Index Option Pricing: Do Investors Pay for Skewness?

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INTRODUCTION

There is ample conceptual and empirical evidence that investors value positive skewness in the distribution of returns. Alderfer and Bierman (1970) demonstrate that individual investment choices can not be explained using only the mean and variance, and that skewness of the return distribution is quite relevant. Borch (1974) shows that portfolio analysis based only on mean-variance methods can be applied without restriction only when all assets have normal return distributions.

The importance of skewness in the return distribution is noted by Beedles (1979) to depend on the investor's attitude toward risk. He points out that many investors are willing to accept a lower expected return on assets having positively skewed return distributions. Robichek (1969) suggests that "investment risk" is unique to the individual investor. What seems "risky" to one investor may not seem risky to another. Cooley (1979) shows that some investors view risk as variability in either direction while others see risk as being associated mainly with the chance on a "bad" outcome. Tsaing (1972) concludes that skewness takes on more importance when the size of the investment is small *relative* to the investor's total wealth.

Kahneman and Tversky (1979) demonstrate that when gains are possible but not probable, most people choose the prospect that offers the larger possible gain (which is not necessarily the prospect with the larger probability-weighted expected value). They contend that the carriers of value are [proportional] changes in the value of the investment, rather than the final states of wealth, and that preference for positively skewed return distributions generally increases with the *absolute* size of the investment.

Based on an extensive empirical study of stock prices, Kraus and Litzenberger (1976) conclude that investors pay a significant price for positive *systematic* skewness in the distribution of returns on stocks. Sears and Trennepohl (1983) also support the concept of value associated with positive skewness. They show that reducing diversification to preserve skewness in the distribution of returns on portfolios of stock options could be considered rational investor behavior. Stephens and Proffitt (1990) demonstrate that positive skewness is a significant explanatory factor in the returns from international mutual funds.

Since positive skewness has been shown to have value to investors, and since the distributions of returns on groups (portfolios) of purchased call index options have

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a high degree of positive skewness, this characteristic may be important in the pricing of index option contracts. At least three rationales can be given for this.

The first is that the arbitrage arguments presented by Merton (1973), which can be used to obtain rational bounds on stock option prices, may not hold for index options. As explained by Ritchken (1987, p. 138), "... it is virtually impossible to establish a portfolio of stocks that exactly reproduces the price of an index, such as the S&P 100." Further, as noted by Stein (1989), given that volatility in the market price of securities is unknown and changing, options are not wholly redundant with their underlying assets but reflect a speculative market in volatility.

The second reason is that index call options may be bought, at the margin, by relatively undiversified investors (speculators) who are "price-takers". The prices these investors are willing to pay are not constrained by rational arbitrage boundaries, but based on their utility of wealth functions, which may include consideration of return skewness. For these traders, purchase of an index call option is not unlike the purchase of a raffle ticket which has a very small probability of an extremely large payoff. Evidence that index option prices do frequently violate the arbitrage boundary is provided by Evnine and Rudd (1985). They also show that market prices of index call options often deviate substantially from prices derived with the binomial option pricing model.

A third reason for including skewness is based on second-degree stochastic dominance. The relative desirability of an investment is a function not only of the maximum possible return (as suggested by Kahneman and Tversky (1979)) but also of the cumulative probability up to any possible level of return. If two distributions (called A and B) have the same mean and same variance, but B has greater positive skewness, then the cumulative area under A will be greater than for B at all levels of return, and B will dominate A. As empirically evidenced by Cooley (1977), investors perceive relatively less risk in return distributions that are positively skewed. Since a pricing equilibrium between two return distributions would require that the one with more perceived risk offer higher expected (mean) return, positive skewness should be inversely related to the mean return.

Following from the above, the purpose of this research is to investigate empirically whether skewness is a significant factor in the expected returns from investments in index call options.

DEVELOPMENT OF THE REGRESSION MODEL

Kraus and Litzenberger (1976) extend the mean-variance capital asset pricing model to incorporate the effect of *systematic* skewness in the return distributions of securities. Assuming the rate of return on the market is nonsymetrically distributed, they write an equation for the equilibrium excess mean rate of return on a risky portfolio (R_j) substantially as:

$$R_j = b_1\beta_j + b_2\gamma_j, \tag{1}$$

where

$\beta_j = E[R_j - \bar{R}_j)(R_M - \bar{R}_M)]/E(R_M - \bar{R}_M)^2$
$= \sigma_{jM}/\sigma_M^2$, the market beta or systematic standard deviation of the j^{th} portfolio,

$\gamma_j = E[(R_j - \bar{R}_j)(R_M - \bar{R}_M)^2]/E(R_M - \bar{R}_M)^3$
$= m_{jMM}/m_M^3$, the market gamma or systematic skewness of the j^{th} portfolio,

σ = the standard deviation of returns,
m = the skewness of returns,
$b_1 = (dW/d\sigma_W)\sigma_M$, the market price for beta reduction,
$b_2 = (dW/dm_W)m_M$, the market price of gamma, and M represents the "market".

