An Examination of Basis Risk Due to Estimation

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I. INTRODUCTION

Hedging in futures markets provides a means of shifting the risk of spot price changes within an economy. Participation in these markets exposes hedgers to the risk of variation in the time pattern of spot and futures prices. Thus hedging programs are described as substituting basis risk for price risk.[1] This article develops a measure of the basis risk which results from a nonstationary covariance matrix summarizing the rates of change of spot and futures prices. This is termed basis-estimation risk. Basis risk from this source is shown to be related to the degree of participation in the futures market and variation in the second-moment estimators used to determine the optimal risk-minimizing hedge ratio. A method for calculating this basis risk is introduced and empirical evidence is produced to demonstrate the significance of basis-estimation risk in hedging operations.

This perspective on hedging operations is distinct from previous research in this area. Previous research utilizing estimators for risk-minimizing hedge ratios essentially regards the covariance matrix of spot and futures price changes as stationary.[2] Hedge effectiveness measures such as the R^2 measure of Ederington (1979) and the Risk-Return Measure of Howard and D'Antonio (1984) are developed under this condition.[3] The model proposed here emphasizes the role of a nonstationary covariance matrix in the determination of basis risk.

Bawa, Brown, and Klein (1979) study the effect of estimation risk on security valuation models and conclude that estimation risk tends to decrease investment in

[1]Cootner (1967) refers to this substitution as specializing the risk faced by hedgers. This approach emphasizes managerial incentives to replace exogenously determined sources of risk with sources of risk which fall within the scope of managerial control.

[2]Research examining the covariance matrix for currency futures includes Grammatikos and Saunders (1983) who explore alternative procedures to estimate hedge ratios. Their evidence suggests these covariance matrices are not stationary. In addition, Eldridge (1984) demonstrates that volatility of futures price changes apparently responds to parallel contracts simultaneously traded on other exchanges.

[3]For a review of hedge effectiveness measures utilizing the R^2 measure in currency futures, see Dale (1981) and Hill and Schneeweis (1982).

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risky assets. Similarly, the results of this article show that hedgers lessen use of futures contracts when the covariance of futures and spot price changes intertemporally. Cecchetti, Crumby, and Figlewski (1988) consider the problem of variation in the covariance matrix in addition to incorporating expected returns from hedged positions. Their approach finds hedge ratios for Treasury bonds hedged with T-bond futures considerably below those using the usual hedge ratio approach. The evidence from this study suggests that the practice of underhedging relative to the regression hedge ratios is consistent with consideration of the problem of time variation in the distribution of spot and futures prices.

Castelino (1989) develops a model for basis volatility which incorporates the declining amount of basis as contracts approach delivery. Thus basis volatility is conditional on time to delivery. The model predicts these conditional variances decline with time to delivery. The model of this article differs in that the focus is on the unconditional variance of basis.

II. THE LINKAGE BETWEEN HEDGE RATIOS AND BASIS RISK

The value of a hedge is determined by the price of the liability in the spot market and N futures contracts.[4] Letting S represent the spot price and F the futures price, Ederington (1979) develops the optimal risk-minimizing hedge ratio under the assumption of zero speculative demand.[5] Assuming a stable covariance matrix, the optimal number (N^*) of futures contracts is

$$N^* = -\frac{\sigma_{fs}}{\sigma_f^2} \tag{1}$$

where σ_f^2 is the variance of the percentage price changes of futures contract prices over the hedge period and σ_{fs} is the covariance of spot and futures percentage price changes. This ratio gives the optimum number of futures contracts for a one-unit commitment in the spot good. The negative sign in (1) implies a short position in futures (or selling futures), committing the hedger to future delivery to hedge a long position in the cash market. Increases (decreases) in the value of the liability are offset by decreases (increases) in the value of the futures contract.

Estimation of N^* is available by regressing rates of change for a series of spot prices on rates of change for a time-matched series of futures prices. Use of this estimator for N^* assumes the *ex ante* pattern of spot and futures prices is adequately described by the *ex post* pattern of these prices.

