# On Valuing Complex Interest Rate Claims

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## I. INTRODUCTION

As interest rates become more volatile, financial managers are more aware of the need to consider interest rate uncertainty in their decisions. This increased awareness results in a growing demand for financial mechanisms capable of transferring risk between parties in a transaction. For many years interest rate futures contracts were the only method available to accomplish such goals. Over the last decade, however, a number of new instruments, such as exchange traded debt options and over-the-counter products such as forward rate agreements, swaps, caps and floors have been introduced. The phenomenal growth of the over-the-counter market is evidenced by the fact that the notional principal underlying the swap, cap and floor market is rapidly approaching one trillion dollars.

In most cases over-the-counter products are tailor-designed risk management products that consist of a host of contingent claims whose payouts are based on interest rates drawn from a variety of points across the yield curve. To value these complex claims it is often necessary to decompose them into a portfolio of simple put and call options. Fundamental then, to any valuation model for complex interest rate claims, is a simple model for valuing simple put and call options along the yield curve. Central to this issue is the incorporation of uncertainty into term structure models in a satisfactory way. There are currently three different approaches used to incorporate uncertainty into the yield curve process and to value interest rate claims.

This article reviews these approaches and identifies the approach that is most useful for pricing portfolios of claims whose values are contingent on interest rates drawn from different points along the yield curve. This approach, referred to as the term structure constrained approach (TSC) is then expanded upon. Examples are provided to illustrate the methodology. The first example investigates the pricing of interest rate futures and forwards. The futures price of a claim that contains an embedded quality option is also valued. Finally, more complex claims are analyzed and

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defined on binomial lattices with finer partitions. The pricing of multi-period cap on interest rates which guarantees that the rate on a loan at any payment date is either the short rate or the cap rate, whichever is lower, is discussed. Prices of floors and interest rate swaps are derived also. The final example investigates the pricing of an option on a swap. This contract, referred to as a swaption, gives the counterparty the right to pay a fixed rate (put swaption) or the right to receive a fixed rate (call swaption) at some future date. Such contracts can be used in conjunction with bond issues to hedge the exposure on callable debt.

## II. A TAXONOMY OF INTEREST RATE CLAIM MODELS

The usual approach to pricing an interest claim begins with a continuous trading economy driven by a finite number of exogenously specified stochastic variables. In most applications there are one or two underlying sources of uncertainty. Single state models usually take the short rate process as the state variable, while two state models include a long rate. Given these processes, together with a theory of the term structure, discount bond prices can be derived and then claims contingent on these prices can be fairly valued. Examples of this approach include Courtadon (1982), Vasicek (1977), Brennan and Schwartz (1979) and others. Rather than take the interest rate process as exogenous, Cox, Ingersoll and Ross (1985) provide a model where the interest rate process is derived in an equilibrium framework. Since these models are equilibrium models they contain parameters that depend on preferences and estimation problems emerge.

If the assumptions of any equilibrium model are correct, the theoretical bond prices should exactly match observed prices and all contingent claims would be fairly priced. However, most equilibrium models induce yield curves that have restrictive shapes. As a result all theoretical and observed bond prices do not match. Since certain debt option prices are based off theoretical rather than observed prices, the resulting option prices are misspecified. Even if the theoretical price of a particular discount bond matches the observed price, the theoretical prices of short term put and call options generated from the model may not satisfy put-call parity conditions (1986). As a result the value of any complex claim consisting of portfolios of calls and puts based on interest rates drawn from across the yield curve is likely to be erroneous and inconsistent with no riskless arbitrage.

To overcome the difficulty imposed by having preference-dependent parameters that have to be estimated, a second approach uses the bond price as the underlying state variable. Unfortunately, specifying the stochastic process of a discount bond in nontrivial. Ball and Torous follow this approach (1983). Their model includes a Brownian bridge process in the pricing dynamics. While this assumption ensures that the bond price converges to its face value, it creates difficulties for the arbitrage argument. More recently, Schaefer and Schwartz (1987) consider a very simple model where the volatility of prices is limited to a function of the underlying bond's duration. The advantage of this approach is that the resulting prices do not depend on preferences. However, like the equilibrium-based models, short term call and put options on long term discount bonds do not satisfy put-call parity conditions since short term bonds never enter into the analysis. If this approach is used to independently price many options on many bonds, the resulting collection of prices would admit arbitrage. Thus, like the equilibrium-based approach this approach also is not very useful for valuing sets of contingent claims.

