Stock Price Volatility: Some Evidence from an ARCH Model

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A series of sharp breaks in stock prices beginning in 1987 and continuing into 1990 have heightened concern about stock market volatility. The perception of an increasingly volatile market has led to a search for the culprit. Presently, a considerable number of fingers are pointing at programmed trading.[1] In response, the Chicago Mercantile Exchange, Chicago Board of Trade and New York Stock Exchange have investigated suspected market manipulation by program traders and all three exchanges have installed special electronic systems that allow them to track transactions occurring between the spot, futures and options markets. Most recently, the NYSE proposed to restrict program trading on days when the Dow Jones Industrial Average moves up or down by 50 points or more during a trading session. The rationale given by the NYSE is that "The Exchange and other Market Centers have been concerned that sophisticated trading strategies related to program trading may create excess volatility. . . , and may in fact constitute a threat to the viability of American capital markets."[2]

Despite the public outcry concerning increasingly volatile stock prices, the scientific evidence regarding this issue is far from conclusive. Some investigators find no evidence of increased volatility in stock prices since the advent of stock index futures trading in April 1982.[3] Other studies report mixed results.[4] None finds an unambiguous increase in the variance of stock prices that can be associated with program trading.

Unfortunately, the above results are difficult to interpret because they are not well integrated with important general conclusions that have emerged from other

[1]Programmed trading is an investment strategy that involves trading on small and short-lived price differences for the same group of stocks in the spot, futures, and options markets. For further discussion see Black and Scholes (1973), Cinar (1987), Cornell and French (1983), Figlewski (1984), Fortune (1989), Modest and Sundaresan (1983), and Stoll and Whaley (1987).

[2]See for example, Torres (1990) and Torres and Salwen (1990).

[3]See, for example, Fortune (1989), Davis and White (1987), and Santoni (1987). Stoll (1988) finds a statistically significant increase in the variance of stock prices on the days futures and option contracts expire ("Triple Witching Days"). However, this expiration day effect is economically small on average.

[4]See Harris (1988), Jones and Wilson (1989), and Edwards (1988).

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studies of the behavior of speculative prices. These studies indicate that the rates of return (percentage changes in price) implicit in the time series of stock prices are serially uncorrelated and well described by a unimodal symmetric distribution with fatter tails than the normal.[5] However, while the returns are serially uncorrelated, they are not independent. In particular, large percentage changes in price tend to be followed by large changes (of either sign); while small changes tend to be followed by small changes.[6] This suggests that usual measures of stock price volatility are temporally dependent (heteroskedastic). Consequently, a meaningful comparison of price volatility across different time periods can only be made if the comparison controls for this time dependence.[7] Without this control, the investigator can not be certain that observed differences in volatility between different time periods are not simply a fortuitous result of the temporal dependence. Previous studies concerning the effect of stock index futures trading on cash market volatility do not control for this effect although some have noted its presence in the data.[8]

This article accounts for the heteroskedasticity by applying an Autoregressive Conditional Heteroskedastic (ARCH) model to the variance of the rate of return implicit in the time series of stock prices and tests to determine whether the parameters of the model change significantly subsequent to the introduction of trading in stock index futures.[9]

The notion underlying the ARCH model is that the variance of the time series (daily percentage changes in stock prices in this paper) is subject to change. In particular, large changes are followed by large changes and small changes are followed by small changes so that the change in period t is conditional on the changes in period t-1, t-2, etc. However, the sign of the period t change is unpredictable. A version of the model is given in equations (1)-(3) below.[10]

$$(x_t x_{t-1}, \ldots, x_{t-k}) \backsim N(O, \sigma_t^2) \tag{1}$$

$$\sigma_t^2 = f(x_{t-1}, \ldots, x_{t-k}) \tag{2}$$

Equation (3) assigns parameters and specifies restrictions to the function expressed in (2) that guarantee positive conditional and unconditional variances.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{k} \alpha_i x_{t-i}^2 \tag{3}$$

$$\alpha_0 > 0, \qquad 0 \leq \alpha_i < 1$$

The model has the property that the conditional variance may move over time as a function of past realizations of x. Furthermore, if the variance of the x series is governed by a process like that of eq. (3), the unconditional density function will have

[5]Mandelbrot (1963) and Fama (1965) are the seminal works. See Bollerslev (1987) for a more recent discussion. The result holds for price changes sampled at high frequency such as daily or weekly. This is important because the studies concerning the impact of futures trading on the variance of cash prices in the stock market typically examine daily and/or weekly returns.

