# Risk Premia and Price Volatility in Futures Markets

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## INTRODUCTION

One of the major economic functions of futures markets is a risk-sharing role.[1] Firms hedge their planned sales of products by selling futures contracts or hedge their purchases of inputs by buying futures contracts. Besides hedgers, there are speculators in the futures markets who seek profits from price changes. If speculators are risk-averse as a whole in the futures markets, hedgers must pay a risk premium to speculators as a compensation for the risk-sharing in the futures markets. This theory is developed by Keynes (1930) and Hicks (1946). They argue that the futures price for a storable commodity should normally be slightly below the expected future spot price to induce speculators to assume some risk. The difference between the futures price and the expected future spot price is a risk premium that they call "normal backwardation." The theory is based on the assumption that hedgers, as a whole, are net short.

Early studies on risk premia in commodity futures markets adopt Keynes' and Hicks' original assumption that hedgers as a whole are net short. So they view the risk premium as returns to speculators who continuously hold net long positions. These include the studies of Telser (1958) and Dusak (1973).

The assumption that speculators continuously hold net long positions is relaxed by several researchers, in particular, Cootner (1960) and Carter, Rousser, and Schmitz (1983) (CRS). CRS respecify Dusak's study by taking into account that speculators can be either net short or net long. Assuming that risk parameters are stochastic and a function of speculators' actual net position, they find that estimates of systematic risks are not zero.

Financial support from the Columbia Futures Center is gratefully acknowledged. We would like to thank Jinook Jeong and two anonymous referees for their helpful comments.

[1]Another major economic function of the futures market is a price discovery role. Futures price for a commodity represents all the information about the future cash price. So, economic agents make plans and decisions by looking at the futures price.

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The Journal of Futures Markets, Vol. 11, No. 2, 165–177 (1991)

CCC 0270-7314/91/020165-13$04.00

The availability of data on traders' commitments enables researchers to allocate a risk premium in futures markets between speculators as a whole and hedgers as a whole. Houthakker (1957) first utilized the data on traders' commitments to estimate the profits of speculators and hedgers. Although he does not perform an explicit test of whether there exist risk premia in the futures markets, his study indicates that speculators obtain risk premia in return for their risk-bearing.

Since Houthakker's study, researchers have been decomposing speculators' profits into risk premia and profits due to forecasting ability. Rockwell (1967) distinguishes speculators' rewards for risk-bearing from those of forecasting ability. He defines risk premium as the return earned by a hypothetical speculator who follows a naive strategy of being long when hedgers as a whole are net short and short when hedgers are net long. He finds that the rate of risk premium is 2.3% per year on the average for all markets and that the risk premium is negative for 11 of 25 markets.

Employing nonparametric statistical methods, Chang (1985) examines the existence of positive profits to speculators in wheat, corn, and soybean futures markets. He finds that there is a reward to a hypothetical speculator defined by Rockwell for risk-bearing. He also tests whether or not large speculators' profits are combined rewards for both risk-bearing and superior forecasting ability. He finds that the large speculators are rewarded for both risk-bearing and forecasting ability in wheat futures market while they are rewarded for only risk-bearing in corn and soybean futures markets.

This study attempts to measure the magnitudes of the risk premia which are allocated between hedgers and speculators. Like previous studies, the inaccessibility of data on traders' commitments allows consideration of only large hedgers and large speculators.

## DATA

This study uses futures contracts of six commodities: wheat, corn, oats, soybeans, soybean oil, and soybean meal futures which are traded at the Chicago Board of Trade (CBOT); and five foreign currencies: Canadian dollar, British pound, Deutsche mark, Japanese yen, and Swiss franc, which are traded at the International Monetary Market (IMM) of the Chicago Mercantile Exchange (CME). There are five different contracts depending on delivery month per year for the six commodities and four for the five foreign currencies.

Data on futures prices for the six commodities are from the Chicago Board of Trade Commodity Data Tape. For the 30 commodity futures contracts, closing price quotations are used. Data on futures prices for the five foreign currencies are collected from the data tape provided by the Center for the Study of Futures Markets at Columbia University. For the 20 futures contracts of foreign currencies, settlement price quotations are used.