A third approach, developed by Ho and Lee (1986), for pricing interest rate claims is the term structure constrained approach (TSC). It resolves many of the above difficulties because it utilizes full information provided by the currently observable yield curve. Rather than exogenously specifying the process of a state variable such as short or long rates, Ho and Lee use information on the term structure, together with the requirement of no riskless arbitrage, to impose constraints on the parameters of the stochastic process of the entire yield curve. In their model the entire yield curve fluctuates randomly over a binomial lattice, and parameters are constrained such that all theoretical discount bond prices match observed prices. Since the model produces correct prices for straight discount bonds it also provides correct answers for portfolios of such bonds and the set of option prices generated are internally consistent and free of arbitrage possibilities. Unlike the other two approaches, this approach is, therefore, extremely useful for valuing complex claims that can be decomposed into a portfolio of simple claims drawn from across the yield curve.

Though ingenious, the Ho-Lee model does not prevent negative interest rates. As a result, the model often indicates that early exercise of American calls on discount bonds is optimal. Moreover, the volatility of yields to maturity are independent of time and the yield curve shifts in any time period are all parallel. As a result, the model is not particularly useful for valuing claims involving exchanges of payments based on different rates drawn from the yield curve. For example the value of a variable for variable swap can be replicated by a simple portfolio of discount bonds.

To overcome these difficulties Ritchken and Boenewan show how to curtail one of the two parameters in the Ho-Lee model to avoid negative interest rates (1989). Pedersen, Shiu, and Thorlacius consider a model that induces shifting shapes in the yield curve (1989). Bliss and Ronn (1989) provide a state-dependent trinomial model of intertemporal change in the short rate. Heath, Jarrow, and Morton (1988) provide a continuous time model in which negative interest rates do not occur. Their model requires as input the yield curve and a volatility function. Finally, Jamshidian (1989) provides a closed-form TSC model for European debt options when the short rate follows a mean reverting Gaussian interest rate process. This article sets forth a simple discrete-time binomial-based TSC model and illustrates how this model can be used to value several different claims.

## III. TERM STRUCTURE CONSTRAINED MODELS

Let $D_0(t)$ be the time zero discount function. In each period the function can move to one of two possible states. Let $D_{ij}(t)$ be the discount function in period $i$ and state $j$. From vertex $(i,j)$ the price of a $t$-period discount bond can move "up" to $D_{i-1,j+1}(t-1)$ or "down" to $D_{i+1,j}(t-1)$. The index $j$ thus counts the number of "up" moves in $i$ periods.

To prevent riskless arbitrage over the binomial lattice Ho and Lee show that a set of pseudo-probabilities $\{q_{ij}\}$, must exist such that the local expectations hypothesis holds with respect to these probabilities. That is prices must satisfy the equation.

$$D_{ij}(t) = D_{ij}(1)[q_{ij}D_{i-1,j+1}(t-1) + \bar{q}_{ij}D_{i-1,j}(t-1)] \tag{1}$$

where $\bar{q}_{ij} = 1 - q_{ij}$.

Define $c_i = D_{i,j+1}(1)/D_{ij}(1)$ as the spread parameter at vertex $i$. Recursive calculations using (1) lead to

$$D_{ij}(t) = k(i,j;\, t-1)\left\{\prod_{\ell=1}^{t-1} D_{i+\ell,j}(1)\right\} \tag{2}$$

where

$$k(i,j;\, t-1) = q_{ij}\left\{\prod_{\ell=1}^{t-2} c_{i+\ell,j}\right\} k(i+1,j+1;\, t-2) + \bar{q}_{ij}k(i+1,j;\, t-2)$$

and $k(i,j;\,0) = 1$.

From eq. (2), prices of similar bonds in adjacent states can be compared. Specifically,

$$\frac{D_{i,j+1}(t)}{D_{ij}(t)} = \left\{\prod_{\ell=1}^{t-2} c_{i+\ell,j}\right\}\frac{k(i,j+1;\, t-1)}{k(i,j;\, t-1)} \tag{3}$$

Equation (2) also allows one to relate the prices of discount bonds with different maturities at any vertex. Specifically,

$$\frac{D_{ij}(t+1)}{D_{ij}(t)} = D_{i+t,j}(1)\frac{k(i,j;t)}{k(i,j;t-1)} \tag{4}$$

For $i = 0, j = 0$ eq. (4) reduces to

$$D_{t0}(1) = F_0[t,t+1]\frac{k(0,0;t-1)}{k(0,0;t)} \tag{5}$$

where $F_0[\mathrm{t}, \mathrm{t}+1]$ is the forward function. Equation (5) constrains the future one-period bond prices according to the current forward function. Any TSC model on a binomial lattice is fully specified once the $\{q_{ij}\}$ and $\{c_i\}$ values are established. If $q_{ij} = q$ and $c_i = c$, then the model reduces to the Ho-Lee model. Specifically, eq. (5) reduces to

$$D_{i0}(1) = \frac{F_0[i,i+1]}{qc^i + \bar{q}}$$

Further,

$$D_{ij}(1) = D_{i0}(1)c^j = \frac{F_0[i,i+1]c^j}{qc^i + \bar{q}}$$

which is the Ho-Lee model. Unfortunately this simple two parameter model allows negative interest rates and has the property that in any future time period the yield curves are exactly parallel. As a result, any complex interest rate claim that has a payout in a period based on exchanges of rates along the yield curve (plus or minus basis points) can be replicated by an appropriate portfolio of pure discount bonds.

