Put-Call-Futures Parity and Arbitrage Opportunity in the Market for Options on Gold Futures Contracts

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I. INTRODUCTION

The Commodity Exchange Inc., long the reigning market for gold futures contracts, introduced gold futures options in October of 1982. Immediate, sustained interest in the new contracts created a liquid market for options on COMEX gold futures contracts.[1]

Trading in gold futures options occurs in close proximity to trading in the underlying gold futures contracts, which facilitates the expeditious construction of futures/futures options positions. Market liquidity, the close proximity of trading, and the nearly instantaneous availability of price information should serve to mitigate opportunities for arbitrage profits.

The purpose of this study is to ascertain whether arbitrage profits were available to a floor trader utilizing a trading strategy based on the put-call-futures parity relationship by examining a trading strategy within an environment consistent with actual market activity. Simulated trading profits are examined to discover whether they represent deviations from market equilibrium conditions which could have been exploited by an actual trading strategy, or whether they can be explained by other factors.

The earliest examinations of the put-call parity were conducted by Stoll (1969) and by Gould and Galai (1974) in the over-the-counter market for equity options. Both studies relied on closing price data and produced inconclusive results regarding market efficiency.

The advent of exchange-traded put options on the CBOE in 1977 allowed Klemkosky and Resnick (1979) to construct data sets comprised of virtually simultaneous transactional prices for fifteen stocks and their options for one day each month over the period from June 1977 to June 1978. The put-call parity relationship was adjusted to compen-

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[1]Daily volume in COMEX gold futures options averaged 6,455 contracts over the period of study.

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sate for dividend payouts. Those hedges for which the possibility of rational early exercise existed for the granted option were removed from consideration.

Although regression analysis failed to indicate that put and call price relationships were inconsistent with the hypothesis of option market efficiency, a large number of apparent outliers led those authors to conduct an even more rigorous study. When Klemkosky and Resnick (1980) amended their 1979 study by recognizing the time lag between the identification and implementation of a market position, and by an *ad hoc* increase in transaction costs to account for the bid-ask spread, profit opportunities were effectively eliminated.

A recent article by Ball and Torous (1986) utilizes closing and settlement prices to examine parity relationships for options on three distinct futures contracts for the period from October 1, 1982 to March 1, 1985. Even though bid-ask spreads are not considered as part of the transaction costs borne by floor traders, only a small fraction of the put-call-futures hedges violate the boundaries imposed by the model. However, close conformity of closing and settlement prices to the relationship specified by the model is not surprising. When trading is thin, exchange settlement committees often specify settlement prices according to the very model these authors were testing. The use of contemporaneous, transactional price data would have produced more realistic results.

A transactional data test of the put-call-futures parity for Treasury bond futures options is undertaken by Jordan and Seale (1986). Rigorous *ex post* and *ex ante* tests of the joint hypotheses of model validity and market efficiency lead then to conclude that profits of potential economic significance are generally confined to floor traders. Further, these potential profits are found to be systematically related to violations of the model's assumptions.

Utilizing transactions data, Bodurtha and Courtadon (1986) demonstrate that efficiency tests of put-call parity rational pricing boundaries in the foreign currency options market are sensitive to both transaction costs and nonsynchronous prices.

This article is structured as follows. Section II presents the put-call-futures parity model and a discussion of the probable effects of violations of the model assumptions. Section III presents a trading strategy designed to counteract the effects of these violations in actual markets. Section IV describes the data and the methodology, and reports the results of the study. Section V offers conclusions.

II. THE MODEL

The parity relationship between futures contracts and their options relies on the axiom that, in an efficient market, a riskless combination of financial instruments can yield no more than the risk-free rate of return. When both lending and borrowing at the riskless rate are possible, forward contracts can be combined with European put and call options to produce certain cash flows. In equilibrium, the price of any of these instruments can be determined by substituting the prices of the other two instruments into the parity equation,

$$C(F, T, E) - P(F, T, E) = (F - E)e^{-rT}, \qquad (1)$$

where F is the forward or futures price, E is the options' exercise price, r is the riskless rate of return, T is the time to option expiration, and $P(F, T, E)$ and $C(F, T, E)$ are the respective put and call option prices. The parity relationship applies to European options on futures contracts when profits and losses on the futures are realized only at expiration of the contracts.[2] Table I shows the initial and terminal cash flows resulting from both short and long hedges of futures and futures options contracts.[3]

Exchange-traded futures options are not European so that a positive probability of early exercise for these options always exists. When interest rates are not deterministic, futures and forward prices may be unequal because gains and losses in the futures market must be settled at the end of each trading day. Additionally, when borrowing rates exceed lending rates, hedges that require an initial cash outlay are less desirable than those which do not.

