The Relationship Between the Volatilities of the S&P 500 Index and Futures Contracts Implicit in Their Call Option Prices

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I. INTRODUCTION

The volatility of the stock market is a matter of great concern to investors. The high level of market volatility has attracted regulatory attention since the crash of October 19, 1987. The stock market is believed to be more volatile now than it has been in the past. Investor surveys conducted after the crash suggest that the increased volatility of the market and the ensuing loss of investor confidence has caused many investors to stay out of the market.[1] Some recent estimates of market volatility are based on inter-day price movement.[2] Volatility measures that take into account the long-term future prospects of the market are desirable measures from the perspective of long-term investors.

Volatility as contained implicitly in the prices of options on a market index provides one measure of anticipated variability of market returns for the longer term. Such a volatility measure incorporates the *ex-ante* estimates of market participants and is therefore a valuable indicator of expected variability. In an analogous manner, stock index futures options provide a means of measuring market participants' *ex-ante* assessment regarding average volatility of futures prices changes on a daily basis.[3]

The implicit volatility of the market index is analyzed in a number of recent studies. Poterba and Summers (1986) analyze whether or not volatility changes persist for the implied volatility of the S&P 500 index. They conclude that the effect of shocks are not persistent. Much of the observed variation in stock prices is therefore due to factors other than fluctuating risk premia brought about by volatility changes. Franks and Schwartz (1988) investigate the impact of variables such as corporate leverage, trading

We wish to thank Keith Fairchild and two reviewers of this *Journal* for their many helpful comments. We are responsible for all the remaining errors. Misra's research was supported in part by a research grant from the Dean, College of Business at UTSA.

[1]See Power (1988). Edwards (1988) also points out that increased volatility results in a loss of investor confidence.

[2]Edwards (1988) computes the daily close-to-close volatility and high-low volatility of the market index. Grossman (1988) conducts a study of market volatility and trading volume. He uses different measures of inter-day price movement and finds no relationship between volatility and volume.

[3]The volatility of index return is subsequently referred to as volatility of the index. Similarly, the volatility of index futures price changes is referred to as the volatility of the index futures.

The Journal of Futures Markets, Vol. 10, No. 3, 273–285 (1990)

CCC 0270-7314/90/030273-13$04.00

volume, interest rates, inflation, oil prices, and exchange rates on the changes in the implied volatility for the Financial Times Stock Exchange Index (FTSE). They show that leverage, volume, and exchange rates explain volatility changes. While the implicit volatility of the index and the stock index futures has been studied individually, the relationship between these two volatility measures has not been empirically analyzed.

Modest and Sundaresan (1983) and Brenner, Courtadon, and Subrahmanyam (1985) indicate that if the futures (or forward) price of the index is related to the spot price by the "cost-of-carry" model, then, in the absence of transactions costs, interest rate risk, and dividend uncertainty the volatilities of the index and the index futures should be identical. Under these conditions, the implied volatility obtained from the index and index futures options should be equal. Realistic inclusion of stochastic interest rates, dividend yield, marking-to-market, transaction costs, and taxes may invalidate the cost-of-carry model. Whether the equality relationship between the two implicit volatility measures actually holds is, therefore, an empirical issue. Under the assumption of option market efficiency, unequal implied volatilities may imply also that the cost-of-carry model is not a correct specification of futures pricing.

One hypothesis, attributed to Samuelson (1965), states that futures price volatility increases as the contract nears maturity. This hypothesis assumes that futures prices are unbiased estimators of the future spot prices and that the spot price follows a first order autoregressive process. With a deterministic relationship between volatility and maturity, the optimal hedge ratio is time varying. This hypothesis is tested with the implied volatility of futures by Ball and Torous (1986) using gold, sugar, and Deutsche Marks futures options and by Park and Sears (1985) using S&P 500 and NYSE futures options. Ball and Torous find mixed support of the hypothesis and Park and Sears obtain results contrary to the hypothesis.[4] Finally, aggregate leverage changes may explain inter-day volatility changes (Christie (1982)). This hypothesis is tested by Franks and Schwartz with the FTSE implied volatility data.

