

Basis Risk and Optimal Decision Making for California Feedlots

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Most agricultural production is characterized by a lag between the time the production decision is made and the time the output actually reaches the market. Hence, the actual cash price that will be received is unknown when the production decision is made. For many commodities, futures markets exist in which a producer may hedge all or part of the production to reduce price risk. Numerous economic studies have determined optimal production and futures market positions under these circumstances using various criteria or assumed objectives of the firm.

If it is assumed that the producer minimizes the variance of his income, an optimal hedging ratio can be prescribed. This concept was developed by Ederington (1979) who drew on the original works of Johnson (1960) and Stein (1961). Empirical estimates of this ratio of the covariance between spot and futures prices and the variance of futures price are frequently and easily obtained by regressing spot on futures price using OLS. Hence, the estimate of the minimum variance hedge ratio is often referred to as the β coefficient. Discussion about the empirical estimation of β is abundant in both the economics and finance literature.

Rather than simply regressing spot on futures price to obtain an estimate of β , Working (1953) proposed, for a number of reasons which will be discussed later in the article, that first differences in spot prices should be regressed on first differences in futures prices. Although Peck (1975) basically agreed with Working, she suggested that β should be estimated using data based on unanticipated changes in spot and futures prices. Hence, the difference between spot price and producers' forecasts of spot price should be regressed on the difference between futures price and its forecast to determine an estimate of β . Grant and Eaker (1984) and Overdahl and Starleaf (1986) are among those who have used this approach as well.

Regardless of how β is estimated, it will not in general be the optimal hedge ratio when it is assumed that the producer maximizes the expected utility of profit. Heifner (1972) shows that β is the optimal hedge ratio if a linear mean-variance expected utility

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function is assumed and the expected profit from holding a futures position equals zero (i.e., futures markets are unbiased). Benninga, Eldor, and Zilcha (1984) show that β is optimal in a general expected utility framework if it is assumed that futures markets are unbiased and spot price is a linear function of futures price. Hence, only under very restrictive assumptions will β be the optimal hedge ratio in a general expected utility framework.

Batlin (1983), using a linear mean-variance utility function framework, illustrates that although β is not the optimal hedge ratio, it can be useful in determining comparative static results. Hence, the easily estimated β coefficient can still be helpful to producers in choosing optimal production and futures market positions. For example, the magnitude of β will indicate whether it is optimal for a firm to increase or decrease its position in the futures market as input prices or expected output price rise.

The work in this article contributes to the literature in a number of ways. First, Batlin's model is reexamined in a general utility framework showing nearly identical results. Although β is not the optimal hedge ratio, it is still useful in determining production and marketing decisions. Secondly, the theoretical results are illustrated in an empirical application to the California fed cattle industry. Specifically, the β coefficient is estimated at three different locations in California using the approach suggested by Peck. A number of different expectations models are used to estimate producers' forecasts of spot and futures prices. The results using the different expectations models are compared and implications on comparative static results are discussed.

1. THE THEORETICAL MODEL

For a producer hedging in a futures market in which basis risk is present, the decision making problem can be specified as follows. In the first time period, inputs are purchased at known prices p_1 and the producer hedges f units of output at the known futures price p_1^f . In the second period when production is complete, output is sold at the spot price prevailing in the region. The producer offsets his position in the futures market by buying back f units of output at futures price $\bar{p}_2^f$. The future spot price $\bar{p}$ and the second period futures price are random variables when the producer is making output and hedging decisions in the first period. Basis risk arises due to the deviations between the random second period spot and futures prices.

The producer is assumed to possess a von Neumann-Morgenstern utility function U defined on profit π . Optimal output and hedging decisions are determined by maximizing expected utility of profit. Hence, the objective function of the producer is to

$$\text{maximize } EU(\bar{\pi}) = EU[\bar{p}F(X_1) + (p_1^f - \bar{p}_2^f)f - p_1X_1] \quad (1)$$

where

- $\bar{\pi}$ = profits
- $\bar{p}$ = price of output in period 2
- X_1 = vector of input quantities
- f = quantity of output hedged in the futures market in period 1
- p_1 = vector of input prices
- p_1^f = futures market price in period 1 for contracts with maturity in period 2
- $\bar{p}_2^f$ = futures market price in period 2 for contracts with maturity in period 2
- $F(X_1)$ = the production function where $F'(X_1) > 0$ and $F''(X_1) < 0$.