Two crucial assumptions of the model, those of zero probability of rational early exercise and zero cash flows between hedge inception and expiration, are inconsistent with the market environment. The probable effect that relaxing these assumptions has on market price relationships is discussed below.

A. The Assumption of No Early Exercise

Following Merton's (1973) analysis, the put-call-futures parity model itself provides a rationale for the early exercise of either the put or the call futures option. Arbitrage arguments set the respective lower boundaries for American put and call futures options as

$$P(F,T,E) \geq \max(0, E - F), \tag{2}$$

and

$$C(F,T,E) \geq \max(0, F - E). \tag{3}$$

From eq. (1) it can be shown that the lower boundary for the put option is violated whenever

$$C(F,T,E) < (E - F)(1 - e^{-rT}). \tag{4}$$

[2]See Moriarty, Phillips, and Tosini (1981) and Whaley (1986) for the assumptions and development of the put-call-futures parity model.

[3]The popular terminology for describing a long hedge is "conversion;" a short hedge is called a "reverse conversion." We follow the convention established by Klemkosky and Resnick (1979) and (1980).

Table I
PUT-CALL PARITY FOR FUTURES OPTIONS

	Initial Investment	Value at Expiration[a]	
Long Hedge Strategy		$F_1 < E$	$F_1 \geq E$
Sell a Call	C	0	$-(F_1 - E)$
Buy a Put	$-P$	$E - F_1$	0
Long One Futures Contract	0	$F_1 - F_0$	$F_1 - F_0$
Net Cash Flow	$C - P$	$E - F_0$	$E - F_0$
Short Hedge Strategy			
Buy a Call	$-C$	0	$F_1 - E$
Sell a Put	P	$-(E - F_1)$	0
Short One Futures Contract	0	$F_0 - F_1$	$F_0 - F_1$
Net Cash Flow	$P - C$	$F_0 - E$	$F_0 - E$

[a]The subscripts 0 and 1 denote values at time 0 and time 1 (expiration).

If eq. (4) obtains, the arbitrage condition specified by eq. (2) is violated and the put will be exercised. Similarly, the call will be exercised if

$$P(F,TE) < (F - E)(1 - e^{-rT}). \tag{5}$$

Since the granted option may be exercised at any time during the hedge, the put-call-futures parity for American options must be expressed by the bounding inequalities,

$$E - F \geq P(F,T,E) - C(F,T,E) \geq (E - F)e^{-rT}. \tag{6}$$

Even when these arbitrage conditions are not violated, practical considerations may serve to make premature exercise prudent. Since thin trading exists for deep-in-the-money options, traders wishing to liquidate positions in these options may prefer to do so by exercising them. The trader would receive the difference in the option exercise price and the futures settlement price, and acquire a futures position which could be reversed in the more liquid futures market where the bid-ask spread is likely to be smaller.

The possibility of early exercise of the granted option abates the riskless nature of the parity pricing relationship. As a consequence, the model specified by eq. (1) is only strictly valid for European futures options.

B. The Assumption of Zero Interim Cash Flows

Black (1976) shows that the right-hand side of eq. (1) is the price of a forward contract. Although futures and forward contracts are similar and have often been treated as equivalent in the literature, these contracts are not identical and may exhibit different prices.[4] This pricing difference results from the daily settlement of profits and losses in the futures market. With respect to put-call-futures parity, the daily settlement characteristic of futures contracts may cause interim cashflows to vary substantially even though the initial cash flow and the cumulate cash flow over the hedge are known with certainty. The stochastic nature of these cash flows makes it impossible to determine at the initiation of the hedge whether there will be a net gain of interest earned or an opportunity loss of interest forgone over the course of the hedge.

