

# The Efficiency of the U. S. Dollar Index Futures Market

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## INTRODUCTION

The U. S. Dollar Index (USDIX) is composed of ten major foreign currencies and expressed in an index form relative to the base value of 100 as of March 1973. The unique feature of the USDIX is the weighting of the component foreign currencies included in the index, and the method by which the index is computed. In contrast to other indexes, the USDIX is computed as in *inverse value weighted geometric* average of ten major foreign exchange (FX) rates, whereby each FX rate receives a different weight in the index as determined by the Federal Reserve Board in 1973. These weights reflect the relative importance of each foreign country in trading with the U. S. since 1973. Although there have been tremendous changes in international trade since 1973, such as the emergence of Japan and West Germany as the dominant countries in international trade, the weights of the component FX rates in the USDIX remain the same.

USDIX futures contracts have been traded on the Financial Instrument Exchange (FINEX), a division of the New York Cotton Exchange since November 20, 1985. The USDIX futures contracts are traded in a three-month cycle where delivery is on the third Wednesday of the expiring contract month. The futures contract size is \$500 times the contemporaneous USDIX spot value. Any futures contracts held to expiration are subject to cash settlement, based on the difference between the USDIX value and the USDIX futures price times \$500 on the last day of trading for an expiring contract.

As an *inverse geometric average index*, the USDIX suffers from a peculiar problem, namely, an *upward bias* to the index relative to what is obtained with arithmetic averaging. Another nonstandard property of the USDIX emanates from the fact that as an inverse geometric index, it is expressed in "European terms" (i.e., FX rates per U. S. dollars) rather than quoted in the conventional "American terms" (i.e., U. S. dollars per FX rates). As a result, the USDIX rises when the dollar appre-

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ciates in value, and declines when the dollar depreciates. The *upward bias* embedded in the USDX is documented first by Redfield (1986) who uses a rule-of-thumb methodology to estimate the magnitude of the USDX drift which he refers to as the "volatility premium." Relying on Redfield's intuition, Eytan, Harpaz, and Krull (1988), hereafter EHK, use the Black and Scholes (1973), Merton (1973), and Black (1976) contingent claims methodology to explore the determinants and magnitude of the USDX *upward drift*. They show that the USDX drift factor depends not only on the volatility of the index, but on the weighted average volatility of the component *FX* rates. EHK also contend that the upward drift factor embedded in the USDX affects the pricing of its derivative futures contracts. More specifically, they also provided preliminary empirical estimates of the drift's magnitude and conjecture (but do not conduct any statistical tests) that the failure of the market to account properly for the upward drift factor in the pricing of USDX futures may cause market inefficiency.

The objective of this study is twofold: to examine whether investors in the market place incorporate the correct *upward drift* in the pricing of USDX futures contracts; and, to use the daily settlement prices of USDX futures contracts traded on the FINEX for the period November 20, 1985 through December 31, 1988 to test for the efficiency of the FINEX market by employing the EHK futures pricing formula as a benchmark. The rest of the article is organized in three sections. The methodology for the proposed statistical tests and the description of the data set are presented in the second section. The results of the empirical tests are discussed in the third section. The last section contains the conclusion.

## METHODOLOGY

### The Proposed Statistical Tests

EHK (1988) proposed a USDX futures formula that shows that the *theoretical* USDX futures price is given by

$$F_e(t, T) = I(t) \exp[(\bar{r} - r_{us} + \delta)(T - t)], \quad (1)$$

where  $F_e(t, T)$  is the USDX futures model price in period  $t$  for expiration in period  $T$ ;  $I(t)$  is the contemporaneous USDX spot value as of period  $t$ ;  $\bar{r} = \sum_{i=1}^{10} w_i r_i$  is the weighted average foreign risk free rate of interest;  $r_i$  and  $w_i$  are the  $i$ th country risk-free rate of interest and its weight in the index, respectively;  $r_{us}$  is the U. S. riskfree rate of interest;  $T - t$  is the time left to expiration of the futures contract; and  $\delta$  is the (positive) "drift" parameter (which is unobservable) which corrects for the upward bias effect due to the (inverse) geometric configuration of the USDX. For the empirical tests, one needs to estimate the *implied* (drift parameter),  $\delta_a$ , which can be extracted from (1) as follows:

$$\delta_a = \ln[F_a(t, T)/I(t)]/(T - t) - (\bar{r} - r_{us}), \quad (2)$$

where  $F_a$  is the *historical* USDX futures price. While the *implied delta* (2) is derived from the EHK futures pricing formula, the "true" delta is unfortunately unobservable. To conduct tests for market efficiency it is necessary to develop an alternative estimate of the "true" delta, the *historical delta*.