The properties of implied volatility measures (of S&P 500 Index options and S&P 500 Index Futures options) are analyzed in this study. Hypotheses regarding maturity and leverage on volatility are examined. Finally, the relationship between the implied volatility of the index and index futures is examined. The behavior of the index and index futures markets and their relationships before and after the market crash of October 1987 are investigated, also. The following section reviews the implied volatility literature; Section III describes the data and methodology; Section IV presents the results of the tests of various hypotheses; and Section V contains a summary and conclusion.

II. VOLATILITIES IMPLIED IN OPTIONS PRICES

Option prices may contain information not contained in stock prices. The Black and Scholes (1973) formula for options, the Black (1976) formula for futures options, and the Garman and Kohlhagen (1983) formula for foreign currency options have been used to estimate the volatility or other parameters of the underlying process.[5] The volatility of futures prices implied in their options have been examined by Ball and Torous and by Park and Sears. The futures options model of Black is used here in the context of the S&P 500 futures call options.

[4]Anderson (1985) conducts a direct test of the Samuelson hypothesis using an extensive sample of futures price data. He finds that the changing variance of futures prices are due primarily to seasonal factors and secondarily to maturity effects. Anderson's sample, however, does not include stock index futures data.

[5]The Chiras and Manaster (1978) study of implied volatility, Manaster and Rendleman (1982) study of implied stock prices and, Peterson and Tucker (1988) study of implied foreign currency prices, provide some examples where options prices are used to estimate various parameters of the underlying process.

$$C_f = e^{-rt}[F \cdot N(d_1) - X \cdot N(d_2)] \tag{1}$$

$C_f \equiv$ price of call option on the S&P futures contract
$F \equiv$ S&P futures price
$X \equiv$ the call option's exercise price
$r \equiv$ the riskfree rate of return
$t \equiv$ the time to maturity of the option
$\sigma_f^2 \equiv$ the instantaneous variance of $\ln(F_t/F_{t-1})$
$N(\cdot) \equiv$ cumulative normal density function
$d_1 \equiv \{\ln(F/X)/(\sigma_f \cdot \sqrt{t}) + 0.5 \cdot (\sigma_f \cdot \sqrt{t})\}$
$d_2 \equiv d_1 - (\sigma_f \cdot \sqrt{t})$

The Black-Scholes formula adjusted for continuous dividends is a pricing formula for the S&P 500 index option.

$$C_s = e^{-dt} \cdot S \cdot N(d_1) - e^{-rt} \cdot X \cdot N(d_2) \tag{2}$$

X, r, t and $N(\cdot)$ are as defined before

$C_s \equiv$ the price of the call option on the index
$d \equiv$ continuous dividend yield
$S \equiv$ the value of the S&P 500 index
$\sigma_s^2 \equiv$ the instantaneous variance of $\ln(S_t/S_{t-1})$
$d_1 \equiv \{\ln(S/X)/(\sigma_s \cdot \sqrt{t}) + (r - d + 0.5\sigma_s^2) \cdot (\sqrt{t}/\sigma_s)\}$
$d_2 \equiv d_1 - (\sigma_s \cdot \sqrt{t})$

The cost-of-carry model relates the price of the futures contract to the underlying index. Under nonstochastic interest rates and dividend yields:

$$F = S \cdot e^{(r-d) \cdot t} \tag{3}$$

If eq. (3) is true and $r - d$ is nonstochastic, then σ_s and σ_f would be identical.[6] Stoll and Whaley (1988) argue that realistically the cost-of-carry relationship is violated.[7]

Direct computations of the volatilities can be undertaken from the index and the index futures data. The quality of estimates from time series data is suspect, especially in view of market conditions since the crash. The option pricing models (1) and (2) allow for imputation of the volatilities from the option data. As a practical matter, the Black model and the Black-Scholes model describe option pricing well.[8] An analysis of the relationship between volatilities of the index and the index futures is conducted therefore with the *Implied Standard Deviation* (ISD) obtained from the options data.

