge is unwound at this point, it earns exactly the risk-free interest rate over the period, r_0, which is what it is designed to do.

In the next period, the hedge is rebalanced, using $1/RF_2$ as the tailing factor. The cash price goes up another 3 points and the future rises by 1.90. In addition, inter-

Table I
FUTURES PRICED BY COST OF CARRY, HEDGE TAILED BY $1/RF_{t+1}$

	Beginning ($t = 0$)	First Mark to Market ($t = 1$)	Hedge Horizon ($t = H$)	Futures Expiration ($t = T$)
Spot Interest Rates				
1-month	$r_0 = 1.0000$	$r_1 = 1.1000$	$r_2 = 1.2001$	
2-month	1.0500	1.1500		
3-month	1.1000			
Rate to futures expiration	$RF_0 = 1.0334$	$RF_1 = 1.0231$	$RF_2 = 1.0120$	$RF_3 = 1.0000$
Rate to hedge horizon	$RH_0 = 1.0211$	$RH_1 = 1.0110$	$RH_2 = 1.0000$	
Prices				
Cash price	$P_0 = 100$	$P_1 = 102$	$P_H = 105$	$P_T = 100$
Futures price (cost of carry)	$F_0 = 103.34$	$F_1 = 104.36$	$F_H = 106.26$	$F_T = 100$
Tailed Hedge—Long 1 unit cash, h units futures				
Value of cash position	100	102	105	100
Hedge ratio $h = -1/RF_{t+1}$	−0.9774	−0.9881	−1.0000	
Variation margin flow		−1.0000	−1.8780	6.2601
Interest			−0.0110	−0.0347
Cumulative cash flow from futures		−1.0000	−2.8890	3.3364
Value of hedged position	100	101.0000	102.1110	103.3364
Current period hedge return (%)		1.0000	1.1000	1.2001
Cumulative hedge return (%)		1.0000	1.0500	1.1000
Untailed Hedge—Long 1 unit cash, short 1 unit futures				
Value of position	100	100.9769	102.0651	103.2900
Current period hedge return (%)		0.9769	1.0777	1.2001
Cumulative hedge return (%)		0.9769	1.0273	1.0848

est at the risk-free rate r_1 must be paid on the 1.00 dollar variation margin outflow from the previous period, leading to a further outflow of −0.0110.

The total value of the hedged position is now 102.1110. Because it is properly tailed, it earns the period 1 riskless rate of 1.1000% during the second period, and its realized rate of return over the entire hedge period is exactly 1.0500%, which was the 2-period interest rate at the outset.

Carrying the hedge to futures expiration would lead to performance exactly as anticipated at the outset. This example illustrates that when the future is priced according to the cost of carry relation, tailing the hedge is necessary, and it leads to a hedged position that earns the riskless interest rate in every period.

For comparison, the results of an untailed 1 for 1 hedge are shown at the bottom of the table. Because the hedge does not adjust for the financing cost of the variation margin outflow in the first period, it underperforms, earning a return more than 2 basis points below the 1.05% earned on the tailed hedge.

It is important to note that the cost of carry relationship does not need to hold at every point for the correctly tailed hedge to lock in a certain payoff. It is enough for the future to be at its cost of carry value only at the hedge horizon. Letting $M = F_0 - P_0 RF_0$ denote the deviation of the futures price at time zero from its cost of carry value, i.e., the initial mispricing of the future, eq. (9) produces:

$$\Pi = \frac{F_0}{RF_H} = P_0 RH_0 + \frac{M}{RF_H}, \tag{12}$$

This result follows from the substitution of the cost of carry relationship $F_H = P_H RF_H$ into (9).

One can conclude that where the cost of carry holds at the hedge horizon, *RF* is the correct factor to use in tailing the hedge. The intuition behind this result starts with the fact that the cost of carry reflects the financing cost involved in holding the cash instrument to futures expiration. The cost of carry formula allows one to specify a deterministic relationship between futures and cash prices. Thus, the tailing factor *RF* from the cost of carry formula simultaneously accomplishes two objectives when used as the hedge ratio: it both eliminates uncertainty and takes account of financing costs.

INTEREST RATES AND TIME HORIZONS FOR A BASIS TRADE

The cost of carry relationship allows the hedger to specify what the ratio F/P will be at the hedge horizon even though the individual prices are not known in advance. However, carrying cost pricing does not hold well in all futures markets or at all times. In some cases, a trader may feel more confident about making a specific prediction for the *basis*, i.e., the price difference $F - P$, at the hedge horizon rather than assuming that the cost of carry will hold. This is more likely in cases where the future is currently significantly "mispriced" and the hedge horizon is short.

To explore this case, consider tailing the hedge using the cumulative interest factor to the hedge horizon instead of futures expiration. That is,

$$h_t = -\frac{1}{RH_{t+1}} \tag{13}$$

Substituting this into the payoff function of eq. (7), the following equation involving the futures–cash basis results:

$$\Pi = F_0 - F_H + P_H = (B_0 - B_H) + P_0 \tag{14}$$

where $B_0 = F_0 - P_0$ is the basis at time 0 and $B_H = F_H - P_H$ is the basis at the hedging horizon.