The *historical drift parameter*,  $\delta_h$ , as proposed by EHK, is computed from historical variances,

$$\delta_h = \left[ \sum_{i=1}^{10} w_i \sigma_i^2 + \sigma_I^2 \right] / 2 = [\bar{\sigma}^2 + \sigma_I^2] / 2, \quad (3)$$

where  $\sigma_i^2$  and  $\sigma_j^2$  are the variances of the rate-of-change on  $i$ th  $FX$ , and the contemporaneous USD $X$ , respectively; while  $\bar{\sigma}^2$  is the weighted average variance of the representative  $FX$  included in the index. Clearly the historical delta ( $\delta_h$ ) must be positive. The implied and historical deltas are both estimates of the "true" delta. If the market for the USD $X$  futures contracts is efficient and the EHK model is correct, then the implied drift factor ( $\delta_a$ ) will equal the true delta, and also be an efficient and unbiased estimator of the subsequent volatilities on the component currencies and the USD $X$  contract.

In addition, there should be no systematic difference between  $\delta_a$  and  $\delta_h$ , and if  $\delta_a \neq \delta_h$ , no mechanical trading rule should produce profits after transaction costs. Note that while  $F_a$  is the historical USD $X$  futures price,  $F_e$  is the USD $X$  model price from (1) using the historical delta,  $\delta_h$ , in (3) as an estimate for the "true" but unobservable delta.

An explicit test for the efficiency of the FINEX market is based on the strategy proposed by EHK (1988). This strategy calls for either long-long or short-short positions in the USD $X$  futures contracts and its component  $FX$  forward contracts. Specifically, on each day  $t$  for which the USD $X$  futures contract is incorrectly valued ( $\delta_a \neq \delta_h$ ), one long (short) position in the USD $X$  futures contract is combined with 10 long (short) positions in each of the underlying forward contracts, for undervalued (overvalued) contracts, respectively. The investment in the  $i$ th  $FX$  forward contract is  $w_i[F_a(t, T)/f_i(t, T)]$  where  $f_i(t, T)$  is the  $i$ th currency forward price at time  $t$  for fast delivery at time  $T$ . Finally, all the positions are offset at expiration.<sup>1</sup> The combined profit  $\pi(t, T)$  of the long-long (short-short) hedge portfolio over the contract life  $(t, T)$  is given by:

$$\pi(t, T) = (I(T) - F_a(t, T)) + \sum_{i=1}^{10} [w_i I(T)/f_i(t, T)](S_i(T) - f_i(t, T)), \quad (4)$$

where  $S_i(T)$  is the spot exchange rate of currency  $i$  at the time  $T$ . Clearly, for the short-short arbitrage strategy the profit is given by  $-\pi(t, T)$ . In the absence of margin requirements, the initial investment in the hedge portfolio is zero, and given the null hypothesis that the USD $X$  futures market is efficient, one expects the arbitrage profits  $\pi(t, T)$  in (4) (for all contracts) to be zero.

In the first set of tests, incorrectly valued portfolios are identified as follows. For a particular contract  $j$  and for each day  $t$  over its life span the delta ratio  $G_{j,t} = (\delta_h/\delta_a)_{j,t}$  as well as the mean value,  $\mu_G$ , and the standard deviation,  $\sigma_G$ , are computed for all contracts from day 1 until day  $t - 1$ . If the USD $X$  futures contract is persistently mispriced, then the mean delta ratio,  $\mu_G$ , will be different from 1.0. Therefore, to test for the degree of mispricing of a particular contract  $j$ , its delta ratio,  $G_j$ , is compared to 1.0. In addition, to reduce potential sample sensitivity-related problems the range of  $1 \pm \sigma_G$  is used.

