Option Pricing with Futures-Style Margining

Derming Lieu

The Chicago Mercantile Exchange and the Chicago Board of Trade are petitioning the Commodity Futures Trading Commission (CFTC) to repeal a long-standing regulation requiring an investor to pay the total value of an option premium when purchasing a commodity option. Under the current "stock-style" option system the option buyer or "long" must pay the entire premium when the transaction is initiated. No further payments are required. The premium is credited to the account of the option seller or "short" who must keep it posted as margin. The option seller also must put up risk margin to cover potential adverse market moves in his obligation. If the option increases in value, the short must deposit additional funds into the account. These funds, however, are not transferred to the long, who must exercise or sell the option to realize any increase in its value. By contrast, if the option value decreases, the short may withdraw any excess funds from its account.

Under the proposed futures-style margin system, both the long and the short position holders would post risk-based original margin upon entering their option positions. During the life of the option, the option value would be marked to market daily, and gains and losses would be paid and collected on a daily basis just like futures positions are maintained today.[1]

The CFTC published a notice of petition for rulemaking in the Federal Register[2] of March 17, 1989. In the Federal Register, the CFTC identified a number of benefits arising from futures-style margining of options. The CFTC also raised some concerns with respect to futures-style margining. A significant source of confusion, the CFTC pointed out, could be the differences in option pricing which could result from futures-style margining. Option premiums potentially would be higher under a futures-style system because shorts would demand a higher price to compensate for the loss of interest on the full premium and longs would be willing to pay a higher price because they would be gaining such interest income. But exactly how the proposed system would change the equilibrium option pricing formula is unclear.

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[1] However, a futures-style option is unlike a futures contract in that the long side of the option position can exercise the option either on any business day (if American) or at option expiration (if European).

[2] The proposed rule change affects only options on futures contracts. Options on physical (cash) are not affected. The CFTC rule change should be distinguished from the proposed SEC rule change on equity options that was published in the Federal Register of June 21, 1989.

Derming Lieu is a Financial Economist in the Research Division at the Chicago Mercantile Exchange.

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This article uses arbitrage-free conditions to derive new pricing equations for futures-style options. It is found that the new formula under the proposed new system is not only simpler than the old formula, it also eliminates the early exercise problem that is associated with the old formula.

This article is organized as follows. An arbitrage-free pricing model for futures-style options using techniques similar to Black and Scholes [2] and Black [1] is set forth in the following section. A new put-call parity under the new system is proved as well. Section II compares the stock-style option model and futures-style option model. Section III presents three numerical examples to illustrate the new features of futures-style option. Section IV concludes the study.

I. THE MODEL

In their seminal article, Black and Scholes [2] use arbitrage-free conditions to develop an equilibrium option-pricing model for stock options. Black [1] applies the same principle to price stock-style European call options on futures. $F(t)$ is denoted here as the underlying futures price at current time t for the commodity that is to be settled on time T or later.[3] Black [1] assumes that the instantaneous futures price change relative is described by the stochastic differential equation.

$$dF/F = \mu dt + \sigma d\,W(t)\,, \tag{1}$$

where $W(t)$ is a standard Gauss-Wiener process at time t, μ is the expected instantaneous price change relative of the futures contract, and σ is the instantaneous standard deviation of the futures price. Using arbitrage-free conditions Black [1] derive the following differential equation:

$$(1/2)c_{FF}\sigma^2F^2 + c_t - rc = 0\,, \tag{2}$$

where c is the price of a European call option on a futures contract, r is the risk-free interest rate, and the subscripts denote first partial derivatives. Employing a suitable boundary condition, Black [1] solves eq. (2) for the following formula for the value of a call option on a futures contract:

$$c(t) = \exp(-r\tau)\,[F(t)N(d_1) - XN(d_2)]\,, \tag{3}$$

where T is the expiration date for options,

X is the exercise price, and
N is the cumulative standard normal distribution function,
$d_1 = \{\ln (F/X) + (½)\sigma^2\tau\}/\sigma\sqrt{\tau}\,,$
$d_2 = d_1 - \sigma\sqrt{\tau}\,,$
$\tau \equiv T - t\,.$

With a different boundary condition, the partial differential eq. (2) can be used to derive the formula for the value of a European put option on a futures contract:

$$p(t) = \exp(-r\tau)\,[-F(t)N(-d_1) + XN(-d_2)]\,. \tag{4}$$

[3]For physical delivery futures, the expiration date of futures options is always earlier than the settlement date of its underlying futures. For cash settlement futures, the expiration dates of both futures and options could be the same. The difference in expiration date for futures and options does not affect the result in this article.

