The Probability Distribution of Futures Prices in the Foreign Exchange Market: A Comparison of Candidate Processes

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I. INTRODUCTION

There is an increased interest in the statistical properties of asset prices generated in part by the application of various economic models to determine the optimal allocation of assets and the pricing of options. In addition, tests of market efficiency are dependent upon the stochastic behavior underlying asset prices. While much of the existing literature on asset prices focuses on the properties of spot prices, researchers are applying similar analyses to futures prices.

The purpose of this study is to compare various stochastic processes to determine which candidate process best describes the change in the futures price of foreign exchange rates. Previous studies addressing this issue include Cornew, Town, and Crowson (1984), So (1987), and Hall, Brorsen, and Irwin (1989). These articles make use of the class of stable distributions and estimate the parameters using daily data. The important empirical significance of these articles deals with the issues of non-normality and time varying parameters.

The extant research, relating to foreign exchange rates, suggests that the unconditional distribution of futures price changes, measured at high frequency (daily or weekly), has a greater area in the tail relative to the normal distribution. However, at a relatively low frequency (monthly or quarterly), the unconditional distribution appears to fit the normal distribution fairly well.[1]

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[1]The research by Hall, Brorsen, and Irwin (1989) suggests that the monthly difference in the foreign exchange futures price is close to normality. This is inferred by the estimate of the characteristic exponent of the stable Paretian distribution, which is close to the value for the normal distribution. Boothe and Glassman (1987) report that the quarterly difference in the foreign exchange spot price is close to normality.

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The observed leptokurtosis for high frequency data has been explained by two competing hypotheses. One hypothesis maintains that the change in futures prices can be described by a stable distribution. This class of distributions exhibits leptokurtotic behavior; being more peaked at the mean and having a thicker tail than the normal distribution.[2] An important characteristic of the stable distribution is that the variance is infinite except for the normal distribution which is a special case of the stable class of distributions.[3] Also, the parameters which describe the kurtosis and skewness of a stable distribution are constant under addition of independent observations. This offers a stability-under-addition test.

The alternative hypothesis allows for a finite variance by considering a mixture of distributions with finite variance, or allowing for time-varying parameters. A distribution with time-varying moments is one possible way of generating more mass in the tails than the normal. The observed leptokurtosis can be explained by a normal distribution with time varying mean and/or scale parameter. In the case of daily changes in spot exchange rates, the results by Boothe and Glassman (1987) support the view that the class of distributions remains unchanged while the parameters of the distribution may vary over time.

Not explicitly contained in the above hypotheses is the condition that the stochastic process accounts for the observed tendency of low frequency changes in the futures price to approach a normal distribution. In the case of the stable Paretian distribution, the sum of independent stable random variables do not converge to the normal, unless the characteristic exponent is equal to 2 (the normal case) for each stable random variable. Such a case is trivial since the underlying distribution is already normal. Additional evidence against the stable distribution is the failure of the stability-under-addition property to hold. Examining various commodity futures, the results by Hall, et. al. (1989) suggest that the characteristic exponent converges to 2, for the change in futures prices. With this consideration in mind, this article eliminates the stable Paretian distribution as a candidate stochastic process.

The question of whether the data is generated by an independently distributed process requires consideration as well. Mandelbrot (1963) hypothesized the possible dependence of the random variables contributing to the long tails. Mandelbrot's (1963) and Fama's (1965) analyses of stock price changes report that large (small) changes tend to be followed by large (small) changes, of random sign. This dependence also appears to affect foreign exchange rates. The evidence by Hall, et. al. (1989) suggests that dependence in the distribution can be attributed to the existence of serial correlation in the variance.[4]

This article addresses the issues of independence, nonnormality and time varying parameters, and implicitly includes the constraint on the normality of futures price changes under temporal aggregation. This article considers only the hypothesis involving mixtures of distributions with finite variance or time varying parameters. The candidate processes considered are: (1) the ARCH process, (2) the compound normal distribution, (3) the mixed diffusion-jump process, and (4) the Student *t*-

[2]Application of the stable distribution to asset prices is initially attributed to Mandelbrot (1963) and Fama (1963, 1965). Mandelbrot notes that the variance of the change in the log of stock prices does not appear to have a limiting value, which motivates the use of a distribution with infinite variance.

[3]The normal distribution arises when the characteristic exponent is equal to 2. The characteristic exponent describes the behavior of the tails of the distribution.

[4]Hall, et al. (1989) reached this conclusion by examining the randomized historical data of various futures commodities.

distribution. These candidate processes assume independence, with the exception of the ARCH process. Results on the weekly change in the spot exchange rate are presented to enable comparison to related work in the literature.

The stochastic processes considered above can all be linked to the manner in which the flow of information affects asset prices. A theoretical formulation based on information flow effects is established in Anderson and Danthine (1983). They demonstrate how demand and supply uncertainty for commodity futures influence the variance of futures prices. In their article, the volatility of futures prices depends on the amount and type of information arriving at different points in time. They point out that their result does not imply any particular relationship with respect to the term to maturity. In a subsequent article, Anderson (1985) tests the determinants of commodity futures price volatility and finds that seasonality effects are predominant, while maturity effects are secondary. The seasonality effect discovered by Anderson (1985) captures the arrival of new information. This line of research emphasizes that it is the flow of information which is crucial and that this information flow need not exhibit a specific relationship to the time to maturity. In view of this, this article characterizes the type of information flow environment associated with each candidate process.

II. THE RELATIONSHIP BETWEEN SPOT AND FUTURES PRICES

Casual observation reveals that spot and futures prices tend to move in tandem with the innovations in the spot market driving the innovations in futures prices. Thus, one may suggest that the distribution of futures prices is not significantly different from that of spot prices. However, theoretically, futures prices differ from spot prices making it desirable to investigate whether there are any differences between the two asset prices.

Cox, Ingersoll, and Ross (1981) show that the futures price of foreign currency at time t, with maturity date T, denoted by $F(T, t)$, is the present value of the payoff $S(T)\ \Pi_{k=t}^{T-1}(1 + r_k)$; where $S(T)$ is the spot price of the foreign currency at T, and r_k is the one-period spot interest rate.