Traders' market positions of being long or short are available from *Commitments of Traders in Commodity Futures* published by the Commodity Futures Trading Commission (CFTC). This publication reports traders' commitments over time, which are divided into three groups: (large) hedging, (large) speculative, and nonreporting. Traders whose commitments in any one futures contract exceed the reporting limit are required to report their entire position to the Commodity Futures Trading Commission, which classifies futures commitments as either hedging or speculative. The above publication reports traders' commitments which are classified according to a commodity, not a delivery month within a commodity. This study, however, needs traders' commitments on a delivery month within the commodity futures. To solve this problem it is assumed that all delivery months within a commodity futures have the same percentage of open interest.[2]

For the commodity futures, the sample periods are from January 1976 through December 1984, excluding the year 1982. For the foreign currency futures, the sample periods are from January 1977 through September 1988, again excluding 1982. The year of 1982 is excluded from the sample because the CFTC discontinued the reports of traders' commitments during that year.

Since the data on large hedgers' (speculators') net positions in a futures delivery month within a commodity and the data on prices of this futures contract are obtained, the rate of return to large hedgers (speculators) as a whole for month $t$ is calculated as follows.

$$\pi_t = (p_{h,t} - p_{h,t-1})/p_{h,t-1} \text{ if large hedgers (speculators)}$$

as a whole are net long.

$$\pi_t = (p_{h,t-1} - p_{h,t})/p_{h,t-1} \text{ if large hedgers (speculators)}$$

as a whole are net short.

where $p_{h,t}$ is the price of a futures contract at the end of month $t$, which matures at time $h$. The results of this study are interpreted as those of large speculators as a whole and large hedgers as a whole.[3] These are interpreted also as those of a representative speculator (hedger) who follows a simple strategy of being long when large speculators (large hedgers) as a group are net long and short when they are net short.

## THE HYPOTHESES

The first hypothesis to be tested is whether or not the representative large hedger consistently loses money in the futures markets, that is whether or not the large hedger's average profit is significantly negative. The second hypothesis to be tested is whether the representative large speculator consistently makes a profit in the futures markets, that is whether or not the large speculator's average profit is significantly positive. The first hypothesis and the second hypothesis are not identical because there are also small traders (hedgers and speculators) in the futures markets. The third hypothesis to be tested is whether the representative large speculator makes a profit by a simple strategy of being in the opposite position of the representative large hedger or by a better strategy. For this test, it is convenient to adopt the hypothetical speculator who follows a simple strategy of being long when large hedgers are net short and short when large hedgers are net long as Rockwell defines. So, a test is conducted to determine whether or not the representative large speculator makes more than the profit which the hypothetical speculator makes. The excess of the representative large speculator's profit over the hypothetical speculator's profit, on average, implies that the representative large speculator is us-

[2]This assumption is identical to the assumption by Chang (1985). Houthakker (1957) and Rockwell make a similar assumption. In their studies, traders' commitments on a commodity futures are classified according to trading groups and delivery months. So they assume that the distribution of total open interest according to trading groups and according to delivery months is statistically independent to obtain traders commitments on each delivery month within a commodity futures.

[3]The market shares of large hedgers, large speculators, and small traders are about 40% to 60%, 10% to 20%, and 30% to 40% on the average, respectively.

ing a better strategy than the simple one. It indicates that large speculators earn additional profits at the expense of small traders since large speculators have access to better information.

## EMPIRICAL RESULTS

The results of tests for the representative large hedger and speculator are reported in Table I. The representative large hedger's average profit is significantly negative for most of these futures contracts. The results support the evidence that the repre-