[6] $F_t = S_t \cdot e^{(r-d)t}$. Lagging by one period, $F_{t-1} = S_{t-1} \cdot e^{(r-d)\cdot(t+1)}$. Take the ratio, log and variance of both sides. The $\sigma_f^2 \equiv \text{Var}\{\ln(F_t/F_{t-1})\}$ equals $\sigma_s^2 \equiv \text{Var}\{\ln(S_t/S_{t-1})$ if $(r - d)$ is nonstochastic. Implied volatility of index option and futures option should be identical under this condition since Black-Scholes's formula (2) is identical to Black's formula (1). If r and d are stochastic, then $\sigma_f^2 = \sigma_s^2 + \sigma_d^2 + \sigma_r^2 + 2[\sigma_{sd} + \sigma_{sr} + \sigma_{rd}]$.

[7] The role of dividends complicates the issue. The cost-of-carry model for futures prices does not hold with uncertain cash dividends. Ball and Torous point out that if investors' expectations about future dividends change sufficiently smoothly, then Black's futures options formula holds even if the cost-of-carry model does not obtain. Brenner, Courtadon, and Subrahmanyam show by numerical solutions that an American call option on futures is worth more than a call option on the cash instrument due to the early exercise potential inherent in American options. Intermediate dividend payments on the cash instrument, however, narrow the difference between the call option price on the cash and the futures instrument.

[8] See Whaley (1982), Ball and Torous (1986), and the references cited in Park and Sears (1985), in support of the practical applicability of both the Black and the Black-Scholes models.

III. DATA AND METHODOLOGY

Data

The daily closing prices of the S&P 500 Index Call Option were collected for the 13 month period of June 1, 1987, through June 30, 1988 from the *Wall Street Journal*. Index options and index future options contracts expire in March, June, September, and December. Call price data for 5 contracts (September and December 1987; March, June, and September 1988) was collected.

The closing prices of seven call options corresponding to a specific maturity were collected daily. At least four of the options were out-of-the-money. Option prices less than $1 were not included in the sample since the minimum tick requirement may bias the price. Options were excluded if they violated arbitrage conditions: (a) the price of an option is less than its intrinsic value, or (b) the price of a lower strike price call option is less than the price of a higher strike option. The number of options used was therefore less than the requisite seven options on some days. A day was treated as missing if a minimum of four call option prices were not available.

A total of 2,513 call option prices were collected. During the two weeks following the market crash of October 19, 1987, the prices reported in the *Wall Street Journal* usually violated arbitrage conditions. Therefore, data for the period from October 16, 1987 to October 30, 1987, was excluded. Options within six days of maturity were excluded also since the implied volatility estimates for options close to maturity are not reliable (Ball and Torous (1986, p. 868)).

The closing price of the S&P 500 Index was obtained from the *Wall Street Journal*. The T-Bill yield was obtained from the *Wall Street Journal* for a T-bill maturing at the same time (within five days) as the option. The dividend yield of the S&P 500 Index was obtained from *Barron's*.

The prices of S&P 500 Futures call options maturing at the same time as the Index call options were collected using criteria similar to that of the S&P 500 Index call options. The prices of five futures call options were collected daily corresponding to each maturity. Options priced less than $1 were not used due to the minimum tick bias. If at least three call options prices were not available during a day, that day was treated as a missing observation. The prices of 2,680 call options on the S&P 500 Index Futures were collected. As with the index options, observations during the period October 16, 1987 to October 30, 1987, and options within six days of maturity were deleted. Daily closing prices for S&P 500 Futures with maturities corresponding to the options were collected from the *Wall Street Journal*.