First-order conditions are

$$\frac{\partial EU(\bar{\pi})}{\partial X_1} = E(U'(\bar{\pi})(F'(X_1)\bar{p} - p_1)) = 0 \quad (2)$$

$$\frac{\partial EU(\bar{\pi})}{\partial f} = E(U'(\bar{\pi})(p_1^f - \bar{p}_2^f)) = 0 \quad (3)$$

where it is assumed that utility increases as profits rise, $U'(\bar{\pi}) > 0$ and the producer is risk averse, $U''(\bar{\pi}) < 0$.

Reduced form solutions for output and hedging are derived in the Appendix and are given by

$$F(X_1) = (\bar{\pi}_s + \bar{\pi}_f \beta_f)/D \sigma_s^2 \quad (4)$$

$$f = (\bar{\pi}_f + \bar{\pi}_s \beta_s)/D \sigma_f^2 \quad (5)$$

where the following definitions have been used:

$$\bar{\pi}_s \equiv E(\bar{p}) - \frac{p_1}{F'(X_1)}$$

$$\bar{\pi}_f \equiv p_1^f - E(\bar{p}_2^f)$$

$$\gamma(\bar{\pi}) \equiv -\frac{EU''(\bar{\pi})}{EU'(\bar{\pi})}$$

$$\text{and } \beta_f = \frac{\sigma_{sf}}{\sigma_f^2}, \quad \beta_s = \frac{\sigma_{sf}}{\sigma_s^2},$$

$$D = \gamma(1 - \rho^2), \quad \rho^2 = \beta_s \beta_f.$$

In the above equations $\bar{\pi}_s$ is the expected marginal profit from production, and $\bar{\pi}_f$ is the expected marginal profit from hedging. The notation σ_s^2 , σ_f^2 , and σ_{sf} represents the variances of spot and futures prices and the covariance between these prices, respectively. The gamma (γ) term is a measure analogous to the Arrow-Pratt measure of absolute risk aversion. If the gamma term is assumed to be constant, the underlying utility function is characterized by constant absolute risk aversion. Batlin derived nearly identical results assuming a linear mean-variance utility function. The only difference in his results was that γ was the actual Pratt-Arrow coefficient of risk aversion. Thus, the results presented here under more general assumptions support the robustness of Batlin's original work. In addition, an expression for unhedged output can also be defined.¹

$$z \equiv F(X_1) - f = \left(\frac{\bar{\pi}_s(1 - \beta_f)}{\sigma_s^2} - \frac{\bar{\pi}_f(1 - \beta_s)}{\sigma_f^2} \right) / D \quad (6)$$

These reduced form solutions reveal that both output and hedging decisions will depend on variances and covariances of spot and futures prices, the producer's risk aversion, along with expected returns from production and from the futures market. Clearly, optimal output and hedging decisions are interdependent in a model of decision making with basis risk. Also, the optimal hedge ratio is not equal to β_f , the hedge ratio defined by Ederington. However, as will be elaborated in the following section, it is apparent that β_f and β_s play an important role in determining the signs of the comparative static results.

¹Equations (4) through (6) are very similar to those derived by Batlin. Hence, a detailed derivation is not presented in the article but will be supplied on request from the authors.

II. COMPARATIVE STATIC RESULTS

The reduced form solutions for output and hedging can be used to derive comparative statics results for shifts in exogenous parameters. Batlin's results were similar. This article concentrates the analysis on shifts in spot and futures prices as well as input prices. The objective is to highlight the crucial role that easily estimable covariances and variances play in determining the signs of these comparative static results. Table I summarizes the responses of optimal output, the amount hedged, and unhedged output to changes in input prices and the current futures price. Shifts in expected spot price and the expected futures price are examined also.

A. Input Prices

As intuition suggests, if input prices are higher and all other prices and parameters remain the same, the costs of production increase and this leads to an unambiguous de-