Hodrick and Srivastava (1987) incorporate the Cox, Ingersoll, and Ross (1981) analysis into an intertemporal asset pricing model, similar to Lucas (1982) and derive the following relationship:

$$F(T, t) = E_t[S(T)] + \mathrm{cov}_t[(Q_{t,T})(\Pi_{k=t}^{T-1}(1 + r_k)), S(T)] \tag{1}$$

where $E_t[S(T)]$ is the conditional expectation of $S(T)$ at time t, $\mathrm{cov}_t[x, y]$ is the conditional covariance of the random variables x and y, and $Q_{t,T}$ denotes the intertemporal marginal rate of substitution of dollars between time t and T.

The link between spot and futures prices is provided by (1) which indicates that the equilibrium futures price is the sum of the expected future spot price and a conditional covariance term which represents a risk premium. From (1) one can see also that the distribution of futures prices may differ from the spot price distribution when the risk premium is time-varying, which is often suggested in the literature (e.g., Domowitz and Hakkio (1981)). However, if the spot price follows a random walk and the risk premium component does not exist, then the spot and futures prices belong to the same class of distributions.

Based on the above link between spot and futures prices, this study is concerned with the conditional as well as the unconditional distribution of futures and spot prices. Distinguishing between the unconditional and conditional distribution is particularly important when heteroscedasticity is present. For example, if heteroscedasticity exists, then models testing market efficiency have to account for this factor. Standard hypotheses tests are inappropriate in that case. The distinction between unconditional and conditional distributions is also crucial when the variable of interest is not an independent random variable. For example, if the variable exhibits dependence, then the model must make use of the conditional distribution to find the optimal futures hedge which varies with time (e.g., Cecchetti, Cumby, and Figlewski (1986)). Alternatively, if independence holds, then it is appropriate to consider the unconditional distribution and, given a sufficiently long time series of data, make use of existing option pricing models such as Merton (1976).

III. MODEL CANDIDATES

In general, the candidates used in this article are derived as a mixture of distributions. As noted by Blattberg and Gonedes (1974), the mixture of distributions can be thought of as involving a subordinated stochastic processes. Let $z(s)$ and $h(s)$ denote stochastic processes defined by the time interval equal to a length of s. If another stochastic process is defined such that $x(s) = z(h(s))$, then the process $x(s)$ is said to be subordinate to the process $z(s)$ with the process $h(s)$ being a directing process. In this study, the random variable $z(s)$ denotes the change in the futures price of a foreign exchange rate over the time interval s. Blattberg and Gonedes (1974) interpret the random variable $h(s)$ as representing a change in the economic environment, or a change in the information set over the time interval s. For example, $h(s)$ may represent the change in the variance of $z(s)$. The distribution of $h(s)$ provides the mixing of the distributions for $z(s)$.

A summary of the various models to be tested follows. In each case, the random variable $x(t)$ denotes the percentage change in the spot or futures price from period $t - 1$ to t, computed by $\ln S(t) - \ln S(t - 1)$ or $\ln F(T, t) - \ln F(T, t - 1)$, where T denotes the maturity date. For the futures prices, the time series does not hold constant for the term to maturity; but looks at a given maturity range which does not exceed 90 days. The construction of the data series is similar to Cornew, et. al. (1984), and Hall, et. al. (1989) allowing for a longer time series and minimizing the differences in maturity which is not at issue here.

ARCH in Disturbances (ARCH-D)

The basic ARCH-D model developed by Engle (1982) can be summarized as follows

$$Y(t) = x(t)\beta + \epsilon(t) \tag{2}$$

$$\epsilon(t) \,|\, I(t - 1) \sim N(0, \sigma^2(t)) \tag{3}$$

$$\sigma^2(t) = \alpha_0 + \sum_{i=1}^{P} \alpha_i \epsilon^2(t - i) \tag{4}$$

where $I(t - 1)$ is the conditional information set. The variable $Y(t)$ is equal to the change in $\ln S(t)$ or $\ln F(T, t)$, while $x(t)$ is a $1 \mathrm{x} k$ vector which may include: a constant term, lagged dependent variables, and other exogenous variables. The vector β is $k \mathrm{x} 1$ and contains the coefficients of $x(t)$. The second moment for the ARCH process exists when $\Sigma\alpha_i < 1$. The variance for the unconditional distribution is then given by, $\mathrm{Var}(Y(t)) = \alpha_0/(1 - \Sigma\alpha_i)$. The ARCH process has a limiting normal distribution under temporal aggregation as noted by Diebold and Nerlove (1986).

A distinguishing feature of (4), from the mixture of distributions discussed below, is that it is nonrandom.[5] Furthermore, while Y(t) need not exhibit any serial correlation, the observations are not independent random variables because of (4). The other three candidate processes all assume independence. Tests are performed for linear and nonlinear independence of observations similar to Hsieh (1988, 1989).

The autocorrelations for the change in the log of the spot rate and the log of the futures price are computed and the results are set forth in Tables I and II. Since heteroscedasticity is at issue, the standard errors are adjusted as suggested by

Table I
WEEKLY CHANGE IN THE LOG OF THE SPOT PRICE AUTOCORRELATION COEFFICIENTS (HETEROSCEDASTICITY-CONSISTENT STANDARD ERRORS)