These two European option formula, although subject to early exercise modification for American options, are widely used in the futures industry.[4] The fomula, however, is not applicable under the proposed futures-style margining system even for European options because of the mark-to-market nature of the options. In the following, an approach similar to Black [1] to derive equilibrium pricing equations for futures-style options is used.

Like Black [1] a frictionless ideal market with no transaction costs in the options and futures is assumed. The call price function c can be written as:

$$c(t) = c(F(t), t). \tag{5}$$

In the spirit to the Black-Scholes formulation, focus is on the possibility of creating a riskless hedge by forming a portfolio V composed of futures-style call options c and its underlying futures F. Since the value of a futures contract is zero at inception and under futures-style margining the value of a call option is also zero at time t, the value of this hedge portfolio, V, is zero at t.

$$V(t) = 0. \tag{6}$$

However, the change in the value of the hedge is not zero. The change in the value of the hedge, dV, can be expressed in the following equation:

$$dV = a\,dF + b\,dc, \tag{7}$$

where a is the number of futures contracts and b the quantity of calls purchased, dF and dc denote infinitesimal change for F and c.

Like Black [1], it is assumed that the instantaneous futures price change relative is described by the stochastic differential equation (1). Ito's lemma can be employed to express dc in (5) as

$$dc = c_F\,dF + c_t\,dt + (1/2)c_{FF}\sigma^2F^2\,dt. \tag{8}$$

Substituting the expression given for dc in (8) for dc in (7) yields:

$$\begin{aligned} dV &= a\,dF + b[c_F\,dF + c_t\,dt + (1/2)c_{FF}\sigma^2F^2\,dt], \\ &= (a + b\,c_F)dF + b[c_t + (1/2)c_{FF}\sigma^2F^2]dt. \end{aligned} \tag{9}$$

Note that the only stochastic term in the expression for dV is dF. All other terms are strictly deterministic. To make dV riskless:

$$a + bc_F = 0. \tag{10}$$

Indefinitely many solutions exist to eq. (10). One simple solution is to set $a = 1$ and $b = -1/c_F$. This yields:

$$dV = (-1/c_F)[c_t + (1/2)c_{FF}\sigma^2F^2]\,dt. \tag{11}$$

Because the hedge is riskless and requires no initial capital, to eliminate arbitrage opportunities its return must be zero. This implies:

$$dV = 0. \tag{12}$$

[4]Whaley [8] provides an analytical approximation for an American call option on a futures contract. He finds that only when the option is considerably in-the-money does the early exercise premium account for a significant proportion of the price of the option.

Substituting (11) into (12) defines a differential equation for the value of the futures-style call option:

$$(1/2)c_{FF}\sigma^2F^2 + c_t = 0. \tag{13}$$

The differential equation must conform to the boundary conditions:

$$c(F(T), T, X)) = \text{Max}(0, F(T) - X), \tag{14}$$

where T is the date when the option expires. The differential eq. (13) is a simplified version of the eq. (2) in Black [1] with $r = 0$. The solution of this kind of differential equation is well documented in Black and Scholes [2, p. 643] and Merton [6, p. 166]. Applying the same principle of change of variable to (13), the equilibrium price for futures-style call option is:

$$c(t) = F.N(d_1) - X.N(d_2), \tag{15}$$

where $d_1 = \{\ln (F/X) + (½)\sigma^2\tau\}/\sigma\sqrt{\tau}$,

$d_2 = d_1 - \sigma\sqrt{\tau}$ and

$\tau \equiv T - t$.

Futures-style margining also changes the put-call parity for options on futures. The new relation between a call and a put is established as Property A.

Property A: Under futures-style margining, the relationship between a put and call option on a futures contract now becomes[5]:

$$p(F(t), X, T) = c(F(t), X, T) + X - F(t). \tag{16}$$

Proof: Refer to Appendix A.

