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current high level of volatility or should a forecast volatility be used to estimate a hedge ratio appropriate for the insurance horizon? Hill, Jain, and Wood (1988) show that unexpected changes in volatility are the most important risk factor in estimating the cost of portfolio insurance.

The second reason for focusing research on stock index futures is because understanding the futures price series can yield important insights into the nature of the true but largely unobservable stock market returns process. Broadly based stock indexes exhibit a large amount of positive day-to-day correlation that is caused in part by noncontemporaneous price changes by the component stocks in the index.² The stock index futures price series exhibits little if any dependence. The lack of autocorrelation in the stock index futures and the fact that the futures price frequently leads the index is shown by Stoll and Whaley (1988) and others. Due to transaction costs, the theoretical cost-of-carry futures model only describes the approximate relationship between the spot and the futures with each price series exhibiting different characteristics. Thus, the futures price series becomes an attractive expedient for the real but unobserved stock market process. By using options on futures contracts, the complication of incorporating dividend information into the option model is avoided since the futures price already contains the market's assessment of dividend payouts over the life of the futures contract. No precise relationship between the index and the futures is assumed, and no assumptions need be made concerning dividends and interest rates.

Since stock market volatility is shown to be strongly mean reverting,³ the volatility implied in option prices becomes an attractive surrogate to use in modeling market volatility. This is because the volatility implied in option prices is an estimate of market volatility at a particular point in time and, therefore, lends itself to time series analysis. The implied standard deviation (ISD) technique, originally proposed by Latane and Rendleman (1976), proves to be a useful estimator when working with market volatility. Schmalensee and Trippi (1978) and Chiras and Manaster (1978) both find ISD to be a better predictor of future market volatility than historic volatility measures.⁴

One problem that arises when dealing with stock index futures and futures options is that the spot and futures markets close at different times. To avoid noncontemporaneous closing prices, and other difficulties at the close, opening prices are used in this analysis. Parameters for the mean reversion model are generated from time series analysis of the volatility implied by stock index futures contract opening call option prices.

Akgiray (1989) uses several time series models, including the GARCH model, to model and forecast stock market volatility using the Center for Research in Security Prices (CRSP) value-weighted and equal-weighted indexes. The GARCH model provides the best forecast of market volatility in the 1963 and 1986 time frame.

This study uses 1986 and 1988 for the sample period. This provides the opportunity to test the models in a period which includes extreme shifts in volatility and to analyze the usefulness of the GARCH methodology on stock index futures prices.

²Positive autocorrelation will cause observed variance to understate the variance of the uncorrelated series (the true but unobservable index). See Scholes and Williams (1977), Roll (1981), and Perry (1985) for discussions of this point.

³The term "strongly mean reverting" implies that once market volatility departs from its long-run level it returns to that level rather quickly.

⁴The high-low variance estimator developed by Parkinson (1980) and combined with ISD by Beckers (1983) is an attractive alternative to the traditional estimates of variance but is not included in this study.

Parameters for the Garch model are generated from time series analysis of the period just prior to the forecast period.

Since the two modeling approaches are quite different, the expectation is that the models will perform differently in a forecasting role. The expectation is correct. The GARCH model is only marginally better than the naive forecast used for a benchmark, while the mean reversion model outperforms the benchmark by a considerable margin. The mean reversion model is found to be the best modeling approach for the sample considered.

THE MODELS

The Mean Reversion Model

The stochastic volatility option modeling literature assumes that the market return (S) and the instantaneous standard deviation (V) satisfy the following stochastic differential equations:

$$dS = \mu S dt + V S dw,$$

$$dV = \delta V dt + \theta V dz,$$

where dw and dz are Wiener processes with correlation r . The variables μ , θ , and δ are parameters that may depend on S , V , and t . The drift parameter of the volatility process, δ , is assumed to follow a mean reversion process;

$$\delta = a + bV,$$

where a and b are constants and $a > 0$ and $b < 0$. The expected volatility at time t is given by

$$E[V_t] = e^{bt}[V_{t-1} - MRL] + MRL, \quad (1)$$

where MRL is the mean reversion level, $-a/b$. The volatility of the volatility, θ , is the standard deviation of V . This mean reverting model (MRM) is the same as the model considered by Cox, Ingersoll, and Ross (1985) for the short-term rate of interest. This model is used also by Hull and White (1988) and Wiggins (1987) in simulating results for the stochastic volatility option pricing model (SVOPM). The MRM (1) is used to estimate the parameters of the stochastic volatility process, as represented by ISD, and to generate out of sample forecasts.

