Premiums on Stock Index Futures-Some Evidence

Swati Bhatt
Nusret Cakici

I. INTRODUCTION

Since the introduction of stock index futures (SIF) contracts in April 1982, researchers have been interested in the relationship between the index futures price and the underlying index. The question asked is whether arbitrage opportunities exist in these markets. Is the futures contract priced such that traders can buy/sell the futures contract and take an opposite position in the underlying stocks to make a riskless return that exceeds the risk-free rate of return?

The size of transaction costs determines whether potential arbitrage opportunities are profitable. Transactions costs include brokerage commissions and market impact costs of trading. While the former is relatively easy to measure, the latter varies by the characteristics of the particular arbitrageur (e.g., visibility). The issue of measuring transaction costs is considered here by posing the question slightly differently. The mispricing series gives the maximum level of transaction costs that would prevent profitable arbitrage. The size and direction of the mispricing series is what is important. Individual traders can then determine whether, based on their respective transactions costs, the mispricing is economically significant or merely statistically significant. Previous research generally suggests that when the futures contract is mispriced it is selling at a discount relative to its theoretical price (Figlewski (1984); Cornell and French (1983(a)) and (1983(b)); Modest and Sunderesan (1983); and Eytan and Harpaz (1986)). This research finds that the opposite is true.

Using a more extensive data set than any of the articles cited in Section II below (except MacKinley and Ramaswamy, (1988), who have a different focus), the nature of the discrepency between the theoretical futures price and the actual futures price is examined. It is found that the mispricing is usually small but positive. By examining the mispricing series by year, from 1982 through 1987, and by contract, the existence of a premium is found in most cases. At each point in time the two nearest maturity contracts are compared and it is found that the mispricing, usually a premium, is larger for the longer maturity contract compared with the nearest maturity contract.

Moreover, the mispricing is not random, but rather systematically related to the dividend yield and the time-to-maturity, both important parameters in the relationship be-

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Swati Bhatt is an Economist with the Federal Reserve Bank of New York.

Nusret Cakici is an Assistant Professor of Finance at Rutgers University in New Brunswick.

The Journal of Futures Markets, Vol. 10, No. 4, 367–375 (1990)
 CCC 0270-7314/90/040367-09$04.00

tween the theoretical index futures price and the actual futures price. The mispricing series is regressed against dividend yield and time-to-maturity and the coefficients are found to be significantly different from zero, indicating a nontrivial correlation between the degree of mispricing and these parameters. The longer maturity contracts have larger coefficients, implying a stronger correlation.

II. THEORY

A. Background

In perfect markets, the availability of a substitute or replicating portfolio for the derivative security guarantees that any price misalignment is removed by the actions of arbitrageurs. The carrying cost relationship predicts that in a world with costless arbitrage, the price of the forward contract is higher than the current level of the index by an amount equal to the risk-free interest rate minus the dividend yield on the index portfolio. In reality, with transaction costs and execution delays the arbitrage trades may not be so easily accomplished. Consequently, there should exist a band around the theoretical price so that the actual price can move freely within this band without inducing arbitrage trades. This pricing relationship has been extended to futures contracts by most researchers.

Eytan and Harpaz (1986) study the Value Line Composite Index (VLCI) and find that actual futures prices may fall short of the spot index because of the special properties of the VLCI. In particular, the VLCI cannot be duplicated by a self-financing portfolio. Cornell and French (1983(b)) and Modest and Sunderesan (1983) find that SIF prices violate the theoretical bounds; in particular, the theoretical futures price is higher than the actual price. Cornell and French attribute this discrepency to the particular tax structure. All profits on SIF are taxed in the current fiscal year. A stock portfolio, on the other hand, offers the investor a "timing option." An investor can deduct losses at short-term capital gains tax rates if the value of the portfolio declines; if it goes up the investor may extend his holding period to take advantage of lower long-term capital gains tax rates. Thus, SIF must be priced below their theoretical value by an amount equal to the value of this option. However, valuing this option is a complicated problem and, in view of the recent tax reform, the effect of this timing option may not be large.