Methodology

Volatility of the underlying process which is implicit in option prices can be computed by various methods. Implied volatility was computed by using two approaches in this article. A technique used by Whaley (1982) and applied by Park and Sears in the context of S&P 500 Futures Options was employed. The Whaley approach yielded an ISD estimate that minimizes the sum of squared error between actual and computed option prices on any day for a set of call options with different exercise prices but having a common maturity.[9] Thus, for example, on August 24, 1987, three different ISDs were

[9]The market price $C_j (j = f \text{ or } s)$ is expanded by Taylor series around a formula price based on some σ° (when the true volatility is σ):

$$C_j = C(\sigma^\circ)_j + \delta C_j/\delta\sigma^\circ \cdot (\sigma - \sigma^\circ) + \ldots \text{ higher order terms} + \epsilon_j$$

Higher order terms are ignored. An estimate $\hat{\sigma}$ of σ is found by applying OLS iteratively, until improvements in the estimate of σ are less than a tolerance level, K. The tolerance level, K, is set equal to 0.0005. Iteration terminates when $|(\hat{\sigma} - \sigma^\circ)/\sigma^\circ| < K$. Convergence is generally achieved within 2 or 3 iterations.

computed corresponding to options (on both index and index futures) maturing in September 1987, December 1987, and March 1988. To conduct some of the tests, a common ISD for a day (across various maturities) was required. Weighted ISD (WISD) for a day was computed by the method used in Chiras and Manaster (1978) were ISDs of individual options are weighted by the price elasticity of options with respect to their ISD. Thus, one WISD estimate was obtained corresponding to August 24, 1987.[10]

IV. RESULTS

Descriptive

Descriptive statistics of the sample data are presented in Table I. On average, the sample options are out-of-the-money (the mean index and index futures prices are lower than the corresponding average exercise prices for the respective options). The riskfree interest rate (5.9%) is greater than the dividend yield (3.3%). Both the series are relatively stable over the period as evidenced by their low levels of volatility.

The ISD of the index and index futures are computed and paired by date and contract maturity (312 paired ISD observations).[11] All the option price data for the September 1987 contract is prior to the crash. Observations for the December 1987 contract are both before and after the crash. The March 1988, June 1988, and September 1988 contracts reflect only post crash data. ISDs of the S&P 500 futures near maturity contract are plotted on a daily basis in Figure 1. The figure is indicative of the great changes in volatility experienced by the market during the period.

The statistics pertaining to the index and index futures ISDs are shown in Table II. On average, the index futures ISDs (σ_f) are greater than those of the index (σ_s). Both series exhibit comparable levels of variability. The mean and standard deviation for each series are substantially greater after the crash than during the pre-crash period. The relationship between the volatilities of the index and the index futures is further explored in a following section.

The ISD estimated in this study may be compared to the volatility estimates for early 1983 as reported by Park and Sears. Their ISDs are of the order of 0.04 per month for the S&P 500 index futures options. The ISD estimated in this study is of the order of 0.22 per year in May and June of 1988, when the market uncertainty induced by the crash of 1987 appears to have subsided. It appears that the ISD of index futures is significantly less now than it was during the introductory stage of these contracts in early 1983.

Time Series Properties of WISD

Daily weighted average ISD (WISD) series are analyzed to study the persistance of the shocks to volatilities. WISD for the index and for the index futures ($WISD_s$ and $WISD_f$,

[10]The ISD of an option i is obtained as σ_i. The price elasticity of the option with respect to its ISD is; $\tau_i \equiv \{(\delta C_i/\delta\sigma_i) \cdot (\sigma_i/C_i)\}$. Given a set of options with different maturities and exercise prices, the Weighted ISD (WISD) corresponding to a day is computed by weighting the ISDs by their price elasticity. $WISD = [\{\Sigma\, ISD_i \cdot \tau_i\}/\{\Sigma\, \tau_i\}]$.

[11]σ_s should be computed by using the realized average dividend yield over the life of the contract. Following Klemkosky and Resnick (1979) we use the contemporaneous dividend yield for the following reasons: (i) In view of relatively stable dividend yields during the sample period, it is assumed that recent dividend yield provides the best estimate of future dividend yield and; (ii) the ISD estimate is relatively insensitive to dividend mis-estimation. Simulations indicate that a 20% mis-estimation of dividend yield results in approximately 1% mis-estimation of σ_s in the same direction. We thank a referee of the *Journal* for pointing out that σ_s estimates may be biased when contemporaneous dividend yield data is used.