If  $\delta_a$  is too low (high), then the actual futures price is too low (high),  $G$  will be high (low) and the USD $X$  futures price undervalued (overvalued). Specifically, contract  $j$  on day  $t$  is defined as being undervalued ( $UV$ ) if  $G_{j,t} > (1 + \sigma_G)$ ; overvalued ( $OV$ ) if  $G_{j,t} < (1 - \sigma_G)$ ; and correctly valued ( $CV$ ) if  $(1 - \sigma_G) \leq G_{j,t} \leq (1 + \sigma_G)$ . For each and every incorrectly valued ( $IV$ ) contract the arbitrage strategy stipulated by (4) is applied to estimate the profit ( $\pi$ ) in dollars per contract. If the arbitrage profits are significantly different from zero, the hypothesis that the FINEX

<sup>1</sup>It should be emphasized that the hedge position is a long-long or short-short hedge rather than the conventional long-short hedge, because the USD $X$  futures price is quoted in "European terms" whereas the component  $FX$  forward prices are expressed in "American terms."

market is efficient can be rejected. In other words, this procedure examines whether *historical* and *implied* deltas differ substantially, and whether trading profits can be earned when such large differences occur.

An alternative and consistent method for identifying incorrectly valued contracts is to compute the incorrect-valuation coefficient based on the *futures price* ratio rather than on the *delta* ratio. Similarly, for a particular contract  $j$  and for each day  $t$  over its life span the ratio  $V_{j,t} = (F_a/F_e)_{j,t}$  is computed, where  $F_a$  and  $F_e$  are the *historical USDX futures price* and the corresponding *model price which incorporates the historical delta in (3)*, respectively. In addition, the mean value,  $\mu_V$ , of the  $V$  ratio, and the standard deviation,  $\sigma_V$ , across all contracts preceding the evaluation day  $t$  are computed. Similar to the delta investment decision rule, contract  $j$  on day  $t$  is defined as being undervalued (*UV*) if  $V_{j,t} < (1 - \sigma_V)$ ; overvalued (*OV*) if  $V_{j,t} > (1 + \sigma_V)$ ; and correctly valued (*CV*) if  $(1 - \sigma_V) < V_{j,t} \leq (1 + \sigma_V)$ . Thus, if the  $V$  ratio is "too high" ("too low") then the USDX futures contract is overvalued (undervalued).

## DATA

The data for this study<sup>2</sup> consist of daily USDX spot and nearby futures settlement prices as well as the component currencies' daily spot and forward settlement prices for the period November 20, 1985 (inception date) through December 31, 1988.<sup>3</sup> It should be emphasized that 1985 includes only 26 trading days.

The daily USDX spot values represent the settlement prices at 2:40 p.m., the close of the USDX market. Individual daily *FX* spot and forward prices are the settlement prices as of 1:00 p.m.<sup>4</sup> This study uses only nearby futures contracts since they are the most liquid and actively traded. The nearby contract usually accounts for at least 75% of the volume and open interest. In total, the number of nearby contracts examined is 13, and the number of observations (or trading days) is 739.

As indicated by (1) the estimation of the USDX futures model price requires the estimation of the drift factor (delta) and the interest-rate differential ( $\bar{r} - r_{us}$ ). The historical delta has been estimated from (3). The most recent 40 daily observations preceding the evaluation date used to estimate the variance of the rate of change of the spot *FX* rate of country  $i$ ,  $\sigma_i^2$ , and the variance of the contemporaneous spot USDX,  $\sigma_f^2$ . The interest rate differential ( $\bar{r} - r_{us}$ ) is estimated using the (assumed) Interest-Rate Parity Theorem (IRPT) as follows:

$$\bar{r} - r_{us} = \sum_{i=1}^{10} w_i r_i - r_{us} = \sum_{i=1}^{10} \hat{w}_i \ln[S_i(t)/f_i(t, T)]/(T - t), \quad (5)$$

where  $S_i(t)$ , and  $F_i(t, T)$  are the spot *FX* rate, and the contemporaneous forward price of country  $i$  as of period  $t$ , respectively. This estimate for the interest rate differential ( $\bar{r} - r_{us}$ ) is used also for computing the implied delta ( $\delta_a$ ) in (2).

<sup>2</sup>The data were provided by Richard Jacobs and June Furlan of the FINEX.

<sup>3</sup>As implied by the magnitude of the riskless arbitrage profit reported in the following section, employing settlement rather than bid or ask prices, and incorporating transaction costs would not change the main conclusions regarding the efficiency of USDX futures contracts.

<sup>4</sup>It should be noted that this lack of synchronization may cause an additional random error.