Due to the marked-to-market nature of futures-style options, a "rollover" portfolio that changes the number of shares of the component security according to the discount rate is needed to prove Property A. The rollover portfolio need not be held to the options' expiration date. If the rollover portfolio is dissolved on or before the expiration date the relation established in the proof remains. Thus, the new put-call parity is true not only for European options, but for the American options as well.

The pricing equation for futures-style put options on futures is obtained by substituting eq (15) into (16):

$$\begin{aligned} p(t) &= F(t).N(d_1) - X.N(d_2) + X - F(t), \\ &= F(t)[N(d_1) - 1] - X[N(d_2) - 1)], \\ &= -F(t)N(-d_1) + XN(-d_2). \end{aligned} \tag{17}$$

Observe that the only difference between the Black model (3) and (4) for stock-style options and model (15) and (17) for futures-style options is that the discount factor $\exp(-r\tau)$ disappears from the new model and interest rates drop out of the futures-style option formula completely. This makes sense because it is no longer necessary to consider borrowing to pay for an option premium, or investing premiums from short options. Because the discount factor is less than one, i.e., ($\exp(-r\tau) < 1$), both call and put prices are higher. This is expected because the writer no longer earns interest on the premium.

[5] By way of contrast, the put-call parity for options on futures with stock-style margining is

$$p(F(t), X, T) = c(F(t), X, T) + \exp(-r\tau)[X - F(t)].$$

II. COMPARISON OF THE OPTION-PRICING FORMULA UNDER DIFFERENT MARGINING SYSTEM

In addition to having a new put-call parity, futures-style margining produces the following properties for options on a futures contract.

Early exercise is a problem for the Black call and put model (3) and (4) for stock-style options.[6] Under futures-style margining, however, early exercise is not a problem for either American call options or for American put options. This assertion is proved in Property B and Property C.

Property B: The call premium formula (15) under futures-style margining always exceeds the call's intrinsic value for any positive τ.

Proof: For out-of-the-money and at-the-money call options, the options' intrinsic value is zero. Since the call premium is always positive with positive time to expiration date,[7] the statement is trivial in these two cases. For in-the-money call options, i.e., $F(t) > X$, it is defined that $a \equiv (F/X)$ and $y(\tau) \equiv [c(t)/X]$, then

$$y(\tau) = aN(d_1) - N(d_2)$$

Taking derivative with respect to τ:

$$\begin{aligned} y'(\tau) &= aN'(d_1)d_1'(\tau) - N'(d_2)d_2'(\tau), \\ &= [\exp(-d_1^2)a\sigma]/[2(2\pi)^{1/2}\sqrt{\tau}] > 0 \end{aligned}$$

Thus, $y(\tau)$ is a strictly increasing function of τ. And for $\tau > 0$:

$$y(\tau) > \lim_{\tau \to 0} y(\tau)$$

[6] Refer to Whaley [8] for a detailed discussion.

[7] The value of a call in eq. (15) is also the result of a positive integration; therefore, the call premium must be positive. To prove this assertion, note that the same call pricing formula can be derived alternatively by computing the call's expected terminal value using the argument of risk-neutrality as suggested in Harrison and Kreps [4]. In this ideal world, the equilibrium rate of return to all assets that require positive capital must equal the risk-free rate r. A security that requires no capital will have zero as its drift term. If the drift term is not zero for such a security, there will exist an arbitrage opportunity. One then considers a rollover of futures contracts. The futures contract requires no capital outflow throughout its life except on the expiration date. Thus, the futures price dynamics in this world must reduce to:

$$dF/F = \sigma\, dW(t).$$

Similarly, a rollover futures-style options also require no capital outflow throughout its life except on the expiration date. The expected average rate of growth in the value of the option must also be zero. That means a futures-style option should be priced according to its expected value. If it is priced otherwise, the option would have a nonzero expected return and there will exist an arbitrage opportunity.

