The Relative Responsiveness to Information and Variability of Storable Commodity Spot and Futures Prices

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INTRODUCTION

Many empirical studies concerning storable commodity spot and futures market performance focus on answering the following three questions: (1) Is the response to information of the spot price greater than, less than, or the same as that of the futures price (Working (1949), Gray (1960), Stein (1962), Weymar (1965))?; (2) Is the variance of futures price changes positively related, negatively related, or unrelated to the contract's time to maturity in the dynamic (i.e., longitudinal) sense[1] (Miller (1979))?; and (3) Is the ratio of the variance of futures price changes to the variance of spot price changes, positively related, negatively related, or unrelated to the futures contract's time to maturity in the static (i.e., cross sectional) sense and/or dynamic sense[2] (Grauer (1976), Rutledge (1976), Westerfield (1977), Castelino (1981), Milonas (1984), Anderson (1985))? These issues are significant for a number of reasons. The significance of each is briefly discussed in turn.

Knowledge of the relative response of spot and futures prices to information is useful in devising a hedging strategy. To illustrate how this knowledge is used, consider the case of a hedger who desires to minimize risk. In general, to determine the proper hedge in such a case, the hedger needs to know both the particular type of uncertainty to be resolved (assuming the relative response of spot and futures prices differs across information types) and the relative response of spot and futures prices associated with the particular information type. For the hedge, it does not matter if the information is under

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[1]In the dynamic or longitudinal sense, it is a function of the contract's time to maturity in that time is passing for a contract with a fixed maturity date.

[2]In the static or cross sectional sense, the ratio of the variance of futures price changes to the variance of spot price changes is a function of the time to maturity in that at a point in time futures contracts with different maturity dates are considered.

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or over-anticipated as long as the uncertainty-type and the relative response of spot and futures prices associated with the uncertainty-type are properly anticipated. For example, suppose it is known that a government estimate concerning the upcoming corn harvest is to be released (i.e., the uncertainty to be resolved concerns future period production). Similarly, the government announcement may concern the likely future supply of T-Bonds or GNMA mortgages (the likely supply of T-Bonds and GNMA mortgages may be inferred from information regarding the likelihood of deficit reduction legislation such as the Gramm-Rudman legislation and housing starts, respectively). If the estimate is surprisingly high (low) the spot and futures price will fall (rise). Moreover, assume the futures price will respond twice as much as the spot price to this type of information and this will be the only type of uncertainty to be resolved. In such a situation, the hedger will take a futures position which is half as large as the spot position. Then, even if the estimate is high or low, the value of the hedger's portfolio is the same. In contrast, suppose the government estimate concerns the current period corn harvest (i.e., the uncertainty to be resolved concerns current period production). Similarly, the government estimate may concern the current budget deficit or housing supply. Also, assume that the response of the futures price is half as much as that of the spot price with regards to this type of information. In this scenario, the hedger takes a futures position which is twice as large as the spot position. Where there is more than one source of fundamental uncertainty, the hedger needs to know both the variances (and covariances) of the fundamental sources of uncertainty and the responses of spot and futures prices to the different sources of uncertainty to devise the proper hedge.

The relationship between futures price change variability and the contract's time to maturity has important implications regarding futures contract risk. First, if speculators are risk-averse and few in number, then the "keynesian" or "traditional" risk premium, which is a direct function of the futures price change variance, should be related to the time to maturity in the same way as the futures price change variance. Second, ceteris paribus, a speculator's position in a futures contract should be related to the contract's time to maturity in the opposite way that the risk of the futures position is related to the contract's time to maturity. Third, the variance of futures price changes is one of the key factors used by regulators and exchanges in the setting of futures price limits and margins. The price limits and margins are set to reduce the likelihood of default on a contract. If futures price variability is strongly related to the contract's time to maturity, then regulators and exchange officials may desire to make futures price limits and margins functions of the contract's time to maturity. Fourth, one of the most important determinants of the value of an option is the variability in the price of the underlying asset. Therefore, to properly price options on futures contracts it is important to understand the determinants of the futures price variability.

While it is important to understand how the volatility of futures prices varies dynamically as the contract approaches its maturity date, empirical tests directly examining the issue are destined to be inconclusive. Because there are sources of heteroskedasticity in futures price changes other than the contract's time to maturity in the dynamic sense, direct empirical tests of question two will be inconclusive because the results are sample-specific. Therefore, on an empirical basis, this issue must be addressed indirectly.

One approach to providing indirect evidence regarding the relation between futures price change variability and the contract's time to maturity in the dynamic sense is to determine the relation between the ratio of the variance of futures prices to the variance of the spot price and the futures contracts' time to maturity (i.e., one approach examines question three as suggested in Stein (1979)). The spot price volatility is used to control for the sources of futures price heteroskedasticity other than the contract's time to maturity. The approach assumes that spot and futures prices are impacted by the same fundamental causes of heteroskedasticity and to the same extent, except for their differences in times to maturity. Using this approach, it is concluded that if the ratio of the futures price variance to the spot price variance is an inverse function of the contract's time to maturity, then assuming that the spot price variance is constant, the futures price variance increases as the contract approaches its maturity in a dynamic sense. Therefore, by answering question three this approach seeks to provide a conditional answer to question two.

These issues are significant in the study of market efficiency as well. Specifically, early efficient market models (Samuelson (1965) and Stein (1979)) imply that near futures prices respond more to information than far futures prices; the variance of immediate one period price changes for a futures contract increase as the contract dynamically approaches its maturity date; and the ratio of the variance of the futures price change to the variance of the spot price change, for immediate one period price changes, is less than one and an inverse function of the futures contract's time to maturity.

The findings of the above empirical studies, however, are not uniform across commodities and/or time periods for a commodity. This suggests that one of the following or some combination of the following is true: these models are misspecified; the empirical tests are misspecified; markets are efficient at some times and inefficient at other times; and the empirical results are a consequence of statistical chance.

This study assumes that the previous inventory-hedging and nonmarket-based models examining these issues are misspecified and offers an alternative equilibrium inventory-hedging model to elucidate, evaluate, and extend the views previously expressed on these issues. It is also found that the ratio of the futures price variance to the spot price variance varies depending on the types of uncertainty to be resolved; in particular, "announcement" periods are identified as predictable periods for which this ratio is low relative to "control" periods.

In addition to addressing the three issues mentioned at the outset of the article, the model determines the variance of one period spot price changes. It is important to understand the determinants of spot price variability for the implications spot price variability has regarding spot market risk premia and option on spot asset pricing models.

I. REVIEW OF THE LITERATURE

A. The Relative Response of Spot and Futures Prices to Information

Hypotheses regarding the relative responses of spot and futures prices to two kinds of information are stated in the storable commodities literature. The types of information are unanticipated expectational revisions concerning future period net consumption demand and unanticipated current period net consumption demand information, where net consumption demand equals consumption demand less production supply.