Yamey (1971) defines basis risk in terms of intertemporal variation of the difference between spot and futures prices. The hedge ratio of Johnson (1960), Ederington (1979), and Franckle (1980) seeks to minimize the predictable portion of basis fluctuation. Unanticipated variation in basis replaces price risk with basis risk. Hedge ratios calculated from *ex post* price series expose investors to the basis risk of changes in the fundamental relationship between spot and futures prices. To see this, note that N^* is determined by the ratio of two second-moment operators which

[4]Alternatively, Smith and Stulz (1985) examine the incentives for firms to base hedge programs on exposure to nonprice risks. This article studies management strategies for controlling exposure from fluctuations in economic values.

[5]With nonzero speculative demand, hedgers adjust positions consistent with *ex ante* appraisals of prices, the size of the futures position depending on risk-adjusted profits from anticipated price changes.

minimize the *anticipated* variation in spot-futures price differences. *Unanticipated* variation in these second-moment operators implies variation about N^*. Intertemporal variation in the pattern of spot and futures prices introduces the need for position revisions during the life of the hedge program. With costly hedging, hedgers can utilize basis-estimation risk estimates to form assessments of probable hedge-rebalancing costs and choose between hedge contracts to lower these costs. For example, risk-adjusted expected returns from a far contract may exceed those of a nearby contract, suggesting that returns from the far contract offer the lower cost hedging instrument.[6] Following Cootner (1967) avoidance of basis risk from this source improves the terms of the trade off between price risk and basis risk.

This should not be taken as implying that basis risk is completely determined by basis-estimation risk. Basis risk is any *unanticipated* variation in basis of which changes in the covariance matrix make up one source. The significance of a nonstationary covariance matrix is examined by defining B as a function of the elements of the covariance matrix which minimize anticipated variation in basis as in equation (1) and examining the implication of changes in the elements of this function. Denoting the function as

$B = f(\hat{\sigma}_{fs}, \hat{\sigma}_f^2)$ and using a Taylor-series, its initial value is expanded to obtain:

$$B \approx B_0 + (\sigma_{fs} - \hat{\sigma}_{fs}) \frac{\delta B}{\delta \sigma_{fs_{\hat{\sigma}_{fs}}}} + (\sigma_f^2 - \hat{\sigma}_f^2) \frac{\delta B}{\delta \sigma^2_{f\,\hat{\sigma}_f^2}} \tag{2}$$

where the subscript 0 refers to the estimator of N^* as in eq. (1). The terms on the right-hand side of (2) having derivatives are of interest. Each gives the impact on predicted basis variation from deviations of past covariances of spot and futures returns or deviations of past variances of futures returns. The expression makes the point that changes in these elements of the covariance matrix impact basis variation. To obtain further insight, an expression for variation in basis from estimation due to these changes is developed. Ignoring higher order terms, rearranging, squaring both sides and taking expectations gives the intertemporal variation of the risk-minimizing hedge ratio expressed in (1).

$$\begin{aligned} \mathrm{VAR}(B) = {} & \mathrm{VAR}(\sigma_{fs}) \left[\frac{1}{\sigma_f^2} \right]^2 + \mathrm{VAR}(\sigma_f^2) [-\sigma_{fs} \sigma_f^{-4}]^2 \\ & + 2\,\mathrm{COV}(\sigma_{fs}, \sigma_f^2) \frac{1}{\sigma_f^2} [-\sigma_{fs} \sigma_f^{-4}] \end{aligned} \tag{3}$$

The notation VAR and COV refer, respectively, to the variation and covariation of the second-moment estimators contained in N^*. Substituting from (1) into (3) gives an expression for basis risk due to changes in the covariance matrix; that is basis-estimation risk is given by

$$\mathrm{VAR}(B) = \frac{1}{\sigma_f^4} [\mathrm{VAR}(\sigma_{fs}) + (N^*)^2\, \mathrm{VAR}(\sigma_f^2) + 2N^*\, \mathrm{COV}(\sigma_{fs}, \sigma_f^2)]. \tag{4}$$