Table I
DISTRIBUTIONAL STATISTICS OF THE S&P 500 INDEX AND INDEX FUTURES CALL OPTIONS (JUNE 1, 1987 THROUGH JUNE 30, 1988)

	Call Price of Index Options C_s	Call Price of Index Futures C_f	Exercise Price of Index Options X_s	Exercise Price of Futures Options X_f	S&P 500 Index S	S&P 500 Index Futures Prices F	Maturity in Days t	Annualized Riskless Rate r	Annualized Dividend Yield d
Mean	\$9.26	\$9.62	\$288.42	\$286.18	\$280.85	\$282.44	49.17	0.0593	0.0330
Standard Deviation	6.89	5.50	30.94	32.73	31.08	31.77	25.67	0.0042	0.0043

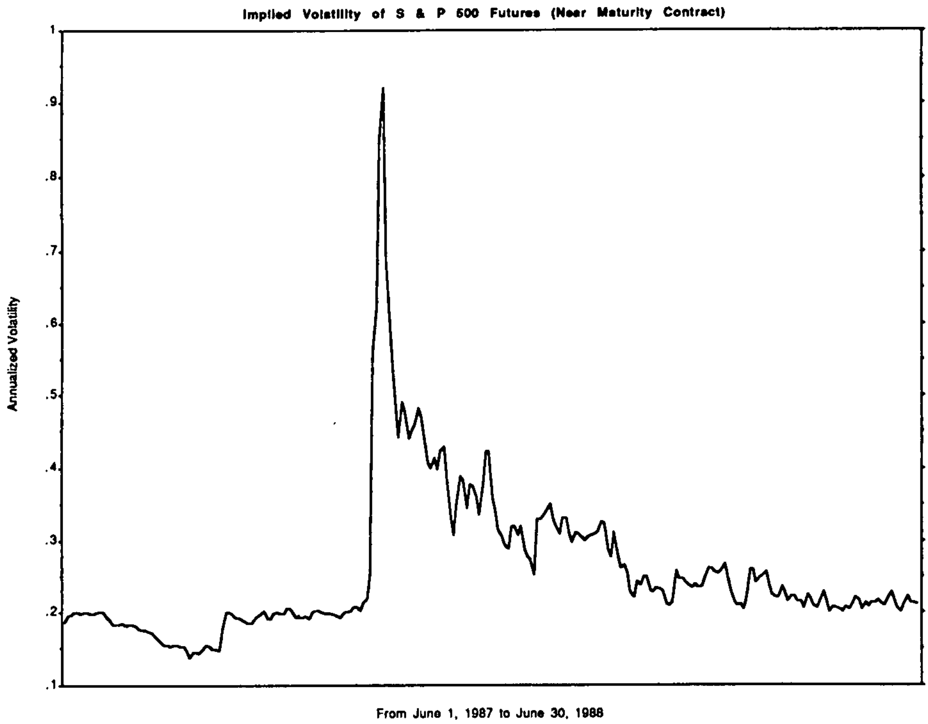

Figure 1
Implied Volatility of S&P 500 Futures (Near Maturity Contract)

respectively) are autoregressive of degree 1 [AR(1)] prior to the crash. Index futures volatility series exhibit a strong downward trend after the crash as shown in Figure 1. The volatility of the index exhibits similar downward trend after the crash.

Table II
DISTRIBUTION OF THE IMPLIED STANDARD DEVIATIONS FOR THE S&P 500 INDEX (σ_s) AND INDEX FUTURES (σ_f)

	Implied Standard Deviation of the Index σ_s	Implied Standard Deviation of the Index Futures σ_f
Mean (The Entire Sample)	0.2275	0.2375
Standard Deviation (The Entire Sample)	0.0633	0.0688
Mean		
Before the Crash	0.1832	0.1856
After the Crash	0.2476	0.2612
Standard Deviation		
Before the Crash	0.0259	0.0188
After the Crash	0.0545	0.0559

The presence of a strong trend suggests that removing the trend in the $WISD_s$ and $WISD_f$ series is required for further analysis of the data. Thus, taking the difference (first order or higher) of the post-crash series is investigated. The need for differencing is further confirmed by fitting the autoregressive model of degree 3 [AR(3)] to the post crash series of WISDs.[12]

The autocorrelation functions of the first differences of $WISD_s$ and $WISD_f$ suggest that second differencing is not necessary. Nevertheless, the unit roots method of Fuller (1976) and Dickey and Fuller (1979) is employed to examine the need for second differencing. The tests indicate that second differencing is not necessary for either of the WISD series.[13] The first difference of $WISD_s$ is an AR(3) process, while that of $WISD_f$ is an AR(4) process. The results are reported in Table III.

