Hedge Ratios under Inherent Risk Reduction in a Commodity Complex

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An extensive body of recent research in futures markets deals with determining optimal hedge ratios or minimum variance hedge ratios for decision makers seeking to reduce risk on a single commodity. The standard approach involves the construction of a portfolio model of commodity stocks and futures contracts.[1] However, many firms, governments, and decision makers face multiple risks simultaneously. Therefore, recent literature addresses how such decision makers might use the portfolio model to determine optimal hedges in multiple futures contracts simultaneously (e.g., Bond, Thompson, and Geldard (1985), Thompson and Bond (1987), Benninga, Eldor, and Zilcha (1985), Alexander, Musser, and Mason (1986), and Peterson and Leuthold (1987)).[2] None of these studies, however, derives hedge ratios within the constraints of a fixed production relationship which relates the products or commodities being hedged.

The purpose of this article is to extend this literature by deriving a model to generate simultaneous minimum risk hedge ratios and to apply the model to the soybean complex where inputs and outputs have a fixed relationship with each other. Determining hedge ratios on related inputs and outputs independently of each other ignores the inherent risk reduction due to the positive price correlation among these products. It is shown that simultaneous hedge ratios for multiple and related risks are quite different from those determined when each individual risk is considered separately.

MODELING THE SOYBEAN COMPLEX

Soybean processors buy soybeans to produce and sell soybean oil and meal. Prices of all three items, beans, meal, and oil, have become more variable over the last couple of decades, increasing the risk faced by soybean processors (USDA (1984)).

[1]Among the many examples are Rolfo (1980), Peck (1975), Ederington (1979), and Hill and Schneeweis (1982).

[2]Early examples of the theoretical development of optimal hedge ratios in the case of multiple cash goods and multiple futures contracts include Heifner (1972) and Anderson and Danthine (1980). Researchers have recognized for some time that a true model is more complex than the two-asset case.

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The Journal of Futures Markets, Vol. 10, No. 5, 497–504 (1990)
 CCC 0270-7314/90/050497-08$04.00

Processors also face the risk that the prices of the three items will not move together. Since they operate on small processing price margins, it is important that they manage their price risks effectively.

Soybean processors buy and sell each of these three commodities in the current cash market, forward cash market, and the futures and options markets. These markets present numerous pricing and risk management combinations. Hieronymus (1977), pp. 231–232, presents a small sample of these alternatives. Furthermore, processors can create many additional pricing alternatives by choosing various time horizons among futures and options contracts and various premiums and discounts among forward contracts. For simplicity, only futures markets used as a forward pricing mechanism are analyzed in this study.

Determining the optimal hedge ratio for each of these three commodities independently can be quite inefficient because the three prices are generally positively correlated. That is, because of the positive price correlations, opposing market positions inherently reduce risks. Thus, the management of multiple risks simultaneously could lead to quite different hedge ratios than when examining each risk independently. Multiple risk management could be more efficient, reducing costs and risks to soybean processors.

The difference between the combined weighted values of the two products, oil and meal, and the value of the input, soybeans, is called the processing margin, or the "crush." As noted above, there are several crushing margins available for the processor to accept and the margins change daily, creating risks. A 60-pound bushel of soybeans usually yields about 48 pounds of soybean meal and 11 pounds of soybean oil. The remainder is waste. This production relationship is utilized in determining the multiple hedge ratios.[3]

For this study the entire hedging period is divided into two stages. The sequence of cash and futures activities is illustrated in Table I. The first stage covers the period from production planning up to the point at which processing actually begins. During this period, hedges are placed simultaneously on soybeans, soybean meal, and soybean oil. When production begins, the necessary cash soybeans are purchased, and any soybean hedge is lifted. After processing, the cash soybean meal and soybean oil are sold in the cash market and any associated futures position is liquidated. The second stage is assumed to be one week, but the first stage varies from one to fifteen weeks.

[3]This article is not concerned with whether a particular crush is profitable or not to the processor, but in how processors manage the risk of the crushing margin.