Equation (4) serves as an estimator for variation in the hedge ratio over the estimation interval. This approach is general, not requiring any presumptions about the stochastic processes governing these price series. Given a stationary covariance matrix, no variation about the second-moment estimators expressed in Eq. (1) exists. This means that risk induced by a nonstationary covariance matrix is nil be-

[6]See, for example, Eldridge (1986).

cause basis fluctuations are adequately predicted; the quality of the prediction provides the hedger with a route to avoid this source of risk.

It can also be shown that utilizing the standard error of the regression coefficient used to estimate (1) as a measure of basis risk relies on a stationary covariance matrix. The sample variance of the regression coefficient is given by

$$\sigma^2(\hat{\beta}) = \frac{\sigma_e^2}{\sigma_f^2}.$$

where σ_e^2 = variance of residuals from the regression (MSE). Both components of the standard error depend on a stationary covariance matrix. Thus, basis risk measured by the standard error implicitly assumes a stationary covariance matrix.

Ederington (1979) proposes use of the regression R^2 as a measure of hedging effectiveness. The intuition of this approach recommends its use. As R^2 increases, variation in futures prices increasingly mirrors variation in spot prices. This measure is given by

$$R^2 = \hat{\beta}^2 \frac{\sigma_f^2}{\sigma_s^2}.$$

Again, a nonstationary covariance matrix implies that this estimator for *ex ante* hedging effectiveness may overstate the effectiveness of the hedge. As variation in the components of (1) rises, the probability of encountering large changes in the basis rises, resulting in greater risk for the hedged position.

The hedging implications of a nonstationary covariance matrix are examined by determining the sensitivity of the hedge ratio to changes in the covariance matrix. A second ratio is defined which summarizes basis-estimation risk. By comparing this ratio to N^* further insight is gained regarding the relation between price risk and basis-estimation risk.

Taking the derivative of (4) with respect to N* gives

$$\frac{\delta \text{ VAR}(B)}{\delta N^*} = \frac{2N^* \text{ VAR}(\sigma_f^2)}{\sigma_f^4} + \frac{2 \text{ COV}(\sigma_{fs}, \sigma_f^2)}{\sigma_f^4}. \tag{5}$$

Equation (5) determines the sensitivity of the risk-minimizing hedge ratio to changes in the covariance matrix. If (5) is positive (negative), basis-estimation risk is increasing (decreasing) with changes in the covariance matrix. Setting (5) to zero and solving for N^* gives an expression which summarizes this source of basis risk, this quantity N^{**} is denoted as

$$N^{**} = -\frac{\text{COV}(\sigma_{fs}, \sigma_f^2)}{\text{VAR}(\sigma_f^2)}, \tag{6}$$

so that eq. (5) can be re-written as

$$\frac{\delta \text{ VAR}(B)}{\delta N^*} = 2\left[\frac{\text{VAR}(\sigma_f^2)}{\sigma_f^4}\right](N^* - N^{**}). \tag{7}$$

The sign of (7) is determined by the difference in two ratios, N^* which minimizes price risk and N^{**}. Note that N^{**} is not a hedge ratio despite the familiarity of its form. Its value is determined by N^* and, therefore, N^{**} cannot be a choice variable. More properly, it serves as a reference value within (7) denoting the importance of the hedger's sensitivity to changes in the covariance matrix. If $N^* = N^{**}$, (7) is zero implying that basis-estimation risk is minimized with the hedge ratio which minimizes price risk. Exposure to price risk is minimized using the usual hedge ratio, a strategy which also minimizes this source of basis risk.