Ritchken and Sankarasubramanian (1989) propose a more flexible model that prevents negative interest rates, allows for changing shapes in the yield curve, and has the property that single-period yields are mean reverting. In their model interest rates are bounded below by specifying a function, $u(t)$ that constrains the maximum deviation of bond prices from the observed forward function. $(u(t) - 1)100$ is the maximum deviation in percentage terms of short term bond prices from forward prices. The function $u(t)$ can reasonably be expected to be concave. Mean reversion of short rates is established by assuming the pseudo-probability values $q_{ij}$ depend on the current level of the short rate. Specifically when the short rate rises (or falls) significantly, there is pressure for the rate to revert towards the long run average short rate. Empirical evidence suggests that interest rates have this property (1985). A two parameter functional form for $q_{ij}$ that has this property is given by

$$q_{ij} = e^{-\alpha[r_{ij}(1)]^{\beta}} \qquad \begin{Bmatrix} \alpha \geq 0 \\ \beta \geq 0 \end{Bmatrix} \tag{6}$$

Figure 1 indicates the form of the function. As $\alpha$ increases and $\beta$ decreases the function steepens indicating stronger mean reversion. If $\beta = 0$, then $q_{ij} = q$ and the Ho-Lee Model obtains. The bounding function and the parameters of mean reversion uniquely determine the stochastic evolution of the discount function over a binomial lattice in a way that overcomes the deficiencies of the Ho-Lee model.

The model is best illustrated by an example. Assume the current yield curve is given by $r_0(1) = 6\%$, $r_0(2) = 7\%$, $r_0(3) = 8\%$, and $r_0(4) = 8.9\%$. Further, assume that the bounding function is given by $u(1) = 1.015\%$, $u(2) = 1.017\%$, and $u(3) = 1.018\%$. To specify the mean reversion function requires estimates of any two points on the curve. At the median short rate, the probability of an upmove is 0.5; at the 99$^{\text{th}}$ percentile, the probability is 0.01. In the example, the median short rate is set at 6.056% and the 99$^{\text{th}}$ percentile at 30%. From these two points the mean reversion parameters are uniquely determined as $\alpha = 18.99$ and $\beta = 1.18$.

Given the above information, the entire stochastic process of the yield curve can be constructed. First, the current discount function is computed from the yield curve. Then given $u(1)$, the largest single-period bond price in period 1, $D_{10}(1)$ can be computed. Specifically

$$D_{10}(1) = D_0(1)u(1) = 0.95754717$$

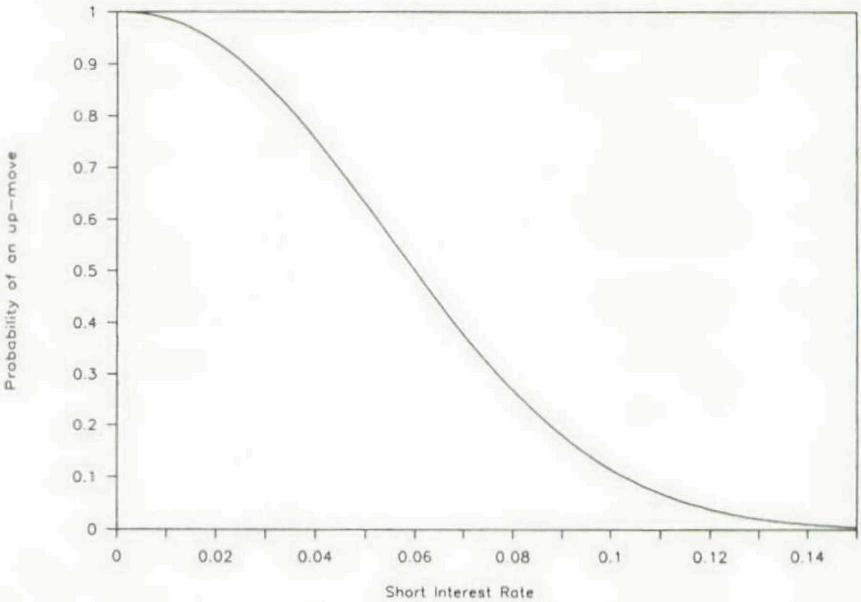


**Figure 1**
The Mean Reversion Function.

Moreover, from (6), $q_0 = 0.50325279$. In addition,

$$D_0(2) = D_0(1)[q_0 D_{11}(1) + \bar{q}_0 D_{10}(1)]$$

Substituting the known values and solving yields $D_{11}(1) = 0.89455275$. The ratio of single-period bond prices in period 1, $c(1)$ is