Modest and Sunderesan (1983) explain the futures discount by arguing that arbitrageurs selling short seldom obtain full use of the proceeds of their short sales. The short seller must pay dividends on the shares borrowed. If the proceeds of the sale are not available for investment in the money market, the futures price has to be at a sufficient discount before arbitrageurs will initiate trades. However, arbitrageurs rarely use short positions in index futures arbitrage because of the uptick rule, weakening the force of this argument.

Figlewski (1984) attributes the existence of this discount to a temporary disequilibrium, "a transitory phenomenon" caused by lack of knowledge about the workings of these markets, especially the marking-to-market convention. The implication is that the discount should disappear entirely, or at least be reduced, over time. This is precisely the result obtained by Billingsley and Chance (1988), using settlement prices of SIF on the Friday of each week from 1982 through 1986. Saunders and Mahajan (1988), using SIF data from October 1982 through September 1984, support the efficiency of futures markets, finding that since the SIF market has matured, price discrepencies have shrunk.

MacKinley and Ramaswamy (1988) study SIF transactions data in conjunction with minute-by-minute quotes of the index. However, they take as given the "substantial and sustained deviation in futures prices from their theoretical values." Their focus is on the stochastic behavior of this discrepency. The authors theorize that a given futures contract should have a larger mispricing the longer the maturity of that contract due to: the effect of unanticipated changes in dividends, unanticipated interest earnings or costs, and tracking errors from employing an incorrect basket of stocks in the arbitrage portfolio. MacKinley and Ramaswamy do not link this theory with any prediction as to the *direction* of the mispricing. In fact, their results show that, on average, futures contracts sell for a premium over the theoretical price.

B. Arbitrage Relationships

Suppose that $\overline{F_{T,t}}$($F_{T,t}$) represents the theoretical (actual) value of a futures contract with expiration date T valued at time t; and that S_t represents the value at time t of the underlying basket of stocks or the index value. A potential arbitrageur can consider the following strategy in the index futures and stock market: (a) Borrow S_t at a risk-free interest rate, r; (b) Buy a basket of stocks for S_t and continuously reinvest the dividends, D; and (c) Sell $\overline{F_{T,t}}$ futures contracts short.

In a perfect market with zero taxes, transaction costs or market impact costs, the existence of riskless and profitable arbitrage would ensure that

$$\overline{F_{T,t}} = S_t \exp[(r - D)(T - t)] \tag{1}$$

With transaction costs, this relationship may not hold precisely; $F_{T,t}$ will vary within some band determined by transaction costs (defined generally).

As in MacKinley and Ramaswamy, the mispricing series is defined as z_t:

$$z_t = \frac{F_{T,t} - S_t \exp[(r - D)(T - t)]}{S_t \exp[(r - D)(T - t)]} \tag{2}$$

The percentage error, z_t, reflects the profit possible via arbitrage. The arbitrage trading rule is defined as: (a) Buy futures contracts worth $F_{T,t}$ and sell the underlying stocks, S_t, short if $z_t < 0$ and (b) Sell $F_{T,t}$ short and buy a basket of stocks for S_t by borrowing the required funds at the risk-free rate, r, if $z_t > 0$.

Let C define transactions costs as a percentage of S_t. Then the trading rule becomes modified as follows: (a′) Buy the futures contract and sell the underlying stocks short if $z_t < 0$ and $|z_t| > C$ and (b′) Sell the futures contract short and buy the underlying stocks by borrowing if $z_t > 0$ and $|z_t| > C$.

Note that C is not time-subscripted, implying that transaction costs are constant over time or that the band within which $F_{T,t}$ can move freely is fixed over time.

The hypotheses examined in this article are: [1] $z_t = 0$, [2] z_t is equal or insignificantly different across different maturity contracts at any given point in time, and [3] z_t is random and unrelated, in any systematic way, to time-to-maturity and divident yield.

The trading rule described above implies a buy and hold arbitrage strategy. At expiration, the futures price is identically equal to the spot value of the index, so that arbitrage profits are locked in at the time the arbitrage portfolio is constructed. Another arbitrage strategy is one where the position is closed out prior to maturity of the contract. In this case, aside from the uncertainty surrounding the futures price prior to maturity, there are additional transactions costs of closing out the position prematurely. However, this strategy does not alter the general trading rule stated above; it simply alters the bands within which no trade is initiated since transaction costs are higher.