## EMPIRICAL RESULTS

### Historical vs. Implied Deltas

The statistical properties of the actual and implied delta, as computed from (3) and (2), respectively, are presented in Table I, and illustrated in Figure 1. The values of both the historical drift factor ( $\delta_h$ ) and the implied delta, ( $\delta_a$ ) are positive and statistically significant for the whole time period 1985–88 as well as for each of the individual years. The statistical significance level is much higher for the  $\delta_h$  than for the  $\delta_a$ . This difference reflects the lower dispersion for  $\delta_h$  than for  $\delta_a$ , which can be seen very clearly in both Figure 1 and the gap between the maximum and minimum values reported in Table I. This discrepancy is consistently lower for  $\delta_h$  than for  $\delta_a$ . Over the whole time period investigated, the mean value for the historical delta ( $\delta_h$ ) is 1.19% as compared with a mean value of 2.58% for the implied delta ( $\delta_a$ ). Note that if the ratio of the two mean values is lower (greater) than 1, it may indicate overvaluation (undervaluation) of the USDX futures contracts. A closer inspection of the intertemporal behavior of the delta ratio reveals that it consistently increases

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delta ratio, the mean  
-  $F_e$ ) also indicates  
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the whole time period  
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for 1985, 1986, 1987,  
 $F_a/F_e - 1$ 100, is de-  
-1987 period, as con-  
88.

$a$ ) and the price ratio  
the 26 days of 1985,  
1988. The test results  
which characterize the  
ruct a profitable risk-  
bnormal profits is in-

the "historical price" ( $F_a$ ). The statistical character  
presented in Table II. Consistent with the behavior of the  
price ratio ( $F_a/F_e$ ) (as well as the mean price difference  $F_a$   
*undervaluation* in 1985 followed by *overvaluation* in 1986 and  
*correct* valuation in 1988. The mean value of ( $F_a - F_e$ ) for t  
1985–88 is 0.0879 (and is statistically different from zero) wh  
years the mean values are -0.2012, 0.0990, 0.1686, and 0.025  
and 1988, respectively. The incorrect valuation coefficient,  
picted in Figure 2 and shows many positive values in the 1985  
trasted with random fluctuations around the zero value in 19

It is apparent from the behavior of both the delta ratio ( $\delta_h/$   
( $F_a/F_e$ ) that the market for USDX futures is *undervalued* for  
*overvalued* in 1986 and 1987, while almost *correctly* valued in  
indicate that, given the incorrect valuation phenomenon w  
USDX futures market in 1986 and 1987; it is possible to const  
less arbitrage strategy. The ability of this strategy to generate  
vestigated next.

Table I  
 STATISTICAL PROPERTIES OF HISTORICAL VS. IMPLIED DELTAS ( $\delta_h$  AND  $\delta_a$ )\*  
 IMBEDDED IN THE UDSX FUTURES PRICES (NOVEMBER 20, 1985–DECEMBER 31, 1988)