Table I
COMPARATIVE STATIC RESULTS

Input Price		
$\frac{\partial F(X_1)}{\partial p_1} = -\frac{1}{DF'(X_1)\sigma_s^2} < 0$		
$\frac{\partial f}{\partial p_1} = -\frac{\beta_s}{D\sigma_f^2} > 0$	as β_s	$\frac{<}{>} 0$
$\frac{\partial z}{\partial p_1} = -\frac{(1 - \beta_f)}{DF'(X_1)\sigma_s^2} > 0$	as β_f	$\frac{>}{<} 1$
Current Futures Price		
$\frac{\partial F(X_1)}{\partial p_f^1} = \frac{\beta_f}{D\sigma_s^2} > 0$	as β_f	$\frac{>}{<} 0$
$\frac{\partial f}{\partial p_f^1} = \frac{1}{D\sigma_f^2} > 0$		
$\frac{\partial z}{\partial p_f^1} = -\frac{(1 - \beta_s)}{D\sigma_f^2} < 0$	as β_s	$\frac{>}{<} 1$
Expected Output Price		
$\frac{\partial F(X_1)}{\partial E(\bar{p})} = \frac{1}{D\sigma_s^2} > 0$		
$\frac{\partial f}{\partial E(\bar{p})} = \frac{\beta_s}{D\sigma_f^2} > 0$	as β_s	$\frac{>}{<} 0$
$\frac{\partial z}{\partial E(\bar{p})} = \frac{(1 - \beta_f)}{D\sigma_s^2} < 0$	as β_f	$\frac{<}{>} 1$
Expected Futures Price		
$\frac{\partial F(X_1)}{\partial E(\bar{p}_2^f)} = \frac{-\beta_f}{D\sigma_f^2} < 0$	as β_f	$\frac{<}{>} 0$
$\frac{\partial f}{\partial E(\bar{p}_2^f)} = \frac{-1}{D\sigma_f^2} < 0$		
$\frac{\partial z}{\partial E(\bar{p}_2^f)} = \frac{(1 - \beta_s)}{D\sigma_f^2} > 0$	as β_s	$\frac{<}{>} 1$

crease in the optimal level of output. The effect of higher input prices on the amount hedged, however, depends on the sign of the covariance between spot and futures prices. If spot and futures prices are positively correlated ($\sigma_{sf} > 0$), β_s will be positive; and higher input prices lead to a decrease in the amount hedged. Unhedged output also responds to changes in input prices. If spot prices are more volatile than futures prices ($\beta_f > 1$), hedging will decline more than output leading to an increase in unhedged output for the short hedger.

B. Current Futures Price

The impact of higher current period futures prices on optimal producer decisions when all other prices and parameters remain the same will again depend on the relative volatilities of spot and futures prices. If spot and futures prices are positively correlated, β_f will be positive; and higher current futures prices lead to increased output. Also the amount hedged is related directly to changes in the current futures price. Unhedged output also is related directly to the current futures price but only when futures prices are more volatile than spot prices; that is, β_s exceeds one. When β_s is less than one, the opposite result holds.

C. Expected Spot Price

Higher expected output prices lead to increased output when all other parameters and prices remain constant, a result consistent with models of firm behavior under uncertainty proposed by Sandmo (1971) and Batra and Ullah (1974). Also, if spot and futures prices are positively correlated, the amount hedged will increase when expected output price is higher. Unhedged output will decline when the expected spot price rises only if spot prices have a greater variance than futures prices ($\beta_f > 1$).

In contrast, Batlin has noted that comparative static results will be different with no basis risk and are dependent no longer on the covariance between spot and futures prices. In this instance the amount hedged will be lower as the producer expects higher spot prices. In turn, unhedged output will increase as the expected spot price is larger.

D. Expected Futures Price

If expected futures prices are higher but all other prices and parameters remain the same, optimal output is smaller if spot and futures prices are positively correlated since β_f is positive. Clearly, in this instance, returns from short hedges will be lower; and hence, the producer will hedge less. The effects on unhedged output again depend on the magnitude of a beta coefficient. Unhedged output will decline as expected futures prices rise when β_s exceeds one.

III. ESTIMATION METHODOLOGY

The importance of the β_s and β_f terms is now apparent. The magnitude of these beta terms clearly plays a crucial role in determining, a priori, the signs of the comparative static results. Estimates of betas derived from regression relationships of spot and futures prices are common in the analysis of both agricultural and financial futures. Two types of approaches have been proposed to estimate the beta coefficient: the price levels model and the price changes model.

In the price levels model, β_f is estimated by regressing spot prices against futures prices:

$$P_t = \alpha + \beta_f P_t^f + \epsilon_t.$$

Brown (1986) pointed out that such an approach overlooks the fact that hedging is designed to reduce the risk of price changes. Econometrically, the procedure will violate assumptions of the OLS model if the distributions of the prices are not stationary. Although spot and futures price levels may be highly correlated, the hedger will, in reality, be interested in correlations between spot and futures price changes.