III. EMPIRICAL RESULTS

A. Data

The futures and stock index data (over 1200 observations), both obtained from the Chicago Mercantile Exchange, are daily closing values from April 21, 1982 through June 19, 1987. The stock index closing values represent the value of the S&P 500 index at closing which is 4 p.m. EST. The futures closing value is the value of the nearest maturity *and* next nearest maturity at the closing of the Chicago Mercantile Exchange, which is 4:15 p.m. EST. While a 15-min difference in closing time exists, it is not expected to significantly affect the results. The 3-month S&P 500 index futures contract has a maturity from 1–90 days, while the 6-month contract has a maturity from 91–180 days.

The annualized daily dividend yield (i.e., the dividend yield on the basket of stocks for the number of days to maturity remaining on the futures contract), D, is calculated from the dividend yield on the value-weighted New York Stock Exchange index from the CRSP (Center for Research in Securities Prices) tape. While it would be preferable to use the dividend yield on the S&P 500 index, the latter is not easily available. Moreover, other authors (e.g., MacKinley and Ramaswamy) have recently made a similar substitution calculated as follows:

$$D = \left[\prod_{i=1}^{N} (1 + d_i)\right]^{252/N} - 1 \tag{3}$$

where N is the remaining days to maturity and d_i is the daily dividend yield obtained from the CRSP tape. The daily dividend yield is the actual rather than the expected yield. Replacing the latter by the former implies perfect foresight, which is an assumption made by a number of previous researchers, including MacKinley and Ramaswamy. The one-month Treasury bill rate is used to approximate the riskless rate, r. It would be theoretically correct to use that Treasury bill rate corresponding to the maturity of each contract. However for the time frames considered, the various short-term rates are likely to be fairly similar and the discrepency created by using the one-month Treasury bill rate relatively insignificant.

B. Results

The price discrepency between the actual futures price and its theoretical value is small, on average. Table I shows that, overall, for the near maturity futures contract the discrepency is 0.20% while the discrepency for the further maturity contract is 0.42%. Both these contracts reflect a small premium over the entire sample period. Table I also shows that the premium is larger for the longer maturity than the near maturity contract. The null hypothesis that z_t is zero and that z_t is identical across contracts of different maturities is rejected at the 5% level. Moreover, the one-tailed t-test designed to test the hypothesis that the shorter maturity contract has the greater mispricing is rejected at the 5% level for the overall sample and for the years 1982, 1984, 1985 and 1986. This result is distinct from MacKinley and Ramaswamy's observation since they look only at the nearest maturity contract at each point in time, whereas both the nearest and the next nearest maturity contracts are considered here.

The year-by-year results also show a small premium for every year for both contracts, except 1982 for the further maturity contract which is mispriced by (has a discount of) −0.35%. The highest premium for the near maturity contract is 0.48% in 1984. The

Table I
MISPRICING SERIES: OVERALL AND YEAR-BY-YEAR

	Overall Mispricing					
	N	Mean	Std. Dev.	Min.	Max.	t-Statistic
3-month contract	1273 (−7.71)[b]	0.2006	0.5572	−3.7290	2.6817	12.86[a]
6-month contract	1211	0.4268	0.8770	−4.9987	2.9958	16.94
			Year-By-Year			
1982						
3-month contract	172 (−2.18)	0.1055	0.9408	−3.7290	2.6817	1.47
6-month contract	172	−0.3549	1.1689	−4.9987	1.9010	−3.98
1983						
3-month contract	245 (5.07)	0.2289	0.5259	−1.5434	1.5203	6.81
6-month contract	245	0.0594	0.7175	−1.9925	1.7450	1.30
1984						
3-month contract	246 (−9.94)	0.4848	0.4791	−0.4436	2.0859	15.90
6-month contract	246	0.9754	0.6081	−0.3191	2.9958	25.14
1985						
3-month contract	246 (−14.94)	0.2213	0.4345	−0.5771	1.5629	7.99
6-month contract	246	0.9960	0.6872	−0.5304	2.4169	22.74
1986						
3-month contract	248 (−7.57)	0.0028	0.3475	−0.8276	1.2178	0.13
6-month contract	248	0.2906	0.4936	−0.9425	1.6444	9.28
1987						
3-month contract	116 (−1.26)	0.0583	0.2959	−0.7795	0.6961	2.12
6-month contract	54	0.1168	0.2483	−0.3031	0.7635	3.46

[a]The null hypothesis that the average mispricing is zero is rejected at the 5% level in every year except 1982 and 1986 for the near maturity contract and 1983 for the longer maturity. The decision rule is: reject H_0 if $|t| > 1.96$.

