Pricing Cross-Currency Options

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INTRODUCTION

A cross-currency option is an option in which the currency of the exercise price is different from the currency used to price the underlying asset. A simple example is a call option to buy 100 Yen at an exercise price of .90 dollars Canadian. This can be considered as a conventional option if the price of Yen is in Canadian dollars. Alternatively, it can be viewed as a cross-currency option if the price of Yen is measured in $US, but the exercise price is in Canadian dollars. The value of this option depends on both the $US value of the Yen and on the $US value of the Canadian dollar. Looking at the option in this second way means that the volatilities implied by the prices of exchange traded options can be used to estimate the volatility of the cross-currency option.

This option is sold by financial institutions to firms which have cash flows in two currencies, neither of which is the US dollar. As an example, General Motors assembles cars from Japanese parts for sale in Canada. The firm is concerned not only about the US dollar value of the Yen or the Canadian dollar, but also about the Canadian dollar value of the Yen. To hedge these currency risks, firms buy over-the-counter cross-currency options from financial institutions. The financial institutions need to be able to price and to hedge these options. To do this requires an estimate of the volatility of, for example, the Canadian dollar value of the Yen.

A more complicated cross-currency option is the Nikkei Put Warrant (NPW) Series II issued by Bankers Trust Bank of Canada in April 1989. The exercise price is 270.535 Canadian dollars and the underlying asset is the Nikkei Index of 225 stocks. This can be viewed as a conventional option if the Nikkei index is denominated in Canadian dollars. Viewed as a cross-currency option, the underlying asset is the Yen value of the Nikkei index, with the exercise price in Canadian dollars. The value of the NPW depends on three prices: (i) the Yen value of the index; (ii) the $US value of the Yen; and (iii) the $US value of the Canadian dollar.

This article provides a general solution to the problems involved in using the prices of exchange traded options to estimate the volatility of the asset in the currency of the exercise price. Latané and Rendleman (1976) and Beckers (1981) demonstrate the superiority of implied volatilities over historical volatilities in predicting the variability of future stock prices. Since very few cross-currency options

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are traded on exchanges, volatilities are difficult to infer directly. For the Canadian dollar option on the yen, prices of $US options on the Yen and Canadian dollar are needed. For the NPW, the volatility estimate requires a price of an exchange traded option on the Nikkei average in Yen, as well as prices of $US exchange traded currency options on the Yen and on the Canadian dollar. Currently, there are Nikkei index options (in Yen) traded on the Osaka exchange, and Yen and Canadian dollar options traded in Philadelphia.

There is no widely-cited work which deals specifically with cross-currency options. There are several articles which deal with currency options. Biger and Hull (1983), Garman and Kohlhagen (1983), Giddy (1983), and Grabbe (1983) all extend the Black-Scholes (1973) formula to price European options on a currency. The extension is an adaptation of Merton (1973), and is similar to Black's (1976) model for options on futures contracts. For the currency option, a holder of a foreign currency is paid interest at the foreign rate. This is analogous to a dividend yield, so that Merton's result can be used. Shastri and Tandon (1986) use this assumption to price American currency options by extending the Geske and Johnson (1984) model. Pitts (1984) challenges the Biger and Hull results by exploring the consequences of a stochastic foreign interest rate. His result does not address the questions raised in pricing cross-currency options.

The problem of pricing an option which depends on several underlying assets is explored as a general equilibrium model by Garman (1976) and Cox, Ingersoll, and Ross (1985). Boyle (1988) develops a general lattice approach to the problem. Fortunately, as shown below, the difficulties of implementing these models is much reduced for cross-currency options. Stulz (1982) develops expressions for the prices of options on the maximum or the minimum of two assets. He obtains an expression for the volatility which has the same form as the expression for the volatility derived in this article. However, a cross-currency option is not an option on the maximum or minimum of two assets. The difference is confirmed in the prices of the options. The price of a European cross-currency option is given by one of the variants of Black-Scholes listed above. The price of a European option on the maximum or minimum is very different from a variation of Black-Scholes. There is no *a priori* reason to expect the volatilities of the maximum or minimum to be the same as the volatility of the asset expressed in the foreign currency.