Therefore, to eliminate arbitrage opportunities, the current price for futures-style call option must be the expected terminal value:

$$\begin{aligned} c(t) &= E[c(T)], \\ &= \int_X^\infty (F(T) - X)L'(F)\,dF, \end{aligned}$$

where L is the cumulative log-normal distribution function for $F(T)$ and $F(T)$ follows

$$dF/F = \sigma\, dW(t).$$

The integration can be solved very easily using the lognormal theorem from Smith [7, p. 16] and yields a solution to the call pricing problem the same as eq. (15).

Thus, it is proved that the value of a call in eq. (15) is also the result of a positive integration. It is easy to prove that the result must be positive because of the positive probability that the call might be in-the-money at the terminal date.

But $$\lim_{\tau \to 0} y(\tau) = a.N(\infty) - N(\infty) = a - 1$$

Thus it is proved that $y(\tau) > a - 1$ for $\tau > 0$. Using the fact that $y(\tau) = c(\tau)/X$ and $a = F(t)/X$, the inequality can be converted to

$$c(\tau) > F(t) - X, \qquad \text{for } \tau > 0\,.$$

Property C: Under futures-style margining the put premium formula (17) always exceed the put's intrinsic value for any positive τ.

Proof: For at-the-money and out-of-the-money put options, their intrinsic value is zero. Since the put premium is always positive with positive τ, the statement is trivial in these two cases. For in-the-money put option, i.e., $F(t) < X$, the new put-call parity (16) can be used

$$p(F(t),X,T) = c(F(t),X,T) + X - F(t)\,.$$

Because $c(F(t),X,T) > 0$ for positive τ, we immediately have

$$p(F(t),X,T) > (X - F(t))\,,$$

where $(X - F(t))$ is the intrinsic value for in-the-money put option.

One of the consequences from Property B and Property C is that premature exercising of an American futures-style option is never optimal. Because funds are never tied up in the form of inaccessible premiums, there is never an incentive to exercise a futures-style option early. If an option is never exercised prematurely and if it is also margined like a futures contract, the option is more like a futures contract than a conventional stock-style option.

Property D: Futures-style margining turns a European option on a futures contract into a futures contract on an option (on a futures contract)[8].

Proof: $c(t)$ is denoted as the call price at time t of a stock-style European option on a futures contract and $c_*(t)$ as the call price of a similar option on a futures contract with futures-style margining. Both options expire on time T. From eqs. (3) and (15):

$$c_*(t) = \exp(r\tau)c(t)\,,$$

where $\tau = T - t$.

Now consider a futures contract that delivers the corresponding stock-style call option on a futures contract at expiration time T. Note that both the futures on the option contract and the futures' underlying stock-style option contract expire at time T. The price of such a futures contract on an option at time t is denoted as $F_c(t)$. Observe that the underlying stock-style option does not produce dividend and there is virtually no storage cost associated with an option contract other than the decay of an option's time value that is impounded in the price of an option. Also, note that an option can be sold short freely, Thus, perfect conditions exist to allow one to relate the price of such a futures contract to its underlying cash price, i.e., the price of a stock-style call option as:[9]

$$F_c(t) = \exp(r\tau)c(t)\,,$$

which is the same as $c_*(t)$, the price of a call option with futures-style margining.

Thus, it is established that the price of a futures contract on a stock-style call option must equal the price of a call option with futures-style margining before expiration time T. The same reasoning can be applied to prove the case for the futures contract on a put option.

[8]A referee of this Journal pointed out that this property was originally argued by Duffie [3], p. 285.

[9]The arbitrage-free equation can be obtained easily using put-call parity for stock-style options.

Because the theoretical price of a futures contract on a stock-style option equals that of a futures-style option at any time t that is before the expiration date T, and because early exercise of a future-style option is never optimal, it is proved that a futures contract on a stock-style option and a futures-style option are virtually indistinguishable.

The effect on European stock-style put prices of a decrease in the time to maturity (the option theta) is ambiguous.[10] However, with futures-style margining, the sign of the option theta is clear.

Property E: With future-style margining, both the call option theta (Θ) and put option theta are equal and positive.

Proof: Taking partial derivative of eq. (16) with respect to τ, $c'(\tau) = p'(\tau)$. In the proof of Property B, $y'(\tau) = [c'(\tau)/X] > 0$. Since $X > 0$, $\Theta \equiv c'(\tau) = p'(\tau) > 0$.