Table I
THE HEDGING PLAN

Stage	Date	Production Activities	Cash Transactions	Futures Transactions
1	$t = 0$	Plan production		Buy soybeans
				Sell soybean meal
				Sell soybean oil
2	$t = 1$	Start processing	Buy soybeans	Sell soybeans
	$t = 2$	Market products	Sell soybean meal	Buy soybean meal
			Sell soybean oil	Buy soybean oil

The producer's returns, R, from both the cash and futures transactions are:

$$R = P_{22}X_2 + P_{32}X_3 - P_{11}X_1 - C + (P_{41} - P_{40})X_4 - (P_{52} - P_{50})X_5 - (P_{62} - P_{60})X_6 \quad (1)$$

where:

P_{it} = price for commodity i at time t ($t = 0, 1, 2$),
X_i = market position of commodity i (converted to pounds),
i = 1: soybeans cash, 4: soybeans futures,
2: soybean meal cash, 5: soybean meal futures,
3: soybean oil cash, 6: soybean oil futures,
C = fixed processing cost

and where the production relationship is assumed, $X_1 = X_2 + X_3$, through converting everything to pounds; that is 59 pounds of soybeans = 48 pounds of soybean meal plus 11 pounds of soybean oil.[4] All prices are converted to dollars per pound.

Substituting $X_2 + X_3$ for X_1 in eq. (1) above, one can rewrite (1) as:[5]

$$R = (P_{22} - P_{11})X_2 + (P_{32} - P_{11})X_3 - C + (P_{41} - P_{40})X_4 - (P_{52} - P_{50})X_5 - (P_{62} - P_{60})X_6. \quad (2)$$

To simplify the derivation and analysis, D's are used to represent price differences as follows:

$$D_2 = P_{22} - P_{11}, \quad D_3 = P_{32} - P_{11}, \quad D_4 = P_{41} - P_{40},$$
$$D_5 = P_{52} - P_{50}, \quad D_6 = P_{62} - P_{60}.$$

Equation (2) can be reexpressed as:

$$R = D_2X_2 + D_3X_3 + D_4X_4 - D_5X_5 - D_6X_6 - C. \quad (3)$$

Cash positions are assumed known and fixed. Soybean processors use futures contracts to minimize risk where risk is measured by the variance of returns, $\mathrm{Var}(R)$. The minimization problem can be formalized as:

$$\underset{X_4, X_5, X_6}{\mathrm{MIN}} \ \mathrm{Var}(R).$$

The futures positions, X_4, X_5, X_6 are decision variables. The solution of this hedging problem is (see the Appendix for details):

$$X_4 = [T_4(1 - H_{56}H_{65}) + T_5(H_{54} - H_{64}H_{56}) + T_6(H_{64} - H_{54}H_{65})]/T$$
$$X_5 = [T_4(H_{45} - H_{46}H_{65}) + T_5(1 - H_{64}H_{46}) + T_6(-H_{65} + H_{45}H_{64})]/T \quad (4)$$
$$X_6 = [T_4(H_{46} - H_{45}H_{56}) + T_5(-H_{56} + H_{46}H_{54}) + T_6(1 - H_{45}H_{54})]/T$$

where:

$$T = 1 + H_{46}H_{54}H_{65} + H_{64}H_{45}H_{56} - H_{45}H_{54} - H_{46}H_{64} - H_{56}H_{65}$$
$$T_4 = -X_2H_{24} - X_3H_{34}$$

[4]The one pound of waste is removed for convenience and to allow equality in the equation.
[5]It is assumed also in the calculations that if $X_1 = 1$, then $X_2 = 48/59$ and $X_3 = 11/59$. This weights prices accordingly.

$$T_5 = X_2H_{25} + X_3H_{35}$$
$$T_6 = X_2H_{26} + X_3H_{36}$$

and the H's in eq. (4) are estimated regression coefficients from the following equation:

$$D_i = a + H_{ij}{}^*D_j + e \qquad (5)$$
$$i = 2,3,4,5,6; \qquad j = 4,5,6; \qquad \text{and } i \neq j$$

and e is the error term.

The procedure is to estimate all the H's in eq. (5) and then insert the results back into eq. (4) to determine the hedge ratios. The solution to the simultaneous equations allows for all the cash and futures prices to interact.[6] In addition, the solution is general and can be applied to any production process; i.e., the size of X_4, X_5, and X_6 is sensitive to the production proportions assumed for X_2 and X_3 (here $X_2 = 48/59$ and $X_3 = 11/59$) because they affect the T's. Equation (4) remains the same regardless of the production process and assigned weights.