The magnitude of the difference between futures and forward prices remains unsolved. Cornell and Reinganum (1981) fail to uncover statistically significant difference between the two prices. In their examination of the Treasury bill futures market, Elton, Gruber, and Rentzler (1984) find that the average interest loss resulting from daily settlement is not significantly different from zero.

Nevertheless, a trader who adopts a buy-and-hold strategy similar to either strategy (described in Table I) does not hold a riskless market position. The extent to which this departure from the model's assumptions affects relative put, call, and futures prices is an empirical question.

III. THE TRADING STRATEGY AND THE OPERATIONAL MODEL

The put-call-futures parity of eq. (1) implicitly assumes a buy and hold strategy where the put-call-futures hedge, once initiated, is held to expiration. In actual markets, such a strategy generates hedges which are risky because they are exposed to the possibility of early option exercise and daily cash settlement. However, both of these sources of risk can be avoided by employing an active trading strategy that reverses market positions the same day they are initiated.

[4]See Jarrow and Oldfield (1981); Cox, Ingersol, and Ross (1981); Richard and Sundareson (1981); and French (1983) for explanations of the difference and the relationship between forward and futures prices.

Suppose that a trader enters similar long and short hedge positions on the same day. Since these hedges must consist of opposite positions in futures and futures options contracts, they will offset each other. The proceeds from the long hedge, $E - F_0$, and the short hedge, $F_0 - E$, will be received immediately rather than at the expiration of the option contracts. Since no market position exists at the end of the trading session, no cash investment would be required.

Unless disequilibrium conditions and/or thin trading persists, hedge positions can be entered and reversed on the same day. Since option exercise is an overnight transaction, the possibility of early exercise of the granted option is eliminated if the hedge is reversed on the day it is initiated. Different borrowing and lending rates are also unimportant since no investment is necessary unless a market position exists at the end of the trading session. It further follows that daily settlement of futures gains and losses during the hedge is no longer an important factor.

Deviations from equilibrium conditions must exist for the trading strategy to be viable; but must not persist for long periods of time. However, while market positions which can be readily reversed are essentially risk-free, hedges subject to a market environment characterized by thin trading and/or persistent disequilibrium are exposed to risk. The association between risk and potential hedge profits is analyzed in the following section.

If the put-call-futures parity model is to be utilized as a trading rule, it must first be adjusted to include transactions costs. For a long hedge the operational put-call-futures parity model is

$$C_b(F,T,E) - P_a(F,T,E) - D - (F_a - E)e^{-rT} \leq 0. \tag{7}$$

For a short hedge the operational model is

$$P_b(F,T,E) - C_a(F,T,E) - D - (E - F_b)e^{-rT} \leq 0. \tag{8}$$

D represents direct trading costs such as exchange memebers' clearing fees. The subscripts, a and b, denote that purchases occur at ask prices and sales occur at bid prices.

Clearly, eqs. (7) and (8) create a range within which combinations of contemporaneous options and futures prices can fluctuate without creating arbitrage opportunities. However, if observed price combinations fall outside the boundaries imposed by the operational models, opportunities to achieve economic profits may exist.

IV. DATA, METHODOLOGY, AND EMPIRICAL RESULTS

A. The Data

Tests for violations of the put-call-futures parity relationship utilize timed, transactional data for gold futures, and gold futures options traded on the COMEX for the period of May 14, 1984 through November 9, 1984. During this period, options were traded on five gold futures contracts: August, October, and December of 1984, and February and April of 1985. The data compiled by the COMEX consist of the time and price for each price-changing transaction in the concurrent gold futures and futures options markets. These accurately timed transaction prices provided by the COMEX facilitate the construction of a synchrony of combinations of options and futures contracts. A total of 1,716 synchronous combinations of put, call, and futures transactions were recorded over the period of study.

The ask-yield quotation for the Treasury bill with a maturity date closest to the option exercise date serves as a proxy for the risk-free interest rate. These yield quotations, obtained from *The Wall Street Journal*, were converted to continuously compounded rates of return.