**Table I**
**MEAN PROFITS OF LARGE HEDGERS AND SPECULATORS**

| Commodity | Maturity | Large Hedgers' Profits | Large Speculators' Profits |
|---|---|---|---|
| Wheat | March | −14.83 (6.34) | 35.64 (5.55) |
| | May | −13.75 (6.10) | 38.23 (4.98) |
| | July | −11.73 (5.70) | 33.38 (4.81) |
| | Sept. | −15.05 (5.58) | 36.98 (4.50) |
| | Dec. | −8.95 (5.28) | 34.66 (4.25) |
| Corn | March | −17.49 (4.29) | 19.02 (4.23) |
| | May | −18.21 (4.24) | 21.01 (4.12) |
| | July | −20.24 (4.58) | 21.46 (4.52) |
| | Sept. | −19.84 (4.76) | 21.83 (4.68) |
| | Dec. | −18.64 (4.39) | 19.96 (4.34) |
| Oats | March | −15.49 (6.52) | 16.07 (6.90) |
| | May | −7.21 (6.42) | 12.16 (6.51) |
| | July | −17.20 (7.24) | 11.16 (7.06) |
| | Sept. | −8.96 (7.19) | 14.11 (7.31) |
| | Dec. | −11.47 (6.59) | 13.54 (6.74) |
| Soybeans | March | −15.49 (6.52) | 26.48 (6.19) |
| | May | −17.09 (6.27) | 27.13 (5.95) |
| | July | −17.20 (7.24) | 27.69 (6.95) |
| | Sept. | −19.45 (6.95) | 26.86 (6.70) |
| | Dec. | −17.87 (6.07) | 20.70 (5.99) |
| Soybean meal | March | −7.95 (6.64) | 25.85 (6.19) |
| | May | −7.73 (6.66) | 24.54 (6.26) |
| | July | −10.74 (7.35) | 23.37 (7.08) |
| | Sept. | −10.60 (7.25) | 23.67 (6.93) |
| | Dec. | −6.89 (6.17) | 18.12 (5.97) |
| Soybean oil | March | −13.51 (7.54) | 29.35 (7.12) |
| | May | −13.56 (7.14) | 31.24 (6.63) |
| | July | −12.57 (7.30) | 29.34 (6.86) |
| | Sept. | −15.27 (8.19) | 32.10 (7.68) |
| | Dec. | −10.60 (6.96) | 23.93 (6.66) |
| Canadian dollar | March | −6.33 (1.31) | 7.42 (1.26) |
| | June | −4.09 (1.23) | 5.80 (1.17) |
| | Sept. | −5.35 (1.17) | 6.01 (1.14) |
| | Dec. | −4.63 (1.20) | 5.62 (1.16) |

Table I (Cont.)

| Commodity | Maturity | Large Hedgers' Profits | Large Speculators' Profits |
|---|---|---|---|
| British pound | March | −18.54 (2.75) | 18.72 (2.73) |
| | June | −18.04 (2.84) | 18.00 (2.84) |
| | Sept. | −17.93 (2.98) | 18.07 (2.97) |
| | Dec. | −19.64 (2.97) | 20.91 (2.88) |
| German deutsche mark | March | −13.18 (3.52) | 15.02 (3.45) |
| | June | −17.36 (4.04) | 16.64 (4.41) |
| | Sept. | −13.33 (3.48) | 13.09 (3.49) |
| | Dec. | −13.55 (3.44) | 13.83 (3.43) |
| Japanese yen | March | −17.12 (3.11) | 17.29 (3.10) |
| | June | −19.79 (3.33) | 16.56 (3.55) |
| | Sept. | −17.31 (3.11) | 15.10 (3.26) |
| | Dec. | −14.64 (4.40) | 14.43 (4.41) |
| Swiss franc | March | −13.84 (3.49) | 11.44 (3.57) |
| | June | −16.16 (3.48) | 14.90 (3.54) |
| | Sept. | −14.01 (3.68) | 11.49 (3.77) |
| | Dec. | −15.17 (3.63) | 13.13 (3.71) |

*Note:* The time span is January 1976 through December 1984 for the six commodities and January 1979 through September 1988 for the five currencies, excluding the year 1982. The number of observations is 96 for the commodity futures and 123 for the currency futures.

Figures in parentheses are standard errors (SE). Mean and SE $\times\ 10^3$

sentative large hedger consistently loses money in the futures markets. This indicates that large hedgers pay some risk premium in the futures markets.

The representative large speculator's average profit is significantly positive for futures contracts. This evidence furnishes an explanation on why large speculators continue to remain in the futures markets. The above findings are consistent with Chang's (1985) even though his study is limited to only wheat, corn, and soybeans futures.

This study provides information about the magnitudes of the profits of the representative large hedger and speculator which Chang does not consider. The large speculator's average profits are higher than the large hedger's average losses in all commodity futures markets except the corn futures market. In the wheat futures market, the average rates of the large speculator's profits are from 3.33% to 3.82% per month while the average rates of the large hedger's losses are from 0.87% to 1.51% per month. In contrast, in the corn futures market, the average rates of the large speculator's profits are from 1.90% to 2.18% per month and the average rates of the large hedger's losses are from 1.75% to 2.02% per month. In the other commodity futures markets (oats, soybeans, soybean meal, and soybean oil), the average rates of the large speculator's profits are from 1.81% to 3.21% per month and the average rates of the large hedger's losses are from 0.69% to 1.95% per month.