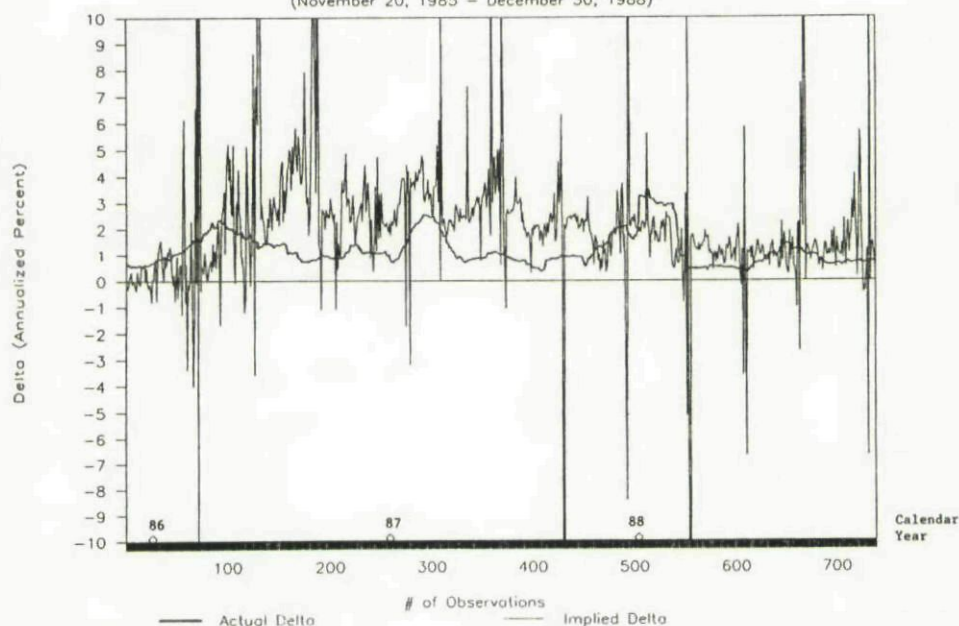
|            | Mean   | Sdv    | Min     | Max    | SE     | t-value | N   |
|------------|--------|--------|---------|--------|--------|---------|-----|
| 1985-8     |        |        |         |        |        |         |     |
| $\delta_h$ | 0.0119 | 0.0066 | 0.0033  | 0.0353 | 0.0002 | 49.09   | 739 |
| $\delta_a$ | 0.0258 | 0.0949 | -0.5985 | 1.5787 | 0.0035 | 7.39    | 739 |
| 1985       |        |        |         |        |        |         |     |
| $\delta_h$ | 0.0006 | 0.0005 | 0.0053  | 0.0080 | 0.0001 | 58.42   | 26  |
| $\delta_a$ | 0.0002 | 0.0036 | -0.0084 | 0.0064 | 0.0007 | 0.22    | 26  |
| 1986       |        |        |         |        |        |         |     |
| $\delta_h$ | 0.0129 | 0.0042 | 0.0069  | 0.0234 | 0.0003 | 46.64   | 232 |
| $\delta_a$ | 0.0370 | 0.1162 | -0.1119 | 1.3484 | 0.0076 | 4.85    | 232 |
| 1987       |        |        |         |        |        |         |     |
| $\delta_h$ | 0.0122 | 0.0057 | 0.0038  | 0.0255 | 0.0004 | 33.52   | 245 |
| $\delta_a$ | 0.0304 | 0.1096 | -0.5986 | 1.5787 | 0.0070 | 4.34    | 245 |
| 1988       |        |        |         |        |        |         |     |
| $\delta_h$ | 0.0113 | 0.0089 | 0.0033  | 0.0353 | 0.0006 | 19.40   | 236 |
| $\delta_a$ | 0.0129 | 0.0456 | -0.4857 | 0.2678 | 0.0030 | 4.35    | 236 |

\*The historical and implied  $\delta_h$  and  $\delta_a$  are computed from eqs. (3) and (2), respectively.

Mean = Mean value; Sdv = Standard deviation; Min = Minimum value; Max = Maximum value; SE = Standard error of the mean; t-value = Mean/SE;  
 N = Number of observations.

## Time Behavior of USDX Futures' Deltas

(November 20, 1985 – December 30, 1988)



**Figure 1**

Historical and implied deltas are estimated from eqs. (3) and (2), respectively.

### Trading Test Results

The long-long trading strategy stipulated by (4) is applied for each day in which an incorrectly valued (*IV*) contract, (i.e., undervalued (*UV*) or overvalued (*OV*)), is found. Note that incorrect valuation is based on the difference between the implied and the historical  $\delta$ 's. If a particular USDX futures contract is undervalued (overvalued), a long (short) position is taken in the USDX futures contract and 10 long (short) positions in the contemporaneous component *FX* forward contracts. The positions are offset at expiration. The trading profits in dollars per contract ( $\pi$ ) are presented in Table III, and are plotted as a frequency distribution in Figure 3. Over the entire sample of 739 daily observations, 165 contracts are identified as overvalued, and 65 contracts as undervalued. Only 2 contracts are found incorrectly valued for 1988. For the whole period of 1985–88, the mean trading profit is \$194.25 and significantly greater than zero. The percentage of positive  $\pi$ 's obtained for all the incorrectly valued contracts is 76%. A similar proportion is found for both the undervalued and overvalued contracts over the entire time period. The frequency distribution of the trading profits per contract is depicted in Figure 3.