B. Transaction Costs

The operational model presented by eqs. (7) and (8) requires that opportunities for arbitrage profits exceed both direct costs resulting from commissions and fees, and indirect market impact costs of executing transactions. For an exchange member, direct transactions costs are nominal, consisting of a clearing fee of \$.75 for each newly initiated option or futures contract held until the close of the trading session. The market impact costs implied by floor traders' bid-ask spreads, however, are consequential but unobservable since they are embodied in transaction prices.

Phillips and Smith (1980) demonstrate that a trading strategy designed to capitalize on market mispricing will tend to select observed transactions which occurred on the "wrong" side of the bid-ask spread, *i.e.*, transactions which were orginally initiated by the desire to sell will often be selected for purchase by a model which identifies them as being undervalued. This selection bias will produce apparent trading profits which contain, but may not exceed, transactions costs implied by bid-ask spreads.

It is often impossible to ascertain from casual observation of the data whether a transaction occurred at a bid or an ask price. Patterns in successive transactional prices suggest that certain transactions are initiated by one party's desire to purchase or to sell, but thin trading or volatile prices can render this method inaccurate and imprecise.[5]

Roll's (1984) procedure estimates the effective bid-ask spread in an informationally efficient market from a series of successive prices. In addition to assuming that markets are frictionless and informationally efficient, Roll assumes that the probability distribution of observed price changes is stationary over short intervals of time. Under these conditions, Roll estimates the effective bid-ask spread by the following equation:

$$s = 2(-\text{cov})^{1/2} \tag{9}$$

where "s" is the effective bid-ask spread and "cov" is the first order serial covariance of successive price changes.[6]

When Roll's analysis is applied to a data set consisting of price-changing transactions only, the equation for estimating the effective bid-ask spread becomes,

$$s = (-\text{cov})^{1/2} \tag{10}$$

Since the bid-ask spread is estimated from actual price-changing transactions, s is the effective spread and not necessarily the spread offered by the scalper.[7] Diametric, contemporaneous market orders may be crossed by traders who have equal incentive to transact and who do so without utilizing the services of a scalper. Additionally, scalpers may oblige floor brokers by accepting larger positions than they actually desire. In such a situation, the scalper may choose to immediately liquidate part of his position at a zero profit. These zero profit or "scratch" trades could result in market buy (sell) orders being

[5]A method for imputing whether a transaction occurred at the bid or the ask price can be found in Bhattacharya (1983, pp. 168–70).

[6]Implicit to the analysis is the assumption that successive transaction types, buy orders and sell orders, arrive with equal conditional probabilities of .5. Choi, Salandro, and Shastri (1988) show that if positive serial in transaction types exists, Roll's procedure produces downward biased estimates of the effective bid-ask spread.

[7]Scalpers are a specialized group of floor traders who fulfill a market-making function by continuously quoting bids and offers against which orders of off-floor traders traders can be executed. Silber (1984) provides a fascinating, detailed analysis of market-maker behavior in the futures markets.

filled at bid (ask) prices, the opposite of what might normally be expected. These types
A regression model is constructed to provide *ex ante* estimates of the effective bid-ask spreads relevant to each hedge position. Equation (10) is applied to 88 randomly selected intraday series of put and call transactional price changes. Effective bid-ask spreads are not estimated for nine intraday series which exhibit positive covariances. These series are removed from the analysis. Each of the remaining 79 time series consist
of transactions serve to make the effective bid-ask spread smaller than the actual price spread quoted by a scalper on the floor of the exchange.
of at least 21 price changing transactions; the largest number of transactions in any one series is 102.

Theoretically, the effective bid-ask spread varies inversely with the degree of competition between floor traders and directly with the risk they assume.[8] While risk, measured by the variance of intraday price changes, exhibits a significant positive relationship with the size of the estimated bid-ask spread, its calculation requires *ex post* price data. Since such data is unavailable to a trading strategy which must estimate the bid-ask spread prior to effecting a market position, the option price is used as a proxy for risk. The price of a futures option is an excellent risk proxy because higher priced futures options are more greatly affected by a given change in the price of the underlying futures contract than are their lower priced counterparts. These options are therefore associated with a greater variance of serial price changes. Since a nonlinear relationship exists between futures and futures option prices, the option price, P, and the square of the option price, P^2, are utilized as proxies for risk.

