Price Forecasts and Interest Rate Forecasts: An Extension of Levy's Hypothesis

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INTRODUCTION

In a recent article in this Journal, Levy (1989) shows that if investors can forecast interest rates one period in advance over the duration of the futures contract, forward and futures prices must be equivalent.[1] Hence the condition that interest rates be deterministic is not a necessary condition for price equivalence. His argument relaxes the condition postulated by Cox, Ingersoll, and Ross (1981), Richard and Sundaresan (1981), and Jarrow and Oldfield (1981). Blenman (1988) demonstrates in a continuous-time framework that the deterministic interest rate condition is neither necessary nor sufficient.

Futures and forward prices are usually functions of sets of state variables that include interest rates and asset prices. In the special case where the interest rate is the only state variable and the term of the interest rate matches the time to maturity of the contract, a one-to-one and onto mapping exists. Knowledge of the interest rate implies knowledge of the contract price and the risks associated with either interest or price forecasts are equivalent.

In cases where there are additional state variables to consider or the term of the interest rate is less than the time to maturity of the contract, this equivalence no longer holds and nothing can be said on an *a priori* basis about the relative risks associated with price and interest rate forecasts. The latter set of circumstances apply in the case of Levy (1989).

This article extends the results of Levy (1989). It is shown that the ability to accurately forecast futures contract prices also forces the same equivalence.[2] This, in

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[1]In principle this requires the hedger to forecast the entire structure of interest rates one period in advance. The reliance on futures price forecasts which is the central focus of this article seems to impose less onerous restrictions. Futures price forecasting can at times be an easier proposition than forecasting interest rates, especially if market conditions dictate that limit moves occur.

[2]In fact much weaker conditions are possible. The same end-of-hedging period aggregate cash flows can be generated with imprecise prior period price forecasts and an accurate terminal period price forecast. This is shown in Appendix A. It is therefore postulated that a hedging strategy based on the ability to accurately forecast the one period interest rate applicable to the end of the hedging period must exist, that will also force equivalance provided there is a unique mapping between interest rates and prices.

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conjunction with that of Levy (1989), indicates that knowledge of either future prices or interest rate distributions should facilitate the construction of riskless hedges.[3] In the alternative approach modelled here, the hedger designs a strategy for futures contract holding based on expected futures price variations. This strategy differs from the usual Cox, Ingersoll, and Ross (1981) and Levy (1989) strategies where the number of futures contract held per time period is a function of prevailing interest rates or bond prices.[4]

Here the strategy is based on replicating the cash flow of a forward contract without any initial injection of outside funds. Contract selection in each period is based upon expected futures price variation. At the end of every period existing futures contract positions are liquidated and the proceeds are invested in bonds whose maturities coincide with that of the futures contract. The results presented here are robust to a wide range of forecasting precision on intermediate futures prices, provided that the minimal condition of being able to forecast the terminal price at contract maturity is met.

From a practical standpoint this hedging strategy adds another element to the hedgers arsenal of strategies. In any event the strategy developed here indicates that weaker conditions than those specified by Levy (1989) are consistent with forward and futures price equivalence. Forecast errors associated with prior periods hedging can be nullified providing that in the terminal period, the futures prices forecast is correct. It is therefore demonstrated that perfect forecasts in every period are not necessary. The hedger can live with some amount of imprecision on earlier forecasts and optimally adjust his futures position in the final period. Such behavior by hedgers in this setting would be sufficient to force equality between forward and futures prices.

II. THE MODEL

$S(n)$ is the spot price of the underlying asset at time n.
$G(t, n)$ is the time-t price of a forward contract that matures at time n.
$F(t, n)$ is the time-t price of a futures contract that matures at time n.
$B(t, n)$ is the time-t price of a zero-coupon bond that matures at time n and pays \$1 with certainty.

By definition, $1/(B(t,n)) = \Pi_{j=t}^{n-1} 1 + r_{j,1}^{t}$ where $r_{j,1}^{t}$ is the one period interest rate for time period j based on information available at time t. If interest rates are stochastic then for $j > t$, $r_{j,1}^{t}$ is not known with certainty. A forward contract purchased at time 0 generates a payoff of $-G(0, n) + S(n)$. If futures strategies are to replicate the hedging features of a forward contract against price risk, all associated cash flows must be self-financing. It is therefore assumed that: (1) the hedging

[3]This should not be a surprising conclusion. For debt futures like T-Bill futures, knowledge of the applicable discount rate implies knowledge of the futures price. The "riskless" hedges that are addressed here as well as in Levy (1989) protect against price risk by ensuring that assets are obtainable in the future at currently prevailing forward and futures prices. Arbitrage activity is then assumed to force equality between the two type of prices.

[4]Cox, Ingersoll, and Ross (1981) and Richard and Sundaresan (1981) derived futures and forward pricing formulas in a homogeneous-consumers setting. Blenman (1988) provides analogous results in a heteregeneous-consumers economy. All of these formulas flow from a forward strategy requiring an initial injection of funds in one period bonds. The initial investment plus all accumulated flows are continuously rolled over until contract maturity. Futures contract positions are selected based on the current structure of interest rates, liquidated at the end of every period and the associated cash flows are rolled over in one-period bonds until contract maturity.

strategies employed are self-financing; (2) establishing forward and futures positions is costless; (3) all cash flows emanate from established positions; and (4) zero-coupon discount bonds exist at all relevant maturities.