Following from a form offered by Prakash and Bear (1986), eq. (1) can be written in a fashion which allows the *comoments,* rather than the systematic moments, to be specified as the independent variables in the model. This is shown as:

$$R_j = \lambda_1 \sigma_{jM} + \lambda_2 m_{jMM}, \tag{2}$$

where

$\lambda_1 = b_1/\sigma_M^2$ = covariance of asset with market divided by market variance,
$\lambda_2 = b_2/m_M^3$ = coskewness of asset with market divided by market skewness.

For purposes of the empirical investigation, eq. (2) can then be written as an ex-post regression model in the form:

$$r_j = \Theta_0 + \Theta_1 Z_{1j} + \Theta_2 Z_{2j} + e_j, \tag{3}$$

where

r_j = annualized mean excess return for the j^{th} group of options,
Θ_0 = the regression intercept term,
Θ_1 and Θ_2 = regression coefficients for covariance and coskewness,
Z_{1j} = sample covariance of j^{th} option group with the market,
Z_{2j} = sample coskewness of j^{th} option group with the market, and
e_j = error term that follows the normal conventions.

The parameters Θ_0, Θ_1, and Θ_2 can be estimated using the ordinary least squares technique. The results of the empirical analysis should provide evidence of whether covariance and coskewness are significant to investors in the pricing of index options. A standard f-statistic indicates the significance of the model, while t-tests indicate the importance of covariance and coskewness in the analysis.

Assuming that investors in options are averse to risk, it is hypothesized that Θ_1 (related to covariance) should be positive. Following from Cooley (1977), an increase in positive skewness should reduce required return. Accordingly, Θ_2 (related to coskewness) should have a sign opposite that for market skewness.

DATA AND EMPIRICAL ANALYSIS

Data on the level of the S&P 100 index and the closing prices for call options on this index at four levels of the striking price are from the *Wall Street Journal* or from the CBOE for each trading day for the period March 21, 1983 (the first day these options were traded) through April 1988. The data contains more than 14,300 observations where an observation is defined as a call option with a specific exercise price, specific expiration date, and specific number of days to expiration.

The risk-free rate of interest is represented by the daily equivalent yield over the time to expiration on U.S. Treasury Bills having maturity closest to the expiration date of each option contract.

Assuming a position taken in a specific option contract is maintained until the expiration date of that option, the holding period returns in excess of the risk-free rate (R_{ikt}) are calculated for bought index calls as:

$$R_{ikt} = [\text{Max } (0, S_i - X_{ki}) - C_{ikt}]/C_{ikt}, \tag{4}$$

where

S_i = index level at expiration of the i^{th} option (i indicates month of maturity = 1 to 55),
X_{ki} = exercise price of the k^{th} level of option i (k indicates relative exercise price = 1 to 4),
C_{ikt} = closing price of the t^{th} observation of the k^{th} level of option i (t indicates *relative* days to expiration = 1 to 60).

Returns on the market in excess of the risk-free rate (R_{Mit}) over the same holding period as for the associated options are calculated as if the investor held a portfolio of securities that exactly matched the performance of the S&P 100 index.[1] This is shown as:

$$R_{Mit} = (S_i - S_{it})/S_{it}, \tag{5}$$

where

S_{it} = the closing index level on the day of purchase of the i^{th} option contract.

After calculating the returns on the individual option contracts and on the market, the covariance and coskewness of the return distribution for each specified group (portfolio) of index options are computed and annualized for comparison across holding periods of different length. Using these distribution comoments, the OLS regression of the form specified in eq. (3) is then run.[2] Separate analyses are conducted for groups of options which are in-the-money (ITM) and out-of-the-money (OTM) at the time of purchase, as well as for groups classified by length of the holding period. For each of these analyses, the regression is run both with and without coskewness included. The significance of the Θ_n, the regression coefficients, indicates whether the distribution comoments are important factors in the returns from investments in S&P 100 index options.