The results shown in Table III indicate that shocks to volatility persist for at least three trading days after the crash compared to about one day before the crash. Greater risk aversion of investors after the crash may have accentuated the market's response to information.

Volatility and Maturity

Samuelson's hypothesis states that futures prices exhibit greater variability as the futures contract nears maturity. As noted earlier, Park and Sears found contrary evidence with ISDs of index futures options. A test of the Samuelson hypothesis is conducted with the ISD data. In view of the strong positive correlation between σ_f and σ_s, both of these volatility measures are expected to exhibit a similar relationship with respect to maturity.

Table III
AUTOREGRESSIVE PROCESSES FOR $WISD_s$ AND $WISD_f$
$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + m + \epsilon_t$ WITH $E(Y_t) = E(Y_{t-1}) = \ldots = u$

	Prior to Oct. 15, 1987		After Nov. 2, 1987*	
	$WISD_s$	$WISD_f$	$\delta WISD_s$	$\delta WISD_f$
u	0.1876	0.1882	−0.0013	−0.0016
ϕ_1	0.8550	0.9750	−0.5729	−0.0764
ϕ_2	---	---	−0.3428	−0.3139
ϕ_3	---	---	−0.1809	−0.0880
ϕ_4	---	---	---	−0.1685

*$\delta WISD_s$ and $\delta WISD_f$ refer to the first differences of $WISD_s$ and $WISD_f$, respectively.

[12]Let ϕ_1, ϕ_2, and ϕ_3 be the parameter estimates for the AR(3) model; $\phi(B) \cdot Y_t = m + \epsilon_t$. Where Y_t refer to a data series ($WISD_s$ or $WISD_f$) and B is the backshift operator. The estimates corresponding to the post crash crash series are:

	ϕ_1	ϕ_2	ϕ_3	TOTAL
$WISD_s$	0.4786	0.2869	0.2346	1.0001
$WISD_f$	0.9974	−0.2559	0.2585	1.0000

The estimates for both series sum up to 1, indicating a unit root of +1. This strongly suggests the need for differencing.

[13]The method requires regressing $\delta^2 Y_t$ on 1, δY_{t-1}, and $\delta^2 Y_{t-1}$ where Y_t refers to the data series, δY_t refers to the first difference, and $\delta^2 Y_t$ to the second difference. If the coefficient of δY_{t-1} is not significant, then second differencing may be appropriate otherwise second differencing is not necessary. The coefficient for $\delta WISD_{s,t-1}$ is −1.767 with a t-statistic of −13.3. The coefficient for $\delta WISD_{f,t-1}$ is −1.265 with a t-statistic of −11.5. Both the coefficients are significant at the 1% level compared to the t-type statistics tabulated by Fuller (1976, p. 373). This suggests that there is no need for second differencing of either series.

The regression is corrected for first order autocorrelation, and the results are reported in Table IV.

For the September 1987 contract, the maturity effect is positive but not statistically significant. The December 1987 contract exhibits a strong negative maturity effect before the crash, consistent with the Samuelson hypothesis; but exhibits a strong positive maturity effect after the crash. The March 1988, June 1988, and September 1988 contracts exhibit significant positive maturity effects, contrary to the hypothesis.

In view of the increasing volatility prior to the crash and the declining volatility subsequent to the crash, the maturity effect of the December 1987 contract may be distorted by the crash-induced variation in volatility. The results corresponding to the September 1987, June 1988, and September 1988 contracts are more convincing, as the observations for these contracts are more distant from the crash and are thus less likely to be influenced by the crash. In general, these tests reject the Samuelson hypothesis and the results are consistent with the findings of Park and Sears.