For $N^* > N^{**}$, (7) is greater than zero, implying that basis-estimation risk *increases* with *increases* in N^*. Hedgers may avoid this basis risk by hedging at less than the optimal risk-minimizing level. To illustrate, consider a hedge position using a hedge ratio slightly below N^*. Clearly, the hedger is subject to some amount of price risk. However, because the position holds less futures it becomes less reliant on a stationary covariance matrix so that changes in the covariance matrix are of lesser importance. Thus, in this case, the risk-minimizing hedge ratio adds sensitivity to changes in the covariance matrix. Hedgers equating both sources of risk will avoid exposure to this source of risk by decreasing their participation in futures markets; that is, they will hedge at less than the risk-minimizing hedge ratio.[7] Thus hedgers may prefer to remain exposed to price risk, rather than increase exposure to this source of basis risk.

For $N^* < N^{**}$, (7) is less than zero, implying that basis-estimation risk *decreases* with *increases* in N^*. Avoidance of basis risk requires taking more contracts than implied by the risk-minimizing hedge ratio, exposing the hedge to the risk of futures price changes. This case suggests that it may be advantageous to remain exposed to the basis risk implied by the hedge ratio which minimizes price risk.

This insight suggests a measure of hedging effectiveness which focuses directly on the hedge ratio and its variation through time. The ratio N^{**}/N^* equals one when basis-estimation risk and price risk are jointly minimized, is less than one when this basis risk is increasing at the risk-minimizing point, and exceeds one when this basis risk is declining at this point.

As previously noted, N^{**} depends on the choice of N^*, hedgers seeking to avoid this source of basis risk can adjust their hedges only through N^*. This is illustrated for the case of a hedger seeking to completely avoid this basis risk by adjusting the N^* in eq. (4) until basis risk due to estimation risk is zero. More formally, the amount of adjustment (δN^*) is obtained by replacing N^* in eq. (4) with ($N^* + \delta N^* \delta \text{ VAR}(\beta)/\delta N^*$) and solving for δN^*. Terms including $(\delta N^*)^2$ are set to zero, giving an approximation whose accuracy depends on the linearity of eq. (7). Inspection of (7) demonstrates that the second derivative of (4) with respect to N^* is very close to zero. The amount of adjustment to the hedge ratio is given by

$$\begin{aligned} \delta N^* &= -\frac{\sigma_f^4[\text{VAR}(\sigma_{fs}) + (N^*)^2 \text{ VAR}(\sigma_f^2) + 2N^* \text{ COV}(\sigma_{fs},\sigma_f^2)]}{4[N^* \text{ VAR}(\sigma_f^2) + \text{COV}(\sigma_{fs},\sigma_f^2)]^2} \\ &= -\frac{\sigma_f^4 \text{ VAR}(\beta)}{4[N^* \text{ VAR}(\sigma_f^2) + \text{COV}(\sigma_{fs},\sigma_f^2)]^2}. \end{aligned} \tag{8}$$

Note that the adjustment always reduces the optimal risk-minimizing hedge ratio. Unlike the previous analysis of Eq. (7), Eq. (8) does simply not compare the risk-minimizing hedge ratio to the reference value, N^{**}. Rather, basis-estimation risk is eliminated regardless of the increased exposure to price risk. Equation (8) thus presumes the hedger to be at least equally averse to price and basis risk.

Finally, Eq. (8) demonstrates that as basis-estimation risk increases the number of futures contracts in optimal hedges decreases. Demand for futures contracts by

[7]See Figlewski (1986) pp. 33–35 who reports that hedge ratios are typically adjusted downward to incorporate control over basis risk into the hedge program. This can be viewed as evidence that the usual sign of this derivative is positive.

risk-minimizing hedgers will be affected by the amount of this basis risk. Viewing basis fluctuation as partially caused by the pricing implications of contract specifications, demand for futures can be linked to contract specifications.