Researchers have become aware of the deficiencies of the price levels model and in response have shifted to the use of price changes in the regressions. In the portfolio model of hedging, the futures position is combined with the cash position to minimize losses in value of the cash position. Hill and Schneeweiss (1981) stated that "effective hedging depends on the amount of covariance between value changes of the cash security and the futures."

The price changes model has been formulated two separate ways. First, Working (1953) proposed regressing first differences of spot prices on the first differences of futures prices:

$$(P_{t+1} - P_t) = \alpha + \beta_f(P_{t+1}^f - P_t^f) + \epsilon_t.$$

However, this model is only appropriate if spot and futures prices follow martingales. That is,

$$E(\tilde{P}_{t+1}) = P_t \quad \text{and} \quad E(\tilde{P}_{t+1}^f) = P_t^f.$$

A somewhat different interpretation of the Working model can be obtained from the work of Peck. She noted that risk should be measured as deviations from expectations and not simply as deviations from the current price levels. The only relevant price variability is that which makes the producer's forecast differ from actual output price. As Peck argues, "the crucial variance remaining, however, is that which surrounds that accuracy of the producers' forecasts." Hence, the beta coefficient should be estimated using data based on unanticipated changes in spot and futures prices.

Grant and Eaker (1984), following Peck's reasoning, suggested a variant of Working's approach by specifying more precisely the process of expectations formation. The authors assumed that current futures prices are unbiased predictors of both spot prices and futures prices in later time periods. Under these assumptions, the beta coefficient is estimated by an OLS regression of the unexpected changes in spot prices on the unexpected changes in futures prices:

$$(P_{t+1} - P_t^f) = \alpha + \beta_f(P_{t+1}^f - P_t^f) + \epsilon_t.$$

This method implicitly accepts Peck's argument that estimation of the beta coefficient should measure the unexpected price changes as perceived by producers.

Although Grant and Eaker recognized the importance of a correct specification of expectations, in the presence of basis risk, futures prices would not generally be unbiased predictors of future spot prices. Hence, a model with basis risk must specify also the process producers use to form expectations of future spot prices. However, because producers' subjective forecasts of the mean and other moments of the distribution of the output price for fed cattle are unobservable, these forecasts must be estimated. Numerous methods have been suggested such as adaptive, extrapolative, naive, and rational expectations models. In recent work, Antonovitz and Roe (1986) found that an ARIMA model gave reasonable estimates of fed cattle producer's forecasts of the mean of output price.

This article examines two alternative models to estimate the β coefficients. In the first, the unexpected spot price changes are estimated by the deviation between expected and observed spot prices. Unexpected futures price changes are modelled following Grant and Eaker by assuming that the futures price in period 1 is an unbiased predictor of futures price in period 2. This assumption, consistent with finance theory, argues that

futures positions have no initial or investment value and, hence, do not provide returns on an investment. Thus, the expected rate of return on a futures contract should be zero. This can be specified in the following form:

$$(P_{t+1} - E\tilde{P}_{t+1}) = \alpha + \beta_f(P_{t+1}^f - P_t^f) + \epsilon_t.$$

β_f is simply determined by reversing the dependent and independent variables. That is, the spot price differences are regressed on the deviations in futures prices. The β coefficients were estimated then using three different commonly used expectations models of producers' forecasts of spot prices: ARIMA, naive, and adaptive.

In a second model, the assumption of unbiased expectations of futures prices is relaxed:

$$(P_{t+1} - E\tilde{P}_{t+1}) = \alpha + \beta_f(P_{t+1}^f - E\tilde{P}_{t+1}^f) + \epsilon_t.$$

Once again, the three different expectations models were used to estimate producers' forecasts of both spot and futures prices.

IV. EMPIRICAL ANALYSIS

The theoretical analysis derived and discussed in the previous sections is illustrated by presenting empirical estimates of the beta coefficients for the California fed cattle industry. However, the analysis could be applied easily to other regions and commodities.

There are basically three regions in California where feedlots are located: the Imperial Valley, the Southern San Joaquin Valley, and the Northern San Joaquin Valley. Most of the feeder cattle entering these feedlots originate in California although some are imported from Texas and the Southeast. The cattle are fed from 120 to 140 days in California although this may vary somewhat by location. For illustration purposes, a feeding period of 120 days was chosen. Hence, the feedlot's output and hedging decisions are assumed to be made 4 months prior to the marketing of the fed cattle.