The GARCH Model

The autoregressive conditional heteroscedasticity (ARCH) model introduced by Engle (1982) allows the conditional variance to change over time as a function of past errors. The strength of the ARCH techniques is that the conditional means and variances can be estimated jointly using traditional specified models for economic variables. Bollerslev (1986) extends the ARCH process to GARCH, which allows for a more flexible lag structure. Bollerslev points out that the extension of the ARCH process is very much like the extension of the standard time series process to the general ARMA process.

For daily returns (R_t), the GARCH process of orders p and q , denoted as GARCH (p, q) can be described as follows:

$$R_t | \Omega_{t-1} \sim F(\mu_t, v_t),$$

$$\mu_t = \Phi_0 + \Phi_1 R_{t-1},$$

$$v_t = \alpha_0 + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^q \beta_j v_{t-j}, \quad (2)$$

and

$$\epsilon_t = R_t - \Phi_0 - \Phi_1 R_{t-1}, \quad (3)$$

where $p > 0$ and $q \geq 0$, and the parameters satisfy the conditions $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, $i = 1, \dots, p$, $j = 1, \dots, q$. $F(\mu_t, v_t)$ is the conditional distribution of the variable, with conditional mean μ_t , and variance v_t . Ω_{t-1} is the information set available at time t . For $q = 0$, the process reduces to the ARCH (q) process, and for $p = q = 0$, ϵ_t is just white noise. Bollerslev shows that the resulting GARCH (p, q) model is essentially a stationary ARCH (q) process.

The unconditional mean and variance of a GARCH process are constant while the conditional mean and variance are time dependent. The conditional variance v_t of the return series is large when the squared error term and prior variance are large.

Akgiray (1989) found that a GARCH (1,1) model is adequate for describing the stock market returns process and develops a GARCH volatility forecasting model. Akgiray's forecasting model for variance over a 20-day period is presented below:

$$v_m = \sum_{i=1}^{20} [(1 - (\Phi_1)^i)/(1 - \Phi_1)]^2 [A^{20-i} v_t + \sum_{j=0}^{19-i} \alpha_0 A^j], \quad (4)$$

where $A = \alpha_1 + \beta$ and v_t is given by eq. (2) for $p = q = 1$. In the analysis that follows, the forecast monthly variance is transformed into the forecast daily standard deviation V_{MF} for the 20-day period, therefore, $V_{MF} = \sqrt{(v_m/20)}$.

METHODOLOGY AND DATA

Three years (1986–1988) of transaction data from the Chicago Mercantile Exchange is used in this study. The opening at-the-money option price on the lead-month S & P 500 futures contract is selected each trading day along with the contemporaneous futures contract transaction price and S & P 500 Index level. With only two exceptions all transactions and the reported index level are within one minute of each other. Daily prices are gathered on the lead-month futures contract until that contract has one week until expiration, then prices are gathered on the next available futures contract. For example, on the first trading day of 1986 (1/2/86) the opening call price of the nearest-to-the money call (strike = 210) on the March 1986 S & P 500 Futures Contract is \$6.00. This transaction occurred at 8:33:02 am. The nearest futures transaction price is 211.75 and the Index level is 211.01. Prices on the March 1986 Futures Contract are gathered until March 17. At this point, data on calls on the June, 1986 Futures Contract are gathered until that contract is within one week of expiration. Thus, a continuous sequence of futures prices, index levels, and futures call option prices are gathered over the three-year period. This series of prices represent opening market conditions on a daily basis.

The at-the-money or nearest-the-money call is used because the nearest-the-money call has relatively high trading volume and, when modeled by the Black-Scholes option pricing model, has very little modeling bias or error when volatility is assumed to be stochastic. Hull and White (1988) and Wiggins (1987) both show that the stochastic volatility option pricing model yields results very similar to the Black-Scholes results for at-the-money options. Therefore, the implied standard deviation result of the Black-Scholes model is correct for the stochastic volatility model.