This point of view is rooted in the literature on the pricing implications of contract details. For example, Cox, Ingersoll, and Ross (1989) show that mark-to-market rules have pricing implications which depend on the correlation of interest rates and futures prices. In addition, delivery specifications typically give short positions quality and timing options. Gay and Manaster (1984) and Boyle (1989) model quality options implicit in the delivery specifications of futures contracts, the empirical work of these authors and that of Arak and Goodman (1987) and Benninga and Smirlock (1985) demonstrate these have considerable value. Boyle (1989) studies timing options giving short positions a range of permissable delivery dates. Arbitrage arguments demonstrate that these options have value. Finally, guarantees of contract performance developed by Kane (1980) suggests that futures prices may include risk premia dependant on the perceived quality of guarantees. Bailey and Ng (1989) develop evidence that these premia do increase as the quality of exchange guarantees is perceived to fall.

Each of these valuation impacts have effects on futures prices which are not present in the prices for spot assets. Thus, basis variability to some extent depends on the value changes of contract features. Contract specifications which increase basis fluctuation will tend to reduce demand for alternative hedging products. Equation (8) then becomes a predictor of successful futures contracts, comparing (8) for several hedging alternatives, successful contracts are expected to require smaller adjustments to the risk-minimizing hedge ratio.

III. ESTIMATION OF BASIS RISK

Variation over the life of the hedge can be estimated by past variation in the spot-futures relationship. Following Jenkins and Watts [1969, pp. 171–181] basis-estimation risk can be calculated in the following manner. Defining

$$c_{ij}(u) = \frac{1}{T}\sum_{T=1}^{T-u}(x_{it} - \bar{x}_i)(x_{j,t+u} - \bar{x}_j) \tag{9}$$

as the covariance of two time series for u lags, then the covariance of any pair of these second-moment estimators is approximated by substituting estimates from (9) into

$$\mathrm{COV}[c_{ij}(k), c_{kl}(l)] = \frac{1}{T}\sum_{r=-\infty}^{\infty}\{c_{ik}(r)c_{jl}(r + l - k) + c_{il}(r + l)c_{jk}(r - k)\}. \tag{10}$$

Setting the indices k and l to zero provides a time-t estimate of $\mathrm{VAR}(\sigma_f^2)$, $\mathrm{VAR}(\sigma_{fs})$, and $\mathrm{COV}(\sigma_f^2, \sigma_{fs})$. Substituting these estimates and estimates of the hedge ratio and variance of futures prices into (4) gives an estimate of basis-estimation risk.

IV. SOME ESTIMATES OF BASIS-ESTIMATION RISK

Empirical evidence of the relevence of basis-estimation risk is developed by examining foreign exchange positions hedged with foreign exchange futures contracts. The evidence of Grammitikos and Saunders (1983) suggests the covariance matrix of futures and spot price changes may not be stationary. Futures prices for foreign exchange contracts traded at the Chicago Mercantile Exchange (CME) are from volume one of their *1985 Yearbook*. Daily settlement prices for nearest-to-delivery contracts for the British pound, Canadian dollar, Deutschemark, Japanese Yen, and Swiss Franc are used. Next-to-nearest contracts are used also. The results do not differ substantially from those reported for the near contract, and are not included here.[8] Corresponding spot rates are closing Bid Side prices from interbank quotations also listed in the CME *Yearbook*.

Table I reports sample statistics for the basis of each contract month. Reported means in most cases are significantly different from zero at the 95% level, the exception being the March Canadian dollar contract. Variation in basis is evidenced by the relatively large standard deviation of spot minus futures prices. This result does not conflict with Castelino (1989) who finds less basis volatility for near contracts than for far contracts. The difference is that the volatilities used here are unconditional, those of Castelino (1989) are conditional on an expectation of the basis.

Using the asymptotic standard error of $n^{-.5}$ gives standard error estimates for autocorrelation coefficients which range from .12 to .15. Investigation of these coefficients up to lag 12 provides no evidence of a pattern either within or across different contracts. At the twelfth lag all but one contract month evidences white noise with reasonable levels of significance, indicating a nonstationary covariance matrix of futures and spot market exchange rates.[9] Evidence of a nonstationary covariance matrix motivates further consideration of the economic significance of basis-estimation risk.