THE NATURE OF THE CROSS-CURRENCY OPTION

It is tempting to suppose that a cross-currency option is merely the sum of two or more simpler options. This is not true. Consider, for example, a call option on 100 Yen with an exercise price of .90 $Canadian. At first, it would seem that the payoff of this option is the same as the payoff from a portfolio consisting of a call option on 100 Yen with an exercise price of .72 $US plus .9 of a put on the Canadian dollar with an exercise price of .80 $US. However, the payoffs are the same only if all options expire either in-the-money or out-of-the-money. It is possible for one $US option to expire in-the-money, and the other to expire worthless. In this case, the payoffs are different.

It can be shown that, in general, the value of the portfolio is at least as great as the cross-currency option. Suppose that the stochastic process underlying the cross-currency option is derived from two variables: (i) the price, S_A of the asset and (ii) the price, S_C of the currency C of the exercise price. Usually these prices are in $US. Let S be the price of the asset as measured in currency C. The relation between these prices is given by the requirement that no triangular arbitrage is possible:

$$S = \frac{S_A}{S_C}. \tag{1}$$

This expression means that if, for example, 100 Yen can be exchanged for .72 $US and if one Canadian dollar can be exchanged for $.80 U.S., then 100 Yen can be exchanged for .90 Canadian dollars. No arbitrage implies that this relation will hold to within transactions costs.

Let X be the exercise price of a cross-currency call option. The portfolio suggested above consists of a long call on the asset and X times a long put on the currency. The exercise prices, X_A and X_C, of the $US call and put, respectively, are chosen so that $X = X_A/X_C$. At the exercise date, the $US value of the cross-currency call is

$$S_C\max(S - X,0) = \max(S_A - XS_C,0) = \max(S_A - X_A\frac{S_C}{X_C},0),$$

while the $US value of the portfolio is

$$\max(S_A - X_A,0) + X\max(X_C - S_C,0) = \max(S_A - X_A,0) + \max(X_A - XS_C,0).$$

Because of the relationships between the spot prices and the exercise prices, there are only six possible outcomes at expiry. The details for each of these outcomes is given in Table I. For the four outcomes in which only one $US option expires in-the-money, the portfolio is worth more than the cross-currency option. For the two outcomes in which both $US options expire either in-the-money or out-of-the-money, the portfolio and the cross-currency option are worth the same.

The conclusion is that the portfolio has a higher expected payoff than the cross-currency option. This is consistent with the higher cost of the portfolio. However, the portfolio provides more insurance than is desired. For example, it provides insurance against adverse movements both in the Yen–$US exchange rate and the $US–Canadian dollar exchange rate, while the cross-currency option provides insurance only against the Yen-Canadian dollar exchange rate. Thus the cross-currency option provides insurance which cannot be obtained using $US options.

Table I
PAYOFF COMPARISON: CROSS-CURRENCY VS. PORTFOLIO

The cross-currency option 'C—C' is worth $S_C \max(S - X, 0)$ at expiry. The 'Portfolio' is worth $\max(S_A - X_A, 0) + X\max(X_C - S_C, 0)$. The spot prices are related by $SS_C = S_A$. The exercise prices are related by $XX_C = X_A$. Only six outcomes are possible. 'Difference' is the difference between the value of the portfolio and the cross-currency option. Observe that 'Difference' is never negative.

