Optimal Portfolios for Commodity Futures Funds

B. Wade Brorsen
Louis P. Lukac

Commodity futures funds are an important part of today's futures markets. Organized as limited partnerships, these funds range in size from a few hundred thousand up to several hundred million dollars in capital. With this capital, Brorsen and Irwin (1987) estimated funds held an average of 23% of open interest (one side of the market) in the nearby contract in ten important futures markets in 1984.

Most of the trading by these funds is based on technical trading systems (Irwin and Brorsen (1985)). Lukac *et al.* (1988), Irwin and Uhrig (1984), and others have shown that these technical systems can provide returns greater than zero. Murphy (1986), Irwin and Brorsen (1985), and Irwin and Landa (1987) showed technically traded futures funds would enter an investor's optimal portfolio that included stocks, Treasury Bills, and other assets. However, not all of the research is unanimous. Elton *et al.* argued technically traded commodity funds' returns were poor and that commodity funds did not belong in an optimal portfolio.

Managing risks is an important factor in the success or failure of a futures fund. The risk of technical trading systems was demonstrated by Lukac *et al.* (1988) and by Murphy (1986). They showed the standard deviation of return was quite large and, in fact, over double that of a naive portfolio of equally weighted long positions. Elton *et al.* showed that the commodity funds' standard deviation was over twice that of common stocks, long-term corporate bonds, long-term government bonds, and Treasury Bills. To be a more competitive investment, futures funds need to increase their return relative to their risk.

The purpose of this article is to suggest a means of reducing the riskiness of futures funds by finding an optimal allocation of capital across commodities in a futures fund portfolio. Previous simulation studies and actual futures funds allocate trading capital by varied methods.

In Brorsen and Irwin's (1987) survey, commodity fund managers responded that margin is not a deciding factor in their allocation decision. Eighty-five percent of managers responding said they hold numbers of contracts according to their volatility. Half of these held positions based on variances only, by holding more contracts of less volatile commodities. Thirty-five percent based their allocation on a tradeoff between risk

The authors thank David C. Wilson and Paul Preckel for computational assistance and Scott Irwin, Bill Uhrig, and Deb Brown for helpful comments.

B. Wade Brorsen is an Associate Professor of Agricultural Economics at Purdue University.

Louis P. Lukac is a Trading Strategist with Prudential-Bache in Chicago.

The Journal of Futures Markets, Vol. 10, No. 3, 247–258 (1990)

CCC 0270-7314/90/030247-12$04.00

historical return. The disadvantage of their approaches is that they do not consider the correlations between returns.[1] The remaining 15% of the managers said they held equal numbers of contracts in each commodity traded. Those that adjusted for volatility did not do it the same way and some consisdered their methods proprietary. By optimally allocating capital, futures funds may be able to increase their trading returns while maintaining the same level of risk.

No academic study has attempted to allocate funds across commodities optimally. Bodie and Rosansky's (1980) portfolio consisted of equal dollar amounts invested in each commodity. Lukac and Brorsen (1988) used equal amounts invested in an average "margin" for each commodity. They assumed initial margin to be 10% of contract value for all commodities except Treasury Bills and Eurodollars, which were 0.5%. Finding an optimal allocation across commodities will help academic researchers increase the power of their tests as well as benefit fund managers and investors.

FUTURES FUNDS' OBJECTIVE FUNCTION

Public futures funds are usually organized as limited partnerships.[2] The general partner is usually a brokerage firm that hires the trading advisor. The commodity trading advisor is responsible for all trading decisions. Most funds give the advisor complete control over selecting which commodities to trade and how to allocate funds across commodities.

Fund managers are quite concerned about risk. If the equity is depleted below a specified percentage (usually 50%) in one year, the fund is dissolved. Several funds have indeed been dissolved for this reason. Also, many funds are closed-ended and customers often withdraw money after a month with a large loss (or sometimes after a large gain). Since customers receive monthly reports, trading advisors are concerned with monthly variability.

These funds are usually large enough that they are allowed to put up U.S. Treasury Bills (T-Bills) as margin. The remaining funds are invested also in T-Bills. Thus, the capital invested in the fund earns the T-Bill rate regardless of whether it is used as margin.[3] Brorsen and Irwin (1987) found that public funds averaged investing 27.7% of equity in margins, while Cornew (1986) found that public and private funds invested 19.8%. Thus, funds have plenty of capital available to increase the number of contracts they trade.