Eq. (14) implies that if the hedger can specify what the basis will be at the hedging horizon, uncertainty over the payoff Π is eliminated by using the interest rate to the hedge horizon as the tailing factor. In this case, the certain return on the hedge position is the difference between the basis at the beginning and the basis at the end ($B_0 - B_H$).

This result confirms the intuition just discussed. In particular, the financing cost is combined with something that allows one to eliminate uncertainty in the payoff function. In this case, the basis is known at the hedge horizon, so the interest rate is specified to the hedge horizon. In the previous case, the cost of carry is known so the interest rate is specified to futures settlement, which is the reference point for the cost of carry relationship.

Example 2

Example 2 (Table II) illustrates the case where the future is no longer priced according to the cost of carry model. Instead, the hedger is assumed able to make a forecast of the basis between cash and futures at the hedge horizon ($F_H - P_H$). As suggested above, this might describe the situation faced by a "basis trader" or arbitrageur trading in a futures market that is not fully efficient.

In example 2, the interest rates and cash prices have the same values as in the first example, but the futures are priced differently. The hedger here sees that the future is 5 points above the cash at the outset, and he believes the difference will drop to 2 points by the hedge horizon date, $t = H$. This 3-point basis change over two periods on an initial cash position worth 100 translates to a 1.4889% per period return.

To lock in that return, the trader must tail his hedge in a different way than in the previous example. Since futures are not assumed to follow the cost of carry, there is no point in trying to construct a hedge that will earn the risk-free interest rate assuming the cost of carry held. Instead, the futures hedge is based on tailing only up to the hedge horizon using $h_t = -1/RH_{t+1}$.

In the first period, the basis goes from 5 to 4 points, leading to a variation margin outflow of −0.9891. The one-period realized return on the hedge is 1.0109% which, unlike the previous example, is not directly related to current interest rates. In other words, the returns earned in periods prior to the hedging horizon are uncertain. The margin call is financed at the new one-period riskless interest rate of 1.1000% and the hedge, which is due to be unwound in period 2, is rebalanced to the value of −1.0 (untailed because period 2 will be the final period of the hedge).

In the second period, the cash price goes up again and the futures do too, ending up at the predicted premium of 2 points. This causes a further margin call of −1.0000, plus interest expense of 0.0109 for the funds used to meet the first period margin call. The total value of the hedged position at the end of the hedging horizon is exactly 103.00; i.e., the initial cash price plus the predicted change in the basis. The realized return over the entire hedge period is exactly 1.4889% per period, which is as anticipated. Once again, the untailed hedge can be seen to lead to a shortfall.

The essence of this example is that the hedger (correctly) predicts that the basis will change by a certain amount, but he does not know exactly when. For the cash flows arising from the futures position to add up to the cash price change ($P_H - P_0$) plus the basis movement projected for the holding period, each intermediate cash flow must be carried forward from the date it is received to the hedge horizon at the interest rate for that time interval, RH_{t+1}. Tailing the hedge by

Table II
FUTURES NOT PRICED BY COST OF CARRY, BASIS AT HEDGE HORIZON IS KNOWN, HEDGE TAILED BY $-1/RH_{t+1}$

	Beginning $t = 0$	First Mark to Market $t = 1$	Hedge Horizon $t = H$	Futures Expiration $t = T$
Spot Interest Rates				
1-month	$r_0 = 1.0000$	$r_1 = 1.1000$	$r_2 = 1.2001$	
2-month	1.0500	1.1500		
3-month	1.1000			
Rate to futures expiration	$RF_0 = 1.0334$	$RF_1 = 1.0231$	$RF_2 = 1.0120$	$RF_3 = 1.0000$
Rate to hedge horizon	$RH_0 = 1.0211$	$RH_1 = 1.0110$	$RH_2 = 1.0000$	
Prices				
Cash Price	$P_0 = 100$	$P_1 = 102$	$P_H = 105$	$P_T = 100$
Futures price	$F_0 = 105.00$	$F_1 = 106.00$	$F_H = 107.00$	$F_T = 100$
Tailed Hedge—Long 1 unit cash, h units futures				
Value of cash position	100	102	105	100
Hedge ratio $h = -1/RH_{t+1}$	−0.9891	−1.0000		
Variation margin flow		−0.9891	−1.0000	
Interest			−0.0109	
Cumulative cash flow from futures		−0.9891	−2.0000	
Value of hedged position	100	101.0109	103.0000	
Current period hedge return (%)		1.0109	1.9692	
Cumulative hedge return (%)		1.0109	1.4889	
Untailed Hedge—Long 1 unit cash, short 1 unit futures				
Value of position	100	101.0000	102.9890	
Current period hedge return (%)		1.0000	1.9693	
Cumulative hedge return (%)		1.0000	1.4835	

the inverse of this interest rate factor in each period produces a series of cash flows whose future values on the hedge horizon date H sum to the desired total.