Define the ratio of the spot price response to the futures price response as *RR*. With regards to expectational shocks concerning future period net consumption demand, the literature contains three views regarding *RR*. The first view, initiated by Working (1948), is that *RR* is one. This view is espoused also by the following: (a) Tomek and Gray ((1970), when one excludes cocoa); (b) Gray (1960); (c) Working ((1962), when one excludes imported commodities); (d) Weymar ((1965), when storage demanded is constant or when the marginal net cost of storage is independent of the level of storage); (e) Stein ((1979), when the storage demanded does not respond to the information from the time it is received to the time the futures contract expires); and (f) Castelino and

Francis ((1982)). The second view holds that $0 < RR < 1$. This view is held by Stein (1961) and Weymar ((1965) when the marginal net cost of carry is positively related to the quantity carried). The third view holds that $RR > 1$. This is the Milonas (1984) interpretation of Samuelson (1965).

Only one view is expressed in the literature regarding information concerning current period net consumption demand. This view holds that $RR > 1$. The only one to explicitly model this type of information is Stein (1961), (1979). Samuelson (1965) also has $RR > 1$, but the information type is unspecified in terms of fundamental economic activities. While it is not expressed in the literature that RR, with respect to information concerning current period net consumption demand is one, this view seems to be consistent with the view expressed above by Working (1948), Gray (1960), and Tomek and Gray (1970), in regards to RR when there are revisions in expectations concerning future period net consumption demand.

Thus, there are many contradictory hypotheses regarding the relative responsiveness of spot and futures prices to information which are not well-founded on a theoretical basis. The models used to address this issue do not incorporate the fact that the marginal cost of carry depends on the spot price and interest rate levels. In general, if the marginal cost of carry and the spot price level are positively (negatively) related, then RR is less (greater) than one. Therefore, it is important that the marginal cost of carry be properly specified. Previous nonmarket-based models assume spot prices follow a stationary $AR(1)$ process; while futures prices are the expected value of their respective subsequent maturity date spot prices, conditional on all currently available information. Rutledge (1976), Samuelson (1976), and others demonstrate that the propositions need not hold when the underlying spot price process is not a stationary $AR(1)$ process. Stein (1979), Anderson and Danthine (1983, B), and others demonstrate that futures prices may be biased estimators of their resultant maturity date spot prices. Thus, neither set of assumptions need hold.

On an empirical basis, the issue is unresolved as well. Empirically determining the ratio of the spot price response to the futures price response associated with a specific kind of information is a difficult and subjective matter. It rests on the *untestable* assumption that the observed price changes reflect the specific information-type tested for. Nevertheless, Working (1949), Gray (1960), Stein (1962), Weymar (1965), and Leistikow (1987) claim to have done so. These studies find $RR = 1$ in the case of wheat (Working) and coffee (Gray's conclusion about his own study) and $0 < RR < 1$ in the case of coffee (Working's conclusion regarding Gray's study), foreign exchange (Stein), cocoa (Weymar), and frozen concentrated orange juice (Leistikow), when the information is a revision in the expected level of future period net consumption demand. When the information represents a current period net consumption demand shock, Stein and Leistikow find $RR > 1$.

B. The Determinants of Futures Price Variability and Its Variability Relative to Spot Price Variability

The issue of determining the variance of spot and futures price changes is very similar to determining the response of spot and futures prices to information, except the focus of attention is upon the variances and covariances of the fundamental sources of uncertainty and the responses of the spot and futures prices to information, rather than the realized deviations of the stochastic terms from their respective means and the responses of the spot and futures prices to information.

Empirical studies of the determinants of spot and futures price change variability and relative variability (Rutledge (1976), Grauer (1976), Westerfield (1977), Miller (1979), Castelino (1981), Milonas (1984), and Anderson (1985)) are motivated by the Samuelson (1965) and the Anderson-Danthine (1983, A), denoted *A-D*, models.

Two hypotheses are generated from the Samuelson model: (1) The variance of futures price changes for a particular contract is an inverse function of the contract's time to maturity in a dynamic sense and (2) The ratio of the futures price change variance to the spot price change variance, for immediate one period price changes, is less than one and an inverse function of the futures contract's time to maturity in the dynamic and static senses.[3] However, the innovation terms need not be homoskedastic so that the maintained assumptions for the first Samuelson hypothesis need not be satisfied. This point is discussed first by Rutledge. One source of heteroskedasticity in spot prices is change in the amount of uncertainty being resolved.

Change in the amount of uncertainty being resolved and the type of uncertainty being resolved are what drive the Anderson-Danthine (1983, A), *A-D*, result.[4] In general, *A-D* find that if the bulk of the uncertainty to be resolved concerns consumption demand or if the bulk of the production supply uncertainty is resolved in period three, then the variance of the futures price change increases as the contract approaches its maturity in a dynamic sense. Alternatively, if the bulk of the uncertainty to be resolved concerns production supply and most of the production supply uncertainty is resolved in period two, then the variance of the futures price change declines as the contract dynamically approaches its maturity. The *A-D* observations that the amount of production and consumption uncertainty to be resolved and the mix of the uncertainty types to be resolved varies over time is an important contribution. However, the *A-D* model is somewhat limited.[5]

[3]Stein (1979) derived a market-based analog to the Samuelson model. Stein showed that the variance of futures price changes and the ratio of the variance of the futures price change to the variance of the spot price change are functions of the rate of change of the marginal cost of carry, the rate of change in the marginal risk-aversion, and the slope of the production supply curve minus the slope of the consumption demand curve, in addition to being inverse functions of the futures contract's time to maturity.

[4]The *A-D* model has three periods and formalizes the resolution of uncertainty in a dynamic framework. As in the Samuelson model, the futures price is the rationally expected value of its respective subsequent maturity date spot price. Production supply and consumption demand take place only in period three, the maturity date of the single futures contract. There is a futures markets in periods two and three but the spot market only exists in period three. In periods two and three production supply uncertainty is resolved. In period three consumption demand uncertainty is resolved.

[5]First, it should be noted that their distinction of information by whether the information concerns production supply or consumption demand is a somewhat misleading artifact of their model. In a more general model, uncertainty should be allowed regarding current period production, current period consumption, future period consumption, future period storage, and future period consumption. In this context it is more appropriate to state their result in the following way. If uncertainty is resolved when the futures contract is far from (close to) its maturity date then the variance of the futures price changes decreases (increases) as the contract dynamically approaches its maturity date. Thus, the *A-D* result rests on the dynamic pattern of uncertainty resolution. This is in marked contrast to the first Samuelson hypothesis. The first Samuelson hypothesis takes a stationary information structure as a maintained assumption and derives its result from a mean-reverting spot price process. Therefore, even though the *A-D* model is purported to clarify the dynamic version of the first Samuelson hypothesis, it does not address the problem in the same spirit. Second, the *A-D* model sheds little light on the issues of the relative responsiveness of spot and futures prices to information, and the relative variability of spot and futures price changes, as the only time there is a spot market is at the maturity date; Therefore, the only time for which spot and futures prices can simultaneously respond to information, is at the maturity date. As *A-D* assume the spot and futures prices are equal at the maturity date, *RR* is one at the maturity date. Similarly, *A-D* can only consider the relative variability of period three spot and futures prices. Conditional on period two information, the period three spot price variability and futures price variability are the same, due to the no basis risk assumption. Third, *A-D*'s point concerning the dynamic resolution of uncertainty being a process of nature can be extended by looking at the resolution of uncertainty as being both a process determined by nature and by whether or not the government gathers and releases data to shift some of the resolution of uncertainty from later periods to earlier periods.