Leverage and Volatility Changes

During this period, there was a significant level of corporate takeover activity accompanied by substantial increases in corporate debt levels. At the level of the firm, in a Modigliani and Miller world with no taxes or dividends, the firm's volatility of equity returns increases linearly in its debt-equity ratio. At the aggregate level, the increased leverage effect translates to greater volatility for the market index. Christie shows that the lower bound for elasticity of equity volatility with respect to stock prices is -1. The elasticity may be positive with risky debt and dividends. The relationship between changes in leverage and changes in the index level is tested by a regression as follows:

$$\ln(\sigma_{j,t}/\sigma_{j,t-1}) = \alpha + \beta \ln(R_t) + \epsilon_t \tag{4}$$

Table IV
TESTS OF THE RELATIONSHIP BETWEEN IMPLIED STANDARD DEVIATION AND MATURITY ($\sigma_j = \alpha + \beta t$; t = DAYS TO MATURITY; $j = f$ OR s)[a]

	σ_f	σ_s
Contract	**β Estimate**	**β Estimate**
September 1987	0.02824	−0.04153
	(0.24)	(−0.28)
December 1987		
Pre Crash	−0.08026	−0.10026
	(−3.64)*	(3.52)*
Post Crash	0.48314	0.30124
	(4.56)*	(2.99)*
March 1988	0.67239	0.56704
	(3.55)*	(4.55)*
June 1988	0.32511	0.29009
	(6.33)*	(7.62)*
September 1988	0.23004	0.10587
	(4.05)*	(1.215)

[a]The regression is adjusted for first-order autocorrelation of residuals using the Cochrane-Orcutt method for all contracts except the September 1987 contract, which is adjusted for second-order autocorrelation.

*t-statistics is significant at the 1% level.

where R_t is the change in the index level. The null hypothesis is that if the index volatility changes are explained by capital structure changes, then the β estimate should be greater than -1. Franks and Schwartz contend that if the estimated β is less than -1, then either measurement errors exist or changes in aggregate leverage do not provide the sole explanation for changes in market volatility. If the cost-of-carry model holds, then the same hypthesis is applicable to implied volatility changes for index futures as well.

The results corresponding to regression (4) are shown in Table V. The WISD corresponding to each day is used as the dependent variable and the change in the index level is the independent variable. The null hypothesis is rejected at the 1% significance level for both the index and the index futures. The results suggest that changes in aggregate leverage may contribute to changes in the volatilities of the index and futures but cannot fully explain the intra-day changes in volatilities.[14] These results are consistent with the Franks and Schwartz study of FTSE index volatility.

Relationship between σ_f and σ_s

To examine the empirical relationship between ISDs of the index and the index futures, the ISDs are adjusted for AR(1) and regressed without intercepts. The results are reported in Table VI. There are two types of tests: Panel A of the table reports the tests by contract maturity, and Panel B reports tests for the overall relationship conducted with the WISDs.

The null hypothesis that σ_f equals σ_s cannot be rejected for the September 1987 contract. This contract matured prior to the stock market crash. The null hypothesis of equality between the index and index futures ISDs is rejected at the 1% level for March, June, and September 1988 contracts. The December 1987 contract is analyzed separately for the periods before and after the crash. The null hypothesis for the December contract cannot be rejected before the crash but is rejected at the 1% level after the crash.

A regression of the average ISDs is conducted by obtaining WISD_s and WISD_f. The null hypothesis that WISD_s and WISD_f are equal cannot be rejected before the crash and is rejected at the 1% level after the crash. The relationship between the two ISDs ap-

Table V
REGRESSION OF CHANGES IN WISD AGAINST RETURNS OF THE S&P 500 INDEX [$\ln(\text{WISD}_{j,t}/\text{WISD}_{j,t-1}) = \alpha + \beta \ln(R_t) + \epsilon_t$]ᵃ

	WISD_s		WISD_f	
	Pre-Crash	Post-Crash	Pre-Crash	Post-Crash
Intercept	0.00219	−0.00268	0.001958	−0.00222
	(0.34)	(−0.64)	(0.56)	(−0.56)
β Estimate	−1.87112	−3.04421	−0.30258	−3.54437
	(−2.65)*	(−11.01)*	(−0.98)	(−17.69)*
R^2	0.12	0.53	0.02	0.68

[a] R_t is one plus the return on the S&P 500 Index. The regressions are adjusted for autocorrelation by the Cochrane-Orcutt procedure.
*t-statistics is significant at the 1% level.