Beta coefficients for the three different regions are estimated for fed cattle marketed during April 1980 through April 1986 using monthly spot and futures prices.² The spot and futures prices are those reported weekly by the Bureau of Market News in California, a source readily available to feedlot operators. The spot prices are for slaughter steers and the futures prices are for the live cattle futures contract from the Chicago Mercantile Exchange. Monthly prices for both the spot and futures are formulated averaging the reported weekly prices. The futures prices are for the nearby contract expiring just after the month the fed cattle are marketed.³

A. Expectations Models

As noted above, three different expectations models are used to estimate producers' forecasts of spot and futures prices. The expectations based on the ARIMA model assume that producers know the underlying time series of the monthly spot prices of

²That is, beta coefficients were estimated using 73 monthly observations.

³More specifically, if the producer were making output and hedging decisions in November 1981, the fed cattle would be marketed in March 1982. It is assumed that the April 1982 contract would be used to hedge the cattle. Thus, the price of the April 1982 contract in March is the futures price used in the data series. If decisions were made in December 1981, the cattle would be marketed in April 1982. It is assumed that the June 1982 rather than the April futures contract would be used to hedge because of the increased volatility of futures prices during the delivery month of the contract. Hence, the futures price used in the data series is the price of the June 1982 contract in April. For cattle marketed in May 1982, it is assumed that the June 1982 contract is used to hedge. The price of the June futures contract in May is used in the data series. Cattle marketed in June 1982 would be hedged with the August contract, and so forth.

choice steers and the monthly futures prices of live cattle from 1970 through 1985. Both series of monthly prices over this 15 year period are found to be represented most accurately by a seasonal ARIMA (0,1,0) (0,1,1)₁₂ model.⁴ Moving ARIMA models based on 10 years of data are used to estimate, four months in advance, the mean of expected output and futures prices. Four-month-ahead forecasts for the period April 1980 to April 1986 are generated.⁵

Adaptive expectations is a second model used to approximate producers' forecasts of spot and futures prices. The procedure is one suggested by Bessler (1982) who pointed out that expectations formed in an adaptive manner could be generated with an ARIMA(0,1,1) model. Once again, moving ARIMA models based on 10 years of data are used to estimate producers' expectations of spot and futures prices by obtaining four-month-ahead forecasts.

Naive expectations are generated also. Because current futures price reflects what producers expect futures price to be at a later date, a simple four month moving average of past and current futures prices associated with the appropriate futures contract is used to estimate a naive expectation of futures price:⁶

$$E\tilde{P}_{t+4}^f = \sum_{i=0}^3 P_{t-i}^f / 4.$$

To estimate a naive expectation of spot price, the seasonality of these prices is considered. These expectations are approximated by adding the change in the current year's price level to last year's price:

$$E\tilde{P}_{t+4} = (P_t - P_{t-12}) + P_{t-12+4}.$$

B. Empirical Results

The estimates of β_s and β_f are presented in Tables II and III.⁷ It should be noted that although the estimates of β_s and β_f differ somewhat by location the significance of these coefficients is the same for all regions with one exception. For the Imperial Valley, β_f is significantly different from zero when producers' forecasts of spot and futures prices follow ARIMA processes; while in the Southern and Northern San Joaquin Valleys the estimate of β_f is not significantly different from zero. Thus, there will be no further discussion about differences based on location.

⁴This is the notation used by Pankratz (1983) to represent ARIMA processes with both seasonal and nonseasonal patterns. The first set of parentheses indicates the nonseasonal orders while the second set represents the seasonal orders. Realizations generated by this process have a pattern with a periodicity of twelve months which is indicated by the subscript.

⁵For example, if the producers were making output and hedging decisions in December 1979, the cattle would be marketed in April 1980. Monthly observations from January 1970 through December 1979 are used to forecast the prices in April 1980. Similarly, if producer decisions were made in January 1980, the fed cattle were marketed in May 1980. Monthly observations from February 1970 through January 1980 are used to forecast the prices in May 1980, etc.

⁶For example, as mentioned in footnote 2, if the producer were making output and hedging decisions in November 1981, the fed cattle would be marketed in March 1982. The price of the April 1982 contract in March is the futures price used in the data series. The naive futures price expectation is formed by averaging the prices of the April 1982 contract in August, September, October, and November of 1981.