Lag Length	BP	CD	GM	JY	SF
1	.0155	.0556	.0266	.0557	−.0071
	(.0562)	(.0538)	(.0502)	(.0440)	(.0498)
2	−.0377	.0071	.0344	.1285[c]	.0165
	(.0541)	(.0497)	(.0506)	(.0434)	(.0506)
3	.0221	−.0027	.0012	.0137	.0025
	(.0575)	(.0502)	(.0562)	(.0449)	(.0592)
4	.0855	−.0191	.0243	.0714	.0483
	(.0550)	(.0487)	(.0497)	(.0471)	(.0481)
5	−.0269	−.0381	−.0645	−.0056	−.0350
	(.0580)	(.0472)	(.0469)	(.0501)	(.0485)
6	.0094	−.0427	−.0529	.0107	−.0366
	(.0438)	(.0411)	(.0444)	(.0453)	(.0534)
7	.0888[a]	−.0242	.0435	.0310	.0152
	(.0466)	(.0389)	(.0475)	(.0403)	(.0442)
8	.0272	−.0121	.0546	−.0209	.0547
	(.0487)	(.0431)	(.0456)	(.0411)	(.0462)
9	−.0036	−.0585	.0481	−.0337	.0240
	(.0476)	(.0503)	(.0459)	(.0401)	(.0463)
10	.0481	−.0173	.0247	−.0436	−.0125
	(.0444)	(.0456)	(.0406)	(.0422)	(.0429)
13	−.0171	.0470	.0173	.0730[a]	−.0207
	(.0418)	(.0467)	(.0392)	(.0417)	(.0386)
20	.0033	−.0224	.0025	.0168	.0401
	(.0452)	(.0378)	(.0428)	(.0567)	(.0428)
Adj. Box-Pierce	23.79	28.48	28.37	41.19	21.74
Q(36)	[.9409]	[.8095]	[.8139]	[.2540]	[.9708]

Significance levels for Prob > $|z|$.
[a]10%.
[b]5%.
[c]1%.
Value within [ ] denotes the significance level.

[5]This need not always be the case. Bollerslev (1987) shows that an ARCH-D model with conditionally t-distributed errors can be derived as a subordinate stochastic process. In this case, (2) and (3) would remain the same while the conditional variance in (4) would be a random variable driven by the addition of a prediction error component.

Table II
WEEKLY CHANGE IN THE LOG OF THE FUTURES PRICE AUTOCORRELATION COEFFICIENTS (HETEROSCEDASTICITY-CONSISTENT STANDARD ERRORS)

Lag Length	BP	CD	GM	JY	SF
1	−.0385	.0314	−.0164	.0275	−.0253
	(.0559)	(.0526)	(.0539)	(.0443)	(.0507)
2	−.0674	−.0297	−.0060	.1446[c]	−.0192
	(.0543)	(.0482)	(.0518)	(.0454)	(.0492)
3	.0337	.0118	.0036	−.0423	.0127
	(.0524)	(.0538)	(.0532)	(.0437)	(.0548)
4	.0880	−.0003	.0471	.0730	.0610
	(.0554)	(.0434)	(.0496)	(.0467)	(.0474)
5	−.0631	−.0871[a]	−.0855[a]	−.0302	−.0750
	(.0553)	(.0494)	(.0483)	(.0503)	(.0466)
6	.0474	−.0634	−.0521	.0323	−.0195
	(.0463)	(.0404)	(.0493)	(.0449)	(.0533)
7	.0788	.0242	.0308	.0117	−.0042
	(.0491)	(.0421)	(.0486)	(.0428)	(.0470)
8	.0245	.0230	.0273	−.0410	.0094
	(.0465)	(.0470)	(.0452)	(.0418)	(.0440)
9	.0019	−.0661	.0680	−.0193	.0274
	(.0513)	(.0499)	(.0476)	(.0419)	(.0462)
10	.0397	−.0128	.0198	−.0685[a]	−.0027
	(.0462)	(.0490)	(.0412)	(.0390)	(.0456)
13	−.0135	.0463	.0806[b]	.1541[c]	.0711
	(.0422)	(.0424)	(.0412)	(.0431)	(.0480)
20	−.0138	−.0290	−.0244	.0008	−.0060
	(.0441)	(.0372)	(.0440)	(.0532)	(.0440)
Adj. Box-Pierce	27.46	32.15	29.15	59.41	26.51
Q(36)	[.8459]	[.6525]	[.7838]	[.0083]	[.8759]

Significance levels for Prob > $|z|$.
[a]10%.
[b]5%.
[c]1%.
Value in [ ] denotes significance level.

Diebold (1986) and Hsieh (1988). An adjusted Box-Pierce Q statistic, also used by Diebold (1986), is used to test for linear independence or serial correlation.

The null hypothesis of serial independence for the spot exchange rates is not rejected. With the exception of the Japanese yen, the null hypothesis of serial independence for the futures prices is not rejected. In the case of the yen futures, the rejection of the null appears to be attributable to two conspicuous autocorrelations at lags 2 and 13.[6] The latter lag is associated with the splice point for the terms to

[6]No further filtering of the yen futures is performed. Only the first difference is examined. However, a simple linear filter of the first difference in the log of the yen futures is used and an AR(3) model estimated. This model was selected after conducting a search over AR(p) processes until the significance level of the Box-Pierce Q statistic for 36 lags exceeded 10%. The residuals of the fitted model are then tested. No significant linear or nonlinear dependence is detected which is the crucial issue here.

maturity, that is, when there is a shift in the maturity date. However, this is not peculiar to the yen futures since the spot yen also exhibits a significant value for the autocorrelation at lag 13. The German mark is the only other currency futures which exhibits a significant autocorrelation at the splice point of lag 13. This suggests that the splicing of maturities does not appear to have a significant impact on the first differences.[7]

The test for nonlinear independence is attributed to McLeod and Li (1983) who show that the squared residuals of an ARMA model can be used to construct a test. The autocorrelations of the squared residuals are used to form a Box-Pierce Q statistic which has an asymptotic Chi-square distribution.[8] For both the spot and futures prices, the Canadian dollar and Japanese yen did not reject the null hypothesis of nonlinear independence while the remaining currencies reject the null for both market prices. The results of this test suggests that the ARCH model should perform well for the British pound, German mark, and Swiss franc. It is found that the candidate processes which exhibit independence are still viable since the Canadian dollar and Japanese yen should reject the ARCH model in favor of one of the models which assume independence.

A test for the presence of ARCH effects using the Lagrange multiplier test as given by Engle (1982) is performed as well.[9] Both the spot and futures price change show significant ARCH effects for the British pound, German mark, and Swiss franc. The Canadian dollar exhibits low order ARCH effects, while the Japanese yen rejects the presence of ARCH effects.[10] These results are in accordance with the nonlinear test for independence conducted above.

Finally, a constraint often imposed on the lagged error terms appearing in (4) is that the weights decline the further in time one proceeds. That is, economic agents tend to weight the recent information more heavily. The estimation procedure takes this restriction into account by imposing linearly declining coefficients.