The model minimizes risk (variance). It is assumed that this approach approximates the behavior of decision makers in the industry. However, this model is used in the same way as the portfolio model which is based on risk and returns, solved by maximizing returns subject to risk levels, and assumes the futures markets are unbiased (i.e., there is no speculative component).[7] No known study using recent prices or adequate methodology exists showing biased prices or price inefficiencies in the soybean futures complex.

DATA

Cash and futures prices are once a week (Wednesday) observations from January, 1983 through June, 1988. Cash prices are soybeans: #1, yellow soybeans, Central Illinois; soybean meal: 44% protein, Decatur; soybean oil: bulk oil, Decatur; and futures prices are from the Chicago Board of Trade.

As mentioned above, hedge planning horizons range from 1 to 15 weeks, while the production period remains constant at 1 week. Hedge ratios are estimated in both nearby and distant contracts. The contract selected as nearby is the one which is closest to maturity when the hedge is liquidated. Obviously, it may not be the nearby contract when the hedge is placed, but this procedure eliminates the need to roll hedges among contracts. The contract selected as distant is that contract which is fifth from maturity when the hedge is liquidated. Of course, for any given data the contracts selected vary depending upon the hedging horizon.

RESULTS

The minimum risk hedge ratios for nearby and distant contracts are presented in Tables II and III, respectively. For comparison, minimum risk hedge ratios are calculated for each commodity separately, ignoring intercommodity price correlations. Note that for the nearby contracts all of these simple hedge ratios are very near the value of 1.0, the naive hedge. This confirms empirically what is often presumed in-

[6]Note that basis risks still exist for the processor; i.e., cash and futures prices may not move exactly parallel to each other. This can affect hedge ratios, as it should.

[7]See Sarassoro and Leuthold (1988) for the derivation of this alternative, more complex model.

Table II
SEPARATE AND COMPLEX HEDGE RATIOS: NEARBY CONTRACTS

Commodity		Hedging Horizon (weeks) 1	3	6	9	12	15	Average
Soybeans	Separate	0.95	0.93	0.93	0.94	0.92	0.94	0.94
	Complex	1.01	1.06	0.97	0.97	0.87	0.90	0.96
Soybean Meal	Separate	0.94	0.96	0.92	0.93	0.93	0.89	0.93
	Complex	0.74	0.87	0.84	0.79	0.80	0.76	0.80
Soybean Oil	Separate	0.98	0.95	1.00	0.99	1.03	0.97	0.99
	Complex	0.19	0.23	0.19	0.18	0.17	0.16	0.19
# of Observations:		283	281	278	275	272	269	

tuitively in the grain industry that following a naive hedge is a reasonable strategy.[8] The simple hedge ratios with the distant contracts are also near, but not as close to, one and show more variability across horizons.

After accounting for all the price correlations, the complex hedge ratios differ from the simple, separate hedge ratios. In Table II the complex hedge ratios for soybeans tend to be slightly larger than the separate hedge ratios except for the longer horizons. The hedge ratios for soybean meal all become smaller, and on average they are about 13 percentage points smaller. However, for soybean oil the hedge ratios fall substantially from 99 percent for simple hedges to 19 percent for the complex hedges. This reflects the intercorrelations among prices. To minimize risk soybean processors need hedge only a small portion of the soybean oil if they are

Table III
SEPARATE AND COMPLEX HEDGE RATIOS: DISTANT CONTRACTS

Commodity		Hedging Horizon (weeks) 1	3	6	9	12	15	Average
Soybeans	Separate	0.89	0.85	0.89	0.92	0.87	0.93	0.89
	Complex	0.97	1.11	1.07	1.02	0.89	0.99	1.01
Soybean Meal	Separate	0.94	0.91	0.89	0.92	0.89	0.86	0.90
	Complex	0.69	0.93	0.92	0.83	0.84	0.80	0.84
Soybean Oil	Separate	1.05	1.00	1.13	1.04	1.07	1.12	1.07
	Complex	0.23	0.25	0.21	0.19	0.18	0.18	0.21
# of Observations:		283	281	278	275	268	240	

[8]Other known empirical results for grains are Baldwin, Irwin, Ahmed, and Rye (1988) and Wilson (1984).

hedging soybeans and soybean meal as well. Complex hedge ratios change some, but not substantially, across hedging horizons.