Table II reports hedge ratios calculated by regressing spot price rates of change on futures price rates of change and associated measures of basis risk. Comparing reported regression coefficients to unity can be interpreted as a test for the effectiveness of the dollar-matching hedge strategy.[10] In all but the June and September contracts on the Canadian dollar, each estimated hedge ratio falls within two standard errors of one suggesting that differences between the dollar-matching and risk-minimizing strategies may be nil. Examining the residuals from these regressions indicates significant negative autocorrelation at the first lag suggesting reported standard errors are biased.[11] Thus, the above test procedure may overstate the case for using the dollar-matching strategy.

Intertemporal variation in the risk-minimizing hedge ratio is reported in terms of its standard deviation. Comparing these to the regression R^2 suggests the expected effectiveness of these hedge ratios may be overstated. The R^2 measures do not seem to reflect the importance of variation in the covariance matrix. Large standard deviations imply increasing probability of changes in the risk-minimizing hedge ratio, indicative of substantial basis risk.

Ratios for basis risk from the covariance matrix (N^{**}) are in every case close to zero and less than the risk-minimizing hedge ratio for price risk. From examination of eq. (7) this suggests that this basis risk is increasing with N^*. The appropriate behavior on the part of risk-minimizing managers is to underutilize hedge programs as determined by the risk-minimizing hedge ratio. Empirical work suggests that managers underutilize futures contracts as implied by the usual models of hedging.

[8]These results are available from the authors on request.

[9]See Doukas and Rahman (1987) who find borderline nonstationarity in futures returns based on tests for unit roots.

[10]Brown (1985) for example, computes risk-minimizing hedge ratios for several commodities and compares them to one. A difference in excess of two standard errors is interpreted as evidence for the risk-minimizing hedge ratio.

[11]Not reported, but are available on request.

Table I
SAMPLE STATISTICS FOR BASIS NEAREST-TO-DELIVERY CONTRACT BY CONTRACT

	Mean	Standard Deviation	Autocorrelations*											
			−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−11	−12
British Pound														
March 85	0.004491	0.004423	.27	.03	−.16	−.07	.12	.21	.02	−.01	.00	.16	.12	−.10
June 85	0.005605	0.005559	.19	.32	.32	.26	.29	.22	.22	.34	.33	.17	.23	.06
September 85	0.006587	0.005219	.51	.53	.45	.47	.36	.44	.32	.29	.24	.26	.28	.16
December 85	0.005637	0.004503	.55	.67	.44	.55	.37	.41	.44	.41	.48	.34	.32	.24
Canadian Dollar														
March 85	0.000613	0.002773	.14	.10	−.00	.07	.07	.11	.06	.07	.11	.00	.03	−.12
June 85	0.001110	0.001251	.34	.36	.30	.30	.21	.17	.37	.14	.26	.16	.26	.05
September 85	0.001225	0.001178	.75	.69	.62	.63	.55	.53	.45	.43	.41	.39	.34	.31
December 85	0.000418	0.000754	.12	.24	.20	.14	.03	.18	.04	.21	.10	.17	.03	.10
Deutschmark														
March 85	−0.000874	0.000970	.40	.37	.38	.22	.30	.22	.04	.11	.09	.09	.13	.01
June 85	−0.001113	0.001129	.24	.40	.34	.36	.39	.35	.23	.33	.30	.24	.28	.11
September 85	−0.001125	0.001101	.13	.21	.14	.24	.17	.19	.16	.10	.22	.04	.13	.02
December 85	−0.001806	0.001221	.47	.45	.49	.45	.41	.43	.49	.35	.41	.28	.31	.22
Japanese Yen														
March 85	−0.000998	0.000672	.59	.56	.50	.42	.49	.45	.27	.24	.24	.18	.18	.16
June 85	−0.001263	0.001015	.70	.71	.67	.59	.56	.50	.47	.42	.38	.35	.37	.28
September 85	−0.000832	0.000676	.28	.22	.22	.26	.36	.20	.23	.23	.29	.10	.19	.07
December 85	−0.000729	0.001861	.26	.26	.30	.23	.30	.29	.16	.23	.22	.12	.10	.19
Swiss Franc														
March 85	−0.001272	0.001403	.44	.47	.30	.30	.35	.24	.38	.17	.15	.08	.16	.14
June 85	−0.001616	0.001773	.25	.22	.08	.32	.12	.09	.14	.16	.20	−.03	.09	.05
September 85	−0.001549	0.001351	.34	.31	.24	.30	.27	.24	.19	.16	.18	.04	.16	.11
December 85	−0.002489	0.001506	.53	.48	.53	.38	.41	.42	.48	.41	.30	.29	.21	.12