The model presented in this article extends the Samuelson (1965), (1976), Stein (1979), and *A-D* (1983, A) contributions by examining the variability and relative variability of spot and futures price changes in a more general market-based model. Unlike the earlier studies, the model considers the effect of the interest rate on price variability by its incorporation into the marginal cost of carry function and allows for contemporaneous current period net consumption demand shocks and revisions to the expected level of future period net consumption demand. The model also explicitly incorporates "announcements" in a dynamic sense and can easily examine the impact of comparative static changes in the amount of uncertainty to be resolved regarding future period net consumption demand. Announcements and changes in the amount of uncertainty to be resolved regarding future period net consumption demand affect the variability and relative variability of spot and futures price changes. These factors may account for the lack of uniformity in the findings for the empirical studies of Rutledge, Grauer, Westerfield, Miller, Castelino, Anderson, and Milonas.

II. THE MODEL

A. Introduction to the Model

The model utilizes three markets: a spot market in which there is trading in periods one, two, and three for immediate delivery; a "near" futures market in which there is trading at time one, for delivery in period two; and a "far" futures market in which there is trading in periods one and two, for delivery in period three. Equilibrium is reached in each market. Each futures contract requires delivery of one unit of the underlying asset and the deliverable commodity matches the grade and location of the commodity trading in the spot market. It is assumed there are no transactions costs. It follows from the preceding assumptions that the maturity date spot and futures prices will equal each other.

There are four groups of participants in the markets: producers, inventory-carriers, futures market speculators, and consumers. Producer and consumer optimizing behavior is not derived in the article. Their production supply and consumption demand functions for the spot asset are exogenuously given. To illustrate the applicability of the model consider the major grains such as corn and soybeans. Both are planted in May and harvested in October/November. Hence, it is a short-run model. It is assumed that these producers do not participate as hedgers in the futures markets. These producers do not hedge their production for various reasons. Farmers have little down-side risk due to the government price support programs. Producers may face quantity uncertainty such that hedging is unattractive to them.[6] Also, the producer may be a government which does not concern itself with hedging its production, as in the cases of T-Bonds and currencies. As for "consumers" (or "processors"), who would be the long hedgers, for simplicity, like in other models (e.g., Stein (1979)), it is assumed that the net hedging pressure is of the type derived from the inventory-carrier's hedging pressure and the consumers' hedging positions are not explicitly modeled.

The remaining description of the market structure is introduced as needed to solve the model. The model is solved as far as is necessary to answer the questions listed at the outset of the article: (1) Is the responsiveness to information of the spot price greater than, less than, or the same as the responsiveness of the futures price?; (2) Is the variability of futures price changes an inverse function of the futures contract's time to ma-

[6]Empirically, it is found that farmers participate minimally in futures markets and when they do participate it is mainly as speculators, not as hedgers. See the Commodity Futures Trading Commission *Report on Farmer's Use of Futures Markets and Forward Contracts*, Washington, 1978.

turity in a dynamic sense?; and (3) Is the ratio of the variance of futures price changes to the variance of spot price changes an inverse function of the futures contract's time to maturity in a dynamic sense? The impact of announcements on the relative variability of spot and futures price changes is also examined. In section B, the optimization problems of the representative inventory-carrier and futures market speculator are described. In section C, three tasks are accomplished: period two and three prices are determined. All of the results concerning the relative responsiveness to information, variability, and relative variability of spot and futures prices are demonstrated. Structure sufficient to solve the entire model in terms of the exogenous variables is presented. The appendix to the article discusses conditions sufficient to ensure economically sensible results.

B. The Inventory-Carrier's and Speculator's Optimization Problems

By assumption there are n_I identical inventory-carriers and n_S identical futures market speculators in the model. The representative inventory-carrier and futures market speculator are perfect competitors. Each is assumed to maximize a linear function of the mean and variance of third period wealth. This mean-variance framework assumes that either the market participant has constant absolute risk-aversion or has preferences that are defined only on the mean and variance of terminal wealth. That the CAPM is not applicable is justified by assuming that the speculator and inventory-carrier are endowed with an aptitude or production process that involves taking large nondiversified positions in the futures and spot markets, respectively, and are either unwilling or unable to sell equity shares in their production processes.

The inventory-carrier's period three wealth is assumed to be

$$W_{3I} = W_{1I}R^2 + RS_1\left[P_2 - RP_1 + b - \frac{c}{2}S_1\right] + S_2\left[P_3 - RP_2 + b - \frac{c}{2}S_2\right] - RX_{21I}[P_2 - F_{21}] - RX_{31I}[F_{32} - F_{31}] - X_{32I}[P_3 - F_{32}] \quad (1)$$

where:

W_{1I} is the inventory-carrier's wealth at time one. It is assumed that W_{1I} is sufficiently large such that wealth is positive for each period.

R is one plus the one period riskless interest rate.

F_{ji} is the price of the futures contract which requires delivery in period j, observed in period i.

P_i is the period i spot price. Recall $P_i = F_{ii}$ from the no basis risk assumption.

S_i is the individual inventory-carrier's carry-out from period i. It is chosen by the inventory-carrier for $i = 1, 2$.

b is a positive constant and represents the negative of the intercept of the noninterest component of the marginal cost of carry function.

c is a positive constant reflecting increasing marginal costs of carry.

The $S_i[P_{i+1} - RP_i + b - \frac{c}{2}S_i]$ term represents the profit on inventory purchased in period i and carried to period $i + 1$, with S_iP_{i+1} representing the revenue from the carrying, S_iP_i representing the cost of purchasing the inventory and $S_i[(R - 1)P_i - b + \frac{c}{2}S_i]$ representing the cost of carrying the inventory from period i to period $i + 1$. The period i one period marginal cost of carry with respect to the quantity carried is therefore $(R - 1)P_i - b + cS_i$.

X_{jiI} is the short position in futures contracts which require delivery in period j, taken in period i by the inventory-carrier.

$X_{jiI}[F_{j(i+1)} - F_{ji}]$ represents the loss on a short position in futures contract j taken in period i and held to period $i + 1$.

It should be noted that in period i, the prices and positions observed in future periods are stochastic.

The inventory-carrier's problem consists of choosing S_1, X_{21I}, and X_{31I} at time one and S_2 and X_{32I} at time two. The optimal choices at times one and two are interdependent. Formally, the inventory-carrier's problem is to:

(At time two):

$$\max_{X_{32I}, S_2} E_2 W_{3I} - \frac{\alpha_I}{2} \mathrm{Var}_2(W_{3I}) \tag{2}$$

E_i is the expectations operator conditional on period i information,
α_I is the constant absolute risk-aversion of the inventory-carrier, and
$\mathrm{Var}_i(\cdot)$ is the variance $(\cdot)$ conditional on the information available at time i.