[b]The null hypothesis that the near and longer maturity contract have equal mispricing is rejected, for all years except 1987, at the 5% level. The t-statistic for the test is given in parenthesis below the 3-month contract. The decision rule is reject H_0 if $|t| > 1.96$. The one-tailed t-test designed to test the null hypothesis that the longer maturity contract had the larger mispricing has the decision rule: reject H_0 if $t < -1.65$. This hypothesis is rejected at the 5% level for all years except 1983 and 1987.

premium is less in 1985 (0.22%) and is nearly zero in 1986 (0.0028%). However, in 1987 the mispricing increases (0.058%). It is not clear whether the increase in simply a singular event or is part of a renewed trend towards increased positive mispricing. The longer maturity contracts have a maximum mispricing close to 1% in 1984 and 1985 after which it decreases to 0.29% in 1986 and further to 0.116% in 1987.

By grouping the year-by-year results into three roughly equal periods, the null hypothesis that the average mispricing over the sample period is the same for all three periods can be tested. Define period I as consisting of observations from 4/21/82 through 12/31/83, period II having observations from 1/1/84 through 12/31/85, and period III as the remaining observations. The average mispricing in each period and its standard deviation is given in Table II. For the near maturity contract, all periods have an average mispricing that is positive. Period I is significantly less than period II, and period III is significantly less than both periods I and II. In the case of the further maturity contract, period I has an average mispricing that is a discount. Period II has a premium that is significantly larger than that in period III. Thus, in both cases the mispricing tends towards zero in the latest period. Moreover, the standard deviation of the mispricing declines over the three periods for both contracts. The null hypothesis that the variances are equal across the time periods is rejected at the 5% level. These results support Figlewski's thesis that as traders become more knowledgeable about the markets the mispricing should decrease.

Table III describes the mispricing by contract. As in Table I for the near maturity futures contract there are more contracts that sell at a premium (16) than at a discount (5). Except for the June 1982 and June 1983 contracts, the discount is small (0.07% for the December 1985 contract, 0.11% for the September 1986 contract, and 0.07% for the December 1986 contract). The mispricing is larger for the longer maturity contracts and is more often a premium (13 contracts out of 20). There are discounts in the earlier contracts, September 1982 through December 1983, after which only the December 1986 contract has a discount.

It is useful to examine whether the mispricing is random or whether it is systematically related to dividend yield and time-to-maturity. Simple calculations, regressing the mispricing series against the dividend yield and time-to-maturity, are performed. The regression equation is:

$$|z_t| = \beta_0 + \beta_1 D + \beta_2 T$$

where D represents the actual annualized daily dividend yield, T represents time-to-maturity and z_t is the percentage error, as defined in section IIB. Table IV gives the regression results. Both the dividend yield and time-to-maturity are positively and sig-

Table II
AVERAGE MISPRICING OVER THREE TIME PERIODS

	3-month Contract		6-month Contract		
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**	N
I. 1982–1983	0.1780	0.7278	−0.1115	0.9515	417
	(−4.31)[a]	(2.34)[b]	(−21.12)	(2.15)	
II. 1984–1985	0.3531	0.4755	0.9857	0.6483	492
	(11.76)	(2.04)	(18.20)	(1.95)	
III. 1986–1987	0.0205	0.3326	0.2595	0.4640	364
	(3.80)	(4.79)	(−6.77)	(4.20)	

[a]The numbers in parenthesis under the value of the mean are t-statistics calculated to test the null hypothesis that the mean in that period is equal to the mean in the following period.

The null hypothesis that mispricing of both contracts (the near maturity and longer maturity) is the same in all three periods is rejected at the 5% level. The decision rule is: reject H_0 if $|t| > 1.96$.

[b]The numbers in parenthesis under the value of the std. dev. are F-statistics calculated to test equality of variance in that period and the immediately following period. This hypothesis is also rejected at the 5% level. The decision rule is: reject H_0 if $F > 1$.