The average rates of profits (losses) of the large speculator (large hedger) in the foreign currency futures markets are lower than those in the commodity futures markets. The average rates of the representative large speculator's profits are about equal to the average rates of the representative large hedger's losses in all the foreign currency futures markets. In the Canadian dollar futures market, the average rates of the large speculator's profits are from 0.56% to 0.74% per month and the average rates of the large hedger's losses are from 0.41% to 0.63% per month. In the other foreign currency futures markets, the average rates of the large speculator's profits are from 1.14% to 2.0% per month while the average rates of the large hedger's losses are from 1.32% to 1.76% per month.

The results of tests whether the representative large speculator has superior information are reported in Table II. The null hypothesis that the representative large speculator's profit is equal to the hypothetical speculator's profit on the average is rejected for every delivery month of wheat futures at a 1% significance level. This

**Table II**
**TESTS OF SPECULATORS' SUPERIOR INFORMATION**

| Commodity | Maturity | Mean Diff. | SE | $t$ |
|---|---|---|---|---|
| Wheat | March | 20.81 | 8.43 | 2.46 |
| | May | 24.48 | 7.87 | 3.11 |
| | July | 21.65 | 7.46 | 2.90 |
| | Sept. | 21.93 | 7.17 | 3.05 |
| | Dec. | 25.71 | 6.80 | 3.78 |
| Corn | March | 1.52 | 6.02 | 0.25 |
| | May | 2.80 | 5.91 | 0.47 |
| | July | 1.22 | 6.43 | 0.19 |
| | Sept. | 1.98 | 6.68 | 0.29 |
| | Dec. | 1.32 | 6.17 | 0.21 |
| Oats | March | 4.68 | 9.69 | 0.48 |
| | May | 4.95 | 9.15 | 0.54 |
| | July | 3.92 | 9.91 | 0.39 |
| | Sept. | 5.16 | 10.26 | 0.50 |
| | Dec. | 2.08 | 9.42 | 0.22 |
| Soybeans | March | 10.99 | 8.99 | 1.22 |
| | May | 10.04 | 8.65 | 1.16 |
| | July | 10.49 | 10.04 | 1.04 |
| | Sept. | 7.41 | 9.65 | 0.76 |
| | Nov. | 2.83 | 8.53 | 0.33 |
| Soybean meal | March | 17.90 | 9.08 | 1.97 |
| | May | 16.81 | 9.41 | 1.83 |
| | July | 12.64 | 10.21 | 1.23 |
| | Sept. | 13.06 | 10.03 | 1.30 |
| | Dec. | 11.23 | 8.58 | 1.30 |
| Soybean oil | March | 15.84 | 10.37 | 1.52 |
| | May | 17.67 | 9.74 | 1.81 |
| | July | 16.77 | 10.02 | 1.67 |
| | Sept. | 16.83 | 11.22 | 1.49 |
| | Dec. | 13.34 | 9.63 | 1.38 |
| Canadian dollar | March | 1.09 | 1.81 | 0.60 |
| | June | 1.71 | 1.70 | 1.01 |
| | Sept. | 0.67 | 1.63 | 0.41 |
| | Dec. | 0.99 | 1.67 | 0.59 |

Table II (Cont.)

| Commodity | Maturity | Mean Diff. | SE | t |
|---|---|---|---|---|
| British pound | March | 0.18 | 3.87 | 0.04 |
| | June | −0.04 | 4.01 | −0.01 |
| | Sept. | 0.15 | 4.21 | 0.03 |
| | Dec. | 1.27 | 4.13 | 0.30 |
| German deutsche mark | March | 1.84 | 4.93 | 0.37 |
| | June | −0.72 | 5.98 | −0.12 |
| | Sept. | −0.23 | 4.93 | −0.04 |
| | Dec. | 0.29 | 3.85 | 0.07 |
| Japanese yen | March | 0.17 | 4.40 | 0.03 |
| | June | −3.23 | 4.87 | −0.66 |
| | Sept. | −2.21 | 4.50 | −0.49 |
| | Dec. | −0.21 | 6.22 | −0.03 |
| Swiss franc | March | −2.40 | 4.99 | −0.48 |
| | June | −1.27 | 4.97 | −0.25 |
| | Sept. | −2.52 | 5.26 | −0.47 |
| | Dec. | −2.03 | 5.19 | −0.39 |

*Note:* Refer to footnote to Table I.

indicates that large speculators have information which is superior to small traders' in the wheat futures market. Large speculators receive a risk premium from large hedgers but also they make money by having superior information. In the other markets, however, we cannot reject the null hypothesis that the representative large speculator makes as much money as the hypothetical speculator makes on the average. This indicates that large speculators have no superior information and they make profits mainly by risk-bearing particularly in the case of the foreign exchange markets.