The ISD of the call options are then derived by solving Black's (1976) commodity option model for ISD using the Treasury bill yield closest to the expiration date,

time to expiration, option price and futures price.⁵ A Newton-Raphson iterative procedure employing the Manaster and Koehler (1982) starting algorithm is used to solve for the ISD. Historical variances are computed using the log of the price relatives of the futures contract prices and the index.

The volatilities used in this study are shown as standard deviations in terms of percent of daily price change. The daily standard deviation of the open-to-open futures returns for the three years 1986-88 is 1.475%. This equates to an annual standard deviation of 23.4%. The daily standard deviation of the S & P 500 Index is 1.411 and, in annual terms, 22.4%.⁶ The volatility of this period is above average because of the stock market crash in October of 1987.⁷

The first 81 trading days of 1986 are used to develop parameters used in the initial forecasting attempts.⁸ Using this 81-day period as the prior information set, a forecast of the standard deviation of the daily returns for the next 20 trading days is made. Since there are approximately 21 trading days in a calendar month, the forecast period is referred to as a month. Once a period is forecast, it then joins the information set. Successive months are forecast for a total of 33 forecasts. The crash of October, 1987 occurs in month 19.

GARCH Parameter Estimation

The GARCH model is implemented by using nonlinear regression techniques on the past 80 days of data to obtain maximum likelihood estimates of the parameters of (2) and (3). This technique simultaneously estimates the parameters of the GARCH model. Since daily returns on futures contracts exhibit very little autocorrelation, Φ_1 is expected to be insignificant, and this is the case.⁹ The only parameter that is consistently, highly significant (99% level) is β . The value of β ranges between 0.63 and 0.99. The value of α_0 is usually positive, usually small in relation to the forecast variance, and is significant (95% level) in 8 out of the 33 sets of parameters. The value of α_1 does not exceed 0.02 and is significant only two times.

When the GARCH model is run using the overall sample ($n = 700$) the results are highly significant ($R\text{-square} = 0.9911$). But the only parameters that are significant in the overall sample are α_1 and β . The following parameter estimates are obtained from the overall sample:

Parameter	Estimate	T-ratio
α_1	0.01271	9.6932
β	0.98577	266.1446

The combined value of these parameters, $\alpha_1 + \beta$, is 0.9985. A value close to 1.0 results in a persistent estimate of volatility. A persistent estimate means that after

⁵In using Black's model two small and off-setting errors are introduced into the estimate of ISD. Using a European option pricing model instead of the Barone-Adesi and Whaley (1987) American option approximation, causes the estimated ISDs to be about 0.25% high for the options under study. This bias is equivalent to less than 2 cents error in price. By not using a stochastic variance option pricing model, the ISD is underestimated by approximately the same amount.

⁶Transformations from daily volatility to annual volatility are made using 252 trading days for the calendar year.

⁷Edwards (1988) reports a S & P 500 index volatility of 13.6% for the period 6/73-4/82.

⁸The 81-day period is selected as a compromise between having a large number of observations for parameter estimation and having a large number of observations remaining to use for the out-of-sample forecast.

⁹Akgiray (1989) found Φ_1 to be significant, which is to be expected when working with daily stock index returns.

volatility changes, GARCH forecasts the new level to be approximately the same in the future. This is the case with most of the GARCH estimates. It is noteworthy that the only parameters in (2) and (3) which are significant are α_1 and β . This is due to the lack of correlation in daily futures price returns.

Once the parameters are estimated, they are inserted into Akgiray's GARCH forecasting model (eq. (4)) to develop forecasts. The volatility input used in the GARCH forecasting model is the prior month's volatility.

Mean Reversion Model Parameter Estimation

After taking a natural log transformation of eq. (1), linear regression is used to estimate the slope parameter for the mean reversion model. The slope parameters used in the forecasts are estimated using the prior 80 days of data. The estimate of the slope parameter is significant in each period and varies between 0.6189 and 0.9904. The log of the slope parameter becomes b in eq. (1).

Keim and Smirlock (1987) provide close-to-close volatility estimates for S & P 500 futures contracts from 4/82 to 12/86 which indicates a typical daily standard deviation of about 1.10%. Other researchers show that open-to-open stock return variance is 10% to 20% greater than close-to-close variance. Based on these factors, a mean reversion level of 1.20% is chosen for the MRM forecasts. This selection of a MRM is *ad hoc* but has no impact on the mean square error analysis.