The model used in this study can be described by,

$$Y(t) = \beta + \epsilon(t) \tag{5}$$

$$\epsilon(t)\,|\,\mathrm{I}(t-1) \sim \mathrm{N}(0, \sigma^2(t)) \tag{6}$$

$$\sigma^2(t) = \alpha_0 + \phi\sum_{i=1}^{p}(p+1-i)\epsilon^2(t-i) \tag{7}$$

Equations (4) and (7) can be related by noting that $\alpha_i = \phi(p + 1 - i)$, for $i = 1$ to p.

A possible environment generating ARCH effects may involve the release of economic data over a series of weeks which tends to create either highly informative or minimally informative intervals.

[7]The Samuelson (1965) hypothesis, that the variance of the change in futures price increases as the contract nears maturity, could be affecting the results given the splicing of various maturities. However, Samuelson's model assumes a stationary time series. Hsieh and Manas-Anton (1986) do not find favorable evidence for the Samuelson hypothesis and interpret their results as supportive of the hypothesis that the variance is time-varying.

[8]Tables for the autocorrelation coefficients of the squared residuals, other test statistics, and estimates for the various models not reported in this article can be obtained from the authors upon request.

[9]Engle's test is complementary to the Box-Pierce Q test indicated above since the former relies on the partial autocorrelation coefficients while the latter makes use of the autocorrelation coefficients.

[10]For the Canadian dollar futures the number of significant lags does not exceed lag 4, while the spot Canadian dollar shows a significant lag at 1 only. The level of significance for the test statistics are between 5% and 10%.

Compound Normal Distributions

The compound normal distribution is a discrete finite mixture of normal distributions. If independent observations generated by a mixture of normals are summed, then one can expect a convergence to a normal distribution by the central limit theorem.

Results by Hall, et. al. (1989) suggest a mixture of normal distributions.[11] The following discussion of the compound normal distribution assumes that the observations are independent.

Suppose that the random variable $x(t)$ occurs with probability p_i $(i = 1,\ldots,m)$ from a normal distribution $N(x(t) \mid \mu_i, \sigma_i^2)$, with $p_1 + p_2 + \ldots + p_m = 1$, then $x(t)$ has a compound normal distribution with a probability density function defined by

$$f(x(t) \mid \Omega) = \sum_{i=1}^{m} p_i f_i(x(t) \mid \mu_i, \sigma_i^2), \quad -\infty < x(t) < \infty \tag{8}$$

where the parameter vector $\Omega = (p_1,\ldots, p_m, \mu_1,\ldots, \mu_m,\ldots, \sigma_1^2,\ldots, \sigma_m^2)$ and $f_i(x(t) \mid \mu_i, \sigma_i^2)$ is the normal density function for the ith distribution. The mean and variance of $x(t)$ are given by

$$E(x(t)) = \sum_{i=1}^{m} p_i \mu_i$$

$$\text{Var}(x(t)) = \sum_{i=1}^{m} p_i[(\mu_i - E(x(t)))^2 + \sigma_i^2]$$

The density function is symmetric if $\mu_i = \mu$, or $p_i = 1/m$ and $\sigma_i^2 = \sigma^2$, for all $i = 1,\ldots, m$. The function may be unimodal if $\mu_i = \mu$ for all i, or have several modes, up to m, depending on the relative values of the parameters.

Akgiray and Booth (1987) note that the use of the compound normal distribution may not be efficient if one is able to exactly partition the sample into subsamples corresponding to the appropriate parent distribution. Thus, each subsample would be investigated separately since the distributional form is dependent on a specified point in time. However, if such a partitioning is not known *a priori*, then it may be desirable to characterize the environment generating $x(t)$ by (8). In this case, the probabilities $p_1,\ldots, p_m$ do not depend on a given point in time.

The environment corresponding to this model assumes that there are a finite number of distinct sources of shocks. For example, one could imagine that fiscal policy shocks and monetary policy shocks are the fundamental exogenous forces which independently drive exchange rate changes. Alternative sources of disturbances would include productivity shocks or specific commodity price shocks.

Mixed Diffusion-Jump

The mixed stochastic process consists of the sum of two random variables. One term is Brownian motion, the other is an independent compound Poisson process exhibiting a random distribution for the jump amplitudes. This study assumes that the jump amplitude is normally distributed, similar to Akgiray and Booth (1987, 1988).[12] While the mixing distribution involves a discrete process, the mix-

[11]However, Hall, et. al. (1989) indicate that the price changes are not independent and hypothesize that the probable cause is because the variance of the mixing distribution is serially correlated.

[12]Jarrow and Rosenfeld (1984) assume a lognormal distribution for the jump amplitude.

ture is not finite as in the compound normal case. Results by Akgiray and Booth (1987), using stock returns, show that the change in prices tends to converge to the normal distribution at a low frequency (monthly changes).

The sample path of the resulting process, for the futures price change, is given by

$$\log(\mathrm{F}(\mathrm{T},t)/\mathrm{F}(\mathrm{T},0)) = (\beta - .5\sigma^2)t + \sigma \mathrm{B}k(t) + \sum_{n=1}^{N(t)} \mathrm{J}(t)_n \tag{9}$$

where

$\mathrm{F}(\mathrm{T},t)$ = the exchange rate futures maturing on date T and purchased at time t,
$\mathrm{B}(t)$ = a standard Brownian motion,
$\mathrm{N}(t)$ = a Poisson counting process with parameter $\tau > 0$,
$\mathrm{J}(t)_n$ = a normal random variable with mean α and variance δ^2 representing the logarithm of one plus the percentage change in the exchange rate at time t caused by the nth jump,
β = the instantaneous conditional expected rate of change, in the foreign exchange rate, per unit of time for the Brownian motion part of the process, and
σ^2 = the instantaneous conditional variance of the rate of change in the foreign exchange rate.