The hedging strategy discussed here entails a constant adjusting of the hedging portfolio with a rectifying adjustment in the terminal stage. This dynamic strategy is not unique and potential hedgers have a wide degree of flexibility in their choice of strategies. Intermediate period contract selection can be altered depending on risk aversion considerations. The general hedging strategy is described below.

At the start of time period k purchase N_k futures contracts. Liquidate the position at time $k + 1$ and invest the proceeds in bonds with $N - k - 1$ periods to maturity. At time $k + 1$ buy N_{k+1} futures contracts. Liquidate this new position at time $k + 2$ and invest the proceeds in bonds with $N - k - 2$ periods to maturity. Repeat this strategy until period $n - 1$ where N_{n-1} futures contracts are purchased and held to contract maturity. A specific example where $k = 0, 1, 2, 3$ and $n = 4$ is illustrated in Table I. Complete details of the derivation of the entries in Table I are provided in Appendix A.

Under a perfect price forecasts assumption, a hedger who chooses to generate constant dollar cash flows in periods 1, 2, 3, and a variable rectifying cash flow in the final period will select the following contract positions.

$$N_0 = \frac{1}{-F(0,4) + F(1,4)}$$

$$N_1 = \frac{1}{-F(1,4) + F(2,4)}$$

$$N_2 = \frac{1}{-F(2,4) + F(3,4)}$$

and

$$N_3 = \frac{-F(0,4) + S(4) - \left(\dfrac{1}{B(1,4)} + \dfrac{1}{B(2,4)} + \dfrac{1}{B(3,4)}\right)}{-F(3,4) + S(4)}$$

At the end of the third period existing futures contracts are rewritten to $S(4)$. N_0, N_1, N_2, and N_3 are all feasible based on the ability to forecast futures prices one

Table I
CASH FLOWS FROM THE FUTURES HEDGE

Start of period k	$k = 0$	$k = 1$	$k = 2$	$k = 3$
Futures contracts purchased	N_0	N_1	N_2	N_3
Futures position cash flow at start of period $k + 1$	1	1	1	$N_3(-F(3,4) + S(4))$
Value of all cash flows at end period 3	$1/B(1,4)$	$1/B(2,4)$	$1/B(3,4)$	$N_3(-F(3,4) + S(4))$

period in advance. In addition N_3 is feasible since the stucture of past interest rates is revealed to the hedger. The accumulated cash flow at the end of period 3 is the aggregate of the entries in row 4 of Table I, namely $-F(0,4) + S(4)$. The futures hedging strategy thus generates a payoff of $-F(0,4) + S(4)$ without any injection of outside funds. A forward contract would generate a payoff of $-G(0,4) + S(4)$. In the absence of arbitrage opportunities $G(0,4) = F(0,4)$ and futures and forward prices must be equal. It is easily verifiable that different choices of contract positions are feasible that would replicate the aggregate cash flow in the terminal period. Hence the hedging strategy is nonunique.

The same cash flows can be generated under less restrictive assumptions. It is shown in Appendix A that the cash flow $-F(0,4) + S(4)$ can be replicated by a futures strategy even if only the terminal contract price is forecast with accuracy. This will of course be the case where the forecast futures price variation may differ from the realized futures price variation in all periods except the ultimate one. In this case, the general strategy generates variable dollar cash flows in each period. In both cases discussed in Appendix A, a hedger has the flexibility of modifying early period futures contract positions at the expense of a possibly more significant adjustment in the terminal period. In general, the longer the hedging period the greater the degree of flexibility accorded to the hedger.

III. CONCLUSION

This article demonstrates that the ability to forecast futures prices can enable a hedger to replicate the cash flows associated with a forward contract by the use of futures contracts and term bonds. The strategy developed here is equivalent to that of Levy (1989) when a one-to-one relation exists between the interest rate and futures price. However, the strategy described here holds even when the one-to-one relation breaks down. These results hold regardless of whether interest rates are stochastic or deterministic. Under the stated assumptions arbitrage activity will force futures and forward prices to be equal.

It is shown that hedgers need not have perfect forecasting ability in every interval over the hedging period. Hence weaker conditions than those specified by Levy (1989) do exist. Finally, the hedging strategy developed here provides another tool which may be applicable in periods when price forecasts are more accurate than interest rates forecasts. If prices are trending up strongly, and barriers like exchange-imposed limit moves are in force then it is possible that price forecasts may be superior to interest rate forecasts simply because of the reduction in the range of permissible variation.