[6]See Mandelbrot (1963); p.418, Bollerslev (1987); p. 542, and Diebold (1989); Chapter 2.

[7]See Diebold (1988); p. 36.

[8]See, for example, Edwards (1988).

[9]For a discussion of ARCH models see Engle (1982), Bollerslev (1987), Diebold (1989), and Greene (1990).

[10]See Greene (1990); p. 416. The model may include any number of past realizations of X_t. See Diebold (1989), pp. 5-6.

fat tails which is a characteristic that is consistent with previous findings for stock price returns.

Once an appropriate ARCH model of stock price returns is estimated, a solution for the implied time series of the conditional variance can be obtained. This yields a meaningful measure of volatility since any uncertainty in price movements that can be removed by conditioning on past observations is economically irrelevant. Stock prices that follow a random walk with ARCH disturbances mean that changes in price can not be forecast but the changing variance can be. This suggests that the ARCH model may be used to obtain time-varying confidence intervals for price changes which, given the random walk, have a zero mean. During periods of high volatility the intervals are large and they shrink during more tranquil times.[11]

THE DATA

Figure 1 plots the first differences of the log of daily closing levels of the S&P500 Composite index.[12] The data are approximately the daily percentage changes in the

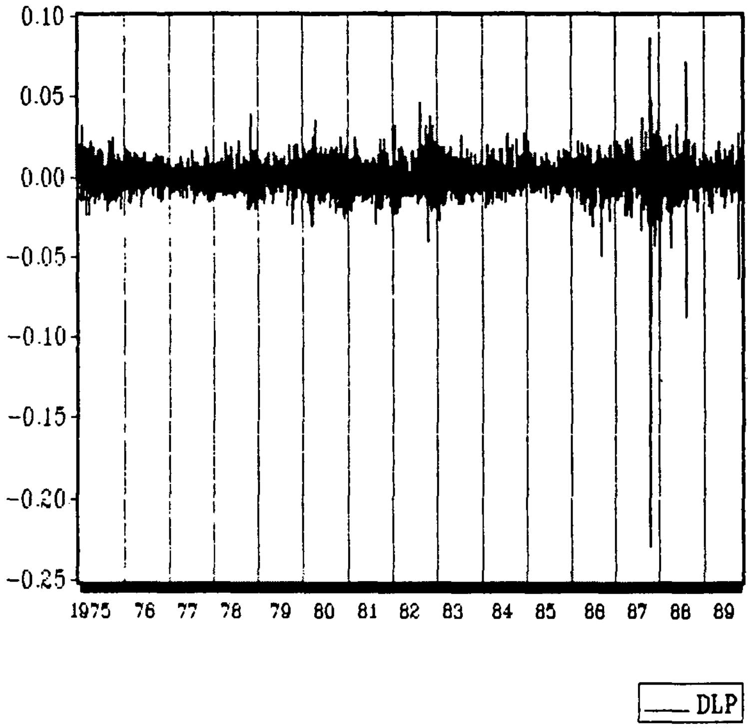

Figure 1
First differences of the log of the S&P 500 daily data, Jan. 1975–Oct. 1989.

[11]See Diebold (1989); p. 36.
[12]The observations are calculated as follows:

$$\Delta \mathrm{Ln} P_t = \mathrm{Ln}(P_t/P_{t-1})$$

where P is the daily closing level of the index.

index. The series begins the first trading day of January 1975 and extends through the last trading day of October 1989. The data in Table I present the mean and variance of the Figure 1 data for each year.

Naive comparison of the variance of daily percentage changes in the index pre-April 1982 ($\sigma^2 = 6.83E\text{-}5$) to post-April 1982 ($\sigma^2 = 1.40E\text{-}4$) suggests greater volatility in the more recent period ($F = 2.05$). It is possible, of course, that the difference in the variance between these periods is a statistical artifact produced by heteroskedasticity which is the issue examined in this article.

While the variance of daily percentage changes in the S&P500 was fairly high in 1982, the variance declined over the next 3 years reaching its second lowest level for the entire period in 1985. The 1987 crash was associated with a sharp increase in volatility that has gradually dampened since then. In fact, price volatility in 1989 was no greater than it was in the two years immediately preceding the advent of stock index futures trading in 1982.