$$c(1) = \frac{D_{11}(1)}{D_{10}(1)} = 0.93421272 .$$

Given the one-period bond prices in period 1, the pseudo probability values can be computed. Specifically

$$q_{10} = 0.61847927; \qquad q_{11} = 0.21797137$$

To compute the one-period bond price in period 2 the highest permissible value, $D_{20}(1)$, is compiled as

$$D_{20}(1) = u(2)D_{10}(1) = 0.97382547$$

Let $c(2)$ represent the ratio of adjacent single-period bond prices in period 2. Then, one can solve the following three equations for $D_{11}(2)$, $D_{10}(2)$, and $c(2)$.

$$\begin{aligned} D_0(3) &= [q_0 D_{11}(2) + \bar{q}_0 D_{10}(2)]D_0(1) \\ D_{10}(2) &= [q_{10} D_{20}(1)c(2) + \bar{q}_{10} D_{20}(1)]D_{10}(1) \\ D_{11}(2) &= [q_{11} D_{20}(1)(c(2))^2 + \bar{q}_{11} D_{20}(1)c(2)]D_{11}(1) \end{aligned}$$

The solutions are:

$$C(2) = 0.92605020; \qquad D_{10}(2) = 0.88983535; \qquad D_{11}(2) = 0.79371432$$

Hence the one-period bond prices in period 2 can be obtained as

$$\begin{aligned} D_{22}(1) &= (c(2))^2 D_{20}(1) = 0.83512251 \\ D_{21}(1) &= c(2)D_{20}(1) = 0.90181127 \end{aligned}$$

and

$$D_{20}(1) = 0.97382547$$

The procedure is now repeated for the third period. First, the highest one period bond price, $D_{30}(1)$ is obtained as $u(3)D_{20}(1) = 0.99135433$. The value for $c(3)$ is obtained by solving the following four equations for $D_{20}(2)$, $D_{21}(2)$, $D_{22}(2)$, and $c(3)$.

$$\begin{aligned} D_{20}(2) &= [q_{20} D_{30}(1)c(3) + \bar{q}_{20} D_{30}(1)]D_{20}(1) \\ D_{21}(2) &= [q_{21} D_{30}(1)(c(3))^2 + \bar{q}_{21} D_{30}(1)c(3)]D_{21}(1) \\ D_{22}(2) &= [q_{22} D_{30}(1)(c(3))^3 + \bar{q}_{22} D_{30}(1)(c(3))^2]D_{22}(1) \end{aligned}$$

and

$$\begin{aligned} D_0(4) = D_0(1)\{&q_0 D_{11}(1)[q_{11} D_{22}(2) + \bar{q}_{11} D_{21}(2)] \\ &+ \bar{q}_0 D_{10}(1)[q_{10} D_{21}(2) + \bar{q}_{10} D_{20}(2)]\} \end{aligned}$$

The solution is given by

$$D_{20}(2) = 0.90605376$$

$$D_{21}(2) = 0.80580831$$

$$D_{22}(2) = 0.69696757$$

$$c(3) = 0.91976966$$

Given the two-period bond prices in period 2, the three-period bond price in period 1 can be computed easily. Figure 2 summarizes the computations to date.