(At time one):

given X_{21I}, X_{31I}, and S_1,
where:

$$\max_{X_{21I}, X_{31I}, S_1} E_1 W_{3I} - \frac{\alpha_I}{2} \mathrm{Var}_1(W_{3I}) \tag{3}$$

given that X_{32I} and S_2 solve (2). The inventory-carrier first solves the time two problem. As stated earlier, the market-participants are perfect competitors. Hence, in their time two maximization, period two prices are taken as deterministic. The first order conditions of this well-known maximization at time two are:

$$E_2 P_3 - RP_2 + b - cS_2 = \alpha_I [S_2 - X_{32I}] \mathrm{Var}_2(P_3). \tag{4}$$

and

$$-E_2 P_3 + F_{32} = \alpha_I [X_{32I} - S_2] \mathrm{Var}_2(P_3). \tag{5}$$

As conditional on period two information, the period two prices are taken as deterministic, $\mathrm{Var}_2(P_3) = \mathrm{Var}_2(P_3 - P_2) = \mathrm{Var}_2(P_3 - F_{32}) = \mathrm{Cov}_2(P_3 - P_2, P_3 - F_{32})$. To save space, only the stochastic part of the arguments for $\mathrm{Cov}_i(\cdot, \cdot)$ and $\mathrm{Var}_i(\cdot)$ are included. As the second order conditions are satisfied, a solution satisfying (4) and (5) will satisfy (3). Conditions are established in the appendix such that an interior solution to (4) and (5) is achieved. Analogs to (4) and (5) are derived in many earlier articles (e.g., Stein (1985)). In (4), the inventory-carrier equates the expected marginal revenue from carrying stock out of period two with the marginal cost of purchasing and carrying the stock out of period two plus an endogenous risk premium. In (5), the inventory-carrier equates the marginal profits in the futures market with an endogenous risk-premium.[7]

For convenience, it is assumed that the futures market speculator does not participate in the spot market. Hence, the speculator's wealth and optimization problems are the same as the inventory-carrier's; except, the "I" subscripts are replaced by "S" and

[7]The i period futures market risk premium is defined as the expected futures price in i periods minus the current futures price, utilizing all currently available information. The i period spot market risk premium is defined as the expected spot price in i periods minus the expected i period cost of carry minus the current spot price, conditional on all currently available information.

$S_S = 0$. Analogous to (5), the time two first order condition for the representative futures market speculator is:

$$-E_2 P_3 + F_{32} - \alpha_S X_{32S} \operatorname{Var}_2(P_3) = 0 . \tag{6}$$

The speculator equates the expected marginal net revenue with an endogenous risk premium.

C. Specification and Solution of the Remainder of the Model

The time three equilibrium is reached when the consumption demanded plus inventory carried-out equal the production supplied plus the inventory carried-in. The time three consumption demand schedule is given as

$$C_3 = \overline{C}_3 - aP_3 + \phi_{C32} + \phi_{C33} . \tag{7}$$

Time three consumption demand has intercept $\overline{C}_3 + \phi_{C32} + \phi_{C33}$, where $\overline{C}_3$ is deterministic and positive and the ϕ_{C3i} shocks are realized in period three and learned about in period i, where $i = 1, 2$. Consumption demand is a downward sloping function of price as $a > 0$.

The time three production supply less storage demand is exogenously determined as

$$Q_3 = \overline{Q}_3 - \overline{S}_3 + \phi_{Q32} + \phi_{Q33} - \phi_{S32} - \phi_{S33} . \tag{8}$$

It is the sum of a deterministic component $(\overline{Q}_3 - \overline{S}_3)$ and shocks $\phi_{Q3i} - \phi_{S3i}$ which are learned in period i.[8]

There are n_I inventory-carriers, each bringing in stock S_2. With these definitions of consumption demand and production less storage demand, the time three market equilibrium is described by (9).

$$A_3 - aP_3 + \phi_{32} + \phi_{33} = n_I S_2 \tag{9}$$

where:

$A_3 = \overline{C}_3 - \overline{Q}_3 + \overline{S}_3$ and

ϕ_{3i} are consumption demand plus storage demand less production supply shocks realized in period three and revealed in period i, defined as $\phi_{C3i} + \phi_{S3i} - \phi_{Q3i}$. For brevity, they are referred to as "net consumption demand" rather than "consumption demand plus storage demand less production supply" shocks.[9]

The distribution of the fundamental $\phi_{\_3i}$ shock terms (where _ can be replaced by C, S, or Q) are impacted by periodic governmental announcements. The announcements are discussed first below followed by a discussion of how the announcements impact on the distribution of the $\phi_{\_3i}$ shock terms.

[8]It should be noted that the results of the model would not be substantively altered if period three production supply less storage demand were endogenously determined as a linearly increasing function of the current period price. It is also noted that the results would also hold under similar conditions, if the period three production supply less storage demand has been modeled as $Q_3 = \overline{Q}_3[1 + \phi_{Q32} + \phi_{Q33}] - \overline{S}_3[1 + \phi_{S32} + \phi_{S33}]$, the period two production as $Q_2 = \overline{Q}_2[1 + \phi_{Q22}]$, period three consumption demand as $C_3 = \overline{C}_3[1 + \phi_{C32} + \phi_{C33}] - aP_3$, and period two consumption demand as $C_2 = \overline{C}_2[1 + \phi_{C22}] - aP_2$.

[9]However, if the shocks are modeled as multiplicative shocks, rather than as additive shocks, then the information must be referred to by specifying the period it refers to and by whether it concerns production supply, consumption demand, or storage demand. In the case of multiplicative shocks, the period to price response can differ for information concerning period three due to differences between $\overline{Q}_3$, $\overline{C}_3$, $\overline{S}_3$. Similarly, the period two price response can differ for information concerning period two due to differences between $\overline{Q}_2$ and $\overline{C}_2$.

The government maintains a list of announcement dates and times prepared well in advance (over a year in some cases) of the actual announcements. It is known that certain statistics will be provided to the public on these dates at the specified times. Examples of such announcements made by the U.S. Government include: GNP, Merchandise Trade, Quarterly Refunding Operations, Housing Starts, and Crop Production Reports. In many cases the law requires private businesses to report their activities to the government. The government then aggregates this information under close security and releases it to the entire public on these prespecified dates. While there are economic incentives for the private sector to gather much of the same data as that contained in the government announcements, the costs of doing so are prohibitive. In practice, the private sector takes a smaller sample than the sample taken by the government. Hence, by expanding the information set regarding future period net consumption demand, government announcements may change the expected value of the future period net consumption demand. Define $\phi_{C32} = \phi^P_{C32} + \phi^G_{C32}$, $\phi_{Q32} = \phi^P_{Q32} + \phi^G_{Q32}$, and $\phi_{S32} = \phi^P_{S32} + \phi^G_{S32}$. The ϕ^P_{32} shocks represent expectational revisions on the basis of the private sector's information set, while the ϕ^G_{32} shocks represent expectational revisions on the basis of the government announcement which are uncorrelated with the revisions made on the basis of the private sector's information.