A closer inspection of Table III reveals that the year by year results are substantially different from each other. In 1985, when the USDX contract began trading, most of the daily observations (18 out of a total of 26) are incorrectly valued. Moreover, these 18 incorrectly valued contracts are all *undervalued*, and *all* of them result in positive trading profits with mean  $\pi$  of \$539.50. The relatively high positive trading profit in 1985 is a result of investors failure to incorporate the drift factor (delta) in the pricing of USDX futures contracts. In 1986, the incorrect-valuation phenomenon is less severe than in 1985, and most of the incorrectly-valued con-

Table II  
STATISTICAL PROPERTIES OF HISTORICAL VS. MODEL USDX FUTURES PRICES— $F_a$  AND  $F_e$   
(NOVEMBER 20, 1985–DECEMBER 31, 1988)\*

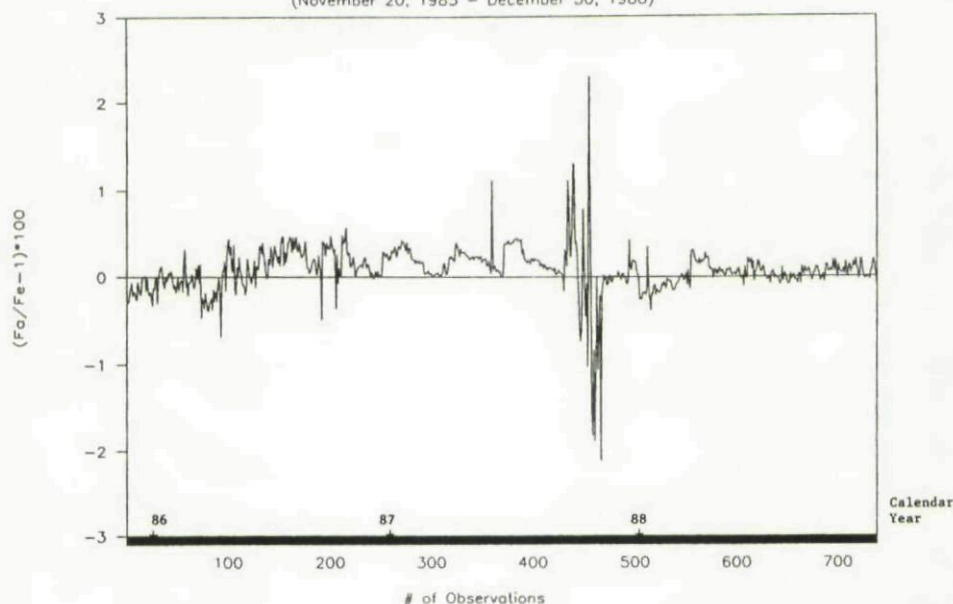
|             | Mean    | Sdv    | Min     | Max    | SE     | t-value | N   |
|-------------|---------|--------|---------|--------|--------|---------|-----|
| 1985–8      |         |        |         |        |        |         |     |
| $F_a$       | 101.51  | 10.28  | 85.66   | 128.86 | 0.3782 | 268.37  | 739 |
| $F_e$       | 101.42  | 10.31  | 85.62   | 129.09 | 0.3793 | 267.39  | 739 |
| $F_a/F_e$   | 1.0008  | 0.0028 | 0.9788  | 1.0230 | 0.0001 | 7.48    | 739 |
| $F_a - F_e$ | 0.0879  | 0.1872 | -0.7752 | 0.6197 | 0.0009 | 12.7711 | 739 |
| 1985        |         |        |         |        |        |         |     |
| $F_a$       | 126.19  | 1.05   | 123.55  | 128.86 | 0.21   | 615.48  | 26  |
| $F_e$       | 126.39  | 1.02   | 123.96  | 129.09 | 0.20   | 632.38  | 26  |
| $F_a/F_e$   | 0.9984  | 0.0010 | 0.9967  | 1.0001 | 0.0002 | -8.40   | 26  |
| $F_a - F_e$ | -0.2012 | 0.1219 | -0.4099 | 0.0142 | 0.0239 | -8.42   | 26  |
| 1986        |         |        |         |        |        |         |     |
| $F_a$       | 112.27  | 5.36   | 104.24  | 125.18 | 0.35   | 319.10  | 232 |
| $F_e$       | 112.17  | 5.48   | 104.05  | 125.09 | 0.36   | 311.60  | 232 |
| $F_a/F_e$   | 1.0010  | 0.0022 | 0.9932  | 1.0057 | 0.0001 | 6.62    | 232 |
| $F_a - F_e$ | 0.0990  | 0.2414 | -0.7752 | 0.6197 | 0.0159 | 6.25    | 232 |
| 1987        |         |        |         |        |        |         |     |
| $F_a$       | 97.15   | 3.64   | 85.66   | 104.84 | 0.2322 | 418.34  | 245 |
| $F_e$       | 96.98   | 3.57   | 85.62   | 104.59 | 0.2282 | 425.04  | 245 |
| $F_a/F_e$   | 1.0013  | 0.0040 | 0.9788  | 1.0230 | 0.0003 | 5.14    | 245 |
| $F_a - F_e$ | 0.1686  | 0.1371 | -0.1618 | 0.4488 | 0.0088 | 19.26   | 245 |
| 1988        |         |        |         |        |        |         |     |
| $F_a$       | 92.74   | 3.37   | 86.30   | 99.32  | 0.22   | 422.16  | 236 |
| $F_e$       | 92.71   | 3.37   | 86.30   | 99.32  | 0.22   | 422.41  | 236 |
| $F_a/F_e$   | 1.0003  | 0.0012 | 0.9961  | 1.0035 | 0.0001 | 3.58    | 236 |
| $F_a - F_e$ | 0.0251  | 0.1059 | -0.3460 | 0.3228 | 0.0069 | 3.64    | 236 |