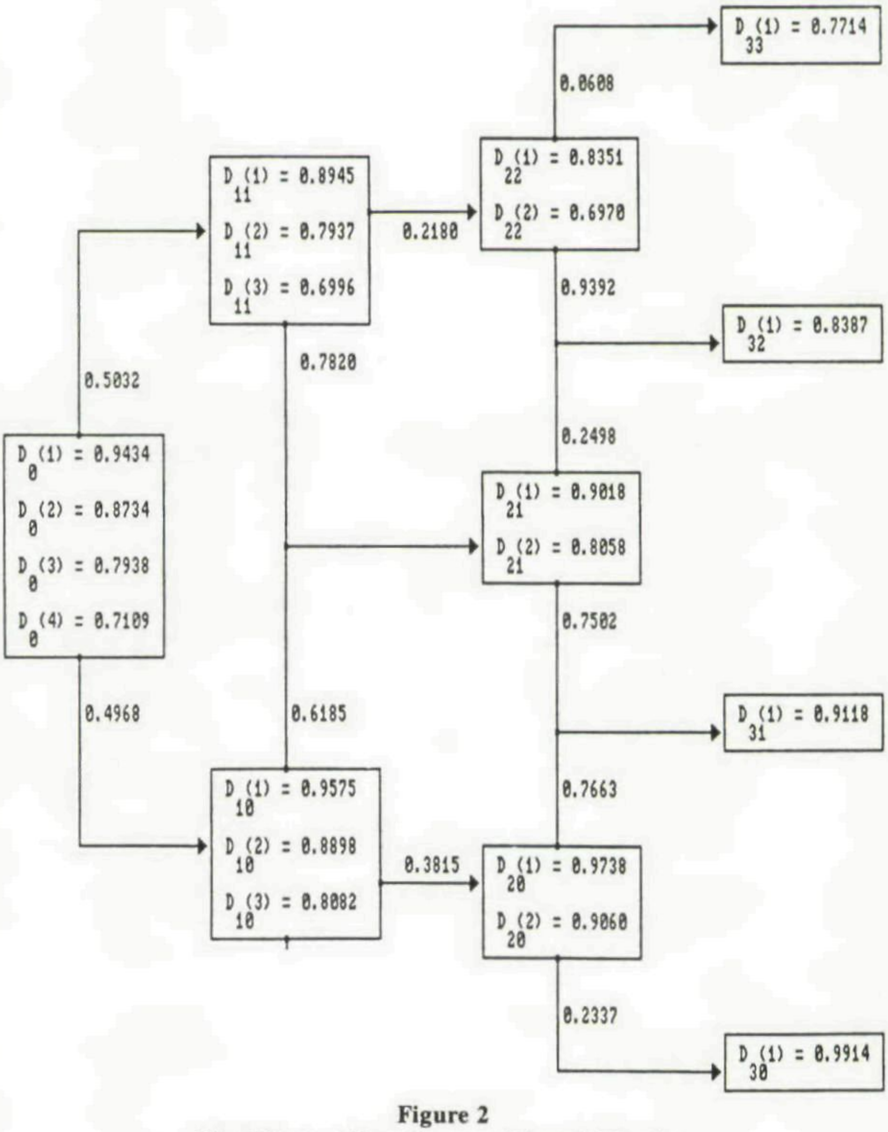


**Figure 2**
The Discount Function on a Binomial Lattice.

## IV. VALUING INTEREST RATE CLAIMS

The valuation of any interest rate claim over the binomial lattice can be performed easily now. To achieve this goal, assume the claim expires in period $n$ and pays the holder $f_{ij}$ in period $i$ and state $j$, for $j = 1, 2, \ldots, i$ and $i = 1, 2, \ldots, n$. Then, following the usual backward recursion techniques, the value of a European style claim at vertex $(i, j)$, $C_{ij}$, is related to its future values as

$$C_{ij} = (q_{ij}(C_{i+1,j+1} + f_{i+1,j+1}) + \bar{q}_{ij}(C_{i+1,j} + f_{i+1,j}))D_{ij}(1) \tag{7}$$

with the boundary condition[1]

$$C_{nj} = f_{nj} \quad \forall j = 0, 1, \ldots, n.$$

The futures price of any claim at node $(i, j)$ can be related also to the futures price of the claim at the nodes $(i + 1, j)$ and $(i + 1, j + 1)$. Let $g_{ij}$ be the futures price of the claim at node $(i, j)$. Clearly, at termination, $g_{n,j}$ equals the spot price for the claim. To establish the appropriate backward recursion technique, consider a portfolio with $H_1$ dollars in discount bonds of maturity $t_1$ and $H_2$ dollars in discount bonds of maturity $t_2$. To replicate the futures contract, the following conditions need to be met

$$H_1 + H_2 = 0$$

$$H_1\left[\frac{D_{i+1,j}(t_1 - 1)}{D_{i,j}(t_1)}\right] + H_2\left[\frac{D_{i+1,j}(t_2 - 1)}{D_{i,j}(t_2)}\right] = g_{i+1,j} - g_{i,j}$$

$$H_1\left[\frac{D_{i+1,j+1}(t_1 - 1)}{D_{i,j}(t_1)}\right] + H_2\left[\frac{D_{i+1,j+1}(t_2 - 1)}{D_{i,j}(t_2)}\right] = g_{i+1,j+1} - g_{i,j}$$

The first condition is that the value of a futures contract is zero as a result of marking to market. The second and third conditions ensure that the portfolio replicates the future payoffs. In conjunction with (1), these imply that

$$g_{ij} = q_{ij}g_{i+1,j+1} + \bar{q}_{ij}g_{i+1,j} \tag{8}$$

The above relationship can be used recursively to obtain the futures price today. For example, consider a highly stylized version of the Treasury Bond futures contract with an embedded quality option that grants the short the right to deliver either a one-period discount bond or a two-period 8% coupon bond at the delivery date in two periods time. Both bonds have a face value of \$1000. For simplicity, assume both conversion factors equal unity. The futures price at the "highest" node at termination can be obtained from Fig. 2, as

$$\text{Min}[835.10, (80 * 0.8351 + 1080 * 0.6970)] = \$819.57$$

Similarly, the values at the other two nodes can be computed to be \$901.80 and \$973.80. Using (8) recursively, the current futures price is \$906.42.