The probability density function of $\ln(\mathrm{F}(\mathrm{T},t)/\mathrm{F}(\mathrm{T},t-1))$ is then

$$f(x(t)\,|\,\Omega) = \sum_{n=0}^{\infty} (\exp(-\tau)/n!)(\tau^n)\mathrm{N}(x(t)\,|\,\mu + n\alpha,\ \sigma^2 + n\delta^2), \tag{10}$$

where $\mu = \beta - .5\sigma^2$ and N(.) is the normal density function with parameter vector $\Omega = (\mu, \sigma^2, \tau, \alpha, \delta^2)$. The density function described by (10) exhibits leptokurtosis if $\tau > 0$ and is skewed for $\alpha \neq 0$. The function is symmetric around μ if $\alpha = 0$. The mean of the distribution is given by $\mathrm{E}(x(t)) = \mu + \tau\alpha$ and the variance is $\mathrm{Var}(x(t)) = \sigma^2 + \tau(\delta^2 + \alpha^2)$. Specifications similar to (9) and (10) are used for the spot exchange rate.

The mixed diffusion-jump model has been applied successfully to stock market prices which display very large changes over a short time interval. Large shocks due to the arrival of certain information are responsible for the sharp jump in the price. These large shifts serve to produce the greater probability in the tail for this distribution relative to the normal distribution. With respect to exchange rate changes, it is possible that such jumps are generated by the random arrival of important information which causes a sharp adverse reaction or an overly optimistic reaction to news.

Scaled-*t*, or Student, Distribution

Observations are assumed to be independent sampling from an identical Student density function $f(x(t)|\Omega)$, where Ω is the parameter vector (δ, c, ν); with δ being the location parameter, c denoting the scale parameter, and ν denoting the degrees of freedom parameter. The density function is expressed as

$$f(x(t)\,|\,\Omega) = \frac{\Gamma[(1+\nu)/2]\nu^{\nu/2}c^{1/2}}{\Gamma(1/2)\Gamma(\nu/2)}[\nu + c(x(t) - \delta)^2]^{-(\nu+1)/2} \tag{11}$$

with $\mathrm{E}(x(t)) = \delta$, for $\nu > 1$ and $\mathrm{Var}(x(t)) = c^{-1}[\nu/(\nu - 2)]$, for $\nu > 2$. In general, all moments of order $n < \nu$ are finite. The Student distribution has fatter tails than the normal, but as $\nu \rightarrow \infty$, the Student density function converges to the normal density. Blattberg and Gonedes (1974) point out that the Student distribution can be derived from a continuous mixture of normal distributions with the same mean but differing scale. In this case, the variance is a random variable with a chi-square distribution.

The environment capable of yielding the Student distribution is essentially an extreme case of the compound normal distribution. The Student distribution is driven by an infinite number of sources of shocks as opposed to the finite number in the compound normal case.

IV. DATA AND ESTIMATION TECHNIQUES

The data consist of weekly spot and futures prices as recorded in the Wall Street Journal. The period of study is from November 1977 through December 1987. Friday closing prices are used except when missing; then, the Monday closing price or the next available price is used.[13] The currencies studied are the British pound (BP), Canadian dollar (CD), German mark (GM), Japanese yen (YN), and the Swiss franc (SF).

This study assumes that the week-to-week change avoids effects attributable to weekends and holidays. It is assumed that the Friday data is representative of the rest of the week because the release of various ecomonic data does not fall mainly on this day. However, the triple witching days may still have some impact which is not accounted for in this study.

To assess the properties of the weekly changes in $\ln S(t)$ and $\ln F(T, t)$ selected descriptive statistics are computed which include the mean, variance, standard deviation, skewness, kurtosis, t-test on the mean for the null hypothesis that it is zero, and Kolmogorov's D statistic for the null hypothesis of normality. For spot and futures price changes, the statistics tend to reject the assumption of normality. The value for Kolmogorov's D has a significance level less than 10% in every case, with the majority of the test statistics being less than 1%. The kurtosis measures are also relatively large, as compared to the normal.

The estimation procedure for each of the candidate models is based on a maximum likelihood technique. Maximization of the log of the likelihood function is obtained by using the DFP algorithm for the ARCH process, mixed diffusion-jump process, and Student distribution. The DFP routine employs numerical first derivatives in its function evaluations. An approximate second derivative matrix is computed to obtain an estimate of the covariance matrix. In the case of the compound normal distribution, the procedure used is the EM algorithm described by Redner and Walker (1984).

When a distribution can be obtained from another distribution through the restriction of some parameter values, a test for the better model can be based on the likelihood ratio. When this is the case, the models are said to be nested. Nested comparisons of models are needed to select the number of finite distributions for the compound normal process and the lag length for the ARCH process. However, when the models are nonnested, the comparison must be conducted differently. Tucker and Pond (1988), in examining the spot rate, compare the Student distribu-

[13]In those instances where quotes in the Wall Street Journal were illegible, data from Barrons are used. The spot prices are from the Foreign Exchange listing. In one instance, the spot price for the Canadian dollar is from the Philadelphia Exchange for foreign currency options. The futures prices are the International Money Market closing prices.

tion and compound normal processes using a likelihood ratio test. But Boothe and Glassman (1987) point out that a likelihood ratio test is inappropriate for comparing these nonnested hypotheses because one cannot impose restrictions on the Student distribution to obtain the compound normal model. The compound normal and the Student distributions are not nested because the compound normal is composed of a discrete mixture, as opposed to a continuous mixture in the Student case, of normal distributions.

The candidate processes being examined in this study are nonnested models which are compared by employing the Akaike Information Criteria (AIC). The value of the AIC is defined by

$$\mathrm{AIC} = -2\ln[\mathrm{l}(x \mid \Omega)] + 2k \tag{12}$$

where $\mathrm{l}(x|\Omega)$ is the value of the model's maximized likelihood function and k is the number of independent parameters in the model. The candidate model which minimizes the value of the AIC is selected as the best fitting model. The AIC value controls for the number of parameters being estimated and can be applied to both nested and nonnested models.[14] The candidate models are compared also according to the Schwarz criterion (SC), used also by Akgiray and Booth (1987), and Tucker and Pond (1988), which tends to give more weight to models with a lower dimension (fewer parameters). The value for the SC is given by

$$\mathrm{SC} = \ln[\mathrm{l}(x \mid \Omega)] - (k/2)\ln(nobs) \tag{13}$$

The candidate model which maximizes the SC value provides the best fit.