Given these data, it is surprising that the notion that program trading contributes significantly to price volatility is so prevalent among investors, traders, exchange officials and government regulators. If it is appropriate to lay the blame for increased price volatility in 1987 and 1988 at the feet of program trading, it is equally appropriate to suggest that program trading is responsible for the relatively tranquil years of 1983–1986 and 1989.

Both Figure 1 and Table I suggest some cycling in the variance of close to close percentage changes in the index implying that the data may follow an ARCH process. In addition, the frequency distribution of daily percentage changes in the index (shown in Fig. 2) gives further indication that ARCH effects may be present

Table I
MEANS AND VARIANCES OF FIRST DIFFERENCES OF THE LOG OF THE S&P500[1]
Daily Data
Jan. 2, 1975–Oct. 31, 1989

Year	Mean	Variance
1975	.00099	.000094
1976	.00070	.000050
1977	−.00048	.000035
1978	.00004	.000063
1979	.00044	.000047
1980	.00085	.000108
1981	−.00035	.000074
1982	.00054	.000132
1983	.00063	.000070
1984	.00006	.000064
1985	.00093	.000041
1986	.00054	.000086
1987	.00008	.000462
1988	.00046	.000168
1989	.00096	.000074
1975–1989	.00042	.000110

[1]The data are approximately percentage changes in the index expressed in decimals.

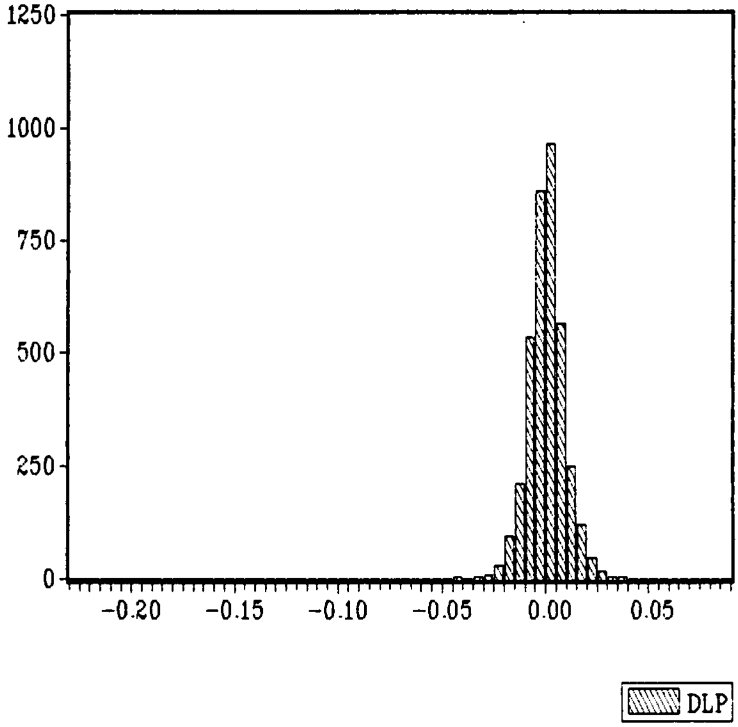

Figure 2
Frequency Distribution first differences of the log of the S&P 500 daily closing data, Jan. 1975–Oct. 1989.

in the data. Despite appearances, the distribution has fat tails (Kurtosis = 75.9) and it fails the Jarque and Bera (1981) test for normality at the 5 percent level.[13]

TESTING FOR ARCH

The above characterization of the behavior of stock prices is consistent with that found in other studies. Furthermore, this behavior is not unexpected if ARCH effects are present in the data.

The regression result presented in Table II is an estimate of the ARCH model specified in eq. (3). The data are squared differences between changes in the log of the daily closing level of the S&P500 and its expected value from the first trading day in January 1975 through the last trading day in October 1989. The Table II estimate includes a dummy variable for October 22 and 23, 1987. On October 22 the NYSE announced that it would begin closing two hours and fifteen minutes early (at 2:00 PM rather than 4:15 PM Eastern time). The shortened trading hours went into effect the next day, October 23. At the same time, the CME shortened its trad-

[13]The test statistic is:

$$N = n[B_1/6 + (B_2 - 3)^2/24]$$

where $B_1 = (u_3/u_2^{1.5})^2$, $B_2 = u_4/u_2^2$ and $u_i = (1/n)\Sigma u_i^r$ $(r = 2, 3, 4)$.
See Kmenta (1986); pp. 266–67 and Jarque and Bera (1981). For the distribution given in Chart 2, the test statistic is 836961 and the critical value is 5.99.