\*The model price  $F_e$  is computed from eq. (1), based on the historical delta in (3).

Mean = Mean value; Sdv = Standard deviation; Min = Minimum value; Max = Maximum value; SE = Standard error of the mean; t-value = Mean/SE; N = Number of observations.

## USDX Incorrect Valuation Coefficient

(November 20, 1985 – December 30, 1988)



**Figure 2**

The incorrect-valuation coefficient is equal to  $(F_a/F_e - 1)100$ , where  $F_a$  and  $F_e$  are the historical and model prices from eq. (1), respectively. Note that the model price  $F_e$  in (1) is based on the historical delta in (3) as an estimate for the true but unobservable delta.

tracts are *overvalued*. The mean profit ( $\pi$ ) on the riskless trading strategy (\$210.95) is significantly positive, and 79% of all  $\pi$  values are positive. This overvaluation phenomenon continued in 1987 with an average significant profit of \$113.80, except that the proportion of positive  $\pi$ 's decreases from 79% in 1986 to 68% in 1987. In contrast to 1985, 1986, and 1987, the mispricing phenomenon is corrected in 1988. This result expresses the intertemporal behavior of the degree of mispricing of the USDX futures contract. The existence of almost no incorrectly valued contracts in 1988 indicates that the USDX futures contracts are fairly priced and as a result no positive riskless trading profits could be generated. Thus, the hypothesis that the USDX futures market is efficient in 1988, cannot be rejected.<sup>5</sup>

## CONCLUSION

Using the EHK (1988) USDX futures pricing formula, several statistical tests are performed to test for efficiency of the USDX futures contracts during the period 1985–88. Using daily settlement prices for nearby futures contracts traded on the FINEX from the inception of the contract 1985 through 1988, the null hypothesis that the USDX futures market is efficient in this time period is rejected. It is shown that positive trading profits can be obtained if one applies a long-long (short-short) arbitrage strategy to undervalued (overvalued) contracts. This trading strategy comprises a long (short) position in the USDX futures contract and ten long (short) po-

<sup>5</sup>An early, but incomplete investigation of the first three months of 1989 also shows no mispricing of the USDX futures contract.

Table III  
TRADING PROFITS IN DOLLARS PER CONTRACT ( $\pi$ )\* ON INCORRECTLY VALUED USDX FUTURES CONTRACTS  
(NOVEMBER 20, 1985–DECEMBER 31, 1988)

