Corporate Taxes and Hedging with Futures

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I. INTRODUCTION

It is well understood that a risk averse utility maximizing producer may have an incentive to hedge against fluctuations in the prices of his outputs by taking short positions in the relevant commodity futures contracts. By constrast, there is no satisfactory understanding of the role of hedging by corporations which are managed to maximize the market value of their operations. Without taxes, if financial markets are informationally efficient, "sufficiently" complete, and free from transaction costs or other barriers to trade, the net present value (NPV) of every transaction in financial markets is zero. Therefore, corporate hedging in financial markets, cannot possibly increase the firm's value unless it affects the firm's operations in the 'real' markets.

In the presence of taxes the firm's financing (capital structure) decisions become relevant (see for example DeAngelo and Masulis (1980)). Hedging by the firm may become relevant in the context of capital structure, since by reducing the uncertainty about its earnings the firm may be able to borrow more and gain an increased benefit from the tax deductability of interest payments. However, this possible hedging motive would not explain hedging by firms which are financed primarily with equity.

The purpose of this paper is to show that with common tax systems there is a rationale for corporate hedging, independent of capital structure considerations. In the U.S.A., as in many other countries, the government does not provide an automatic tax rebate to a firm which experiences a loss. Instead, the loss can be carried forwards or backward for a number of years to offset future or past profits. If the loss is used to offset future profits, the tax rebate is deferred and therefore its present value is reduced. Moreover, since the rebate is contigent on past and future profitability, it may be partly or completely lost. Compared with a tax system which provides automatic tax rebates, the tax system under consideration generates a production inefficiency. Hedging increases the firm's value by alleviating the tax burden and the induced production inefficiency.

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II. THE MODEL

Consider a competitive firm which at time $t = 0$ commits to produce q units of a commodity. At $t = \tau$ production is completed, the output is sold at an uncertain unit price $\tilde{p}_\tau$, and the (certain) production cost, $c(q)$, is paid. It is assumed that $c'(q) > 0$ and $c''(q) > 0$. For the convenience of using well established results from the options pricing theory, it is assumed, without loss of generality, that the dynamics of $\tilde{p}_\tau$ can be described by a geometric Brownian motion. Finally, to separate the effects of hedging from the effect of borrowing or lending by the firm, it is assumed that the firm is financed only with equity, and that interest income from lending is not subject to taxation.

The production decision when the tax system provides an automatic tax rebate in the event of a loss is examined first. Denoting the corporate tax rate by t_c, the after tax profit at $t = \tau$ is given by

$$\tilde{\pi}_\tau = (1 - t_c)(q\tilde{p}_\tau - c(q)).$$

Since the quantity $t_c c(q)$ is riskless, its present vlaue is obtained by discounting at the riskless interest rate r. Denoting the present value of $\tilde{p}_\tau$ by p_0, the *NPV* of production, V_0, is given by

$$V_0 = (1 - t_c)(qp_0 - e^{-r\tau}c(q)). \tag{1}$$

Clearly, the assumed tax system does not introduce any production inefficiency. The firm's optimal output (q^0) is determined via the marginal cost pricing rule $p_0 \exp(r\tau) = c'(q^0)$, exactly as if the firm were tax-exempt.

Taking a short position at $t = 0$ in a commodity futures contract with expiration date τ is equivalent to a committing to deliver a unit of the commodity at $t = \tau$, for a fixed payment $f(\tau)$ (the futures price) which has been determined at $t = 0$. Since no money exchanges hands at $t = 0$, the market value of the position at that time is zero. The payoff at expiration is $(f(\tau) - \tilde{p}_\tau)$, and to avoid arbitrage its present value at $t = 0$ must be zero. That is, the futures price is given by

$$f(\tau) = p_0 e^{r\tau}. \tag{2}$$

It is now apparent that the first order optimality condition can be written as

$$f(\tau) = c'(q^0). \tag{3}$$

The same *NPV* and optimal output would have resulted if the firm had taken positions in the commodity futures contract. It follows that with automatic tax rebates, the firm is indifferent to hedging.