Table II
A TEST FOR ARCH[1]
Daily Data
Jan. 2, 1975–Oct. 31, 1989

$$x_t^2 = \underset{(3.32)}{5.19\mathrm{E}{-5}} + \underset{(5.83)}{.095x_{t-1}^2} + \underset{(7.14)}{.117x_{t-2}^2} + \underset{(3.92)}{.141x_{t-3}^2} + \underset{(2.35)}{.084x_{t-4}^2} + \underset{(5.70)}{.098x_{t-5}^2} - \underset{(3.43)}{.007D1}$$

$R^2 = .050$
$SSR = .002951$
$n = 3730$
$nR^2 = 187.00$

Where:

$x_t^2 = (h_t - \bar{h})^2$
$h_t = \mathrm{Ln}(P_t/P_{t-1})$
$\bar{h} = 1/n \sum_{t-1}^{n} h_t$
P_t = the daily closing level of the S&P500.
$D1$ = 1 on October 22 and 23, 1987 and zero otherwise.

[1] t-statistics in parentheses.

ing hours for stock index futures to correspond to trading on the NYSE and imposed a 30 point daily price limit on the S&P500 futures contract. The dummy variable is included in the estimate to control for the announcement of these changes.

The test for ARCH is a Lagrange multiplier test of the null hypothesis of no ARCH effects. The test statistic is distributed chi-square with K degrees of freedom and is the product of the number of observations in the sample (3742 in the Table II estimate) and the coefficient of determination (R^2).[14] The calculated test statistic is 187.0 which is sufficiently high to reject the null hypothesis at conventional confidence levels.

The Table II results indicate that ARCH effects are present in daily percentage changes in the S&P500 Composite index. The estimate suggests that price variation in the current period is related to price variation that occurred over five previous periods.

ARCH AND THE RANDOM WALK HYPOTHESIS

The Box-Pierce test for serial correlation is a standard method for evaluating the random walk hypothesis for stock prices. For the data examined in this article, the test is designed to determine whether current changes in the S&P500 index are significantly related to past changes. The random walk hypothesis implies that these changes are not correlated (past changes contain no information about current or future changes). However, if ARCH effects are present, the Box-Pierce statistic is biased toward accepting serial correlation even though the series is white noise. This bias is appreciable for the data examined here. The test is applied to the estimated auto correlations of daily percentage changes in the S&P500 index, resulting

[14] See Diebold (1988); p. 19. The null hypothesis that the residuals of this regression are white can not be rejected. The calculated Box-Pierce at 18 lags is 9.66.

in a calculated Box-Pierce statistic of 39.17 through lag 18 which is sufficiently large to reject the null of no serial correlation.

Fortunately, the Box-Pierce test can be modified to eliminate the bias in the presence of ARCH.[15] The corrected test statistic is

$$BP(K)^{ARCH} = n\sum_{\tau=1}^{K}\left\{\left[\frac{\sigma^4}{\sigma^4 + \gamma_x^2(\tau)}\right]\hat{\rho}_x^2(\tau)\right\} \tag{4}$$

where

σ^4 = the squared unconditional variance of the data series, x.
$\gamma_{x^2}(\tau)$ = the autocovariance of the process x^2 at lag τ.
$\hat{\rho}_k^2(\tau)$ = the autocorrelation of the x series to lag τ.

The test statistic is distributed chi-square with K degrees of freedom. Table III shows the results of applying this test to the estimated auto correlations of daily percentage changes in the S&P500 index. None of the corrected test statistics indicate significant correlation at conventional confidence levels. Stock prices, as reflected by the behavior of the S&P500 index, follow a random walk with ARCH effects during the sample period.

THE EVOLUTION OF PROGRAM TRADING AND THE STRUCTURE OF THE ARCH PROCESS

The claim that program trading significantly increases price volatility in the cash market is essentially a hypothesis that this type of trading significantly alters the parameters of the ARCH model. In particular, the hypothesis suggests that program trading raises the intercept term (α_0 in eq. (3)) or that the coefficients of the x_{t-i}^2 process are significantly greater so current period shocks are associated with larger and more persistent after shocks or both.