An increase in the demand for liquidity services as measured by the volume of contracts traded might be expected to decrease competition between scalpers and induce them to increase their bid-ask spreads. However, increased trading volume provides greater economies of scale and decreases the length of time a scalper must hold a market position, thus decreasing the risk he must assume. Since trading is less active in away-from-the-money options, the absolute value of the difference between the options exercise price and the average daily futures price, $|E - F|$, is used as a proxy for the volume of contracts traded. The number of days remaining to expiration, T, is also used as a proxy for volume since trading activity tends to increase as an option contract approaches expiration. Since a negative relationship exists between trading volume and these two proxies, a positive relationship should exist between both $|E - F|$ and T and the estimated bid-ask spread.

The regression equation used to predict estimates of the effective bid-ask spread,

$$\begin{array}{cccccc} s = 6.08 & + .026T) & + .086|E - F| & - .0055P & + 7.8 \times 10^{-6}P^2 \\ (1.50) & (.02) & (.059) & (.0037) & (2.0 \times 10^{-6}) \end{array} \qquad (11)$$

exhibits a highly significant F value of 24.75, and R^2 of .57, and a standard error of 3.20. As expected, the predictions tendered by eq. (11) increase with both option maturity and "away-from-the-moneyness." Spread estimates also increase with risk, as measured by its proxy, option price. Although significance levels are marginal for the proxy variables, the model functions well as a predictor.

The mean esitmated bid-ask spread is \$11.13, while estimated spreads range from \$5. to \$175. in size. Observed market prices are adjusted by these estimated spreads to reflect prices at which options might be expected to be purchased or sold.

[8]See Demsetz (1968), Tinic (1972), Benston and Hagerman (1974), Barnea and Logue (1975), Logue (1975), and Stoll (1978) for previous research focusing on the determinants of the bid-ask spread.

C. Empirical Results

Price changing transactions for similar put and call options occurring within one-minute intervals are selected from the data base. Trading activity is generally greater in the underlying futures contracts than in the corresponding futures options. Consequently, several futures prices often are observed within the time interval created by the options transactions. In such cases, the highest and lowest futures prices are recorded. In those cases where only one or no recorded futures transactions occurred between option transaction times, the next one or two transactional prices, respectively, are recorded. In every case, however, all transactions occurred within a one-minute interval.

Recording two different futures prices for each put-call-futures hedge enables the selection of the futures price most disadvantageous to hedge profitability. This most disadvantageous price is assumed to be the futures ask (bid) price for a long (short) hedge.

Table II illustrates the degree to which observed, contemporaneous futures and futures options prices conform to the theoretical model of eq. (1) which ignores transaction costs. The 1,716 hedges examined produce mean profits of $27.81 per hedge. Long hedges are more numerous and more profitable than short hedges. The 923 long hedges produce mean profits of $30.81 while the 793 short hedges produce mean profits of $24.31.

Large and frequent deviations from the theoretical model are observed in Table II. Whether these departures from theoretical equilibrium conditions represent economic profits depends upon the extent to which they exceed transactions costs.

Transactions costs must be estimated before the operational models of eqs. (7) and (8) can be utilized. Direct costs in the form of clearing fees amount to only $.75 per option or futures contract, or $2.25 per hedge entered.[9] Implicit transaction costs represented by the bid-ask spread are estimated by the regression model, represented by eq. (11). To avoid selection bias, the option prices are adjusted by the full amount of the bid-ask spread in the manner recommended by Phillips and Smith (1980) for *ex post* tests. The results are presented in Table III.

When transaction costs are considered, mean profits decline to $3.31. A total of 995 hedges, over half the sample, are unprofitable. Estimated transaction costs average $24.50 for each hedge.[10] Although mean profits are small for even the lowest cost trader, attention should be focused on those hedges which seem to produce large, exces-

[9] Clearing fees totaling $2.25 are required also to exit a hedge. Since it is assumed that profitable hedges can be reversed by profitable, similar but opposite hedges, only the clearing fees required to enter a hedge are considered.

[10] This figure excludes the futures bid-ask price which is not estimated but accounted for by selecting the futures price most disadvantageous to the hedge.