Two forecasts are produced using the MRM model: one using the prior month's volatility V_{MA-1} as the volatility input V_{t-1} and another using the prior trading day's ISD as V_{t-1} . The time factor used in the MRM forecasts is 20 days. Thus, t in eq. (1) is 20, and the input volatility V_{t-1} reverts toward the MRL for 20 days.

In the *ex post* examination of the sample, the whole ISD series ($n = 756$) is used to estimate the slope parameter for the MRM model. The resulting R^2 , 0.9208, indicates that the model fits the data well. The slope parameter estimate is 0.9519 with a standard error of 0.0110 and a T -value of 86.24. To find b , the log of the slope parameter is taken and this gives -0.04929 . This estimate of b indicates that the mean reversion process has a half-life of 14 trading days.¹⁰ Thus, when volatility deviates from the long-term level, the mean reversion level, it is estimated that the deviation will be halved in 14 trading days or about three weeks.

Benchmark Forecast

Two benchmark forecasts are computed based on the premise that the immediate future will be like the immediate past.¹¹ One benchmark assumes that the next month's volatility will be the same as the prior month ($V_{MF} = V_{MA-1}$). The other benchmark forecast assumes that the next month's volatility will be the same as the most recent daily ISD ($V_{MF} = ISD_t$).

RESULTS

Table I lists the actual daily standard deviation for each forecast month, the monthly forecasts, and the forecast errors. The forecast error is computed by subtracting the actual from the forecast volatility ($V_{MF} - V_{MA}$). The GARCH forecast

¹⁰The half-life, t_h , is computed using $t_h = -\ln 2/b$.

¹¹Other naive forecasts are evaluated and found to be less accurate than the two included in this study. The less accurate naive forecast include forecasts based on long run levels of volatility; both ISD and directly observed levels.

and the two forecasts produced by the MRM model are presented in Table I. The MRM-ISD is the mean reversion model forecast based on ISD and the MRM-OBS is the forecast based on the prior month's standard deviation. The forecasts are made using the prior 80 days to estimate the parameters. The GARCH forecast is made using the α_1 and β parameter estimates while the other parameters are set to zero. This produces the best GARCH forecast of the various alternatives used in this study. When the other GARCH parameters enter the model, the results deteriorate.

The forecasts are analyzed by means of a number of different statistics: mean error (ME), root mean square error (RMSE), mean absolute error (MAE), and mean absolute percent error. These statistics are presented in Table II along with their calculation methods. The benchmark forecast using the prior months observed volatility as the forecast is the better of the two benchmarks in ME and RMSE while the ISD benchmark has the lower MAPE. The mean reversion model using the prior months observed volatility (MRM-OBS) as a forecasting input proves to be the best forecasting model for the overall sample. The GARCH results are only marginally better than the best benchmark.

Pre-Crash Period Forecasts

Since the crash is such a dominant feature in the sample, the forecast results for the 18-month period prior to the crash is analyzed to see how the models perform in a less volatile, more normal period. Table III presents the results of the analysis of the pre-crash period. There are 18 months available in the pre-crash period. These results show that both MRM forecasts are decidedly better than the benchmarks and the GARCH forecast, and that both MRM models produced similar results.

In the pre-crash period, the better benchmark is the one using ISD. This is the same result found by Schmalensee and Trippi (1978) and Chiras and Manaster (1978). The ISD benchmark is markedly better in the overall sample except for the month 20 forecast. This post-crash forecast is made based on an extreme ISD input. This extreme input plus a very slow reversion rate also hurts the performance of the MRM-ISD forecast in that month.

The GARCH results presented in Table III include GARCH-A, which is the GARCH forecast model run with all parameters included. This version of the GARCH model produces better results in the pre-crash period than the model with only α_1 and β included. This is the opposite of the GARCH results for the whole period, where the model with only α_1 and β performs better than the model with all parameters. In the pre-crash period, GARCH-A results are an improvement over the benchmark results using the prior months volatility but are inferior to the benchmark using ISD.