The Table II estimate of eq. (3) assumes the parameters of the equation are the same across the entire sample period. This restriction on the parameters can be tested by identifying a second (unrestricted) model that allows the parameters to

Table III
AUTOCORRELATION COEFFICIENTS AND ARCH CORRECTED BOX PIERCE STATISTICS
First Differences of the Log of the S&P500
Daily Closing Data
Jan. 2, 1975–Oct. 31, 1989

To Lag	Autocorrelation Coefficient	Corrected Box-Pierce Statistic
1	.048	.95
2	−.038	1.45
3	−.019	1.68
6	−.002	4.50
12	.020	6.50
18	−.016	10.90
n = 3740		

[15]See Diebold (1988); pp. 27–28.

take on different values subsequent to the institution of program trading.[16] The unrestricted model is

$$x_t^2 = \alpha_0 + \sum_{i=1}^{K}\alpha_i x_{t-i}^2 + \beta_0 D_f + \sum_{i=1}^{K}\beta_i D_f x_{t-i}^2 \tag{5}$$

where $D_f = \begin{Bmatrix} 0 \text{ for } f < f^x \\ 1 \text{ for } f \geq f^x \end{Bmatrix}$

f^x = The date corresponding to the advent of program trading.

The different specifications of the model can be tested against one another to determine whether the eq. (3) restrictions significantly increase the sum of the squared residuals (*SSR*) of the estimate. The test employed is an F test on the joint hypothesis that $\beta_0 = \beta_1 = \ldots = \beta_k = 0$ with 6 degrees of freedom in the numerator and 3718 in the denominator.

Identifying the advent of program trading is problematic. A futures contract on the Value Line Composite Index began trading in February, 1982 about one month prior to the opening of S&P500 futures trading, but trading in the Value Line Index has never been brisk. Program trading was, however, in full force after July 1984 when a futures contract on the Major Market Index was introduced.

Three tests are conducted to control for this transition period. In the first, $D_f = 1$ for April 1982–October 1989 (the end of the sample) and zero otherwise. In the second, $D_f = 1$ for July 1984–October 1989 and zero otherwise. The third test examines the hypothesis that the parameters of the model are significantly different only during the transition period. In this test $D_f = 1$ for April 1982–July 1984 and zero otherwise.

None of the above three specifications of the unrestricted model produce estimates with significantly lower SSRs than obtained for the restricted version (see Table IV). The calculated F ratios are all well below the critical value of 2.10 so the hypothesis that $\beta_0 = \beta_1 = \ldots = \beta_k = 0$ is accepted. The data examined here are not consistent with the claim that trading in stock index futures increases price volatility in the cash market.

CONCLUSION

This article examines the hypothesis that program trading results in greater price volatility in the cash market for stocks. The data examined reject this hypothesis.

The article tests for the presence of ARCH effects in daily stock price returns, controls for these effects by modeling them, and tests to determine whether the estimated parameters of the model shift subsequent to the institution of program trading. No evidence is found of a shift in the model's parameters. This accords well with most previous work on the issue. In addition, because the model employed explicitly accounts for ARCH effects, the finding is not subject to the criticism that it is a fortuitous result produced by heteroskedasticity.

[16]Engle (1982); pp. 1001–1003 shows that OLS estimates of ARCH models may produce standard errors that are generally greater than maximum likelihood estimation. The gain in efficiency from using a maximum likelihood estimator increases as the parameters of the x^2 process approach one. See Engle (1982); p. 999. Each of the coefficients in the Table II estimate is far closer to zero than to one and their sum is about .5. This plus the size of the sample used in this study are basic to the strength of the results.

Table IV
***F*-TESTS OF MODEL RESTRICTIONS**

$x_t^2 = \alpha_0 + \sum_{i=1}^{K} \alpha_i x_{t-i}^2 + \beta_0 D_f + \sum_{i=1}^{K} \beta_i D_f x_{t-i}^2$	*SSR*	*F*-Ratio
Restricted Model: $D_f = 0$	.002951	
Test 1: $D_f = 1$ for 4/1982-10/1989		
$D_f = 0$ otherwise	.002948	.63
Test 2: $D_f = 1$ for 7/1984-10/1989		
$D_f = 0$ otherwise	.002948	.63
Test 3: $D_f = 1$ for 4/1982-7/1982		
$D_f = 0$ otherwise	.002950	.21

Bibliography

Anderson, Ronald W. (1984, Fall): "Some Determinants of the Votality of Futures Prices," *Journal of Futures Markets,* 331–348.