Table III(a)
MISPRICING BY CONTRACT
3-MONTH

	N	Mean	Std. Dev.	Min.	Max.	t-Statistic
June 82[a]	40	−0.3475	0.6588	−2.0298	0.9182	−3.33
Sept 82	62	0.3180	1.1327	−3.7290	2.6817	2.21
Dec 82	60	0.1494	0.8505	−1.6984	1.7647	1.36
March 83	62	0.5056	0.5204	−0.6655	1.5203	7.65
June 83	62	−0.2247	0.4905	−1.5434	0.5590	−3.61
Sept 83	62	0.0787	0.2782	−0.5643	0.7301	2.23
Dec 83	59	0.4642	0.2962	−0.1851	1.0937	12.03
March 84	60	0.5933	0.4011	−0.1470	1.3033	11.45
June 84	63	0.1943	0.2566	−0.4272	0.6914	6.02
Sept 84	67	0.3256	0.2874	−0.4436	1.0682	9.28
Dec 84	61	0.8326	0.5336	−0.1946	2.0416	12.19
March 85	56	0.6076	0.5938	−0.2240	2.0859	7.65
June 85	67	0.3488	0.3981	−0.3023	1.3501	7.18
Sept 85	62	0.1208	0.3569	−0.5771	0.9377	2.67
Dec 85	60	−0.0739	0.2721	−0.5534	0.5661	−2.11
March 86	61	0.1015	0.3494	−0.6863	0.8281	2.27
June 86	62	0.1607	0.3609	−0.5586	1.2178	3.51
Sept 86	62	−0.1106	0.3372	−0.8276	0.6391	−2.58
Dec 86	62	−0.0740	0.2902	−0.7822	0.7168	−2.01
March 87	61	0.0174	0.2562	−0.5862	0.0961	0.53
June 87	62	0.0687	0.3344	−0.7795	0.6759	1.62

[a]The mispricing on all contracts except March and June 1987 is statistically significant. The decision rule is: reject the null hypothesis of zero mispricing if $|t| > 2.004$ (5% level).

nificantly related to the mispricing for the near maturity futures contracts. The longer maturity contract has overall mispricing positively related to both time-to-maturity and dividend yield. Both coefficients are significant. This result indicates that the longer the time remaining to maturity, the larger the mispricing. MacKinley and Ramaswamy observe a similar effect by studying the mispricing of a single contract over time rather than comparing the mispricing of different maturity contracts at a point in time. The regression results of this research also imply that the larger the dividend yield, the larger the mispricing.

IV. CONCLUSION

Daily closing values of the S&P 500 index and the two nearest maturity futures contracts are used to construct a mispricing series that relates the actual futures price to its theoretical value. The mispricing series can be used to show the *existence* of arbitrage opportunities. However, in the absence of precise information about the size of transaction costs, no statement can be made as to whether the arbitrage is *profitable*. Moreover, calculation of the mispricing is dependent on estimates of the riskless rate, r and the dividend yield, D. If either of these two parameters is significantly misspecified (despite attempts at precision) the results become more difficult to interpret.

Table III(b)
MISPRICING BY CONTRACT
6-MONTH

	N	Mean	Std. Dev.	Min.	Max.	t-Statistic
Sept 82[a]	40	−1.2637	1.1672	−3.8086	0.9761	−6.85
Dec 82	62	−0.1194	1.2324	−4.9987	1.9010	−0.76
March 83	60	−0.0337	0.8487	−2.0041	1.4995	−0.31
June 83	62	0.1927	0.6040	−1.3967	1.3299	2.51
Sept 83	62	−0.6205	0.5544	−1.9925	0.3294	−8.81
Dec 83	62	−0.2067	0.3114	−0.9221	0.4312	−5.23
March 84	59	0.6655	0.3691	−0.0778	1.3576	13.84
June 84	60	0.9144	0.3898	0.1984	1.7450	18.18
Sept 84	63	0.4327	0.2528	−0.3191	0.7991	13.56
Dec 84	67	0.8540	0.3312	−0.0411	1.6604	21.07
March 85	61	1.6622	0.4231	0.7727	2.7026	30.72
June 85	56	1.5560	0.5486	0.4938	2.9958	21.23
Sept 85	67	1.4541	0.3617	0.8240	2.3175	32.90
Dec 85	62	0.8832	0.5778	−0.2180	1.9886	12.03
March 86	60	0.2186	0.4843	−0.5304	1.2600	3.50
June 86	61	0.6533	0.3767	−0.2571	1.3993	13.55
Sept 86	62	0.5065	0.4641	−0.4849	1.6444	8.60
Dec 86	62	0.2870	0.4176	−0.7356	1.2578	5.42
March 87	62	−0.1627	0.2802	−0.9425	0.5624	−4.57
June 87	61	−0.0781	0.2632	−0.4225	0.7635	−2.32