Figure 3 shows the futures price at the various nodes for this contract as well as for the individual bonds.

The drop in the futures price obtained when the deliverable set contains two bonds reflects the value provided to the short in selecting the cheapest to deliver

[1]The value of an American style claim is obtained by computing the maximum of the value of the European style claim as indicated, and the intrinsic value obtained by immediate exercise.

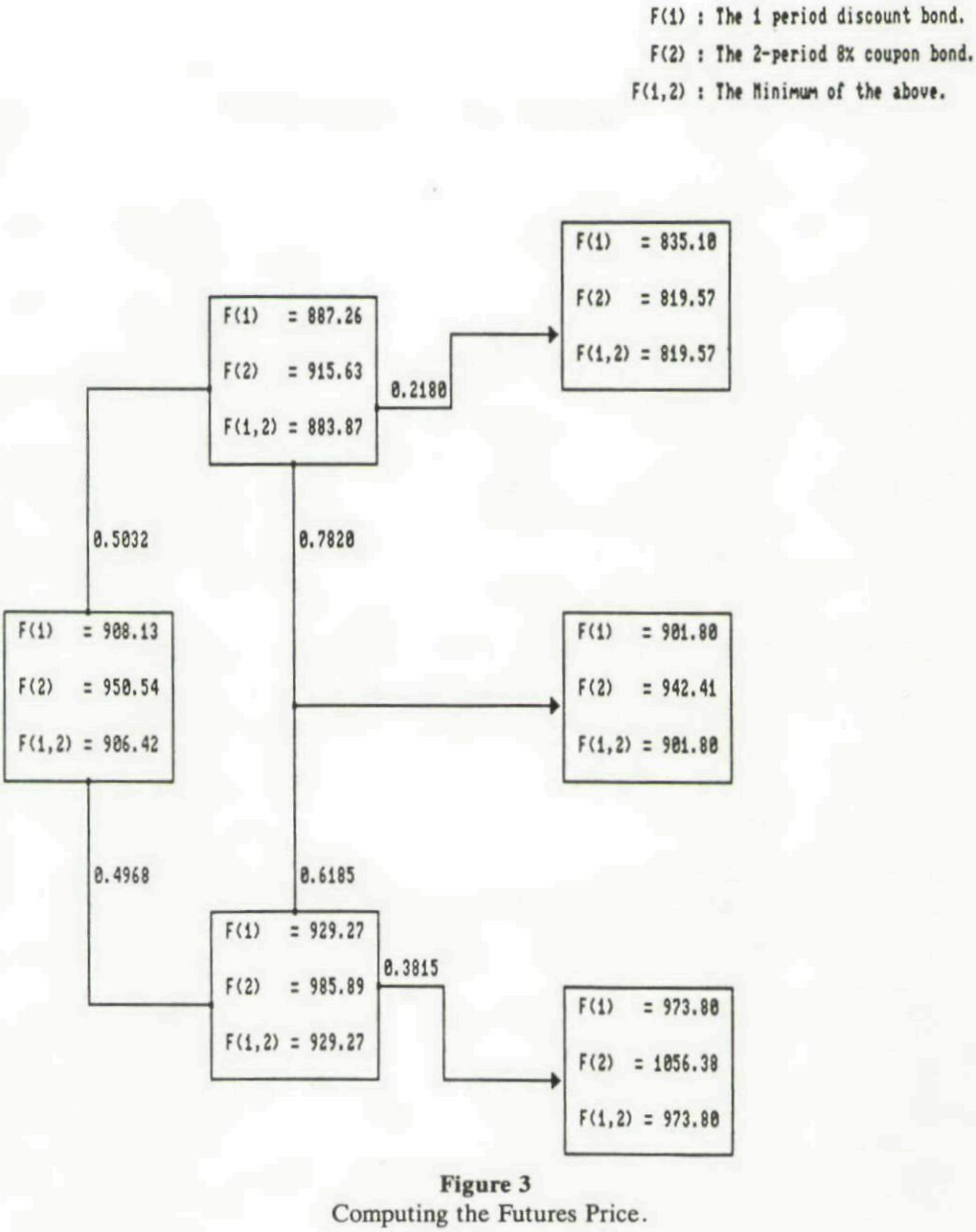


**Figure 3**
Computing the Futures Price.

bond. This drop is the value of the embedded quality option. Using the discount bonds as the reference bond, the value of the quality option is \$1.71. With respect to the 8% coupon bond, the quality option is worth \$44.12.