To see the rationale for hedging in the absence of automatic tax rebates, it is sufficient to examine an extreme case of such a tax system. It is assumed that if a loss occurs it cannot be carried backward or forward. The firm pays taxes on profits but receives no tax rebates for losses. Now the firm's after tax cash flow at $t = \tau$ is

$$\tilde{\pi}_\tau = q\tilde{p}_\tau - c(q) - t_c \max[q\tilde{p}_\tau - c(q); 0],$$

alternatively, using the notation $AC(q)$ for average cost,

$$\tilde{\pi}_\tau = q\tilde{p}_\tau - c(q) - t_c q \max[\tilde{p}_\tau - AC(q); 0], \tag{4}$$

Note that the tax payment in (4) is the rate times the expiration payoff to *q commodity call options,* each with exercise price equal to $AC(q)$. Black (1976) develops the valuation formula for commodity options (assuming that the commodity price follows a geometric Brownian motion). Let σ^2 denote the variance of the instantaneous rate of the commodity price change, and $N(\cdot)$ denote the cumulative standard normal distribution, the value of each commodity call option is given by $W \equiv W(f, \tau, r, \sigma, AC(q))$,

$$W = e^{-r\tau}[f(\tau)N(d_1) - AC(q)N(d_2)], \tag{5}$$

where:

$$d_1 = \frac{\ln\left(\frac{f(\tau)}{AC(q)}\right) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}},$$

$$d_2 = d_1 - \sigma\sqrt{\tau}.$$

From (4) and (2), the NPV of production is

$$V_0 = (qf(\tau) - c(q))e^{-r\tau} - t_c qW. \tag{6}$$

Maximizing V_0 with respect to q, using the fact

$$\frac{\partial W}{\partial AC(q)} = -e^{-r\tau}N(d_2),$$

one obtains

$$f(\tau) = \frac{1 - t_c N(d_2^*)}{1 - t_c N(d_1^*)}c'(q^*), \tag{7}$$

where the superscript (*) indicates optimum values.

Proposition 1: *With the assumed tax system, with restricted loss carry-back/forward (a) a value maximizing a firm which does not hedge in futures markets will produce a quantity which is less than the optimal quantity in the absence of taxes, i.e., $q^* < q^0$; (b) the output quantity declines as the volatility of the underlying commodity price increases, i.e., $\frac{\partial q^*}{\partial \sigma} < 0$.*

Proof: (a) From (7) and the fact $d_1^* > d_2^*$ it follows that $f(\tau) > c'(q^*)$, and therefore $q^* < q^0$. (b) Let $G = [1 - t_c N(d_2^*)/[1 - t_c N(d_1^*)]$, and differentiate eq. (7) with respect to σ to obtain

$$0 = \frac{\partial G}{\partial \sigma}c'(q^*) + [Gc''(q^*) + c'(q^*)\frac{\partial G}{\partial AC(q^*)}AC'(q^*)]\frac{\partial q^*}{\partial \sigma}.$$

It is tedious but not difficult to check that $\frac{\partial G}{\partial \sigma} > 0$, $\frac{\partial G}{\partial AC(q^*)} > 0$. These facts and $c'(q^*) > 0$, $AC'(q^*) > 0$, and $c''(q^*) > 0$ establish the required result. □

To gain an intuitive understanding of proposition 1 and to facilitate a comparison with a tax system which treats profit and loss symmetrically, note that (4) can be rewritten as

$$\tilde{\pi}_\tau = (q\tilde{p}_\tau - c(q))(1 - t_c) - t_c q \max[AC(q) - \tilde{p}_\tau; 0]. \tag{8}$$

The first component of (8) is the after tax profit when the tax system provides an automatic tax rebate for a loss, while the second component is the tax penalty due to the actual lack of such a rebate. Note that this tax penalty is equal to the product of the tax rate and the expiration payoff to q put options with an exercise price equal to the average production cost. Denoting the present value of such a put option by $\hat{W}$, The *NPV* of production is given by

$$V_0 = (qf(\tau) - c(q))e^{-r\tau}(1 - t_c) - t_c q\hat{W}. \tag{9}$$

Equations (9) and (6) are of course identical, one can be obtained from the other one via the put-call parity relationship.