Table II
SUMMARY OF *EX POST* HEDGE PROFITABILITY WHEN TRANSACTION COSTS ARE ZERO

	Number of Hedges with *Ex Post* Profitability						
	<$10	$10–20	$20–30	$30–40	$40–50	$50–60	$60–70
Long hedges	364	129	104	86	57	37	45
Short hedges	291	151	114	80	49	33	17
	$70–80	$80–90	$90–100	>$100	Totals		
Long hedges	23	13	20	54	923		
Short hedges	15	10	11	20	793		

Table III
SUMMARY OF *EX POST* HEDGE PROFITABILITY WHEN TRANSACTION COSTS ARE POSITIVE[a]

	Number of Hedges with *Ex Post* Profitability						
	<$0	$0–10	$10–20	$20–30	$30–40	$40–50	$50–60
Long hedges	529	99	88	63	28	22	24
Short hedges	466	102	81	48	25	25	14
	$60–70	**$70–80**	**$80–90**	**$90–100**	**>$100**	**Totals**	
Long hedges	17	7	3	11	32	923	
Short hedges	10	2	2	4	14	793	

[a]In this table, transaction costs for each hedge consist of the sum of clearing fees of $2.25 and bid-ask spreads estimated by the regression model presented in Table III.

sive profits. A total of 46 hedges, approximately 2.7 percent of the sample, produce profits exceeding estimated transaction costs of $100 or more.

The existence of *ex post* profits, however, is not sufficient evidence for rejecting the hypothesis of market efficiency. Deviations from market equilibrium must persist long enough to allow a trader to recognize and capitalize on them. An *ex ante* test is conducted to discover whether or not profitable *ex post* hedges produce trading signals which could be exploited to yield *ex ante* profits. For the *ex ante* test, each profitable *ex post* hedge triggers the decision to implement a similar position at the next set of simultaneous transactions, providing this occurrs within five minutes.

Of the 431 *ex post* hedges followed within five minutes by a set of contemporaneous transactions, only 214 produce profits in excess of estimated transaction costs. Of the 214 trading decisions triggered by the profitable *ex post* hedges, only 114 produce *ex ante* profits. Mean profits decline from $33.05 for the profitable *ex post* hedges to $8.78 for the *ex ante* hedges. The results of the *ex ante* test are presented in Table IV.

Decisions to execute the majority of the loss producing *ex ante* hedges are triggered by *ex post* hedges which are only marginally profitable. This result is presented in Table V. Of the 100 unprofitable *ex ante* hedge positions, 57 are implemented on *ex post* profit signals of $20 or less. *Ex post* signals greater than $20 produce mean *ex ante* profits of $18.73 while positive *ex post* signals of less than $20 produce *ex ante* losses averaging $2.78 per hedge.

Filter rules are trading rules designed to capitalize on nonrandom price behavior; or, in this case, the nonrandom behavior of price relationships. A filter rule based on

Table IV
A COMPARISON OF THE PROFITABILITY OF *EX POST* WITH *EX ANTE* HEDGES WHEN TRANSACTION COSTS ARE POSITIVE[a]

	Number of Hedges with *Ex Post* Profitability						
	<$0	$0–10	$10–20	$20–30	$30–40	$40–50	$50–60
Ex post hedges	0	54	45	30	21	19	14
Ex ante hedges	100	24	16	23	16	7	8
	$60–70	**$70–80**	**$80–90**	**$90–100**	**>$100**	**Totals**	
Ex post hedges	8	3	2	4	14	214	
Ex ante hedges	3	2	3	1	11	214	

[a]In this table, transaction costs for each hedge consist of the sum of clearing fees of $2.25 and bid-ask spreads estimated by the regression model presented in Table III.