As a second example, consider pricing forward contracts on bonds. The forward price of any claim can be expressed in terms of the current value of the claim and the value of an appropriate discount bond. In particular, the $t$-period forward price, on a claim that is worth $v_0$ today can be written as $v_0/D_0(t)$. For example, forward price that requires delivery of a one-period discount bond in two periods time is given by $D_0(3)/D_0(2)$ which equals \$908.86. Similarly, the forward price that requires delivery of a two period 8% coupon bond in two periods time is given by

$$\frac{80 * D_0(3) + 1080 * D_0(4)}{D_0(2)} = \frac{80 * 0.7938 + 1080 * 0.7109}{0.8734} = \$951.77$$

Now consider the more complex forward price which provides the short with the option to deliver either one of these bonds in two periods time. The underlying claim which pays out the minimum of the two bonds is valued in Figure 4. The forward price for this claim is 792.35/0.8734 = \$907.20.

Observe that all forward prices exceed their corresponding futures prices with the difference reflecting marking to market costs. This is consistent with all theoretical models of futures and forwards on bonds. As with the futures contracts the forward price declines with the inclusion of a second bond in the deliverable set. The size of the drop here equals the value of the quality option embedded in the forward contract. These results are summarized in Table I.

While the above examples are highly stylized, they illustrate the potential application of the interest rate claim model developed. Following is a analysis of more complex claims defined on binomial lattices with finer partitions.

## V. EXAMPLES OF VALUING COMPLEX INTEREST RATE CLAIMS

Figure 5 shows the yield curve for twenty periods. The parameters for the $q(\bullet)$ function are selected such that the median interest rate is 6% and the 99$^{\text{th}}$ percentile is 30%. The bounding function $u(\bullet)$ is selected such that the elasticity with respect to time of the maximum deviation of the spot rate from the forward rate is a constant $a$. Specifically,

$$\frac{d}{dt}(f(t,t+1) - r_{\min}(t,t+1))\frac{t}{f(t,t+1) - r_{\min}(t,t+1)} = a.$$

If $a > 1$, the binomial lattice explodes while $a = 1$ corresponds to a linear growth rate. In these examples, $a = 0.2$.

One must first analyze the prices of caps. According to this agreement in each of the next $n$ periods the holder has the option to exchange the $k$-period rate for a fixed cap rate $K$. In this example $n = 7$, $k = 2$, $K = 900$ basis points and the notional principal is one million dollars. Ignoring default risk, the value of this interest rate claim can be computed as the value of a portfolio of interest rate call options. For the given parameters, the value of the cap is \$7259.35. Table II shows the sensitivity of the value of the cap to changes in the cap rate, $K$.

The table also contrasts these values to the prices of floor contracts which protects the owner from interest rate declines below the level $K$. Such a contract can be viewed as a portfolio of interest rate puts. The final column of the table values a commitment (swap) in which the party is obligated to the exchange in every second period.

A second example considers valuing a *call swaption*. This contract gives the owner the right to enter into a swap in some future period. To value such a claim first requires computing all possible values of the swap at the expiration date of the swaption. These values determine the boundary conditions, and through the normal backward recursion procedure the swaptions can be valued. In this example, the underlying swap involves exchanges in every second period starting from the fourth and ending in the sixteenth. At each exchange the holder is obligated to