Now proposition 1 can be readily explained intuitively. The value of the tax penalty due to the lack of automatic tax rebate for loss ($t_c q\hat{W}$) increases with the level of output (q). This is so for two reasons: (a) increasing production increases the average cost (which is the exercise price of the put option) and, therefore, increases the value $\hat{W}$, (b) the total quantity of put options, and hence their value ($q\hat{W}$), increases. Thus, on an after tax basis, the marginal cost of production (present value) in an economy without an automatic tax rebate for loss is higher than in an economy with automatic tax rebate. This higher after tax marginal cost leads to a lower optimal production level, i.e., $q^* < q_0$. An increase in the volatility of the underlying commodity (σ) increases the value of the put option and the value of the after tax marginal cost and, therefore, reduces the optimal production level.

Now consider what happens if the firm can take n short commodity futures positions, and thus adds to its pretax earnings the payoff $n(f(\tau) - \tilde{p}_\tau)$, resulting in after tax profit

$$\tilde{\pi}_\tau = q\tilde{p}_\tau + n(f(\tau) - \tilde{p}r) - c(q) - t_c \max[(q - n)\tilde{p}_\tau + nf(\tau) - c(q); 0]. \tag{10}$$

To facilitate a comparison with a tax system with automatic rebates, one can rewrite (10) as follows:

$$\tilde{\pi}_\tau = [q\tilde{p}_\tau + n(f(\tau) - \tilde{p}_\tau) - c(q)](1 - t_c) - \tilde{\phi}_\tau, \tag{11}$$

where

$$\tilde{\phi}\tau = t_c \max[(n - q)\tilde{p}_\tau - nf(\tau) + c(q); 0]. \tag{12}$$

is the tax penalty due to lack of automatic tax rebate.

Proposition 2: *A value maximizing producer will hedge by taking n short commodity futures positions at $t = 0$, where $q^0 \geq n \geq c(q^0)/f(\tau)$, and will produce the same output quantity as in the absence of taxes, i.e., $q^* = q^0$.*

Proof: Consider an output $q > 0$ and a hedge n such that $q \geq n \geq c(q)/f(\tau)$. With such a configuration the pre-tax profit is always positive, and the penalty $\phi_\tau = 0$. Therefore,

$$\tilde{\pi}_\tau = [q\tilde{p}_\tau + n(f(\tau) - \tilde{p}_\tau) - c(q)](1 - t_c).$$

Recalling that the present value of a futures position is zero, and using (2), the *NPV* of production is

$$V_0 = [qf(\tau) - c(q)](1 - t_c)e^{-r\tau}, \tag{13}$$

which is independent of n. From (1) and (2), V_0 in (13) is identical to the value of profits in an economy with automatic tax rebates. Therefore, the optimal quantity is q^0, defined by (3). For any other configuration of (q, n), $\tilde{\phi}_\tau \geq 0$, with strict inequality for some $\tilde{p}_\tau \geq 0$. Hence, the present value of ϕ_τ is positive, and from (11)

$$V_0 < [qf(\tau) - c(q)](1 - t_c)e^{-r\tau} < [q^0 f(\tau) - c(q^0)](1 - t_c)e^{-r\tau}.$$

It follows immediately that q^0 and n, $q^0 \geq n \geq c(q^0)/f(\tau)$, constitute an optimal production hedging configuration. □

It is important to recognize that while optimal hedging will reduce the volatility of pretax earnings, it will not necessarily minimize it. The purpose of hedging is not to stabilize earnings, but rather to minimize the present value of the penalty due to the lack of an automatic tax rebate for loss. Indeed, proposition 2 shows that the optimal hedge is not unique and, therefore, a variety of earnings volatilities will be consistent with optimum.