## RISK PREMIA AND PRICE VOLATILITY

An underlying intuition of the normal backwardation theory is that the risk premium is related to the uncertainty of the future price. In the Appendix, the relationship between the risk premium hedgers pay for a futures contract and the volatility of the future futures price is discussed. The hypothesis that the risk premia hedgers pay is positively related to the futures price is tested by estimating the regression equation:

$$r_t = \alpha + \beta\phi_t + \varepsilon_t \qquad (1)$$

where $r_t$ is the risk-premium hedgers pay for a certain futures contract and $\phi_t$ is the variance of the futures price. The average losses of the representative large hedger, estimated earlier, are used for the data of risk premia. As for the variance of the futures price, the results in the Appendix suggest that what is needed is the expected variance. For the expected variance of the price of a futures contract, the variance of its daily price for each month is used as a proxy. Normalization of the variances of the futures price is needed to make the variance comparable across the commodities (and the foreign currencies). The sample variance of the futures price is, therefore, divided by the square of the sample mean for normalization. The normalized sample variance of the futures price (coefficient of variation squared) is estimated as follows:

$$\phi_t = \frac{1}{\mu_t^2(n-1)} \sum_{j=1}^{n} (p_{h,t,j} - \mu_t)^2$$

where

$$\mu_t = \frac{1}{n} \sum_{j=1}^{n} p_{h,t,j}$$

and $p_{h,t,j}$ is the price of a futures contract on $j$ date at $t$ month, which matures at $h$ month.

The empirical results are presented in Table III. Estimates are presented of the coefficient $\beta$ in eq. (1) and its $t$ ratio. Since the data are time-series data, serial cor-

**Table III**
**ESTIMATES OF THE EFFECT OF PRICE VOLATILITY ON RISK PREMIA**

| Commodity | Maturity | $\hat{\beta}$ | SE ($\hat{\beta}$) | $t$ | DW |
|---|---|---|---|---|---|
| Wheat | March | 19.732 | 4.560 | 4.33 | 2.02 |
| | May | 24.507 | 4.658 | 5.26 | 2.00 |
| | July | 24.673 | 4.173 | 5.91 | 1.86 |
| | Sept. | 25.280 | 4.229 | 5.98 | 2.01 |
| | Dec. | 22.582 | 4.080 | 5.54 | 1.79 |
| Corn | March | 41.906 | 3.219 | 13.02 | 1.80 |
| | May | 40.429 | 6.143 | 12.56 | 1.73 |
| | July | 40.113 | 4.094 | 9.80 | 1.91 |
| | Sept. | 48.998 | 4.157 | 11.79 | 1.96 |
| | Dec. | 40.048 | 3.126 | 12.81 | 1.93 |
| Oats | March | 9.207 | 14.256 | 0.65 | 2.13 |
| | May | 0.872 | 14.854 | 0.06 | 2.02 |
| | July | 20.304 | 11.513 | 1.76 | 2.05 |
| | Sept. | 28.993 | 8.302 | 3.49 | 1.93 |
| | Dec. | 31.241 | 9.087 | 3.44 | 2.12 |
| Soybeans | March | 19.956 | 12.129 | 1.65 | 1.67 |
| | May | 26.172 | 10.959 | 2.39 | 1.82 |
| | July | 21.355 | 10.959 | 2.11 | 1.91 |
| | Sept. | 19.478 | 7.387 | 2.64 | 2.26 |
| | Nov. | 21.409 | 7.023 | 3.05 | 2.19 |
| Soybean meal | March | 11.560 | 7.025 | 1.65 | 1.59 |
| | May | 13.488 | 7.652 | 1.76 | 1.64 |
| | July | 23.485 | 4.534 | 5.18 | 1.55 |
| | Sept. | 21.326 | 5.955 | 3.58 | 1.81 |
| | Dec. | 7.802 | 6.369 | 1.23 | 1.81 |
| Soybean oil | March | 13.574 | 2.759 | 4.92 | 1.71 |
| | May | 14.189 | 2.860 | 4.96 | 1.76 |
| | July | 9.928 | 3.773 | 2.63 | 1.74 |
| | Sept. | 11.314 | 2.722 | 4.16 | 1.71 |
| | Dec. | 11.977 | 2.469 | 4.85 | 1.82 |