[14] Introducing interest rate changes and dividend yield changes as additional indepedent variables in regression eq. (4) does not provide any significant coefficients. Results are available from the authors.

Table VI
TESTS OF RELATIONSHIP BETWEEN THE IMPLIED VOLATILITIES

Panel A: By Contract ($\sigma_f = \beta\sigma_s + \epsilon$)[a]

Contract	β Estimate	R^2
September 1987	1.0027 (0.23)	0.997
December 1987		
Before the Crash	1.0042 (−0.034)	0.995
After the Crash	1.0624 (2.80)*	0.994
March 1988	1.0431 (5.74)*	0.997
June 1988	1.0574 (6.77)*	0.997
September 1988	1.0634 (5.46)*	0.997

Panel B: Average Relationship ($WISD_f = BWISD_s + \epsilon$)[a]

	β Estimate	R^2	p[b]
Pre-October 15, 1987	1.006439 (0.84)	0.997	+0.2780
Post-November 2, 1987	1.058078 (8.44)*	0.996	−0.1983

[a]The regression is adjusted for first-order autocorrelation of the residuals using Cochrange-Orcutt method. The t-statistics for the null hypothesis that $\beta = 1$ is given in parenthesis.

*t-statistics significant at the 1% level.

[b]p is the value of the first-order autocorrelation parameter.

pears to have changed after the market crash with the index futures exhibiting greater volatility (≈5.8% more) than the index after the crash.

The volatility of the index and index futures should be equal if the cost-of-carry model holds and interest rates and dividend yields are relatively stable. Prior to the crash, the volatility of the index and index futures was comparable. The relationship between the stock and the futures markets may have undergone a structural change since the crash, with the futures market being more volatile than the spot market. Institutional factors provide a potential explanation for the observed differences in volatility. Trading volume in stock index futures has declined by approximately 33% since the crash.[15] The reduction in futures trading volume likely made the futures market less liquid which, in turn, might have induced greater volatility in futures prices.[16]

[15]The average daily volume in S&P 500 stock index futures was approximately 75,000 contracts in June of 1987 and had dropped to less than 50,000 contracts in May 1988 (*Wall Street Journal*, July 1, 1988, pp. 1, 6). Part of the decline in volume was due to limited use of portfolio insurance, program trading, and reduced stock-index arbitrage activities since the crash.

[16]Franks and Schwartz (1988) show that changes in trading volume, oil prices, unexpected inflation, and capital structure changes explain implied volatility changes in the FTSE index. Similar real variables may explain the differences between the index and futures volatility observed in this study.

V. SUMMARY AND CONCLUSIONS

The properties of the implied volatilities of the S&P 500 Index and the S&P 500 Index Futures obtained from their option prices are examined and their relationship is obtained using data from June 1987 through June 1988. The shocks to volatilities of both the index and the index futures generally do not persist long but they appear to persist for longer periods subsequent to the crash.

The maturity effect on the volatilities is also studied. Volatilities of the index and the index futures increase with maturities in general, contrary to the Samuelson hypothesis but consistent with Park and Sears' findings on the index futures implied volatility in 1983. The intra-day changes in the volatilities of the index and index futures cannot be explained by changes in the aggregate amount of leverage.

The relationship between volatilities of the index and the index futures is found to have changed after the crash. Prior to the crash, the index and index futures volatities appear to be of similar magnitude. Subsequent to the crash, however, index futures are approximately 5.8% more volatile than the index itself. Reduced levels of index futures trading volume and other real shocks may have a bearing on the noted differences in the stock and futures market volatilities.

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