Thus, unlike the theta of a stock-style put, the theta of a futures-style put is positive. For futures options with the same exercise price and the same expiration date, the call and put option thetas are also equal.

The price of an option can be divided into two parts: the intrinsic value and the time value, where the time value measures the amount of the option price that is lost if the option is held to expiration and the underlying security price remains unchanged. The time value of an option measures the riskiness of an option and is the basis of the risk-margin charged to the option traders. A surprising fact is that under futures-style margining the time value for a call and a put option with the same exercise price is always equal.

Property F: the time values for a call and a put option with the same exercise and the same expiration date are always equal.

Proof: By definition, the time value of an option is the option's price minus its intrinsic value (if positive). For $F(t) = X$, by eq. (16) the price of a call is identical to that of a put. Since the intrinsic value for both the call and the put are zero, their time values must be equal.

For $F(t) > X$ the call is in-the-money and

$$c(t) = \text{call's time-value} + F(t) - X$$

Substituting the above equation into eq. (16),

$$p(t) = \text{call's time-value}.$$

When the call is in-the-money, the corresponding put must be out-of-the-money and the put's entire premium is the time value. Thus, the time values of a call and a put are equal. Similar proof can be applied to the case for $F(t) < X$.

Property F implies that the premiums of a call and a put option with the same exercise and the same expiration date are equal after adjusting by the intrinsic value. Thus, the conversion between a call and a put is now very simple for options with the same exercise. One can easily judge whether the premium of an option is out-of-line with that of the same class option by adding the price of an out-of-the-money option, which is the time value of this option class, to the in-the-money-option's intrinsic value.

III. EXAMPLES

The only organized Exchange that currently uses futures-style margining on option contracts is the London International Financial Futures Exchange (LIFFE). However, the model developed here does not apply to option contracts that are traded at LIFFE.

[10]Refer to Jarrow and Rudd [5, p. 210] for proof.

LIFFE trades options on foreign-currency cash not futures as required by the model. LIFFE trades options on interest rate futures. But interest rate futures contracts do not satisfy the distributional assumptions that are demanded by the Black-Scholes type option models. Thus, this proposed model cannot apply to an option on interest rate futures. LIFFE does trade Financial Time Stock Exchange 100 Index (FTSE) futures that can use the model in this paper, Unfortunately, the trading volume in FTSE at LIFFE is virtually nonexistent and trading information for this contract is not reported at all. Thus, no good laboratory currently exists to test the model. Instead, the following numerical examples are used to illustrate the difference between the stock-style margin system and the proposed futures-style margin system.

In each case, it is assumed that an at-the-money call with an exercise price of 270 and sixty days to expiration is purchased. The annualized standard deviation for the futures prices is assumed to be .132 and constant throughout the time. The yearly interest rate is assumed to be fixed at 0.1. Suppose the call option is priced at 2853.43 for stock-style margining and at 2901.37 for futures-style margining.

Example 1: Futures price unchanged. Assume that the futures price remains at 270 on the day 2 and remains the same on the day 59 and 60. Under both stock-style and futures-style margining, the long's loss is limited to the initial premium. But the amount of payments and the timing of the payments differ. Note that the price of a call option with futures-style margining initially differs significantly from that of a call option with stock-style margining. The difference, however, disappears as the expiration date draws very near.

Stock-Style Margining

Long	Short
Day 1-Pays full premium of 2853.43	Day 1-Posts full premium received from long plus initial margin.
Day 2-Pays no additional funds Premium is now = 2820.1	Day 2-May withdraw $33 which is the decrease in value of option position since a day before
Day 2-to 58-Pays no additional funds	Day 2 to 58-May withdraw amount equal to decrease in value of option position
Day 59-Pays no additional funds Premium is now = 449.45	Day 59-May withdraw $186 which is the decrease in value of option position since a day before
Day 60-Option expires valueless Nothing is returned	Day 60-Option expires valueless. Collects $449.45. Initial margin is returned.