⁷The analysis described here assumes that the optimal β coefficients would be known to only one or a small number of producers who do not significantly influence spot and/or futures market prices. If a large number of producers knew the optimal β 's and altered their production and hedging plans enough to influence cash and/or futures prices, a different analysis would be appropriate.

Table II
ESTIMATES OF β_i

Futures Price Expectation:	Southern San Joaquin	Northern San Joaquin	Imperial Valley
Spot Price Expectation: ARIMA			
ARIMA	.072 (.109) [†]	.067 (.115)	.105 (.115)
$H_0: \beta_i = 0$	.664	.578	.914
$H_0: \beta_i = 1$	-8.518**	-8.095**	-7.805**
Unbiased	.387 (.065)	.422 (.069)	.446 (.068)
$H_0: \beta_i = 0$	5.924**	6.141**	6.607**
$H_0: \beta_i = 1$	-9.394**	-8.411**	-8.212**
Spot Price Expectation: Naive			
Naive	.323 (.064)	.365 (.065)	.365 (.060)
$H_0: \beta_i = 0$	5.023**	5.609**	6.037**
$H_0: \beta_i = 1$	-10.527**	-9.772**	-10.517**
Unbiased	.411 (.068)	.446 (.070)	.422 (.068)
$H_0: \beta_i = 0$	6.006**	6.368**	6.390**
$H_0: \beta_i = 1$	-8.616**	-7.896**	-8.758**
Spot Price Expectation: Adaptive			
Adaptive	.078 (.059)	.076 (.060)	.087 (.060)
$H_0: \beta_i = 0$	1.319	1.276	1.448
$H_0: \beta_i = 1$	-15.716**	-15.503**	-15.218**
Unbiased	.447 (.082)	.474 (.084)	.537 (.082)
$H_0: \beta_i = 0$	5.834**	5.619**	6.561**
$H_0: \beta_i = 1$	-6.392**	-6.242**	-5.650**

[†]Standard errors of the estimates of β_i are given in parentheses.

*Indicates significance of a two-tailed t statistic at the .05 level.

**Indicates significance of a two-tailed t statistic at the .01 level.

First, the estimates of β_s and β_f are examined when the forecasts of both spot and futures prices are assumed to follow the same expectations process. It should be noted that the expectation process assumed affects the conclusion of whether β_s and β_f are significantly different from zero or one. If forecasts of both prices are assumed to follow either an ARIMA process or adaptive expectations, both β_s and β_f are found to be significantly less than one but not different from zero. Hence, as input prices rise, production falls, futures position remains the same, and unhedged production falls. When current futures price rises, production remains the same but short futures positions increase while unhedged output falls. Increases in expected output price have the following affect: output rises, futures position remains unchanged, and unhedged production increases. Increases

Table III
ESTIMATES OF β_f

Futures Price Expectation:	Southern San Joaquin	Northern San Joaquin	Imperial Valley
Spot Price Expectation: ARIMA			
ARIMA	.201 (.110) [†]	.181 (.102)	.200 (.098)
$H_0: \beta_f = 0$	1.821	1.769	2.049*
$H_0: \beta_f = 1$	-7.253**	-8.024**	-8.188**
Unbiased	.902 (.139)	.858 (.131)	.883 (.128)
$H_0: \beta_f = 0$	6.475**	6.561**	6.892**
$H_0: \beta_f = 1$	-.706	-1.086	-0.091
Spot Price Expectation: Naive			
Naive	.815 (.164)	.840 (.152)	.939 (.155)
$H_0: \beta_f = 0$	4.985**	5.529**	6.046**
$H_0: \beta_f = 1$	-1.131	-1.053	-.039
Unbiased	.835 (.134)	.826 (.127)	.873 (.132)
$H_0: \beta_f = 0$	6.227**	6.525**	6.591**
$H_0: \beta_f = 1$	-1.228	-1.376	-.957
Spot Price Expectation: Adaptive			
Adaptive	.168 (.121)	.177 (.117)	.174 (.112)
$H_0: \beta_f = 0$	1.394	1.508	1.551
$H_0: \beta_f = 1$	-6.891**	-7.010**	-7.378**
Unbiased	.659 (.119)	.626 (.118)	.670 (.109)
$H_0: \beta_f = 0$	5.561**	5.316**	6.141**
$H_0: \beta_f = 1$	-2.872**	-3.178**	-3.018**

[†]Standard errors of the estimates of β_f are given in parentheses.