Table III (Cont.)

| Commodity | Maturity | $\hat{\beta}$ | SE ($\hat{\beta}$) | $t$ | DW |
|---|---|---|---|---|---|
| Canadian dollar | March | 38.577 | 17.806 | 2.17 | 1.88 |
| | June | 63.618 | 20.699 | 3.07 | 1.94 |
| | Sept. | 61.051 | 45.804 | 3.25 | 1.82 |
| | Dec. | 60.698 | 18.320 | 3.31 | 1.93 |
| British pound | March | 55.995 | 7.079 | 2.14 | 2.03 |
| | June | 56.018 | 6.794 | 2.45 | 2.10 |
| | Sept. | 61.876 | 8.628 | 1.89 | 2.09 |
| | Dec. | 54.943 | 9.145 | 2.58 | 2.19 |
| German deutsche mark | March | 65.683 | 9.875 | 6.67 | 1.71 |
| | June | 74.750 | 5.799 | 12.89 | 1.76 |
| | Sept. | 78.440 | 7.305 | 10.74 | 1.65 |
| | Dec. | 77.550 | 6.648 | 11.66 | 1.77 |
| Japanese yen | March | 79.832 | 10.779 | 7.41 | 2.02 |
| | June | 81.548 | 11.600 | 7.03 | 2.06 |
| | Sept. | 79.477 | 15.543 | 5.11 | 2.11 |
| | Dec. | 76.140 | 17.679 | 4.31 | 2.19 |
| Swiss franc | March | 62.252 | 4.799 | 12.97 | 1.50 |
| | June | 62.034 | 4.031 | 15.38 | 1.79 |
| | Sept. | 64.456 | 4.989 | 12.92 | 1.63 |
| | Dec. | 63.906 | 5.178 | 12.34 | 1.90 |

*Note:* Refer to footnote to Table I.

relation of the error terms is possible. However, the Durbin–Watson (D–W) statistics indicate no serious problem due to the serial correlation of the error terms.

## CONCLUSIONS

To examine the existence of risk premia in the futures markets, a test is performed to determine whether the representative large hedger loses money in the futures markets. Large-sample tests show that the representative large hedger consistently loses money in the futures markets on the average. This indicates that large hedgers pay some risk premium in the futures markets. Tests are performed also to determine whether the representative large speculator makes money in the futures markets. The empirical results show that the representative large speculator consistently makes profit on the average. This is the reason why large speculators continue to remain in the futures markets. Of course, the results do not mean an individual large speculator necessarily makes money in them. It implies, however, that futures contracts are settled in favor of large speculators as a whole.

Another test is performed to examine whether or not large speculators make a profit by having superior information compared to small traders. It is shown that the large speculators are rewarded for risk-bearing as well as for superior information in all wheat futures and a few soybean meal and oil futures. In the other markets, the representative large speculator makes no more than the money which the hypothetical speculator makes. Hence, it is believed that large speculators' profits are mostly due to risk-bearing.

Another test is performed to determine the validity of the hypothesis that the more volatile the futures price is, the more the risk premium hedgers pay for the futures contract. The hypothesis is tested empirically by regressing the *ex post* risk premium the hedger pays on the *ex post* variance of the price of the futures contract. The data on risk premia are obtained by calculating a representative large hedger's losses. The *ex post* variance is estimated by the squared coefficient of variation of the daily prices. The coefficient of variation is used to make the results comparable across the six commodities and five foreign currencies considered. The general conclusion is that the variance of the futures price has a significantly positive effect on the risk premium.

# Appendix

Participants in futures markets are: (1) output hedgers, (2) input hedgers, and (3) speculators. An examination of behavior of the representative agents from each group is conducted.