Also by expanding the information set regarding future period net consumption demand, government announcements impact the variance of the ϕ_{3i} terms. Government announcements shift the resolution of uncertainty from later periods to earlier periods. At time one, the uncertainty regarding period three net consumption demand is $\mathrm{Var}_1(\phi_{32}) + \mathrm{Var}_1(\phi_{33}) = (Y + Z)\ \mathrm{Var}\ \phi$. In period two, $\mathrm{Var}_1(\phi_{32}) = (Z + G)\ \mathrm{Var}\ \phi$ of the uncertainty regarding period three net consumption demand is eliminated, whereby the remaining $\mathrm{Var}_2(\phi_{33}) = (Y - G)\ \mathrm{Var}\ \phi$ is eliminated in period three.[10] Note that Y, Z, and G are constants, where $Y > G \geq$ and $Z > 0$. A comparative static rise in G reflects the effect of a government announcement in period two regarding period three net consumption demand on the amount of uncertainty resolved in periods two and three, regarding period three net consumption demand. Government announcements have two effects. They increase the amount of uncertainty resolved in period two and they diminish the amount of uncertainty resolved in period three. Ignoring the announcement considerations, this treatment of the terminal condition as reflected in (7) and (8) is consistent with that given in Anderson-Danthine (B).

From (9) and the definition of $\mathrm{Var}_2(\phi_{33})$, the variance of the period three spot and futures prices, conditional on period two information is derived as

$$\mathrm{Var}_2(P_3) = \frac{(Y - G)\ \mathrm{Var}\ \phi}{a^2}. \tag{10}$$

The variance of the time three spot (or futures) price (or price change), conditional on time two information depends positively upon the variance of the time three net consumption demand shock and negatively upon the absolute value of the slope of the consumption demand function (The more elastic the consumption demand the less the spot price must change to absorb the shock.).

The time two spot market equilibrium is determined where the consumption demanded plus storage demanded equal production plus carry-in. Similar to that in time three, the time two consumption demand is a stochastic inverse function of the current

[10]However, if the shocks are modeled as multiplicative shocks, rather than additive shocks, then the ϕ, Y, Z, and G must be replaced by ϕ_-, Y_-, Z_-, and G_-, respectively, whenever they appear in the preceding two sentences. In each case, two $_-$ is replaced with C, Q, or S throughout.

period spot price, given as

$$C_2 = \overline{C}_2 - aP_2 + \phi_{C22}. \tag{11}$$

Time two production is given as

$$Q_2 = \overline{Q}_2 + \phi_{Q22}. \tag{12}$$

Period two production is the sum of a deterministic component, $\overline{Q}_2$, plus a stochastic component ϕ_{Q22}, which is revealed in period two.

As there are n_I inventory-carriers in period one as well as in period two, the time two market-clearing condition is given as (13).

$$A_2 - aP_2 + \phi_{22} + n_I S_2 = n_I S_1, \tag{13}$$

where: $A_2 = \overline{C}_2 - \overline{Q}_2$, $\phi_{22} = \phi_{C22} - \phi_{Q22}$, while $E_1\phi_{22} = 0$ and $\mathrm{Var}_1(\phi_{22}) = \mathrm{Var}\ \phi$.

Throughout the article, all the fundamental shock terms are assumed to be independently and symmetrically distributed about zero with finite variances.

As noted above at time two there are current period net consumption demand shocks and expectational revisions concerning period three net consumption demand with variances of Var ϕ and $(Z + G)$ Var ϕ, respectively. While the two different types of information enter randomly and independently, the ratio of their variances is known. The ratio of the variance of expectational shocks concerning period three net consumption demand to the variance of period two net consumption demand shocks, where each is learned at time two, is $(Z + G)$; $(Z + G)$ could be high as a consequence of nature and/or activities of the government.

The informational shocks regarding period three fundamental activities are transmitted to period two prices via the period two storage demand. From (4), (5), (6), (9), (10), and (13) the inventory-carrier's period two storage demand is derived as (14).

$$S_2 = \frac{[A_3 + \phi_{32}] - R[A_2 - n_I S_1 + \phi_{22}] + ba}{\chi} \tag{14}$$

where:

$$\chi = (R + 1)n_I + ca + a\gamma n_I, \qquad \gamma = \frac{\theta(Y - G)\ \mathrm{Var}\ \phi}{a^2}, \quad \text{and} \quad \theta = \frac{\alpha_I \alpha_S}{n_S \alpha_I + n_I \alpha_S}.$$

The "θ" term may be interpreted as the "market-weighted risk-aversion;" it rises with α_I and α_S, while it declines with n_I and n_S.

The inventory-carrier's carry-out from time two is positively related to the following five variables: the deterministic (at time one) component of the aggregate carry-out from time three, period two upward revisions in the expected period three net consumption demand, the deterministic (at time one) component of production in period two, the aggregate carry-in from time one, and the absolute value of the intercept of the noninterest component of the marginal cost of carry. It is negatively related to the following variables: period two net consumption shocks, the deterministic (at time one) component of period three production, the risk-aversion of the market participants and the variance of the marginal profits for the speculator, as reflected in χ and γ, the rate of change in the marginal cost of carry with respect to the quantity carried, as reflected in χ and c, and the interest rate.

Substituting (14) into (13) and rearranging gives the period two spot price.

$$P_2 = \frac{n_I[A_3 + \phi_{32}] + (n_I + ca + a\gamma n_I)[A_2 - n_I S_1 + \phi_{22}] + ba}{a\chi} \tag{15}$$

The second period far futures price is determined from (4), (5), (14), and (15) as (16).

$$F_{32} = \frac{(ac + Rn_I)[A_3 + \phi_{32}] + R(n_I + a\gamma n_I)[A_2 - n_I S_1 + \phi_{22}] - bn_I a(1 + a\gamma)}{a\chi}. \tag{16}$$

The time two spot and far futures prices are both positively related to the following: the deterministic (at time one) component of consumption demand in periods two and three, the deterministic (at time one) component of the aggregate carry-out from time three, upward revisions in the expected period three net consumption demand, and positive period two net consumption demand shocks. The time two spot and far futures prices are both negatively related to the deterministic (at time one) components of periods two and three production and the carry-in for period two. While the spot price is positively related to the absolute value of the intercept of the noninterest component of the marginal cost of carry, the far futures price is negatively related to the absolute value of the intercept of the noninterest component of the marginal cost of carry. Finally, a rise in the interest rate will decrease the spot price but will have an ambiguous effect on the far futures price.

Result 1: The ratio of the spot price response to the futures price response associated with current period net consumption demand shocks is $\gtreqless 1$ depending upon macro and market-specific variables.