*Indicates significance of a two-tailed *t* statistic at the .05 level.

**Indicates significance of a two-tailed *t* statistic at the .01 level.

in expected futures price cause producers to take a smaller short position in the futures market while increasing their uncovered cash position with quantity produced unaffected.

If producers are assumed to have naive expectations of both spot and futures prices, β_f is found to be significantly different from zero but not different from one; and β_s is significantly different from both zero and one. Many of the comparative static results are different as a result. When input prices rise, both production and short futures position fall while unhedged output remains the same. Rises in current futures price result in increases in output and short futures positions and a decrease in uncovered cash position. As expected output price rises, production rises, short futures positions fall, and there is no change in uncovered output. Production and short futures positions fall and unhedged output rises as expected futures price rises.

Next, comparative static results based on estimates of β_s and β_f are examined when it is assumed that producers have unbiased expectations of futures prices (i.e., $p_1^f = E(\bar{p}_2^f)$). First, it should be noted that the theoretical formulation of the comparative static results changes since $\bar{\pi}_f = p_1^f - E(\bar{p}_2^f) = 0$. Hence, there is no change in production, futures position, or unhedged production when either current or expected futures prices change. However, comparative static results remain the same for changes in input or expected output prices. Thus, the only relevant tests of significance are whether β_s is different from zero and whether β_f is different from one.

Regardless of the signs or magnitudes of β_s and β_f , production decreases as input prices rise and increase when expected output price rises. For all expectations models, β_s is significantly greater than zero. Hence, short futures positions fall when input prices rise but increase as expected output price rises. When expectations of futures prices are unbiased but spot price forecasts are based either on an ARIMA process or naive expectations, β_f is found to be not significantly different from one. In this instance, unhedged output does not change when either input or expected output price changes. If spot price expectations follow an adaptive process, β_f is significantly less than one. Unhedged output, in this case, rises when input prices increase but falls as expected output price rises.

V. CONCLUSIONS

The empirical results presented in this article can be used either in a normative/prescriptive manner or from a positive/descriptive viewpoint. In practice, these results can be used to help feedlot operators make better marketing and production decisions. Optimal responses in output and futures positions to changes in input prices, expected and current futures prices, and expected output prices can be determined for feedlots at specific locations. If this analysis is to be used for this purpose, however, it is clear that the choice of the forecast of spot and futures prices may significantly alter the prescribed marketing and production strategies.

Alternatively, the results of this work can be used to gain insight into how producers actually form their expectations of spot and futures prices. For example, if it is observed that feedlots do not change their production and hedging decisions as futures prices change, this may provide evidence to suggest that their futures price expectations are unbiased. Similar observations on other comparative static results can be used to support hypotheses about producers' forecasts of spot prices as well. This may be useful not only in gaining insights into producers' expectations but also in suggesting more accurate methods of forecasting prices to producers.

Appendix

Reduced form eqs. (4)–(6) can be derived by using Stein's Theorem (1973), which was derived independently by Rubinstein (1976). It states that if $\bar{x}$ and $\bar{y}$ follow a bivariate normal distribution and $g(\bar{y})$ is a once-differentiable function of y , then

$$\text{Cov}(g(\bar{y}), \bar{x}) = E g'(\bar{y}) \text{Cov}(\bar{y}, \bar{x}). \quad (\text{A.1})$$

The theorem was extended to variables belonging to the continuous exponential family by Hudson (1978). Thus, if it is assumed that $\bar{\pi}$ and $\bar{p}$ as well as $\bar{\pi}$ and $\bar{p}_1^f$ follow bivariate distributions from the exponential family, Stein's Theorem can be applied to the first-order conditions, eq. (2) and (3), leading to the following explicit solutions for output and hedging:

$$F(X_1) = \frac{\sigma_f}{\sigma_s^2} f + \frac{\tilde{\pi}_s}{\gamma \sigma_s^2} \quad (\text{A.2})$$

$$f = \frac{\sigma_f}{\sigma_f^2} F(X_1) + \frac{\tilde{\pi}_f}{\gamma \sigma_f^2} \quad (\text{A.3})$$

where the terms $\tilde{\pi}_s$, $\tilde{\pi}_f$, γ , σ_s^2 , σ_f^2 , and σ_{sf} are defined in the article. Equations (A.2) and (A.3) can be rearranged and solved for the reduced form equations given by (4) and (5).

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