Ex Post Analysis

To test the models with parameters estimated over a long period of time, an *ex post* analysis is performed. As described above, parameters for GARCH and MRM using the total sample are estimated, and then these estimates are used to make monthly predictions. The results, along with the best benchmark for the overall period, are shown in Table IV. The forecasts using MRM-ISD show the biggest improvement when long-term parameters are used. While the forecasts from MRM-OBS are degraded slightly, the GARCH forecasts are also improved but are still only marginally better than the benchmark forecasts.

Table I
MONTHLY FORECASTS OF S&P 500 FUTURES CONTRACT RETURN, STANDARD DEVIATION (1986-1988)

Month	Actual SD	Forecast			Error		
		GARCH	MRM-ISD*	MRM-OBS**	GARCH	MRM-ISD	MRM-OBS
1	1.0916	1.1404	1.1651	1.1914	0.0489	0.0736	0.0998
2	1.0912	1.0912	1.1523	1.1881	-0.0001	0.0611	0.0969
3	1.1219	1.0896	1.1717	1.1909	-0.0322	0.0499	0.0690
4	0.8393	1.1203	1.1764	1.1923	0.2811	0.3371	0.3530
5	1.6520	0.8403	1.1874	1.1883	-0.8117	-0.4646	-0.4637
6	1.0470	1.6477	1.2000	1.2000	0.6008	0.1530	0.1531
7	0.8718	1.0459	1.1999	1.1999	0.1742	0.3282	0.3282
8	1.1560	0.8711	1.1998	1.1999	-0.2849	0.0438	0.0438
9	0.8603	1.1531	1.1999	1.2000	0.2928	0.3396	0.3397
10	1.3015	0.8577	1.1129	1.1304	-0.4439	-0.1886	-0.1711
11	0.9474	1.2939	1.1779	1.2154	0.3465	0.2305	0.2680
12	1.5674	0.9460	1.1554	1.1508	-0.6214	-0.4120	-0.4166
13	1.8438	1.5671	1.1628	1.2698	-0.2767	-0.6810	-0.5740
14	1.0770	1.8400	1.2013	1.2918	0.7630	0.1243	0.2148
15	0.8387	1.0747	1.1774	1.1850	0.2360	0.3387	0.3463
16	0.5920	0.8384	1.1624	1.1417	0.2464	0.5704	0.5497

17	0.8536	0.5928	0.8473	0.7027	-0.2607	-0.0063	-0.1508
18	1.5726	0.8536	1.1654	1.1173	-0.7191	-0.4073	-0.4554
19	4.3078	1.6236	1.1691	1.2921	-2.6842	-3.1387	-3.0157
20	3.4607	4.2818	5.1095	3.7625	0.8211	1.6488	0.3018
21	2.0738	3.4365	1.3963	1.6378	1.3626	-0.6776	-0.4360
22	2.2800	2.0857	1.2628	1.3000	-0.1942	-1.0172	-0.9799
23	1.1290	2.2663	1.2243	1.2417	1.1373	0.0953	0.1126
24	0.8601	1.1288	1.2029	1.1991	0.2687	0.3428	0.3390
25	1.3837	0.8591	1.2013	1.1694	-0.5246	-0.1824	-0.2143
26	0.7920	1.3820	1.2148	1.2136	0.5901	0.4228	0.4217
27	1.3783	0.7990	1.1909	1.1541	-0.5793	-0.1873	-0.2242
28	1.1842	1.3711	1.1997	1.2004	0.1868	0.0155	0.0161
29	1.0523	1.1880	1.1967	1.1997	0.1357	0.1443	0.1474
30	0.9012	1.0510	1.2000	1.2000	0.1498	0.2988	0.2988
31	0.6626	0.8970	1.1998	1.1998	0.2344	0.5373	0.5372
32	0.8284	0.6618	1.1679	1.1430	-0.1667	0.3395	0.3146
33	0.6552	0.8272	1.1749	1.1569	0.1720	0.5197	0.5016

Note: Error = forecast - actual. Crash occurs in month 19.

*Mean reversion model using ISD as the input V_{t-1} .