The representative agent is assumed to maximize his expected utility. Let the representative agent's utility function be strictly concave and characterized by constant absolute risk aversion. The utility function is given by

$$U(x) = -e^{-\alpha x}$$

where $x$ is the agent's asset and $\alpha$ is the coefficient of constant absolute risk aversion. If $x$ is a random variable with a normal distribution, maximizing expected utility $E[U(x)]$ is equivalent to maximizing

$$E(x) - 0.5\alpha V(x).^{4}$$

Suppose that a representative producer (output hedger) makes his production decision for time $t$ at time $t - 1$ when output price $s_t$ is unknown. The producer can manage his risk by using the futures markets. At time $t - 1$ when production decision is made, he sells at price $p_{t,t-1}$ quantity $z_{t-1}$ of futures contracts which mature at time $t$. At time $t$ he will sell his products $y_t$ at spot price $s_t$ and repurchase the futures contracts $z_{t-1}$ at price $s_t$. At maturity time $t$, $s_t = p_{t,t} + e_t$ where $e_t$ represents basis risks.[5] Hence the producer's profit for time $t$ is

$$\pi_t = s_t(y_t - z_{t-1}) - C(y_t) + p_{t,t-1}z_{t-1}$$

where $C(y_t)$ is the production cost. Assume that the production function is quadratic such that

$$C(y_t) = 0.5cy_t^2$$

The producer's expectation of profit for time $t$, formed at time $t - 1$ is

$$E_{t-1}(\pi_t) = E_{t-1}(s_t)(y_t - z_{t-1}) - 0.5cy_t^2 + p_{t,t-1}z_{t-1}$$

The variance of the producer's profit for time $t$, formed at $t - 1$ is

$$V_{t-1}(\pi_t) = V_{t-1}(s_t)(y_t - z_{t-1})^2$$

[4] See Hanoch and Levy (1969).

[5] The producer can reduce the risks associated with price changes, but he cannot eliminate the basis risks such as quantity risk, quality risk, location risk, and expiration date risk. Here it is assumed that $E_{t-1}(e_t) = 0$ and $p_{t,t}$ and $e_t$ are statistically independent. Hence, $E_{t-1}(s_t) = E_{t-1}(p_{t,t})$ and $V_{t-1}(s_t) = V_{t-1}(p_{t,t}) + V_{t-1}(e_t)$.

Thus, the producer's expected utility for time $t$, formed at time $t - 1$ is given by

$$E(U) = E_{t-1}(\pi_t) - 0.5\alpha V_{t-1}(\pi_t) =$$
$$E_{t-1}(s_t)(y_t - z_{t-1}) - 0.5cy_t^2 + p_{t,t-1}z_{t-1} - 0.5\alpha V_{t-1}(s_t)(y_t - z_{t-1})^2 \qquad (2)$$

where $\alpha$ is the coefficient of the producer's risk aversion. The producer will maximize his expected utility as given by eq. (2). By maximizing expected utility (eq. (2)) with respect to the outputs $y_t$ and futures contracts $z_{t-1}$, one gets the optimal outputs $y_t$ and futures contracts $z_{t-1}$.

$$y_t = \frac{p_{t,t-1}}{c}$$
$$z_{t-1} = \frac{p_{t,t-1}}{c} + \frac{[p_{t,t-1} - E_{t-1}(s_t)]}{\alpha V_{t-1}(s_t)}$$

Multiplying the producer's supply of futures contracts by the number of producers (output hedgers), the aggregate supply of futures contracts by output hedgers is obtained.

$$Z_{t-1} = kz_{t-1}$$

where $k$ represent the number of producers (output hedgers).

A representative producer (input hedger) also manages risks when the input price is uncertain. At time $t - 1$ he obtains a revenue $R(x_t)$ from forward sales and buys at price $p_{t,t-1}$ quantity $w_{t-1}$ of futures contracts which mature at time $t$. At time $t$ he will purchase inputs required $x_t$ at price $s_t$ and sell at price $s_t$ quantity $w_{t-1}$ of futures contracts which mature at time $t$. This input hedger's profit for time $t$ is given by

$$\pi_t = R(x_t) - s_t(x_t - w_{t-1}) - p_{t,t-1}w_{t-1}$$

where $R(x_t)$ is the revenue function with respect to inputs $x_t$. Assume that the marginal revenue with respect to inputs is

$$R'(x_t) = a - bx_t \quad \text{where } a > s_t.$$

The input hedger's expectation of profit for time $t$, formed at time $t - 1$ is

$$E_{t-1}(\pi_t) = R(x_t) - E_{t-1}(s_t)(x_t - w_{t-1}) - p_{t,t-1}w_{t-1}$$

The variance of the input hedger's profit for time $t$, formed at time $t - 1$ is