S_A	S_C	S	Portfolio	C—C	Difference
$> X_A$	$< X_C$	$> X$	$S_A - XS_C$	$S_A - XS_C$	0
$< X_A$	$> X_C$	$< X$	0	0	0
$> X_A$	$> X_C$	$> X$	$S_A - X_A$	$S_A - XS_C$	$X_A\left(\frac{S_C}{X_C} - 1\right)$
$> X_A$	$> X_C$	$< X$	$S_A - X_A$	0	$S_A - X_A$
$< X_A$	$< X_C$	$> X$	$X_A - XS_C$	$S_A - XS_C$	$X_A - S_A$
$< X_A$	$< X_C$	$< X$	$X_A - XS_C$	0	$X_A - XS_C$

VALUING CROSS-CURRENCY OPTIONS

A cross-currency option can be viewed as a conventional option, provided the price of the asset is measured in the currency of the exercise price. This means that the pricing techniques of Black–Scholes (1973) or Cox, Ross, and Rubinstein (1979) can be used. The difficulty in implementing these techniques is in estimating the volatility of the asset.

In this section the volatility is estimated from the prices of options which are traded on exchanges. Volatilities can be inferred from these prices using the Black-Scholes model for European-type options, or standard numerical options for American options for which early exercise is likely (see Shastri and Tandon (1986)).

In the simple case, the price, S, of the cross-currency asset depends on two \$US prices, S_A and S_C. The volatility of S can then be estimated in terms of the volatilities of S_A and S_C, where these three prices are related by eq. (1). Since S is not a sum of the S_A and S_C, the expression for the volatility of S is not a trivial application of the expression for the standard deviation of a portfolio.

To derive an expression for the volatility, assume that the prices S_A and S_C each follow an Ito process:

$$dS_A = \mu_A S_A dt + \sigma_A S_A dz_A \tag{2}$$

and

$$dS_C = \mu_C S_C dt + \sigma_C S_C dz_C, \tag{3}$$

where dz_A and the dz_C are unit-variance zero-mean Wiener processes. Ito's lemma and the expressions (1), (2), and (3) mean that the process followed by the cross-currency price, S, depends on the two stochastic processes, dz_A and dz_C:

$$dS = (\mu_A - \mu_C - \rho_{AC}\sigma_A\sigma_C)Sdt + \sigma_A S dz_A - \sigma_C S dz_C. \tag{4}$$

This expression is consistent with intuition. Suppose S to be the Canadian dollar price of the Yen, and S_A and S_C to be the \$US prices of the Yen and Canadian dollar, respectively. Equation (4) shows that the Canadian dollar value of the Yen will increase faster (the 'drift rate' will be higher) if the \$US value of the Yen increases faster, or if the \$US value of the Canadian dollar increases more slowly. Equation (4) also shows that a random increase in the \$US value of the Yen will increase the Canadian dollar value of the Yen, while a random increase in the \$US value of the Canadian dollar will decrease the Canadian dollar value of the Yen.

The volatility, σ, of S follows from expression (4). The variance, σ^2, is $E(\sigma_A dz_A - \sigma_C dz_C)^2$ or

$$\sigma^2 = \sigma_A^2 + \sigma_C^2 - 2\rho_{AC}\sigma_A\sigma_C. \tag{5}$$

The correlation between dz_A and dz_C is ρ_{AC}.

This technique can be extended arbitrarily. For the case of the Nikkei Put Warrants (Series II), the goal is to estimate the volatility of the Nikkei 225 index as measured in Canadian dollars. The options available are: (i) Yen options on the index; (ii) \$US options on the Yen; and (iii) \$US options on the Canadian dollar. Labelling the prices of the assets underlying these options as S_A, S_B, and S_C, respectively, the relation analogous to (1) is

$$S = \frac{S_A S_B}{S_C}. \tag{6}$$

Using this, the volatility, σ, of S is obtained from

$$\sigma^2 = \sigma_A^2 + \sigma_B^2 + \sigma_C^2 + 2\rho_{AB}\sigma_A\sigma_B - 2\rho_{BC}\sigma_B\sigma_C - 2\rho_{AC}\sigma_A\sigma_C. \tag{7}$$

SUMMARY

The advantage of these models to estimate the volatility of the cross-currency asset is that forward looking market data, in the form of implied volatilities, are used. The shortcoming is that the correlation between the stochastic processes must be estimated.

The estimate of the correlation between the exchange rates can be made using historical data. It is an empirical question whether the correlation between the rates is sufficiently stable for the historical estimate to be an accurate estimate.

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