RESULTS OF THE ANALYSIS

All Observations

For the first cross-sectional analysis, all observations with maturities of seven or more days are classified into groups (portfolios) by days to expiration. With five trading days in each of weeks two through thirteen prior to maturity, this yields 60 groups, each containing approximately 220 observations (55 option expiration months times 4 striking prices). The mean return is the dependent variable in the regression analysis; the covariance and the coskewness are the independent variables. The regression coefficients and the related statistics for this analysis are shown in Table I.

[1]While the changes in the market prices of options on the S&P index follow closely the movement of the underlying index, the returns on investment in these options are not highly correlated with returns on the index. Regressions of several return series (for various subsets of the data) provide correlation coefficients that are generally less than 0.02 and insignificant f-statistics.

[2]While the sixty return distributions for groups of individual option contracts are generally characterized by positive skewness, the distribution of the means of these distributions is approximately normal, as would be expected. The statistics for this distribution of means are: mean = 6.39; minimum = 1.02, maximum = 29.31; standard deviation = 6.92; and skewness = 1.77.

Table I
SUMMARY REGRESSION STATISTICS, ALL OPTION GROUPS

Number of Comoments	Intercept	Coefficients (t-values) (prob > t) Covariance	Coskewness	
One	−6.09	0.517		Adj. R-square = .872
	(−8.63)	(19.92)		f-value = 396.81
	(.0001)	(.0001)		prob > f = .0001
Two	−7.88	0.631	0.051	Adj. R-square = .886
	(−8.61)	(13.48)	(2.84)	f-value = 227.10
	(.0001)	(.0001)	(.0062)	prob > f = .0001

With only covariance included, the model provides an r-square of 0.872 and a significant f-statistic of 396.8. The analysis including coskewness provides a slightly higher adjusted r-square of 0.886 and statistically significant coefficients for both covariance and coskewness. Moreover, based on investors associating negative value with variance and positive value with positive skewness, the signs of the coefficients are in the direction expected.[3]

These results suggest that coskewness is statistically significant to the returns on purchased index call options, but that this factor does not add much explanatory power to the regression model.

ITM Versus OTM Options

To further investigate the conditions under which coskewness is important, the second analysis considers groups of option contracts which are in-the-money (ITM) separately from groups which are out-of-the-money (OTM) at the time of purchase. Summary statistics for the analysis of the sixty groups of ITM options are shown in Panel A of Table II; statistics for corresponding groups of OTM options are shown in Panel B.

With only covariance included, the analysis for ITM options gives an r-square of 0.765 and a significant f-statistic of 192.6. When coskewness is included, the adjusted r-square increases substantially to 0.929. The regression coefficients indicate that both comoments are statistically significant and have the "correct" sign.

Quite a different picture is given with OTM options. The analysis with only covariance gives an adjusted r-square of 0.891. When coskewness is added, the r-square is essentially unchanged. While the expanded model is statistically significant, and covariance is significant at the .01 level, the t-statistic for coskewness has a probability of .755.

While the return distributions for groups of OTM options have a higher degree of coskewness than ITM options, this factor does not appear to be as important to the pricing of OTM options. While this result may be more a function of the statistical properties of the return distributions than of investor utility, a sufficient explanation for these empirical finding is not immediately evident.

[3]During the period considered in this analysis the sign of market skewness is negative most of the time. Thus, the positive sign on the coefficient indicates that return is inverse with market skewness, as expected.

Table II
SUMMARY REGRESSION STATISTICS, ITM AND OTM

Panel A: In-the-money Options

Number of Comoments	Intercept	Covariance	Coskewness	
		Coefficients (*t*-values) (prob > *t*)		
One	−3.99	0.496		Adj. R-square = .765
	(−4.48)	(13.88)		f-value = 192.64
	(.0026)	(.0001)		prob > f = .0001
Two	−7.768	0.709	0.123	Adj. R-square = .929
	(−13.26)	(26.45)	(11.66)	f-value = 388.21
	(.0001)	(.0001)	(.0001)	prob > f = .0001

Panel B: Out-of-the-money Options

Number of Comoments	Intercept	Covariance	Coskewness	
		Coefficients (*t*-values) (prob > *t*)		
One	−5.03	0.421		Adj. R-square = .891
	(−4.48)	(13.88)		f-value = 481
	(.0001)	(.0001)		prob > f = .0001
Two	−5.20	0.429	0.002	Adj. R-square = .889
	(−6.29)	(12.980)	(0.31)	f-value = 236.81
	(.0001)	(.0001)	(.7546)	prob > f = .0001

Different Holding Periods

A further analysis of returns on groups of index options compares subsets for various holding periods (time to expiration). Summary regression statistics for these analyses are shown in Table III. The results for groups of options with holding periods of seven through 46 days are in Panel A. With both comoments included, the analysis provides significant t-statistics and correct signs for all coefficients.