$$V_{t-1}(\pi_t) = V_{t-1}(s_t)(x_t - w_{t-1})^2$$

Therefore, his expected utility for time $t$, formed at time $t - 1$ will be

$$E_{t-1}(U) = E_{t-1}(\pi_t) - 0.5\beta V_{t-1}(\pi_t) = R(x_t) - E_{t-1}(s_t)(x_t - w_{t-1})$$
$$- p_{t,t-1}w_{t-1} - 0.5\beta V_{t-1}(s_t)(x_t - w_{t-1})^2 \qquad (3)$$

where $\beta$ is the coefficient of the input hedger's risk aversion. Maximizing the expected utility (eq. (3)) with respect to inputs and futures contracts will yield the optimal inputs and futures contracts

$$x_t = \frac{a - p_{t,t-1}}{b}$$
$$w_{t-1} = \frac{a - p_{t,t-1}}{b} + \frac{E_{t-1}(s_t) - p_{t,t-1}}{\beta V_{t-1}(s_t)}$$

Multiplying an individual input hedger's demand for futures contracts by the number of input hedgers, the aggregate demand for futures contracts by input hedgers is obtained.

$$W_{t-1} = mw_{t-1}$$

where $m$ represents the number of input hedgers.

A representative speculator will buy futures contracts if he expects that the futures price at time $t$ will be greater than the futures price at time $t - 1$, and sell futures contracts if he expects that futures price at time $t$ will be less than the futures price at time $t - 1$. The speculator's profit for time $t$ is

$$\pi_t = (p_{t,t} - p_{t,t-1})f_{t-1}$$

where $f_{t-1}$ is the speculator's net position of futures contracts. Positive $f_{t-1}$ implies long position while negative $f_{t-1}$ implies short position. The speculator's expected utility for time $t$, formed at time $t - 1$ will be

$$E_{t-1}(U) = E_{t-1}(\pi_t) - 0.5\delta V_{t-1}(\pi_t) = [E_{t-1}(p_{t,t}) - p_{t,t-1}]f_{t-1} - 0.5\delta f_{t-1}{}^2 V_{t-1}(p_{t,t}) \tag{4}$$

where $\delta$ is the coefficient of speculator's risk aversion. Maximizing expected utility (eq. (4)) with respect to future contracts $f_{t-1}$ will yield the optimal number of futures contracts.

$$f_{t-1} = \frac{[E_{t-1}(p_{t,t}) - p_{t,t-1}]}{\delta V_{t-1}(p_{t,t})}$$

Multiplying an individual speculator's demand for futures contracts by the number of speculators, the aggregate demand for futures by speculators is obtained.

$$F_{t-1} = nf_{t-1}$$

where $n$ represents the number of speculators.

The futures market is in equilibrium when the aggregate excess supply of futures contracts by hedgers equals the aggregate excess demand for futures contracts by speculators.

$$Z_{t-1} - W_{t-1} = F_{t-1}$$

The equilibrium price of futures contracts at time $t - 1$ is

$$p^*_{t,t-1} = \frac{DE_{t-1}(p_{t,t}) + ma/b}{D + k/c + m/b}$$

where

$$D = \frac{k}{\alpha[V_{t-1}(p_{t,t}) + V_{t-1}(e_t)]} + \frac{m}{\beta[V_{t-1}(p_{t,t}) + V_{t-1}(e_t)]} + \frac{n}{\delta V_{t-1}(p_{t,t})}$$

since $E(s_t) = E(p_{t,t})$ and $V_{t-1}(s_t) = V_{t-1}(p_{t,t}) + V_{t-1}(e_t)$.

The difference between the expected future futures price $E_{t-1}(p_{t,t})$ and the equilibrium futures price $p^*_{t,t-1}$ at time $t - 1$ is an *ex ante* risk premium.

$$E_{t-1}(r_t) = E_{t-1}(p_{t,t}) - p^*_{t,t-1} = \frac{(k/c + m/b)E_{t-1}(p_{t,t}) - ma/b}{D + k/c + m/b}$$

The *ex ante* risk premium is positively related to the expected variance of the future futures price at time $t$. Speculators ask more rewards for their risk-bearing as they expect that the future futures price is more volatile. So hedgers have to pay more risk premium as the future futures price is expected to be more volatile.

This model suggests that the *ex ante* risk premium is related positively to the basis risk measured by $V[e(t)]$.

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