How does hedging work? The short futures position becomes more profitable as the underlying commodity futures spot price declines, that is, low operating profits are associated with high profits on the futures position, and vice versa. In this model the optimal hedge eliminates any potential operating loss, at the cost of reducing the potential profit. Without taxes, or with a symmetric tax system, the firm is indifferent to such a tradeoff. However, with any tax system without an automatic tax rebate for loss, the reduction of potential profit is less costly (after taxes) than the gain from reducing the potential loss. In addition to this direct tax benefit, because the potential loss is eliminated, the asymmetric treatment of profit and loss is no longer relevant, and the incentive to cut production is eliminated.

III. CONCLUDING REMARKS

It is shown that because of the asymmetric tax treatment of profit and loss, the present value of the firm's tax liability can be viewed as the value of the tax liability under a symmetric tax system, plus a tax penalty whose value is equal to the product of the tax rate, the production level and the value of a European commodity put option with an exercise price equal to the average unit cost of production. Compared to a symmetric tax system, the additional tax penalty implies a higher present value of after-tax marginal cost of production and, therefore, a lower level of production. The larger is the volatility of the underlying commodity (and, hence, the volatility of pre-tax earnings), the larger is the value of tax penalty and the reduction in production.

When a futures market in the underlying commodity exists, optimal hedging increases the value of the firm through two related effects: Given a level of production, the present value of the tax penalty is minimized; the optimal level of production increases and, therefore, the expected pre-tax earnings increase. In the model proposed in this article, hedging eliminates the potential loss and, hence, the tax penalty. As a result, production increases to the level that would prevail in the absence of taxes (or under a symmetric tax system). Obviously, even if the firm does not change its production level, the reduction in the present value of the tax penalty is a sufficient rationale for hedging.

To obtain these ideal results of hedging, the availability of a futures contract whose payoffs are perfectly correlated with the firm's pre-tax earnings is crucial. The availability of such a contract allows for a hedge which eliminates any potential loss and; therefore, the lack of automatic tax rebate for loss becomes inconsequential. The natural question is whether the hedging rationale can be extended to hedging with futures contracts which are only partially correlated with the pre-tax earnings. While a rigorous answer to this question is beyond the scope of this article, an intuitive affirmative answer can be given in a variety of cases.

Suppose that, in contrast to this model's assumption, the production cost function is uncertain due, for example, to fluctuations in the cost of energy resulting from fluctuations in the price of oil. This additional source of uncertainty abolishes the perfect correlation between pre-tax earnings and the output commodity futures contract. However, the potential benefits of hedging can still be fully realized if oil futures can be used to stabilize the cost function. In essence, the model's results apply directly to those cases where there exists some package of futures contracts which is perfectly correlated with the firm's pre-tax earnings.

If only packages of futures contracts which are partially correlated with the firm's pre-tax earnings are feasible, the value of hedging is reduced, but not necessarily eliminated. The difficulty introduced by the partial correlation is that it becomes impossible to guarantee that the firm's operating losses will never coincide with losses on the futures positions. Such incidents contribute to an increase of the present value of the tax penalty due to the lack of automatic tax rebate for losses. However, the likelihood of such incidents declines as the volatility of the futures package declines, and as its correlation with the firm's pre-tax earnings increases. Thus, in principle, it is possible to reduce the present value of the tax penalty by hedging with a package of futures contracts which has a "sufficiently" low volatility and a "sufficiently" high correlation with the firm's pre-tax earnings. The structure of the optimal hedge with multiple futures contracts and the extent to which actual firms can benefit from such hedging are theoretical and empirical questions worthy of further research.

Finally, it should be pointed out that other mechanisms to avoid the consequences of the asymmetric tax treatment of profit and loss exist. For example, the firm may issue equity and invest the funds at the risk-free rate to generate enough profit that virtually eliminates the possibility of loss and the tax penalty associated with it. Similarly, a loss can be sold to another firm that can use the tax deduction via a merger or an effective leasing program. However, these methods entail substantial transaction costs and often are impractical.

Bibliography

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