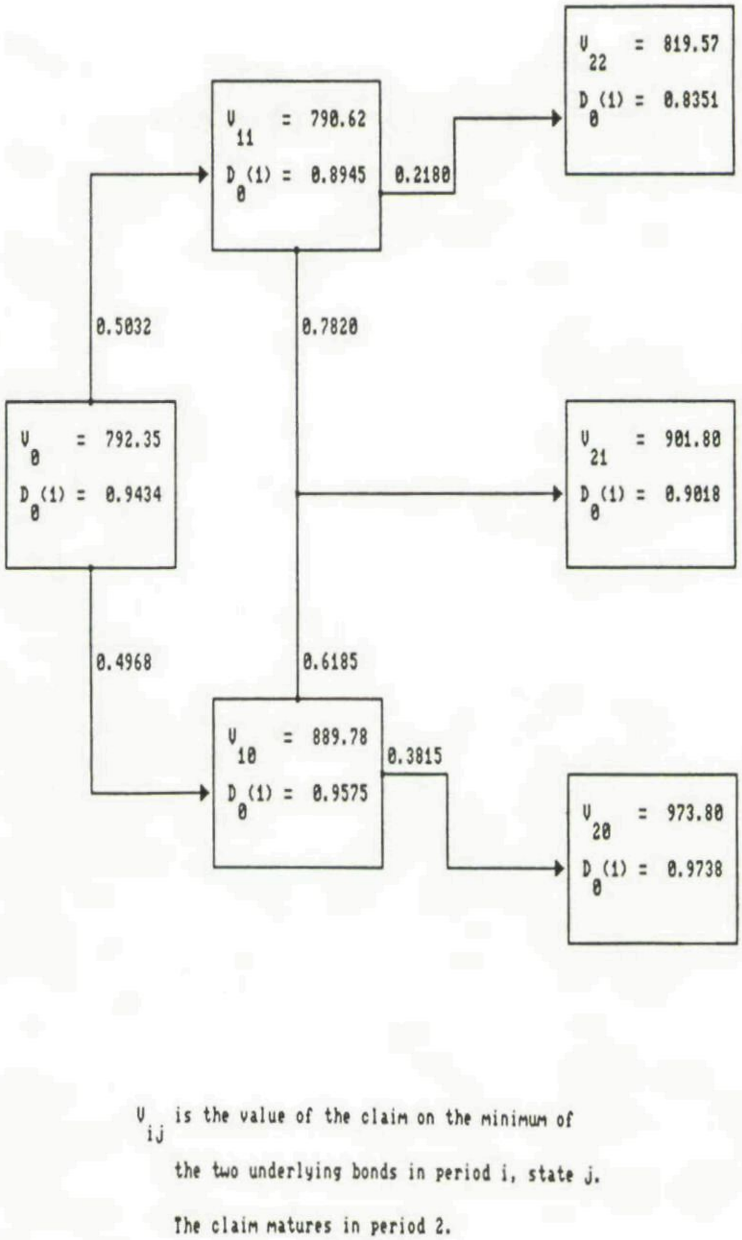


**Figure 4**
Valuing the Minimum of two Bonds.

swap the two-period rate for 800 basis points using a notional principal of a million dollars. The value of a three-period call swaption on the swap can be computed as \$40221.62. The put swaption, on the other hand is worth \$13379.95.

The above examples illustrate the case in which the TSC model values complex interest rate claims in such a way that all information on the yield curve is incorporated.

Table I
**FORWARD AND FUTURES PRICES WITH EMBEDDED QUALITY OPTIONS**

| | Discount Bond | Coupon Bond | Both Bonds |
|---|---|---|---|
| Forward Price | 908.86 | 951.77 | 907.20 |
| Futures Price | 908.13 | 950.54 | 906.42 |
| Marking to Market Costs | 0.73 | 1.23 | 0.78 |

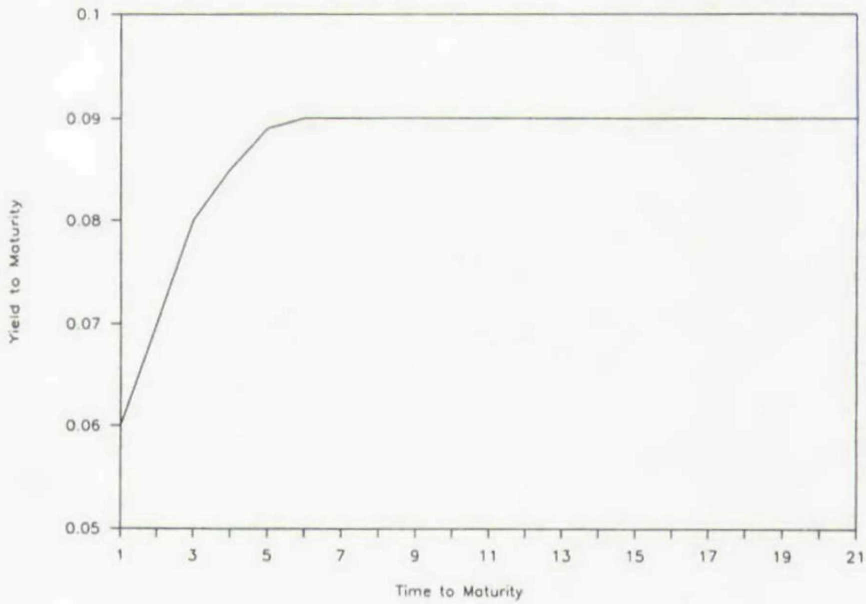


**Figure 5**
Initial Yield Curve

Table II
**SENSITIVITY OF CONTINGENT CLAIMS TO STRIKE PRICES**

| Strike | Value of a Cap | Value of a Floor | Swap |
|---|---|---|---|
| 6 | 189667.40 | 3181.73 | 186485.67 |
| 7 | 147285.09 | 12084.06 | 135201.03 |
| 8 | 106741.22 | 22824.83 | 83916.39 |
| 9 | 72592.83 | 39961.08 | 32631.75 |
| 10 | 44966.44 | 63619.33 | −18652.89 |
| 11 | 2753.60 | 97441.13 | −69937.53 |
| 12 | 13774.50 | 134996.66 | −121222.17 |

**VI. CONCLUSION**

Term structure constrained models for interest rate claims provide option prices which are consistent with current term structure. Valuation of complex term structure constrained American style interest rate claims is nontrivial; closed form solutions are highly unlikely; and numerical methods are usually required to obtain solutions. This article illustrates how a discrete-time term structure constrained model can be used to value complex interest rate claims.

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