Futures-Style Margining

Long	Short
Day 1-Post initial margin Premium = 2901.37	Day 1-Posts initial margin
Day 2-Pays variation margin $35 Premium is now = 2866.33	Day2-Collects $35 which is the decrease in value of option position since a day before
Day 2-58-Pays variation margin	Day 2-58-May withdraw amount equal to decrease in value of option position
Day 59-Pays variation margin $186; Premium is now = 449.6	Day 59-May withdraw $186 which is the decrease in value of option position since a day before

Long	Short
Day 60-Option expires valueless Pays variation margin $449.6 Nothing is returned.	Day 60-Option expires valueless. Collects $449.6. Initial margin is returned.

Example 2: Futures price increases. Assume that the futures price rises above the exercise price to 285. The option matures "in-the-money" by 15 points, or $7,500 per contract. Under both systems, the long's profits are the same. Again, the timing of the payments differs.

Stock-Style Margining

Long	Short
Day 1-Pays full premium of $2853.43	Day 1-Posts full premium received from long plus initial margin
Day 2 to 59-Collects nothing over life of option	Day 2 to 59-Posts additional funds equal to the increase in value of option position over the life of option
Day 60-Option matures with a premium of $7,500 for a gain of $4,646.57.	Day 60-Option matures with a premium of $7,500 for a loss of $4,646.57. Initial margin is returned.

Futures-Style Margining

Long	Short
Day 1-Post initial margin Premium = $2901.37	Day 1-Posts initial margin
Day 2-to 59-Over life of option collects aggregate settlement variation up to $4,598.63	Day 2 to 59-Over life of option pays aggregate settlement variation up to $4,598.63.
Day 60-Option matures with a premium of $7,500. Collect final variation. Margin is returned.	Day 60-Option matures with a premium of $7,500. Pay final variation. Nothing is returned.

Note that the long might also choose to exercise the in-the-money call. Exercising a futures-style option is analogous to taking delivery on a futures position. To receive a cash commodity by taking delivery on a futures contract, the long must pay the settlement price of the futures contract prevailing at the time of delivery. Similarly, to obtain a futures position by exercising an option, the long must pay the settlement price of the option prevailing at the time of exercise. In other words, the long must pay the full premium marked to market on the day of exercise. Under a futures-style margining system, this payment is offset by the various payments received by the long during the life of the option. The differences between this procedure and the exercise of stock-style options are demonstrated in the following example.

Stock-Style Margining

Long	Short
Exercises option	Option is exercised
Receives long fuutres position at strike price of 270 Futures position is marked-to-market by the Clearing House and the long is credited with $7,500 ($500 × (285–270))	Receives short futures position at strike price of 270. Futures position is marked-to-market by the Clearing House and the short is debited $7,500

Options position is closed thru exercise, but risk margin is retained until the futures position is offset.	Options position is closed thru exercise, but risk margin is retained until the futures position is offset.

Futures-Style Margining

Long	Short
Exercises option	Option is exercised
Clearing House debits account for premium settlement price of $7,500	Clearing House credits short with $7,500 settlement of premium
Receive long futures position at strike price of 270 Futures position is marked-to-market by the Clearing House and the long is credited with $7,500 ($500 × (285–270))	Receives short futures position at strike price of 270. Futures position is marked-to-market by the Clearing House and the short is debited $7,500
Options position is closed thru exercise, but risk margin is retained until the futures position is offset	Options position is closed thru exercise, but risk margin is retained until the futures position is offset

Since both the gains and losses are already paid to the respective positions through mark-to-market payments, the need for exercise may appear redundant. However, the procedures are required to assure convergence of the option premium on the amount the option is in-the-money.

IV. SUMMARY AND CONCLUSION

This article derives new arbitrage-free option-pricing equations with futures-style margining. The interest rate factor is completely dropped from the new formula. The early-exercise feature of American futures options no longer matters for the options with futures-style margining. As a result, the value of European futures options and American futures options converge. This study proves that futures-style margining turns an option on a futures contract into a futures contract on a stock-style option. Thus, a futures-style option can be considered more of a futures contract than an option contract.

Futures-style margining calls for a new put-call parity, which makes the conversion from a call option to a put option or an option to its underlying futures much simpler. In fact, under the proposed futures-style margining system, there no longer exists any structural difference between an option on a futures contract and its underlying futures contract. Only the difference in return-loss profile remains. Together with the underlying futures, futures-style options can be used to create a portfolio with desired payout structure associated to the underlying futures.