*Standard errors for autocorrelations are computed as $n^{-.5}$.
These values range from .12 to .15.

Table II

HEDGE RATIOS AND EXPECTED EFFECTIVENESS MEASURES NEAREST-TO-DELIVERY CONTRACT BY CONTRACT

	Hedge Ratio (n^*)	Standard Error	Expected Effectiveness (R^2 measure)	Intertemporal Standard Deviation	Basis Reference Ratio (n^{**})	n^{**}/n^*
British Pound						
March 85	0.9178	0.0677	0.79	0.0938	−0.0611	−0.07
June 85	0.9080	0.0519	0.84	0.0970	−0.0039	−0.00
September 85	0.9646	0.0457	0.88	0.1372	−0.0062	−0.01
December 85	1.0654	0.0389	0.93	0.1720	−0.0156	−0.01
Canadian Dollar						
March 85	0.8300	0.1665	0.33	0.0808	−0.3280	−0.40
June 85	0.7690	0.0681	0.68	0.0456	0.0501	0.07
September 85	0.7423	0.0662	0.68	0.0397	0.0455	0.07
December 85	0.8716	0.0725	0.71	0.0339	−0.2198	−0.25
Deutschemark						
March 85	1.0094	0.0465	0.90	0.0090	−0.0002	0.00
June 85	0.9313	0.0487	0.86	0.0068	0.0502	0.05
September 85	1.0007	0.0581	0.83	0.0099	−0.0635	−0.06
December 85	1.0175	0.0429	0.91	0.0099	−0.0785	−0.08
Japanese Yen						
March 85	0.9725	0.0363	0.93	0.0142	−0.0225	−0.02
June 85	0.9191	0.0487	0.86	0.0108	−0.0108	−0.01
September 85	0.9655	0.0575	0.82	0.0132	−0.0698	−0.07
December 85	0.9057	0.0613	0.79	0.0186	0.0273	0.03
Swiss Franc						
March 85	0.9213	0.0591	0.83	0.0106	−0.0249	−0.03
June 85	1.0000	0.0598	0.83	0.0080	−0.1626	−0.16
September 85	0.9628	0.0421	0.90	0.0130	−0.0371	−0.04
December 85	0.9583	0.0405	0.90	0.0151	−0.0199	−0.02

V. CONCLUSION

This article shows that basis risk is partially determined by intertemporal changes in the covariance matrix of spot and futures prices and participation in the futures market. Since increased participation in the futures markets increases exposure to these changes, hedgers may seek to avoid use of risk-minimizing hedge ratios. Optimal utilization of futures contracts is defined in terms of the risk-minimizing hedge ratio and a second ratio which summarizes basis-estimation risk.

Since the relationship between price changes in the corresponding markets depends on contract terms, contract terms may lead to basis-estimation risk. The role of contract specifications on the determination of basis risk suggests future empirical research examining the success and failure of alternative contract specifications in terms of their basis risk.

A measure of basis-estimation risk is proposed and used to examine this source of basis risk for five foreign exchange contracts traded on the CME. The results reconcile the observation that futures markets are underutilized compared to optimal hedge ratios that do not adjust for basis risk owing to a nonstationary covariance matrix.

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