The results in Table III for distant horizons show a similar, although not identical, pattern as that just described. Relative to the simple hedge ratios, complex soybean hedge ratios: increase, on average, about 12 percentage points; soybean meal hedge ratios drop slightly; and soybean oil hedge ratios drop dramatically to almost the same level as for the nearby contracts. Although there are slight variations, the general nature of the results is not very sensitive to either the hedging horizon or to the contracts selected.[9]

Note that in both Tables II and III at most horizons the sum of the complex soybean oil and soybean meal hedge ratios approximates the size of the complex soybean hedge ratio, which is usually near 1.0.[10] Differences are probably due to basis and margin risks. This feature implies that the processor is (nearby) fully hedged with respect to margin risk rather than individual price-level risk. The fact that soybean oil hedge ratios change so much relative to soybean meal ratios results from the sizes of the cross-correlation coefficients and price sensitivity.

This information can help the soybean processor reduce costs by negating the necessity of holding a large number of futures contracts, especially in soybean oil. The larger futures positions suggested by the independent hege ratios overexposes the processor to price risk relative to the multiple hedge solutions. This additional risk itself can be costly to the firm. Making decisions based on independently determined hedge ratios can be costly and inefficient for the soybean processor.

IMPLICATIONS AND CONCLUSIONS

This article demonstrates the concept of determining simultaneously minimum risk hedge ratios for multiple risks when there is a fixed production relationship among the various risks. The model is applied to soybeans, but it is applicable to cattle feeding, crude oil processing, numerous relationships among financial instruments of various yields and maturities, and to institutions dealing in numerous currencies simultaneously.

The results show that if a soybean processor follows minimum risk hedge ratios for soybeans, soybean meal, and soybean oil independently and without regard to the positive correlation among prices and fixed production relationships, they will have incorrect futures positions. Most notably, they will be highly overexposed in their futures market position for soybean oil relative to solving for the three futures positions simultaneously. That is, the independent hedges ignore internal (inherent) risk reduction occurring because of strong price relationships among the three products. This overexposed futures position increases processor costs and risks due to futures positions which are too large. Hence, the simultaneous determination of minimum risk hedge ratios should increase processor profits through lower costs and less risk exposure, and minimize the risks on these profits.

This article incorporates the constraint into the optimal hedging model of a fixed production relationship between the commodities being hedged. Such a constrained

[9]Studies of this type commonly determine the effectiveness of the hedge through the proportional reduction in variance as reflected in the R^2. However, that measure is not directly calculable in this multiple risk case. It could be compared to the unhedged case as Sarassoro and Leuthold (1988) do. This article compares the hedge ratios utilizing the production relationship with those from single hedge ratios where the fixed proportions among inputs and outputs are ignored.

[10]Remember, soybean hedges are short while oil and meal hedges are long.

model is applicable to many firms, governments, and institutions who face multiple risks among related products or instruments. The next step in model development and empirical application is to relax the assumption of known cash positions to reflect more realistic situations where cash and futures positions are determined simultaneously while continuing to impose the constrained production relationship.

Appendix

The detailed expression of the variance of R is:

$$\begin{aligned}\mathrm{Var}(R) = \sum_{i=2}^{6} X_i^2\sigma_i^2 + 2(&X_2X_3\sigma_{23} + X_2X_4\sigma_{24} - X_2X_5\sigma_{25} - X_2X_6\sigma_{26} \\ &+ X_3X_4\sigma_{34} - X_3X_5\sigma_{35} - X_3X_6\sigma_{36} - X_4X_5\sigma_{45} \\ &- X_4X_6\sigma_{46} + X_5X_6\sigma_{56})\end{aligned} \tag{A.1}$$

where σ_i^2's and σ_{ij}'s are variances and covariances, respectively. The minimum is obtained at the point where the first derivatives of eq. (A.1) with respect to the decision variables X_4, X_5, and X_6 are equal to zero. These first derivatives are:

$$\begin{aligned} d\,\mathrm{Var}(R)/dX_4 &= 2(X_4\sigma_4^2 + X_2\sigma_{24} + X_3\sigma_{34} - X_5\sigma_{54} - X_6\sigma_{64}) \\ d\,\mathrm{Var}(R)/dX_5 &= 2(X_5\sigma_5^2 - X_2\sigma_{25} - X_3\sigma_{35} - X_4\sigma_{45} + X_6\sigma_{65}) \\ d\,\mathrm{Var}(R)/dX_6 &= 2(X_6\sigma_6^2 - X_2\sigma_{26} - X_3\sigma_{36} - X_4\sigma_{46} + X_5\sigma_{56}). \end{aligned} \tag{A.2}$$