The results of the analysis for periods of 47 through 88 days are shown in panel B. This provides a significant f-statistic and significant t-statistics when both comoments are included. The r-square, however, is only about 0.78 as compared to 0.88 for the shorter holding periods. This suggests that coskewness may be more important for groups of options with shorter terms to expiration than for longer-term options.

To further investigate this possible "maturity factor," the analysis is run for periods of 7–30, 7–60, 7–75, 31–60, and 61–88 days. While the smaller sample sizes makes some of these analyses less reliable, the results (not shown herein) suggest that coskewness may be most important for options of intermediate time to expiration, about four to eight weeks.

SUMMARY AND CONCLUSIONS

It is generally agreed, and empirically supported, that positive skewness in the distribution of returns is important in the pricing of securities. The distributions of returns from purchased call options are characterized by a high degree of positive

Table III
SUMMARY REGRESSION STATISTICS, BY MATURITIES

Panel A: Option Maturities of 7–46 Days

Number of Comoments	Intercept	Coefficients (*t*-values) (prob > *t*) Covariance	Coskewness	
One	−4.45	0.486		Adj. R-square = .836
	(−3.30)	(12.18)		f-value = 148.25
	(.0026)	(.0001)		prob > f = .0001
Two	−6.38	0.591	0.043	Adj. R-square = .846
	(−3.77)	(8.21)	(1.73)	f-value = 80.91
	(.0009)	(.0001)	(.0949)	prob > f = .0001

Panel B: Option Maturities of 47–88 Days

Number of Comoments	Intercept	Coefficients (*t*-values) (prob > *t*) Covariance	Coskewness	
One	−2.29	0.264		Adj. R-square = .665
	(−3.64)	(7.54)		f-value = 56.78
	(.0012)	(.0001)		prob > f = .0001
Two	−3.78	0.376	0.062	Adj. R-square = .709
	(−4.27)	(6.30)	(2.24)	f-value = 35.15
	(.0002)	(.0001)	(.0336)	prob > f = .0001

skewness. Since it is not possible to create an exact arbitrage between index options and the underlying asset, as can be done with options on individual stocks, skewness may be an important factor in the pricing of these contracts.

For all wealth-maximizing, risk-averse investors, second-degree stochastic dominance also suggests that skewness should be a factor in the pricing of index options. Since skewness in the return distribution of groups of index options affects the perceived risk of investing in these contracts, this factor should be reflected in the required returns from, and prices investors are willing to pay for, these contracts.

To test the hypothesis that skewness is an important factor in index option pricing, mean returns from investing in specified groups of call options on the S&P 100 index over a 55-month period are evaluated with a regression model derived from the three-moment CAPM offered by Kraus and Litzenberger (1976). This model specifically incorporates both covariance and coskewness of the option return distributions.

When including all observations with time to expiration between seven and 88 days, the model has an r-square of about 0.92 and both comoments are found to be statistically significant. Further, the signs of the regression coefficients are in the direction expected; covariance is positive and coskewness is opposite the sign of market skewness. This is consistent with investors perceptions of risk as presented by Cooley (1977), and with the results for stock portfolios obtained of Kraus and Litzenberger (1976).

For groups of options that are in-the-money at the time of purchase, an adjusted r-square of about 0.93 is obtained. While coskewness is significant, this factor does not increase the explanatory power of the model. The analysis of out-of-the-money groups of options has an adjusted r-square of about 0.89 and provides a significant coefficient only for covariance.

When analyzing subsets of the data classified by days to expiration, it is indicated that coskewness is more important for groups of options with shorter terms to expiration than for longer-term options. This factor may be most important for groups of options with times to expiration of four-weeks to eight-weeks. Based on these results, it can be inferred that coskewness is most significant to investors buying intermediate-term, in-the-money index options.

Since this research is limited to purchased call options, natural extensions include analysis of sold call options as well as of both bought and sold put options on the S&P 100 index.

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