The presidential Working Group on Financial Markets claim in its Interim Report that the current stock-style option margin system was an impediment to the smooth flow of capital during the October market crisis. The Working Group called for the exploration of futures-style margins for options as one mechanism to "reduce cash transfers, simplify settlement systems, and unify clearing."[11] The simplified option-pricing formula for options with futures-style margining derived in this article is a response to their call.

[11]Interim Report of the Working Group on Financial Markets ("Working Group Report") at p. 10 and Appendix D (May 1988).

It must be noted, that the traditional shortcomings associated to the Black-Scholes type option models apply to this model as well. The distributional assumptions made here make the model unfit to value options on interest rate futures. Although interest rate factors are completely dropped from the derivation of the main model, the assumption of a deterministic interest rate is required to derive a new put-call parity and Property D. Also, early exercise is never optimal in the model. But the exercise procedures developed by each Exchange may trigger a case when premature exercise is feasible.

Appendix A

Proof of the new put-call parity (16)

$$p(F(t), X, T) = c(F(t), X, T) + X - F(t).$$

Consider a "rollover" portfolio that contains $\exp(-r(T - t))$ units of long call and $\exp(-r(T - t))$ units of short put on a futures contract with the same exercise price and expiration date, and $\exp(-r(T - t))$ units of short position in the futures contract on date t. On the second day the number of contracts purchased or sold on the options and futures is changed to $\exp(-r(T - t - 1))$. On the third day it is changed to $\exp(-r(T - t - 2))$, and so on.

To understand the nature of the rollover strategy, consider the first day's settlement activity for the long call options. At the beginning of the first day, $\exp(-r(T - t))$ call options are purchased at price $c(t)$. At the end of the day, the contracts are marked to market, and the long registers a gain (loss) of $\exp(-r(T - t))\,[c(t + 1) - c(t)]$. On the second day, the gain is $\exp(-t(T - t - 1))\,[c(t + 2) - c(t + 1)]$, and on the third day, $\exp(-t(T - t - 2))\,[c(t + 3) - c(t + 2)]$. If each of these daily gains (losses) is then invested at the riskless rate until the end of the futures contract life, the terminal value of the rollover long options position will be as follows:

$$\begin{gathered}\exp(-r(T - t))\,[c(t + 1) - c(t)]\exp(r(T - t)) + \\ \exp(-r(T - t - 1))\,[c(t + 2) - c(t + 1)]\exp(r(T - t - 1) + \\ \exp(-r(T - t - 2))\,[c(t + 3) - c(t + 2)]\exp(r(T - t - 2)) + \\ \ldots + \exp(-r)\,[c(T) - c(T - 1)]\exp(r) \\ = c(T) - c(t)\end{gathered}$$

But

$$c(T) = 0 \qquad \text{for } F(T) < X$$

and

$$c(T) = F(T) - X \qquad \text{for } F(T) >= X$$

Thus the rollover call position at terminal date T must have value $= -c(t)$ if $F(T) < X$ or have value $= F(T) - X - c(t)$ if $F(T) > X$. Similar analysis can be applied to rollover put options and futures position to come up with the terminal value of the rollover portfolio.

With the strategy, one can compute the terminal value of each component security and the terminal value of the portfolio as

$$\begin{aligned} &-c(t) - [X - F(T) - p] - [F(T) - F(t)] \\ &= -c(t) - X + p(t) + F(t) \qquad \text{for } X >= F(T) \end{aligned}$$

and

$$\begin{aligned} &[F(T) - X - c] + p(t) - [F(T) - F(t)] \\ &= -c(t) - X + p(t) + F(t) \qquad \text{for } X < F(T) \end{aligned}$$

Note that regardless of the futures price at terminal time T, the portfolio has the same terminal value of $[-c(t) - X + p(t) + F(t)]$. Because it costs nothing to set up the portfolio, to prevent riskless arbitrage opportunities from arising, the terminal values must be zero. Hence, $-c(t) - X + p(t) + F(t) = 0$. And eq. (16) must hold.

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