Table V
TRANSITION MATRIX OF THE PROFITABILITY OF *EX POST* AND *EX ANTE* HEDGES WITH LAGGED OBSERVATIONS OF FIVE MINUTES OR LESS[a]

Number of Hedges with *Ex Ante* Profits	Number of Hedges with *Ex Post* Profitability										
	$ 0–$10 54	$10–$20 45	$20–$30 30	$30–$40 21	$40–$50 19	$50–$60 14	$60–$70 8	$70–$80 3	$80–$90 2	$90–$100 4	>$100 14
< −$100			1								
−$100 to −$90				1						1	
−$ 90 to −$80		1									
−$ 80 to −$70											
−$ 70 to −$60			1			1					1
−$ 60 to −$50	5			1							
−$ 50 to −$40	1	3		1		1					
−$ 40 to −$30	6	3	1	1	2	1					1
−$ 30 to −$20	4	2	2	2	1						
−$ 20 to −$10	5	6	3	3	1	2				1	
−$ 10 to $ 0	11	10	4	2	4	2		1			
$ 0 to $10	6	6	6	2		1	2				1
$ 10 to $20	4	4	3	1	2	2					
$ 20 to $30	5	6	5	2	3				1		1
$ 30 to $40	3	3	2		3	1	3				1
$ 40 to $50	1		2		1	1	1	1			
$ 50 to $60	2			1	1	1	1	1			1
$ 60 to $70				1		1			1		
$ 70 to $80				1						1	
$ 80 to $90	1			1							1
$ 90 to $100											1
> $100		1		1	1		1			1	6

[a]Transaction costs for each hedge consist of clearing fees of $2.25 bid-ask spreads estimated by the regression model presented by eq. (11).

ex post profits can produce *ex ante* economic profits only when disequilibrium conditions persist in the put-call-futures price relationship. The Mann-Whitney-Wilcoxon test procedure is performed to determine whether the difference in median *ex ante* profits is a chance occurrence or attributable to the use of the arbitrary \$20 *ex post* profit filter. Since the samples are relatively large, the normal approximation to the Mann-Whitney-Wilcoxon statistic is employed. From the test statistic, $Z = -3.399$, the P-value is approximated to be less than .001. The hypothesis of equal median returns is therefore rejected, indicating that the application of the filter to *ex post* profit signals produces significantly greater *ex ante* profits.

Such results are unexpected if the sequential profits generated by observed contemporaneous put, call, and futures prices fluctuate randomly about the theoretical equilibrium value of zero. The Spearman rank correlation coefficient is calculated to measure the degree of association between *ex post* profit signals and *ex ante* profits. Additionally, the null hypothesis of independence between *ex post* and *ex ante* profits is tested against the alternate hypothesis that a direct, positive relationship exists between these two variables.

The Spearman coefficient of correlation, R, is .2996, yielding an asymptotic P-value of less than .001. The null hypothesis of independence between *ex post* and *ex ante* put-call-futures price relationships is therefore rejected in favor of the alternative hypothesis of a direct relationship between *ex post* and *ex ante* hedge returns.

Although hedge returns are diminished by the time interval between the identification and implementation of potentially profitable market positions, *ex post* profit signals are useful in securing *ex ante* profits. However, the lack of independence between subsequent sets of put, call, and futures prices has important implications for a dynamic trading strategy designed to counteract the risk associated with violations of the model assumptions.

Under the trading strategy outlined in the previous section, hedge posiitons are entered whenever the conditions imposed by either eq. (7) or eq. (8) are not satisfied. All open hedge positions are reversed when a set of simultaneous transactions indicate a profit opportunity for a hedge in the opposite direction. Any open hedges which can not be reversed by an opposite market position are simply held until expiration.

Hedge profits both before and after transaction costs are recorded in addition to: the number of days a hedge is held before being reversed, the initial cash outlay for hedges held for longer than one day, and the previously defined proxies for volume, $|E - F|$ and T. Table VI presents the summary statistics for these variables.

The mean number of days hedges are held is surprisingly large. Of the 720 hedges, 208 are reversed on the day initiated and 162 are reversed the following day. Many hedges, however, are held for long periods before the options either expire or a profitable set of opposite transactions is encountered. Some hedges are initiated with relatively long times remaining to the option expiration date. Trading in these options, already thin, often becomes nonexistent when the futures price drifts away from the options' exercise price. While reversal transactions can always be initiated, there is no way to predict whether the outcome will be profitable.