V. EMPIRICAL RESULTS

Comparison of the Candidate Models

The values for the AIC and the SC are reported below in Table III, with the latter values contained in parentheses. Table III reveals the poor fit of the normal distribution which exhibits the highest AIC value, relative to the other processes, in every instance.

As expected, the ARCH model does well for the majority of the currencies in the instances where the Lagrange multiplier test for ARCH effects are highly significant and when nonlinear independence is rejected. The AIC value shows a marked difference from the other processes which assume independence, as evidenced by the pound, mark, and franc. When the test for ARCH effects are weak and the nonlinear independence hypothesis is accepted, the data complements this by showing the relatively poor performance of the ARCH model. In particular, the ARCH test statistics for the yen are highly insignificant, which is evident by the high AIC relative to the remaining independence models.

Table IV contains several test statistics on the standardized residuals of the fitted ARCH model. Of particular interest are the tests for normality given by Kolmogorov's D statistic and the Kiefer-Salmon (KS) statistic. Of the three currencies which should be characterized by an ARCH process, only the German mark has a signifi-

[14]The use of a selection criteria for tested hypotheses, as opposed to a likelihood ratio test, should yield a more parsimonious model. One should be able to compare the dependent and independent models since the selection criteria represents a goodness-of-fit measure for comparable joint densities of the first differences in the spot or futures price.

Table III
WEEKLY CHANGE IN THE LOG OF THE FUTURES PRICE
AIC (SC) VALUES

Model\Currency	BP	CD	GM	JY	SF
Normal	−2671.17	−3852.04	−2744.74	−2791.81	−2586.31
(k = 2)	(1325.08)	(1915.51)	(1361.86)	(1385.40)	(1282.65)
ARCH	−2796.84	−3876.39	−2791.44	−2793.75	−2622.15
(k = 3)	(1382.66)	(1922.44)	(1379.96)	(1381.12)	(1295.32)
[# of lags]	[20]	[3]	[5]	[6]	[7]
Compound Normal	−2782.65	−3893.09	−2762.48	−2831.40	−2595.59
	(1365.06)	(1904.51)	(1354.97)	(1373.67)	(1271.53)
[# of distributions]	[2]	[3]	[2]	[3]	[2]
Diffusion-Jump	−2782.60	−3893.33	−2760.68	−2827.31	−2590.93
(k = 5)	(1365.03)	(1920.35)	(1354.07)	(1387.39)	(1269.20)
Student	−2784.46	−3898.69	−2764.91	−2825.78	−2592.74
(k = 3)	(1376.47)	(1933.58)	(1366.69)	(1397.13)	(1280.61)

k denotes the number of independent parameters.[15]

Table IV
STANDARDIZED ARCH RESIDUALS FOR THE
WEEKLY CHANGE IN THE LOG OF THE FUTURES PRICE
TEST STATISTICS

Sample Size: 520 Statistic	BP	CD	GM	JY	SF
Mean (μ)	−.01316	−.01754	.01831	.01284	.00563
Standard Dev.	1.00088	1.00081	1.00079	1.00088	1.00095
$t(H_0: \mu = 0)$	−.2999	−.3995	.4173	.2924	.1282
Prob > |t|	.7644	.6897	.6766	.7701	.8981
Skewness	−.0187	−.8872	.0932	.7123	.3328
KSS	.0376	59.9394[c]	.1250	39.0889[c]	8.5979[c]
Kurtosis	2.1218	4.6246	.6215	3.6973	.3728
KSK	94.5686[c]	451.6988[c]	7.8955[c]	288.5147[c]	2.7719[a]
KS	94.6062[c]	511.6383[c]	8.0205[c]	327.6036[c]	11.3698[c]
D	.0379[a]	.0646[c]	.0455[c]	.0720[c]	.0351

The kurtosis measure is normalized so that a zero value corresponds to the normal distribution.
KSS = Kiefer-Salmon normality test, $\chi^2(1)$ test statistic for skewness.
KSK = Kiefer-Salmon normality test, $\chi^2(1)$ test statistic for kurtosis.
KS = Kiefer-Salmon normality test which consists of the independent tests for skewness (KSS) and kurtosis (KSK). This is a χ^2 test statistic with 2 degrees of freedom.
D = Kolmogorov's D for the null hypothesis of normality.
Significance levels.
[a]10%.
[b]5%.
[c]1%.

cant D statistic at the 1% level. On the other hand, the pound, mark, and franc rejected normality at the 1% significance level according to the KS statistic. The rejection of normality by the German mark is due to the kurtosis measure, as indicated by the significance of the Kiefer-Salmon kurtosis statistic (KSK). Although the British pound is an exception, according to the D statistic, the large kurtosis measure makes it suspect and this is confirmed by the KSK statistic which rejects normality at the 1% significance level. The Swiss franc does not display as serious a problem with the remaining kurtosis in its residuals but does possess an undesirable skewness measure, with the Kiefer-Salmon test (KSS) rejecting the null of zero skewness at the 1% significance level. The other currencies which also have a significant D statistic at the 1% level, the Canadian dollar and Japanese yen, are not expected to be modeled adequately by the ARCH process.

When the ARCH process is not the best fit, the AIC values in Table III indicate that the compound normal distribution is the best fit for the Japanese yen, while the Student distribution is the best fit for the Canadian dollar. When an independent process is selected, the differences among the AIC values for the various independent models are not dramatic.

An examination of the SC values does show a greater distinction in goodness-of-fit values over their AIC counterparts. However, with the exception of the yen, the AIC and SC values pick out the same model. For the yen, the SC value selects the Student distribution over the compound normal which is selected by the AIC value.

The following discussion examines the question of whether a risk premium exists for futures prices and provides some evidence relating to the source of the nonlinear dependence detected for the pound, mark, and franc.