Setting eq. (A.2) equal to zero, dividing them through by σ_4^2, σ_5^2, and σ_6^2, respectively, and rearranging terms, eq. (A.2) can be simplified as:

$$\begin{aligned} X_4 - H_{54}X_5 - H_{64}X_6 &= -X_2H_{24} - X_3H_{34}, \\ -H_{45}X_4 + X_5 + H_{65}X_6 &= X_2H_{25} + X_3H_{35}, \\ -H_{46}X_4 + H_{56}X_5 + X_6 &= X_2H_{26} + X_3H_{36}, \end{aligned} \tag{A.3}$$

where

$H_{ij} = \sigma_{ij}/\sigma_j^2$
$i = 2,3,4,5,6; \qquad j = 4,5,6; \qquad$ and $i \neq j$.

H's can be estimated by regression eq. (5) in the main text. Using Cramer's rule, the solution to (A.3) can be obtained and is given as eq. (4) in the text.

Bibliography

Alexander, V. J., Musser, W. N., and Mason, G. (1986): "Futures Markets and Firm Decisions Under Price, Production, and Financial Uncertainty," *Southern Journal of Agricultural Economics,* 18:39–49.

Anderson, R.W., and Danthine, J. P. (1980): "Hedging and Joint Production: Theory and Illustrations," *Journal of Finance,* 35:487–498.

Baldin, D., Irwin, S., Ahmed, H., and Rye, R. (1988): "Hedging Effectiveness in a Vertical Marketing Channel," *NCR-134 Conference on Applied Commodity Price Analysis, Forcasting, and Market Risk Management,* 359–372.

Benninga, S., Eldor, R., and Zilcha, I. (1985): "Optimal International Hedging in Commodity and Currency Forward Markets," *Journal of International Money and Finance,* 4:537–552.

Bond, G. E., Thompson, S. R., and Geldard, J. M. (1985): "Basis Risk and Hedging Strategies for Australian Wheat Exports," *Australian Journal of Agricultural Economics,* 29:199–209.

Ederington, L. H. (1979): "The Hedging Performance of the New Futures Markets," *Journal of Finance,* 34:157–170.

Heifner, R. G. (1972, April): "Optimal Hedging Levels and Hedging Effectiveness in Cattle Feeding," *Agricultural Economics Research,* 24:25–36.

Hieronymus, T. A. (1977): *Economics of Futures Trading for Commercial and Personal Profit,* 2nd ed., New York: Commodity Research Bureau.

Hill, J., and Schneeweis, T. (1982): "The Hedging Effectiveness of Foreign Currency Futures," *Journal of Financial Research,* 5:95–104.

Peck, A. E. (1975): "Hedging and Income Stability: Concepts, Implications, and an Example," *American Journal of Agricultural Economics,* 57:410–419.

Peterson, P. E., and Leuthold, R. M. (1987): "A Portfolio Approach to Optimal Hedging for a Commercial Cattle Feedlot," *Journal of Futures Markets,* 7:443–457.

Rolfo, J. (1980): "Optimal Hedging Under Price and Quantity Uncertainty: The Case of a Cocoa Producer," *Journal of Political Economy,* 88:100–116.

Sarassoro, G. F., and Leuthold, R. M. (1988): "A Multiple-Risk Optimal Hedging Model: An Example," *NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management,* 296–309.

Thompson, S. R., and Bond, G. E. (1987): "Offshore Commodity Hedging Under Floating Exchange Rates," *American Journal of Agricultural Economics,* 69:46–55.

U.S. Department of Agriculture (1984): "Soybeans--Background for 1985 Farm Legislation," ERS, Agricultural Information Bulletin No. 472.

Wilson, W.W. (1984): "Hedging Effectiveness of U.S. Wheat Futures Markets," *Review of Research in Futures Markets,* 3:64–79.