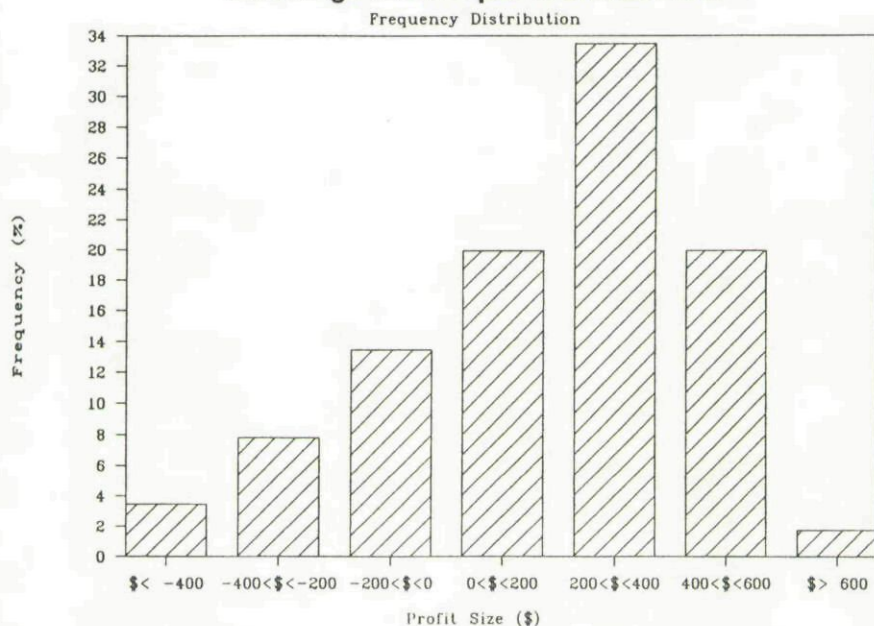
|        | Mean | Sdv   | Min  | Max   | SE  | t-value | N   | %<br>Pos | %<br>Neg |
|--------|------|-------|------|-------|-----|---------|-----|----------|----------|
| 1985–8 |      |       |      |       |     |         |     |          |          |
| IV     | 194  | 303   | -783 | 1,012 | 20  | 9.74    | 230 | 76       | 24       |
| UV     | 264  | 366   | -704 | 1,012 | 45  | 5.82    | 65  | 78       | 22       |
| OV     | 167  | 268   | -783 | 852   | 21  | 7.97    | 165 | 75       | 25       |
| 1985   |      |       |      |       |     |         |     |          |          |
| IV     | 539  | 210   | 194  | 971   | 49  | 10.91   | 18  | 100      | 0        |
| UV     | 539  | 210   | 194  | 971   | 49  | 10.91   | 18  | 100      | 0        |
| OV     | —    | —     | —    | —     | —   | —       | 0   | —        | —        |
| 1986   |      |       |      |       |     |         |     |          |          |
| IV     | 211  | 299   | -783 | 1,012 | 29  | 7.34    | 108 | 79       | 21       |
| UV     | 63   | 348   | -704 | 1,012 | 62  | 1.02    | 32  | 59       | 41       |
| OV     | 986  | 1,522 | 783  | 852   | 175 | 5.65    | 76  | 87       | 13       |
| 1987   |      |       |      |       |     |         |     |          |          |
| IV     | 114  | 276   | -555 | 896   | 27  | 4.17    | 102 | 68       | 32       |
| UV     | 375  | 291   | -136 | 896   | 78  | 4.82    | 14  | 93       | 7        |
| OV     | 72   | 249   | -555 | 554   | 27  | 2.72    | 88  | 64       | 36       |
| 1988** |      |       |      |       |     |         |     |          |          |
| IV     | 290  | —     | —    | —     | —   | —       | 2   | 100      | 0        |
| UV     | 185  | —     | —    | —     | —   | —       | 1   | 100      | 0        |
| OV     | 395  | —     | —    | —     | —   | —       | 1   | 100      | 0        |

\* $\pi$  is given by eq. (4), and it is the dollar profit on the hedge portfolio over contract life.

\*\*Not all values for 1988 are given since only 2 observations were found incorrectly valued, and they both yield highly positive returns.

Mean = Mean value; Sdv = Standard deviation; Min = Minimum value; Max = Maximum value; SE = Standard error of the mean; t-value = Mean/SE; N = Number of observations; IV = Incorrectly valued contracts; UV = Undervalued portfolio contracts; OV = Overvalued contracts; % Pos = Percentage of positive  $\pi$ ; % Neg = Percentage of negative  $\pi$ .

## Arbitrage Profits per Contract (\$s)



**Figure 3**

The trading profits in dollars per contract are computed from eq. (4) for the period November 20, 1985 through December 31, 1988. Note that in 1988 only 2 observations were found incorrectly valued.

sitions in the component forward contracts and offsetting the positions at expiration. The USDX futures contract was clearly *undervalued* in the 26 trading days of 1985 but in 1986 and 1987 it was *overvalued*. In 1988 it was correctly priced. The incorrect valuation is a result of investors not properly accounting for the positive drift parameter (delta) stipulated by the pricing of the USDX futures contracts. The presence of the positive drift factor (delta) is embedded in the USDX futures contracts because of the *inverse geometric* averaging of the ten foreign exchange rates included in the index. Since 1988, investors have properly incorporated the 'true' drift parameter (delta) in their pricing and this has resulted in the correct valuation of USDX futures contracts.

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