[a]The mispricing on all contracts except December 1982 and March 1983 is statistically significant.

Table IV
REGRESSION RESULTS
(DEPENDENT VARIABLE IS ABSOLUTE VALUE OF PERCENTAGE ERROR)
$|Z_t| = \beta_0 + \beta_1 D + \beta_2 T$
WHERE D = ACTUAL ANNUALIZED DAILY DIVIDEND YIELD
T = TIME TO MATURITY

	Parameter	Std. Error	T Stat.	
3-month contract				
Intercept	−0.0802	0.0356	−2.257	$R^2 = 0.1644$
Dividend	8.4659*	0.7550	11.213	
Time-to-Maturity	1.0448*	0.1626	6.424	
(Average dividend yield = 0.04522)				
6-month contract				
Intercept	−0.5615	0.1287	−4.363	$R^2 = 0.0926$
Dividend	23.8054*	2.2721	10.477	
Time-to-Maturity	0.6881*	0.2331	2.952	
(Average dividend yield = 0.04497)				

* = Significant.

Given the above qualification, the analysis indicates the following results. First, the percentage mispricing or error is shown to be small but positive. Second, the percentage error is large, in absolute value, the longer the maturity. That is, the percentage error is larger for the longer maturity contract than the near maturity contract. Third, if the entire sample period is represented by three broad time periods, the later time period demonstrates smaller mispricing than the earlier time periods. This result supports Figlewski's thesis that as traders mature in their knowledge of futures markets, futures prices become more efficient. Fourth, calculating the percentage error by contract, rather than by year, shows that more contracts are selling at a premium, on average, than at a discount. And fifth, the regression results show that the mispricing is positively and significantly related to time-to-maturity and dividend yield for both near and longer maturity contracts, rather than being stochastic over time.

These results stand in contrast to previous research on SIF prices. In particular, S&P 500 index prices are priced overall, at a premium as opposed to a discount with respect to the theoretical price. It remains to be explained why index futures contracts are priced at a sustained premium and not a discount. It may be, though do not proved here, that the parameters r and D may be misspecified. Second, the mispricing may actually represent a risk-adjusted return to arbitrageurs. Third, as suggested by Figlewski, this may simply be a temporary disequilibrium which will be erased over time.

Bibliography

Billingsley, R., and Chance, D. (1988, June): "The Pricing and Performance of Stock Index Futures Spreads," *The Journal of Futures Markets*, 8,3:303–318.

Cornell, B., and French, K. (1983(a), Spring): "The Pricing of Stock Index Futures," *The Journal of Futures Markets*, 3,1:1–14.

Cornell, B., and French, K. (1983(b), June): "Taxes and the Pricing of Stock Index Futures," *Journal of Finance*, 38,2:675–694.

Eytan, T., and Harpaz, G. (1986, September): "The Pricing of Futures and Option Contracts on the Value Line Index," *Journal of Finance*, 41,4:843–857.

MacKinley, C., and Ramaswamy, K. (1988, Summer): "Index Futures Arbitrage and the Behaviour of Stock Index Futures Prices," *Review of Financial Studies*, 1,2:137–158.

Figlewski, S. (1984, July–August): "Explaining the Early Discounts on Stock Index Futures; The Case of Disequilibrium," *Financial Analysts Journal*, 43–67.

Modest, D., and Sunderesan, M. (1983, Fall): "The Relationship between Spot and Futures Prices in Stock Index Futures Markets: Some Preliminary Evidence," *The Journal of Futures Markets*, 3,1:15–41.

Saunders, E., and Mahajan, A. (1988, June): "An Empirical Examination of Composite Stock Index Futures Pricing," *The Journal of Futures Markets*, 8,2:210–228.