This result can be demonstrated by considering the period two spot and far futures prices given in (15) and (16), respectively. The ratio of responses associated with current period net consumption demand shocks is defined as: $(\partial P_2/\partial \phi_{22})/(\partial F_{32}/\partial \phi_{22})$. By inspection, the ratio of the spot price response to the futures price response associated with current period net consumption demand information $\gtreqless 1$ is equivalent to $c \gtreqless (R - 1)n_I(1 + a\gamma)/a$. When the rate of change in the marginal cost of carry is sufficiently large, consumption demand is sufficiently elastic or the interest rate, risk, and risk-aversion are sufficiently small, the spot prices will respond more than the futures prices to current period net consumption demand information. The view prevailing in the literature, that the response of spot prices exceeds the response of futures prices when the information concerns current period net consumption demand, is a special case. *Unlike any of the previously expressed views in the literature, the spot price may respond less than the futures price when the information concerns current period net consumption demand.* Note: In the case of multiplicative disturbances, the production and consumption shocks must be considered separately (i.e., $(\partial P_2/\partial \phi_{Q22})/(\partial F_{32}/\phi_{Q22})$ or $(\partial P_2/\partial \phi_{C22})/(\partial F_{32}/\partial \phi_{C22})$ must be examined). This note applies to result three as well.

Result 2: The ratio of the spot price response to the futures price response, associated with expectational shocks concerning future period net consumption demand is less than one.

Using the period two spot and far futures prices, the ratio of the spot price response to the futures price response associated with expectational shocks concerning future period net consumption demand is defined as $(\partial P_2/\partial \phi_{32})/(\partial F_{32}/\partial \phi_{32})$. By inspection of (15) and (16), this ratio of price responses is $n_I/(ac + Rn_I)$, which is less than one. *The view that the ratio of the spot price response to the futures price response, associated with expectational shocks concerning future period net consumption demand is greater than or equal to one is inconsistent with this model.* In the case of multiplicative disturbances, the production, storage, and consumption shocks must be considered separately (i.e., $(\partial P_2/\partial \phi_{Q32})/(\partial F_{32}/\partial \phi_{Q32})$, $(\partial P_2/\partial \phi_{S32})/(\partial F_{32}/\partial \phi_{S32})$ or $(\partial P_2/\partial \phi_{C32})/(\partial F_{32}/\partial \phi_{C32})$ must be examined). This note applies to result four as well.

Result 3: In the case of information concerning current period net consumption demand, depending on macro and market-specific variables, the ratio of the spot price response to the futures price response either increases, decreases, or stays the same as the contract approaches its maturity date in a dynamic sense.

This result follows from result one and the fact that at time three, the response ratio is one due to the no basis risk condition. In contrast to Samuelson's model, with regards to this type of information, the riskiness of the futures contract relative to the spot (as measured by the relative price response) need not rise as the contract approaches its maturity in a dynamic sense.

Result 4: In the case of information concerning the maturity date net consumption demand, the ratio of the spot price response to the futures price response increases as the futures contract approaches its maturity date in a dynamic sense.

This result follows from result two and the fact that at time three, the response ratio is one due to the no basis risk condition. With regards to this type of information, this model implies the opposite of the Samuelson result, i.e., the riskiness of the futures contract relative to the spot (as measured by the relative price response) declines as the contract approaches its maturity in a dynamic sense. Now consider the variance of immediate one-period price changes.

The variance of the period two spot (or near futures) price is determined from (15), the definition of $\mathrm{Var}_1(\phi_{32})$, the definition of $\mathrm{Var}_1(\phi_{22})$ and the independence of ϕ_{22} and ϕ_{32} as

$$\mathrm{Var}_1(P_2) = \frac{[(n_l + ca + a\gamma n_l)^2 + n_l^2(Z + G)]\ \mathrm{Var}\ \phi}{a^2\chi^2}. \tag{15'}$$

The variance of the second period spot price, conditional on time one information depends upon the following: the absolute value of the slope of the consumption demand function, the slope of the marginal cost of carry function, the interest rate, the market-weighted risk-aversion, the variance of current period net consumption shocks for periods two and three, and the ratio of the amount of uncertainty to be resolved concerning period three net consumption demand to the amount of uncertainty to be resolved concerning period two net consumption demand in period two. The variance of spot prices should be nonstationary. In a comparative static sense, it should be high over periods when there are announcements or periods when Z is high. For the result to hold with respect to announcements, it is assumed that $2\theta\ \mathrm{Var}\ \phi[R(\chi - Rn_l) - n_l(Z + G)] < a\chi$. Hence, if $\theta\ \mathrm{Var}\ \phi$ is small, the spot price variability is greater over announcement periods than it is over control periods. When this term is small, the downward pressure on period two spot price variability due to the diminished uncertainty to be resolved in period three is dominated by the upward pressure from the greater resolution of uncertainty in period two.

The variance of the second period far futures price is derived from (16), the definition of $\mathrm{Var}_1(\phi_{32})$, the definition of $\mathrm{Var}_1(\phi_{22})$ and the independence of ϕ_{22} and ϕ_{32} as (16′).

$$\mathrm{Var}_1(F_{32}) = \frac{[R^2n_l^2(1 + a\gamma)^2 + (Rn_l + ac)^2(Z + G)]\ \mathrm{Var}\ \phi}{a^2\chi^2} \tag{16'}$$

The variance of the period two far futures price is, in a comparative static sense, relatively high over announcement periods and periods when Z is high. This statement holds with respect to announcements, when $2n_l^2\theta\ \mathrm{Var}\ \phi[R^2n_l(1 + a\gamma) - (Rn_l + ca) \times (Z + G)] < (Rn_l + ca)\chi$. The rationale for this condition is analogous to the corresponding statement regarding the period two spot price.

Result 5: Depending on macro and market-specific variables, the variance of the futures price either increases, decreases, or stays the same as the futures contract approaches its maturity in a dynamic sense.

This is analogous to the *A-D* version of the Samuelson hypothesis. From (10) and (16), $\text{Var}_2(P_3) \gtreqless \text{Var}_1(F_{32})$ is equivalent to $\chi^2(Y - G) \gtreqless R^2 n_l^2(1 + a\gamma)^2 + (Rn_l + ac)^2(Z + G)$. If *Y-G* is small, the variance of the futures price declines as the contract dynamically approaches its maturity date. If *R*-1, *Z*, and *G* are small, while *Y* is large, the variance of the futures price rises as the contract dynamically approaches its maturity date.

Result 6: Even if it is assumed that the variance of the spot price is constant, depending on macro and market-specific variables, the variance of the futures price need not increase as the futures contract approaches its maturity in a dynamic sense.

Assuming $\text{Var}_1(P_2) = \text{Var}_2(P_3)$, $\text{Var}_2(P_3) \gtreqless \text{Var}_1(F_{32})$ is equivalent to $(\chi - Rn_l)^2 \gtreqless R^2 n_l^2(1 + a\gamma)^2 + [(Rn_l + ac)^2 - n_l^2](Z + G)$. If *c* is small, the model implies the opposite of the Samuelson result. When $(Z + G)$ and (*R*-1) are small, the model implies the Samuelson result. This is a stronger version of result 5.

Result 7: Depending on the specification of macro and market-specific variables, the ratio of the variance of immediate one period futures price changes to the variance of immediate one period spot price changes are either greater than, less than, or equal to one and need not be an inverse function of the futures contract's time to maturity in a comparative static or dynamic sense.

Result seven follows from result six and the fact that $\text{Var}_1(F_{22}) = \text{Var}_1(P_2)$.