As already noted, the futures price distribution can be distinguished from the spot price distribution given the existence of a risk premium. In general, the empirical evidence of this study suggests that this premium does not enter in a linear fashion given the absence of serial correlation for the first difference of the series. However, the nonlinear dependence that is detected can be attributed possibly to the additive nature of a time varying risk premium component, as opposed to the multiplicative nature of the nonlinearity due to the variance. Following Hsieh (1989), one can distinguish between two types of nonlinearities in the variable $y(t)$:

(1) Additive dependence:

$$y(t) = f(y(t-1),\ldots,y(t-p),x(t-1),\ldots,x(t-q)) + \epsilon(t) \tag{14}$$

(2) Multiplicative dependence:

$$y(t) = f(y(t-1),\ldots,y(t-p),x(t-1),\ldots,x(t-q))\epsilon(t) \tag{15}$$

where $\epsilon(t)$ is an identical and independently distributed random variable with zero mean, and independent of past $y(t)$'s and $x(t)$'s. The function $f(\,)$ is some nonlinear function of the $y(t)$'s and $x(t)$'s for finite p and q.

Given the above formulations, additive dependence indicates that the nonlinearity enters through the mean of the stochastic process. Multiplicative dependence implies that the presence of the nonlinearity is transmitted through the variance, as suggested by the ARCH process. Both additive and multiplicative dependence can influence the random variable $y(t)$ as in the case of the ARCH-in-mean (ARCH-M) model used by Domowitz and Hakkio (1985).

Hsieh (1989) points out that $y(t)^2$ is correlated with its own lags in both the additive and multiplicative cases, which is what the McLeod and Li (1983) test detects. However, the McLeod and Li (1983) test cannot discriminate between the two types of nonlinearity. The distinguishing feature is that under multiplicative dependence

$$E[y(t) \mid y(t-1),\ldots,y(t-p),x(t-1),\ldots,x(t-q)] = 0$$

while additive dependence has

$$E[y(t) \mid y(t-1),\ldots,y(t-p),x(t-1),\ldots,x(t-q)] \neq 0.$$

Hsieh (1989) also provides a third-order moment test to detect the presence of additive dependence, given the null hypothesis of multiplicative dependence. The test statistic exploits the fact that, under multiplicative dependence, $y(t)$ is not correlated with terms such as $y(t - i)y(t - j)$, while additive dependence implies that the variable $y(t)$ is correlated with terms like $y(t - i)y(t - j)$.[16] Thus the null hypothesis of multiplicative dependence makes use of the fact that $E[y(t)y(t - i)y(t - j)] = 0$, while the alternative hypothesis is $E[y(t)y(t - i)y(t - j) \neq 0$, implying additive dependence.

The null hypothesis of multiplicative dependence is tested for $i = j = 10$. The test does not reject the null at the 1% significance level for all the currency futures. This suggests that if a risk premium does exist it is not easily detected. These results imply that one can expect the spot and futures price distributions to be similar.

Comparison with the Spot Exchange Rate

This study estimates the various distributions considered above for the weekly change in the log of the spot exchange rate. Using daily data, studies such as Akgiray and Booth (1988) and Tucker and Pond (1988), tend to support the mixed diffusion-jump model. The results of this study tend to support the ARCH model. Moreover, the results indicate that the diffusion-jump model is never selected as the best fitting model for the currencies examined. Table V below provides a summary of the findings.

The best fitting stochastic process for the various spot exchange rates is identical to the results for the futures prices with respect to the AIC measure. For the ARCH

Table V
WEEKLY CHANGE IN THE LOG OF THE SPOT EXCHANGE RATE
AIC (SC) VALUES

Model\Currency	BP	CD	GM	JY	SF
Normal	−2838.96	−3893.18	−2793.35	−2814.25	−2611.19
($k = 2$)	(1408.97)	(1936.08)	(1386.17)	(1396.62)	(1295.09)
ARCH	−2882.87	−3922.15	−2826.15	−2813.92	−2650.46
($k = 3$)	(1425.67	(1945.32)	(1397.37)	(1391.20)	(1309.47)
[# of lags]	[25]	[2]	[5]	[6]	[7]
Compound Normal	−2867.04	−3950.62	−2814.94	−2928.79	−2628.31
	(1391.49)	(1933.28)	(1381.20)	(1422.36)	(1287.89)
[# of distributions]	[3]	[3]	[2]	[3]	[2]
Diffusion-Jump	−2864.52	−3945.82	−2809.94	−2852.54	−2617.94
($k = 5$)	(1405.99)	(1946.64)	(1378.70)	(1400.00)	(1282.70)
Student	−2867.18	−3954.03	−2812.78	−2851.78	−2623.71
($k = 3$)	(1417.83)	(1961.25)	(1390.63)	(1395.74)	(1296.10)

k denotes the number of independent parameters.[15]

[15]For the compound normal process k is equal to three times the number of distributions minus one. Since the weights attached to the different distributions sum to 1, the adding-up restriction reduces the number of independent parameters by 1. For the ARCH process, the restrictions on the lagged error terms keep the number of independent parameters constant at $k = 3$.

[16]Hsieh (1989) indicates that the product $y(t - i)y(t - j)$ arises after performing a second-order Taylor series expansion of the function $f(\,)$ around zero.

process and the compound normal distribution, the number of lags or the number of mixing distributions is nearly the same. The British pound shows a slightly longer lag length for the spot exchange rate. The compound normal mixtures yield the same number of distributions for both the spot and futures prices. This is to be expected since similar information sets affect both markets.

As with the futures price change, the highly significant ARCH test statistics and the rejection of nonlinear independence for the pound, mark, and franc are reflected by the marked difference in the AIC value for the ARCH model, relative to the other independence models.

Tests of the standardized residuals from the ARCH model, presented in Table VI, show that the Kolmogorov D statistic is statistically significant at the 1% level for two of the three currencies which should be described by an ARCH process. However, the pound, mark, and franc are all rejected by the KS test for normality at the 1% level. Similar to the futures price case, the British pound is the exception for the normality test with respect to the D statistic. However, the large kurtosis measure still indicates that the simple ARCH model may not be adequate for the pound, as evidenced by the KSK statistic rejecting normality at the 1% significance level. The German mark exhibits problems with the remaining kurtosis in its standardized residuals, while the Swiss franc displays some problems with skewness. Further modeling appears to be appropriate to explain the remaining nonlinearities.[17]

When the ARCH model is not selected as the best fitting process, the compound normal is the best candidate for the Japanese yen; while the Student distribution provides the best fit for the Canadian dollar. Among the independence models, the AIC values are more closely related. Finally, note that the AIC and SC measures always select the same model, unlike the futures price case for the yen.