An analysis of the relationships between the variables of Table VI indicates that the hedges exhibiting the greatest potential profit opportunities are also closely associated with factors which mitigate their attractiveness. Table VII presents a matrix of Spearman correlation coefficients for these variable with P-values enclosed in parentheses below each correlation coefficient.

A high degree of association exists between hedge profits and each of the variables considered. Of particular importance, however, is the significant, direct relationship between hedge profits after transactions costs and the number of days the hedge is held.

Table VI
SUMMARY STATISTICS FOR THE VARIABLES AFFECTING HEDGE PROFITABILITY

	Mean	Standard Deviation
Hedge profits before transaction costs	\$97.67	\$62.25
Hedge profits after transaction costs	\$55.60	\$55.69
Holding period (days)	8.23	21.42
Initial cash outlay	\$427.16	\$803.26
Days to expiration, T	36.51	29.54
$\|E - F\|$	\$730.06	\$625.61

Table VII
SPEARMAN RANK CORRELATION COEFFICIENTS FOR THE VARIABLES AFFECTING HEDGE PROFITABILITY

	(1)	(2)	(3)	(4)	(5)
(1) Hedge profits before transaction costs					
(2) Hedge profits after transaction costs	.8892 (.000)				
(3) Holding period (days)	.1571 (.000)	.1274 (.000)			
(4) Initial cash outlay	.1200 (.001)	.0956 (.005)	.4503 (.000)		
(5) Days to expiration, T	.3413 (.000)	.0789 (.017)	.2019 (.000)	.0312 (.201)	
(6) $\|E - F\|$	.2285 (.000)	.1093 (.002)	.4496 (.000)	.6840 (.000)	.1233 (.000)

The inability to reverse a hedge position on the day it is initated exposes the hedge to the risk associated with the daily settlement of futures gains and losses, and to the possibility of early option exercise. Also important is the direct association between hedge profitability and the initial investment required to initiate the hedge position. When borrowing rates exceed lending rates, the riskless rate return may not be a sufficient hurdle rate.

IV. CONCLUSION

This article examines the degree to which simultaneous, transactional gold futures and gold futures options prices conform to the theoretical put-call-futures parity relationship. A trading strategy, developed to counter the effects of violations of the model's assumptions, consistently produces profits in excess of estimated trading costs. The most profitable sets of trades, however, are directly associated with violations of the assumptions of the model.

Even ardent supporters of the hypothesis of market efficiency should not be disturbed by occasional opportunities for arbitrage profits. These exist for several reasons. First, parity in the put-call-futures price relationship requires constant vigilance on the part of traders whose incentive for monitoring this market depends upon occasional departures from the parity price structure. Second, tests of market efficiency can only simulate actual profit opportunities within the constraints of the data collection and reporting process, and the imperfect estimation and adjustment for trading costs and risk.

A portion of the deviation between model and market prices results from necessary violations of the assumptions under which the model was derived. While the worth of an economic model should not be judged by the extent to which its assumptions mirror reality, the violation of model assumptions by actual markets certainly provides a reason for market prices to diverge from model-dictated equilibrium conditions.

Transaction costs are extremely important to any analysis of market efficiency and cannot be ignored. Unfortunately, the most important of the transaction costs accruing to the lowest cost trader, the bid-ask spread, is unobservable and difficult to estimate. Additionally, the factors which are most helpful in explaining the bid-ask spread are those which make the violations of the model's assumptions most important to relative market prices.

Although the put-call-futures parity model is based on the principle of riskless arbitrage, the implementation of a model-based trading strategy involves some degree of risk. Market price relationships may change between the time that profit opportunities are recognized and market positions are established. The magnitude of transaction costs is unknown before the market position is implemented, and market positions held overnight are subject to the risk associated with violations of the model's assumptions. These may well be examples of unsystematic risk, but unsystematic risk is important to traders on the exchange floor.

Only when the effect of interrelated components of risk and transactions costs have been isolated and accurately measured can one conclude that part of the discrepancy between model and market prices is caused by pricing error. The market mispricing component, if it exists, may be impossible to accurately measure because of the difficulty associated with measuring risk and transaction costs. It is very likely, that studies such as this one can serve only to enhance the understanding and appreciation of financial markets rather than to provide unambiguous conclusions regarding their efficiency.

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