Result 8: The ratio of the variance of the futures price change to the variance of the spot price change, for immediate one-period changes, is high in a comparative static sense over announcement periods and periods when *Z* is high.

To demonstrate the result with respect to announcements partially differentiate $\text{Var}_1(F_{32})/\text{Var}_1(P_2)$ with respect to *G*. This result assumes $2n_l\theta$ Var $\phi\{(\chi - Rn_l)[R^2n_l^2(1 + a\gamma)ca - (Rn_l + ca)^2(Z + G)] + (Z + G)R^2n_l^3(1 + a\gamma)\} < a[(Rn_l + ca)^2(\chi - Rn_l)^2 - R^2n_l^4(1 + a\gamma)]$. The result with respect to *Z*, can be demonstrated using (15′) and (16′) and comparing $[\text{Var}_1(F_{32})/\text{Var}_1(P_2)]|_{Z=Z_H}$ with $[\text{Var}_1(F_{32})/\text{Var}_1(P_2)]|_{Z=Z_N}$ where: "*H*" stands for High, "*N*" stands for Normal, and $Z_H > Z_N$. Changing uncertainty regarding future period net consumption demand and announcement considerations may account for why this version of the Samuelson hypothesis, for some commodities, is rejected in some time periods and is accepted in other time periods.

The model is completed by specifying the consumption demand, production supply, and carry-in at time one. The spot market at time one is in equilibrium when the consumption demanded plus storage demand equal the carry-in plus the production supplied. The time one consumption demand is of the same form as in the other periods and is given as (17).

$$C_1 = \overline{C}_1 - aP_1 \tag{17}$$

The time one carry-in is denoted as $\overline{S}_0$, while the production is denoted as $\overline{Q}_1$. Hence, the time one spot market equilibrium is given as (18).

$$A_1 - aP_1 + n_l S_1 = 0 \tag{18}$$

where: $A_1 = \overline{C}_1 - \overline{S}_0 - \overline{Q}_1$.

Having specified the model completely, the period one prices and quantities can be determined in reduced form. Using these values, reduced forms for the period two and three prices and quantities can be determined also. The appendix contains sufficient conditions to ensure economically sensible results.

III. SUMMARY AND IMPLICATIONS

This article theoretically addresses the following questions: (1) Is the response to information of the spot price greater than, less than, or the same as that of the futures price?; (2) Is the variance of futures price changes a function of the contract's time to maturity in the dynamic sense?; and (3) Is the ratio of the variance of futures price changes to the variance of spot price changes as a function of the futures contract's time to maturity in the dynamic and/or static senses?

These issues are examined in the context of a three-period rational expectations model. The model incorporates the following aspects (not previously employed in the analyses of these issues): The model allows trading in spot, near futures, and far futures markets. The model uses the marginal cost of carry as a function of the spot price and interest rate levels. The model allows for changes in the relative amount of uncertainty regarding current and future period net consumption demand. The model incorporates announcements in a dynamic framework and determines the variance of one period spot price changes.

The major new findings of the model include the following: Depending upon macro and market-specific variables, the response of the spot price may be less than that for the futures price, with regards to current period net consumption demand information. The response of the spot price is less than that of the futures price, with regards to information concerning the expected level of future period net consumption demand. In the case of information concerning current period net consumption demand, depending on the specification of macro and market-specific variables, the ratio of the spot price response to the futures price response either increases, decreases, or stays the same as the contract approaches its maturity date in a dynamic sense. In the case of information concerning the maturity date net consumption demand, the ratio of the spot price response to the futures price response increases as the futures contract approaches its maturity date in a dynamic sense. Depending on macro and market-specific variables, the ratio of the variance of the futures price change to the variance of the spot price change is either greater than, less than, or equal to one. The ratio of the variance of the futures price change to the variance of the spot price change is higher over announcement periods than it is over control periods. The ratio of the variance of the futures price change to the variance of the spot price change is higher over periods when there is a relatively large amount of uncertainty being resolved regarding future period net consumption demand than it is over control periods.

It should be noted that the results of the model are contrasted mainly with those of the Samuelson (1965), (1976), and Stein (1979) models. The model presented in this paper is a short-run model. It is unclear whether the Samuelson and Stein models should be interpreted as long-run or short-run models. In fairness to Samuelson and Stein, they may have intended to have their results applied in a long-run context. In either case, it is useful that the contrast be made.

There are many implications for this study. When selecting their hedging positions, hedgers need to consider both the type of uncertainty to be resolved and macro and market-specific variables. For example, the hedging position will likely vary across grain and precious metals markets due to differences in the mixture of the types of uncertainty to be resolved and the size of the rate of change in the marginal cost of carry. It might be expected that the variability of the futures price relative to the variability of the spot price to be greater in the precious metals markets than it is in the grain markets for two reasons. It might be argued that relatively more of the uncertainty being resolved concerns future, rather than current, period net consumption demand for the precious metals markets than for the grain markets. This statement is supported by the fact that the precious metals markets are often distinguished as "expectations" markets. It might also be argued that the size of the rate of change in the marginal cost of carry is lower in the precious metals markets than it is in the grain markets due to spoilage and limited warehousing space considerations. The hedging position selected varies in a dynamic sense as the mix of uncertainty types to be resolved and response ratios varies in a dynamic sense. Leistikow (1989) presents some preliminary work on this issue focussing on the grain, currency, and debt markets for a limited set of announcements. Futures market speculators need to consider the type of uncertainty to be resolved as well as macro and market-specific variables when choosing their positions in the futures markets. Spot and futures risk premia vary over time reflecting time-varying risk. The efficient estimation of futures price change variability and the relative variability of spot and futures price changes, require adjustments for nonstationarity. The nonstationarity arises across (a) announcement and control periods and (b) periods where there are differences in the amount of uncertainty to be resolved regarding future period net consumption demand not due to the presence or absence of announcements. Regulators should consider varying futures margins and price limits in accordance with the discussion regarding futures price variability. Spot and futures option pricing models should incorporate price change heteroskedasticity across (a) and (b) above.