A test to discriminate between multiplicative and additive dependence is performed to see if the nonlinearities detected for some of the spot currencies can be ascribed to the variance. The test assumes $i = j = 10$. Most of the test statistics for the pound, mark, and franc are not significant at the 1% level.[18] In view of (1), these results suggest that there are no additional nonlinearities which would cause the spot price distribution to differ from that of the futures price.

VI. SUMMARY AND CONCLUSION

Previous empirical research on futures prices has not thoroughly addressed the issue of dependence versus independence in price changes.[19] This study finds that dependence exists for futures, as well as, spot prices. However, the results are mixed since the dependence model, represented by the ARCH process, is not the

[17]The ARCH estimates in this study are similar to the results reported by Diebold and Nerlove (1986) and Lastrapes (1989). Both studies examine the weekly change in spot exchange rates using Wednesday to construct their weekly sample. However, this study estimates a much shorter lag for the yen and a much longer lag for the pound than the preceding studies. These differences may be due to different weighing of the lags as in the case of Lastrapes (1989). Diebold and Nerlove (1986) do not identify an optimal lag length for the univariate model of each currency since their objective is to explore multivariate links.

[18]The one exception in which the test is significant at the 1% level is for the Swiss franc when $i = 3$ and $j = 4$.

[19]McCurdy and Morgan (1986) find evidence for foreign currency futures displaying time-varying volatility, but their main hypothesis of interest is in testing whether futures price changes follow a martingale process.

Table VI
STANDARDIZED ARCH RESIDUALS FOR THE WEEKLY CHANGE IN THE LOG OF THE SPOT PRICE
TEST STATISTICS

Sample Size: 520 Statistic	BP	CD	GM	JY	SF
Mean (μ)	−.02327	−.03774	.01754	.00651	.01714
Standard Dev.	1.00069	1.00025	1.00081	1.00094	1.00082
$t(H_0: \mu = 0)$	−.5303	−.8604	.3997	.1482	.3905
Prob > $\lvert t \rvert$	.5961	.3900	.6895	.8822	.6963
Skewness	.0283	−.8192	.1734	.8053	.3293
KSS	.8321	42.6991[c]	1.2518	53.1922[c]	6.6413[c]
Kurtosis	2.0957	5.0934	.7229	4.2445	.4774
KSK	92.1130[c]	545.7362[c]	10.7394[c]	380.7385[c]	4.6062[b]
KS	92.9451[c]	588.4352[c]	11.9911[c]	433.9307[c]	11.2476[c]
D	.0363[a]	.0615[c]	.0452[c]	.0784[c]	.0509[c]

The kurtosis measure is normalized so that a zero value corresponds to the normal distribution.

KSS = Kiefer-Salmon normality test, $\chi^2(1)$ test statistic for skewness.

KSK = Kiefer-Salmon normality test, $\chi^2(1)$ test statistic for kurtosis.

KS = Kiefer-Salmon normality test which consists of the independent tests for skewness (KSS) and kurtosis (KSK). This is a χ^2 test statistic with 2 degrees of freedom.

D = Kolmogorov's D for the null hypothesis of normality.

Significance levels.

[a]10%.

[b]5%.

[c]1%.

best fitting stochastic process for two of the five currencies examined. As a result, one cannot eliminate the models which assume independence.

Evidence on the nature of this dependence indicates that it is attributable to the variance, as opposed to the mean, of the stochastic process. This conclusion is based on a nonlinearity test developed by Hsieh (1989). Dependence in the variance is characteristic of the ARCH process.

This study does not imply that one cannot improve on the simple ARCH model presented above, as evidenced by the excess kurtosis present in the standardized ARCH residuals for spot and futures prices.[20] Bollerslev (1988) suggests the use of a conditional t-distribution as the underlying error term process, as well as a more general specification of the time variation in the conditional variance. Specifically, empirical studies (e.g., Bollerslev (1988)) employing a generalized autoregressive conditional heteroscedastic (GARCH) model find that GARCH models fit some spot foreign currencies better than the ARCH process with conditionally normal errors. Use of the technique outlined by Bollerslev may yield an improved and more parsimonious description of the movement in the conditional variance for spot and futures prices.

With respect to the two currencies which exhibited independence, the Canadian dollar and Japanese yen, it appears that there is a different information process at

[20]Hsieh (1988) examines daily changes in spot exchange rates for the period 1974–1983 and finds that the ARCH model, with a conditional normal distribution, is not able to sufficiently account for the observed leptokurtosis. Lastrapes (1989) examines weekly changes in spot exchange rates and reports similar results.

work. It is difficult to conjecture about the underlying forces which produce this difference in the way information affects the market.

Unlike previous empirical work, the findings of this study do not support the mixed diffusion-jump model as the best fitting model even among the class of models which assume independence. One explanation is that previous research mainly compares models using daily data. This study uses weekly data. The mixed diffusion-jump may have an advantage when using daily data if daily changes exhibit sharper movements than weekly changes, thus favoring the jump component for daily changes. This study provides further evidence that price changes for some spot and futures currencies are not independently distributed which rules out the mixed diffusion-jump process in these instances.

Spot and futures distributions for price changes are compared and the results indicate that they are very similar. This implies that there is not a strong risk premium component present. A time varying risk premium should affect the change in futures prices in an additive manner for the distribution of futures prices to be significantly different from the spot price distribution. This study does not find any evidence for additive nonlinearity in futures prices. However, given the relatively small sample size, the test used to discriminate between additive and multiplicative nonlinearity may not have adequate power to detect a hybrid model which allows for both additive and multiplicative nonlinearities. It would be appropriate to examine an ARCH-M model which can account for the presence of time variation in the mean and variance.

Lastly, additional support for a time varying variance in the foreign exchange market implies that tests for heteroscedasticity should become standard. The search for a better description of time variation in the conditional variance needs more research.

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