Black, Fischer, and Scholes, Myron. (1973, May/June): "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy,* 637–654.

Bollerslev, Tim. (1987, August): "A Conditional Heteroskedastic Time Series Model For Speculative Prices and Rates of Return," *The Review of Economics and Statistics,* 542–547.

Cagan, Phillip. (1981): "Financial Futures Markets: Is more Regulation Needed?," *Journal of Futures Markets,* 1,2:169–192.

Cinar, E. Mine. (1987, January/February): "Evidence on the Effect of Option Expirations on Stock Prices," *Financial Analysts Journal,* 55–57.

Cornell, Bradford, and French, Kenneth R. (1983, Spring): "The Pricing of Stock Index Futures," *Journal of Futures Markets,* 1–14.

Davis, Carolyn D., and White, Alice P. (1987, August): "Stock Market Volatility," Staff Study Number 153, Board of Governors of the Federal Reserve System.

Diebold, Francis X. (1988): *Empirical Modeling of Exchange Rate Dynamics,* Springer-Verlag.

Edwards, Franklin R. (1988, January/February): "Does Futures Trading Increase Stock Market Volatility?," *Financial Analysts Journal,* 63–69.

Edwards, Franklin R. (1988, August): Futures Trading and Cash Market Volatility: Stock Index and Interest Rate Futures," *Journal of Futures Markets,* 421–438.

Engle, Robert F. (1982): "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of U.K. Inflation," *Econometrica,* 50:987–1008.

Fama, Eugene F. (1965, January): "The Behavior of Stock Market Prices," *Journal of Business,* 34–105.

Fama, Eugene F. (1970, May): "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance, Papers and Proceedings,* 383–417.

Figlewski, Stephen. (1984, July): "Hedging Performance and Basis Risk in Stock Index Futures," *Journal of Finance,* 657–669.

Fortune, Peter. (1989, November/December): "An Assessment of Financial Market Volatility: Bills, Bonds and Stocks," *New England Economic Review,* 13–28.

Greene, William H. (1990): *Econometric Analysis,* Macmillan Co., 416–419.

Grossman, Sanford J. (1988, August): "Program Trading and Stock and Futures Price Volatility," *Journal of Futures Markets,* 413–419.

Harris, Lawrence. (1988, October): "S&P500 Futures and Cash Stock Price Volatility," Working Paper #88-1, University of Southern California.

Jarque, C. M., and Bera, A. K. (1981): "An Efficient Large-Sample Test for Normality of Observations and Regression Residuals," Manuscript, Australian National University, Canbera.

Jones, Charles P., and Wilson, Jack W. (1989, November/December): "Is Stock Price Volatility Increasing?," *Financial Analysts Journal,* 20–26.

Kmenta, Jan. (1986): *Elements of Econometrics,* Macmillan.

Mandelbrot, Benoit. (1963, October): "The Variation of Certain Speculative Prices," *Journal of Business,* 394–419.

Modest, David M., and Sundaresan, Mahadevan. (1983, Spring): "The Relationship Between Spot and Futures Prices in Stock Index Futures Markets: Some Preliminary Evidence," *Journal of Futures Markets,* 15–41.

Santoni, G. J. (1987, May): "Has Programmed Trading Made Stock Prices More Volatile?," *Federal Reserve Bank of St. Louis Review,* 18–29.

Stoll, Hans R. (1988, August): "Index Futures, Program Trading, and Stock Market Proceedures," *Journal of Futures Markets,* 391–411.

Stoll, Hans R., and Whaley, Robert E. (1987, March/April): "Program Trading and Expiration Day Effects," *Financial Analysts Journal,* 16–28.

Torres, Craig. (1990, February): "Program Trade Curbs sought by the Big Board," *Wall Street Journal.*

Torres, Craig, and Salwen, Kevin G. (1990, February): "New Hot Line Hunts Program Trading Abuse," *Wall Street Journal.*

Working, Hollbrook. (1960): "A Note on the Correlation of First Differences of a Random Chain," *Econometrica,* 28,4:916–918.