HEDGING INTEREST-BEARING ASSETS

When the underlying asset for a futures contract pays dividends or interest, the cost of carrying a hedged position is reduced by the amount of the cash payout, as is the cost of carry futures price. Do those payouts affect the optimal tailing strategy?

The answer is that they do not. If the futures at the hedge horizon are priced at the cost of carry value (taking the cash payout into account in calculating that value, of course), tailing by the factor $h_t = -1/RF_{t+1}$ still produces the appropriate adjustment. If, instead, the hedger knows what the futures–cash basis will be at the hedge horizon, the correct tailing factor is still $h_t = -1/RH_{t+1}$.

This can be shown fairly easily, by introducing terms relating to the dividends (or coupons) paid up to the hedge horizon and the futures expiration date, DH and DF, respectively. Let DH_t represent the present value at date t of all dividends to be paid between t and H, and let DF_t be defined analogously for the futures expiration date. The cost of carry relationship now becomes:

$$F_t = (P_t - DF_t)RF_t. \tag{15}$$

Following the same reasoning used in eqs. (7) through (9) produces:

$$\Pi = (P_0 - DF_0)RH_0 - (P_H - DF_H) + DH_0RH_0 + P_H \tag{16}$$

This simplifies to eq. (11) because the three terms involving dividends cancel out. The first dividend term is equal to the present value as of H of all dividends paid before H plus those to be paid between H and F, while the second and third terms represent those two parts of the dividend stream separately.

A similar derivation shows that the results for the case in which the basis is known continue to hold when there is a cash payout.

TAILING WHEN THE HEDGE RATIO IS ESTIMATED BY REGRESSION

Frequently the hedge ratio is estimated by running a regression like eq. (17) of cash price changes against changes in the futures price using historical data.

$$\Delta P_t = a + h\Delta F_t \tag{17}$$

Comparing eq. (17) with eq. (3) suggests that if the futures price is determined by the cost of carry, coefficient a in the regression equation should be 0 and h should be equal to $1/(1 + i)^T$, meaning that it is already tailed. In that case, no further tailing is called for.

Unfortunately, things are not that simple. In running the regression eq. (17) over a sample of past data, one combines observations from many periods with different interest rates and times to expiration. If the futures follow the cost of carry exactly during this time period, the regression coefficient still ends up being a kind of average of the appropriate tailing factors that would have applied over the period spanned by the data sample. This means that when the hedge ratio is estimated by regression, it is partly but not completely tailed, and a simple adjustment to correct it is not possible.

TAILING THE HEDGE WHEN FUTURE INTEREST RATES ARE RANDOM

So far, it has been assumed that futures financing rates are known at the outset. If this is not the case, the situation becomes substantially more complicated. An unanticipated change in interest rates introduces uncertainty into the hedge in several places. It changes the relationship between the futures and cash prices because the cost of carry is affected. It affects the financing cost or return on the interim cash flows from variation margin and dividend payouts. Also, there may be correlation between the change in interest rates and the price of the cash security, particularly if it is a bond. An additional result of stochastic interest rates described by Cox, Ingersoll, and Ross (1981), is that, in general, futures and forwards should no longer be priced the same.

It is beyond the scope of this study to analyze the case of tailing under stochastic interest rates. This may be a potential area of difficulty with the analysis. Note, however, that this is a problem shared by all approaches to futures hedging; it is not specific to tailing.

CONCLUSION

Tailing a hedge that uses futures contracts is called for because of the effect of variation margin cash flows. Many basic discussions of hedging ignore the issue entirely, while those that do mention tailing invariably leave ambiguity with regard to the practical details. This study tries to clarify why one should tail a futures hedge and exactly how it should be done.

With respect to the question of why one should tail, an attempt is made to establish which of the two common explanations is correct, that is, whether tailing is called for because futures prices are set by a cost of carry relationship and therefore exhibit larger price changes than the underlying cash instrument, or because of the financing costs (or returns) associated with the variation margin cash flows. It is shown that these explanations are actually the same, because the cost of carry relationship is inherently based on financing costs: The cost of carry futures price is equal to the current spot price plus the cost of financing an inventory of the underlying asset from now until expiration, and these same financing costs also apply to variation margin flows.

The "how to tail" question is found to depend on what the hedger knows about how the futures price and the spot price will be related to one another at the hedge horizon. If the hedger assumes that the futures will be priced at their cost of carry value when the hedge is lifted, the hedge ratio should be reduced by the factor $1/RF_{t+1}$, the inverse of the cumulative interest from the first mark to market until the futures expiration date. If the cost of carry does not hold, but the hedger is able to specify what the futures–cash *basis* will be at the hedge horizon, the correct tailing factor is $1/RH_{t+1}$, the inverse of the cumulative interest only until the hedge horizon.

Note that unlike the practioner's intuition, in neither case is the appropriate interest rate the overnight rate, even though the need to tail is due to financing costs. Nor is the "tail" affected by dividend payouts on the cash security. Instead, as suggested above, in both cases, the correct tailing strategy is a function of the expected relationship between futures and cash at the hedge horizon.

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