Appendix A

CASE 1. PERFECT PRICE FORECAST ASSUMPTION IN ALL PERIODS

Time 0	Cash Flow at Start of time 1 on a Futures Contract	Cash Flows at Start of time 1 on Futures Position
Buy N_0 futures contracts.	$-F(0,4) + F(1,4)$	$N_0(-F(0,4) + F(1,4)) = 1$

The contracts are liquidated at the start of time period 1, and \$1 is invested for 3 time periods. The hedger buys $1/B(1,4)$ bonds since the price of a 3-period bond at the start of time period 1 is $B(1,4)$ where $B(1,4) < 1$. His payoff at the end of period 3 is $1/B(1,4)$.

Time 1	Cash Flow at Start of time 2 on a Futures Contract	Cash Flows at Start of time 2 on Futures Position
Buy N_1 futures contracts.	$-F(1,4) + F(2,4)$	$N_1(-F(1,4) + F(2,4)) = 1$

The contracts are liquidated at the start of period 2 and \$1 is invested for 2 time periods. The hedger then buys $1/B(2,4)$ bonds since the price of a 2-period bond is $B(2,4)$ where $B(2,4) < 1$. His payoff at the end of period 3 is obviously $1/B(2,4)$.

Time 2	Cash Flow at Start of time 3 on a Futures Contract	Cash Flow at Start of time 3 on Futures Position
Buy N_2 futures contracts.	$-F(2,4) + F(3,4)$	$N_2(-F(2,4) + F(3,4)) = 1$

The contracts are liquidated at the start of period 3 and \$1 is invested in 1-time period bonds. The hedger then buys $1/B(3,4)$ bonds and his payoff at the end of period 3 is obviously $1/B(3,4)$. Hence at the start of period 3, the hedger knows that his payoff at the end of the period from his bond position is $1/B(1,4) + 1/B(2,4) + 1/B(3,4)$ and these values are entirely deterministic. The end of period futures price is $F(4,4)$ and based on his forecasting ability he buys N_3 contracts. N_3 can be calculated by choosing the number of contracts such that its derivative cash flow plus all flows from bond positions equal $-F(0,4) + S(4)$. It is explicitly assumed that price convergence between the futures price and the spot price occurs at contract maturity.

At the end of the period the payoff on all outstanding futures contracts plus his bond position is $N_3(-F(3,4) + S(4)) + 1/B(1,4) + 1/B(2,4) + 1/B(3,4)$. This sum simplifies to $-F(0,4) + S(4)$, when N_3 is chosen to be

$$N_3 = \frac{-F(0,4) + S(4) - (1/B(1,4) + 1/B(2,4) + 1/B(3,4))}{-F(3,4) + S(4)}$$

CASE 2: IMPERFECT PRIOR PERIOD FORECASTS AND ACCURATE TERMINAL PERIOD FORECAST

This subsection deals with the possibility that all the forecasts up to the start of period 3 are incorrect. The realized futures price $F(t,n)$ differs from the forecast futures price $F_j(t,n)$ where $j < t$. In case 1 it is implicitly assumed that $F_j(t,n) = F(t,n)$ for all j and t.

Time 0	Cash Flow at Start of time 1 on a Futures Contract	Cash Flows at Start of time 1 on Futures Position
Buy $N_0^* = \dfrac{1}{-F(0,4) + F_0(1,4)}$ futures contracts.	$V_1 \equiv -F(0,4) + F(1,4)$	$V_1 N_0^* \equiv X_1$

The hedger invests in bonds after liquidating his futures position at the start of time 1. $X_1/B(1,4)$ bonds are bought and the resulting cash flow at the end of period 3 is $X_1/B(1,4)$.

	Time 1	Cash Flow at Start of time 2 on a Futures Contract	Cash Flows at Start of time 2 on Futures Position
Buy $N_1^* =$	$\dfrac{1}{-F(1,4) + F_1(2,4)}$ futures contracts.	$V_2 \equiv -F(1,4) + F(2,4)$	$V_2 N_1^* \equiv X_2$

The hedger invests in bonds after liquidating his futures position. $X_2/B(2,4)$ bonds are bought and the resulting cash flows at the end of period 3 is $X_2/B(2,4)$.

	Time 2	Cash Flow at Start of time 3 on a Futures Contract	Cash Flows at Start of time 3 on Futures Position
Buy $N_2^* =$	$\dfrac{1}{-F(2,4) + F_2(3,4)}$ futures contracts.	$V_3 \equiv -F(2,4) + F(3,4)$	$V_3 N_2^* \equiv X_3$

The hedger invests in bonds after liquidating his futures position. $X_3/B(3,4)$ bonds are bought and the resulting cash flow at the end of period 3 is $X_3/B(3,4)$. Note that with perfect intermediate period price forecasts $X_1 = X_2 = X_3 = 1$. The hedger is assumed at the start of the final period to be able to forecast $S(4)$ exactly. Hence he can always select N_3^* such that all accumulated cash flows at the end of time 3 total $-F(0,4) + S(4)$. It is easy to verify that $N_3^* = -F(0,4) + S(4) - (X_1/B(1,4) + X_2/B(2,4) + X_3/B(3,4))/-F(3,4) + S(4)$ is the futures contract position that ensures the desired aggregate terminal cash flow. Note that N_3^* is feasible since the values of the variables X_1, X_2, and X_3 will be known to the hedger at the start of period 3.

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