Appendix

The following assumptions are sufficient for prices, expected prices, storage, expected storage, consumption, and expected consumption to be positive in each period. The one-period marginal cost plus risk premium associated with time one purchase and storage when graphed as a function of S_1, is linear and upward sloping when assumption (A1) holds.

$$a\gamma < 1 \tag{A1}$$

Assumptions (A2) and (A3) require the intercept of the one-period marginal cost plus risk premium associated with time one purchase and storage to be positive; this ensures that the marginal cost plus risk premium associated with time one purchase and storage is positive for positive levels of time one storage.

$$A_3 - RA_2 > 0 \tag{A2}$$

$$(Z + G) > \frac{R(n_I + ca + a\gamma n_I)}{n_I} + \frac{ba^2\chi^3 - A_1 Ra\chi^3}{(A_3 - RA_2)n_I^2\theta \operatorname{Var} \phi(ca + 2a\gamma n_I)} \tag{A3}$$

When graphed as a function of S_1, E_1P_2 is linear and downward sloping. Assumption (A4) requires the intercept of E_1P_2 to be above the intercept associated with the marginal cost of purchasing and carrying the asset for one-period; this assumption when coupled with (A2) and (A3) ensures E_1P_2, S_1, and $E_1S_2 > 0$.

$$(Z + G) < \frac{R(n_I + ca + a\gamma n_I)}{n_I} + \frac{ba^2\chi^3 - A_1Ra\chi^3 + A_3an_I\chi^2 + A_2a\chi^2(n_I + ca + a\gamma n_I)}{(A_3 - RA_2)n_I^2\theta \operatorname{Var} \phi(ca + 2a\gamma n_I)} \tag{A4}$$

Positive period one consumption is ensured by assuming that the marginal cost of purchasing and carrying the stock plus the risk premium for carrying the stock exceeds the expected marginal revenue if all the production plus carry-in for period one were carried out; this is established by (A5).

$$
\begin{aligned}
& ba^2\chi^3 - \overline{C}_1 Ra\chi^3 + (A_3 - RA_2)n_l\theta \operatorname{Var}\phi(ca + a\gamma n_l) \\
&\quad \cdot [R(n_l + ca + a\gamma n_l) - n_l(Z + G)] \\
&\quad + A_3 n_l a\chi^2 + A_2(n_l + ca + a\gamma n_l)a\chi^2 \\
&\quad < ([\overline{Q}_1 + \overline{S}_0]/n_l)\{an_l(n_l + ca + a\gamma n_l)\chi^2 + \chi^3[aRn_l + ca^2]\} \\
&\quad + ([\overline{Q}_1 + \overline{S}_0]/n_l)\theta \operatorname{Var}\phi n_l^2(Z + G)[\chi + ca + 2a\gamma n_l] \\
&\quad + ([\overline{Q}_1 + \overline{S}_0]/n_l)\theta \operatorname{Var}\phi(\chi - Rn_l)[(\chi - Rn_l)^2 + Rn_l^2(1 - a\gamma)] \qquad \text{(A5)}
\end{aligned}
$$

When graphed as a function of S_1, P_1 is linearly increasing. A sufficient condition for $P_1 > 0$ is given in (A6).

$$A_1 > 0 \qquad \text{(A6)}$$

The first period near futures price, period two far futures price and period one far futures price are all linearly decreasing functions of time one storage. Sufficient conditions for them to be positive are (A7), (A8), and (A9), respectively, where S_1 is given by (19).

$$
\begin{aligned}
S_1 = {} & \frac{A_3\{n_l a\chi^2 - n_l\theta \operatorname{Var}\phi(ca + 2a\gamma n_l)[n_l(Z + G) - R(n_l + ca + a\gamma n_l)]\}}{\lambda} \\
& + \frac{A_2\{(n_l + ca + a\gamma n_l)a\chi^2 + Rn_l\theta \operatorname{Var}\phi(ca + 2a\gamma n_l)[n_l(Z + G) - R(n_l + ca + a\gamma n_l)]\}}{\lambda} \\
& + \frac{ba^2\chi^3 - A_1 Ra\chi^3}{\lambda} \qquad \text{(19)}
\end{aligned}
$$

where:

$$
\begin{aligned}
\lambda = {} & a\chi^2[n_l(n_l + ca + a\gamma n_l) + (Rn_l + ca)\chi] + \theta n_l R \operatorname{Var}\phi \\
& \cdot \{(n_l + ca + a\gamma n_l)[Rn_l^2(1 - a\gamma) + (n_l + ca + a\gamma n_l)^2] + (Z + G)n_l^2\chi\}
\end{aligned}
$$

$$
\begin{aligned}
& a\chi^2[A_2(n_l + ca + a\gamma n_l) + A_3 n_l a] + (A_3 - RA_2)n_l\theta \operatorname{Var}\phi(ca + 2a\gamma n_l) \\
& \cdot [R(n_l + ca + a\gamma n_l) - (Z + G)n_l] > S_1 n_l a\chi^2(n_l + ca + a\gamma n_l) \\
& + S_1 n_l\theta R \operatorname{Var}\phi\{[\chi - Rn_l][Rn_l^2(1 - a\gamma) + (\chi - Rn_l)^2] \\
& \qquad + (Z + G)n_l^2(\chi + ca + 2a\gamma n_l)\} \qquad \text{(A7)}
\end{aligned}
$$

$$(ac + Rn_l)[A_3 + \phi_{32}] + Rn_l(1 + a\gamma)[A_2 + \phi_{22}] > bn_l a(1 + a\gamma) + Rn_l^2(1 + a\gamma)S_1 \qquad \text{(A8)}$$

$$
\begin{aligned}
& (A_3 - RA_2)n_l\theta \operatorname{Var}\phi(ca + 2a\gamma n_l)[R^2 n_l(1 + a\gamma) - (Rn_l + ca)(Z + G)] \\
& \quad + a\chi^2[A_3(Rn_l + ca) + A_2 n_l R(1 + a\gamma)] > S_1(1 + a\gamma)n_l^2 Ra\chi^2 \\
& \quad + \theta n_l R \operatorname{Var}\phi\{Rn_l(1 + a\gamma)[Rn_l^2(1 - a\gamma) + (n_l + ca + a\gamma n_l)^2] \\
& \qquad + (Z + G)n_l(Rn_l + ca)[\chi + ca + 2a\gamma n_l]\}. \qquad \text{(A9)}
\end{aligned}
$$

The expected period two far futures and period three spot prices, each conditional on time one information, are positive by (A8), when the ϕ terms are zero.

Period two consumption is positive by (A10).

$$S_1(n_l + ca + a\gamma n_l) + [\chi(\overline{Q}_2 + \phi_{22})/n_l] > A_3 + \phi_{32} - R[A_2 + \phi_{22}] + ba \tag{A10}$$

Expected period two consumption, given period one information, is positive by (A10), where the ϕ terms are zero.

Period three consumption is positive by (A11).

$$\overline{Q}_3 + \phi_{Q32} + \phi_{Q33} - \overline{S}_3 - \phi_{S32} - \phi_{S33} > 0 \tag{A11}$$

Expected period three consumption, conditional on time one (two) information is positive by (A11) when the ϕ_{33} and ϕ_{32} (ϕ_{33}) terms are zero.

A sufficient condition for the period two spot price to be positive is

$$n_l[A_3 + \phi_{32}] + (n_l + ca + a\gamma n_l)[A_2 + \phi_{22}] + ba > (n_l + ca + a\gamma n_l)n_l S_1 . \tag{A12}$$

Period two storage is positive when (A13) is satisfied.

$$A_3 + \phi_{32} - R[A_2 - n_l S_1 + \phi_{22}] + ba > 0 . \tag{A13}$$

A sufficient condition a positive period three spot price is given by (A14).

$$(\chi - n_l)[A_3 + \phi_{32}] + Rn_l[A_2 + \phi_{22}] > n_l ba + Rn_l^2 S_1 - \chi\phi_{33} \tag{A14}$$

The expected period three spot price, conditional on period two information is positive by (A14) when the ϕ_{33} terms are zero.

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