**Mean reversion model using the prior months volatility V_{MA-1} as V_{t-1} .

Table II
ANALYSIS OF FORECASTS OF MONTHLY STANDARD DEVIATIONS* (1986–1988)

Model	ME	RMSE	MAE	MAPE
$V_{MF} = V_{MA-1}$	0.0145	0.6985	0.4790	0.3486
$V_{MF} = ISD_t$	-0.0627	0.8053	0.4738	0.3172
GARCH**	0.0136	0.6886	0.4742	0.3461
MRM-ISD	-0.0105	0.7210	0.4357	0.3131
MRM-OBS	-0.0378	0.6373	0.3926	0.3064

*Where

$$ME = (1/33) \sum_{M=1}^{33} E_M$$

$$RMSE = [(1/33) \sum_{M=1}^{33} E_M^2]^{1/2}$$

$$MAE = (1/33) \sum_{M=1}^{33} |E_M|$$

$$MAPE = (1/33) \sum_{M=1}^{33} |E_M/V_M|$$

Note: Error (E_M) = $V_{MF} - V_{MA}$.

**Only GARCH parameters α_1 and β are used in the forecast.

Table III
ANALYSIS OF FORECASTS OF MONTHLY STANDARD DEVIATIONS,
PRE-CRASH PERIOD ($N = 18$)

Model	ME	RMSE	MAE	MAPE
$V_{MF} = V_{MA-1}$	-0.0243	0.4333	0.3585	0.3166
$V_{MF} = ISD_t$	-0.1764	0.4029	0.3228	0.2692
GARCH*	-0.0256	0.4322	0.3578	0.3157
GARCH-A**	0.0087	0.4081	0.3415	0.3058
MRM-ISD	0.0273	0.3277	0.2672	0.2542
MRM-OBS	0.0350	0.3262	0.2830	0.2721

*Only GARCH parameters α_1 and β are used in the forecast results presented in this row.

**All parameters of the GARCH model are used in the forecast results presented on this row.

Table IV
EX POST ANALYSIS OF MONTHLY STANDARD DEVIATIONS (1986–1988)

Model	ME	RMSE	MAE	MAPE
$V_{MF} = V_{MA-1}$	0.0145	0.6985	0.4790	0.3486
GARCH	0.0174	0.6728	0.4706	0.3452
MRM-ISD	-0.0935	0.6610	0.3975	0.2972
MRM-OBS	-0.0650	0.6500	0.4127	0.3082

SUMMARY AND CONCLUSIONS

The mean reversion model of eq. (1) provides an accurate description of the stock index futures returns volatility process as represented by implied standard deviation. A high R^2 results when ISD is modeled using eq. (1). The accuracy of the mean reversion model in modeling ISD also translates into a superior forecasting model. The results of the out-of-sample forecasting tests and the *ex post* analysis show that the MRM model provides the best forecast. This result supports Bollerslev's (1990) conjecture that modeling with ISD could potentially lead to methods superior to GARCH in assessing estimates of volatility in financial data.

The version of the MRM model using the prior month's volatility, MRM-OBS, provides the best forecast in the out-of-sample forecast test covering the entire sample period. The version of the mean reversion model that uses the prior day's ISD, MRM-ISD, also performs well except for one monthly forecast. Table III shows that when the pre-crash period is examined, the superiority of the MRM model is clear and the results from the total sample are degraded by the crash. The mean reversion model also has flexibility, since the mean reversion level can be modified to reflect the circumstances of the estimation period or to provide a Bayesian input into the modeling process.

The GARCH model does not perform well in its forecasting role. This is because it tends to provide a persistent forecast. This type of forecast is effective only during periods when changes in volatility are relatively small. Based on the analysis presented, it is concluded that the mean reversion model is the best of the two models in representing the volatility process of returns on futures contracts on the S & P 500 Index. The power of the GARCH model is ineffective on stock index futures prices because of the lack of correlation in the returns.

A simple and effective implementation of the MRM model is to use the long-run slope parameter estimate and either ISD or an estimate of observed standard deviation over the past 10 or 20 days. For example, if three-week forecasts are made, then input volatility reverts half the distance to the selected mean reversion level. If an MRL of 1.20 is used, and the recent ISD is 1.00, then the MRM-ISD forecast for the next three-week period would be 1.10.

An area for future research is a test of high-low variance estimators in the mean reversion model. Beckers (1983) shows that a high-low variance estimator combined with ISD is more effective at forecasting variance than traditional close-to-close variance estimators.

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