Lukac and Brorsen (1988) found portfolio returns were not significantly different from a normal distribution. Thus, it is reasonable to assume mean-variance analysis applies. Since trading returns have little or no correlation with other assests (Irwin and Brorsen (1985); Lukac *et al.* (1988)), standard deviations are a resonable measure of risk and fund managers are assumed to maximize the Sharpe's Ratio. Schwager (1984) argues that Sharpe's ratio has several drawbacks: (1) failure to distinguish between intermittent and consecutive losses, (2) it varies depending on the time interval evaluated, (3) failure to distinguish between upside and downside fluctuations, and (4) failure to distinguish between retracements in unrealized profits versus retracements from "trade entry date" equity. His criticisms (1) and (3) imply returns may be autocorrelated and positively skewed. These criticisms are likely much more serious for individual commodity returns since Lukac and Brorsen (1988) found individual commodity returns were not distributed normally, but portfolio returns did not differ significantly from a normal distribution.

[1]Some may have subjectively considered correlations by treating very similar commodities as a group.
[2]See Irwin and Brorsen (1985) for a detailed description of public futures funds.
[3]In some funds, the general partner keeps part of the the interest.

The standard Sharpe's Ratio (SR) is

$$\mathrm{SR} = (\mathrm{M} - r)/\mathrm{SD} \tag{1}$$

where M and SD are the portfolio mean and standard deviation and r is the risk-free rate. Since the fund earns the risk-free rate on all equity, the mean return to the fund is:

$$\mathrm{M} = \mathrm{T} + r \tag{2}$$

where T is the net trading returns from the portfolio. Substituting eq. (2) into eq. (1) gives:

$$\mathrm{SR} = \mathrm{T}/\mathrm{SD} \tag{3}$$

which is the formula used in this article.

A fund manager will select the same allocation regardless of risk preferences. Risk preferences will determine how much a fund is leveraged. The allocation of funds across commodities that maximizes the reward to risk ratio of the net trading returns (eq. (31)) is

$$\text{maximum } c'x/(x'\Omega x)^{0.5} \tag{4}$$

$$\text{subject to } \sum_{i=1}^{n} x_i = 1, \text{ and}$$

$$0 \leq x_i \leq 1 \qquad i = 1, \ldots, n$$

where c is an $n \times 1$ vector of expected trading returns, x is the vector of percentages allocated to each commodity, n is the number of commodities available to trade, and Ω is the variance-covariance matrix of returns.

Equation (4) is a nonlinear objective function. The problem was solved using the nonlinear programming algorithm developed by Murtagh and Saunders. Other means of solving this problem are available, but this approach is more flexible since it can conveniently handle inequality constraints. If such an algorithm is not available, quadratic programming could be used to define the possibilities frontier and then the optimum point could be found graphically.

The next question is what to use for the estimates of c and Ω. The first alternative that we consider is the historical mean and variance-covariance matrix. But, using historical means ignores what Dreman (1979) calls the "regression to the mean effect." This argument is based on the fact that even if all commodities have the same expected return, in a small sample, some will have higher means than others. Lukac and Brorsen (1988) found returns from trading only the commodities with the highest historical returns were not significantly higher than the average across all commodities.

The second approach used assumes one can not predict which market will be "trending" and thus expected returns may be equal after adjusting for differences in variability. Therefore, an alternative estimate of the mean which assumes each commodity has the same return to risk ratio is considered and, thus, c is estimated at

$$\hat{c} = \lambda\hat{\sigma} \tag{5}$$

where c is the vector of estimated means, λ is the constant assumed for the return to risk ratio, and $\hat{\sigma}$ is the vector of historical standard deviations. The value chosen for λ makes no difference since changes in λ result in linear changes in the objective function.

Brorsen and Irwin (1987) found that 50% of futures fund managers held larger positions in commodities with lower volatility. Some of the fund managers stated they held positions with "equal volatility across commodities." For equal variances, let $x_i^* = \sigma_i^{-1}$ and then force the x_i to sum to one by normalizing by the sum of the x_i^*, $\Sigma_{i=1}^{n} x_i^* =$

$\Sigma^n_{i=1} \sigma_i^{-1}$. This means that the standard deviation of the returns from all contracts held of commodity i equals the standard deviation of the returns on all contracts held of commodity j. This approach, unlike the previous approach, does not rely on the assumption that correlations are predictable. It is also simpler. This may be a rational approach since Elton and Gruber (1973) demonstrated the difficulty in estimating correlation coefficients for stocks. Ignoring the information in historical correlations, therefore, may be justified. The third approach considered is:

$$x_i = \sigma_i^{-1} / \sum_{i=1}^{n} \sigma_i^{-1} \tag{6}$$

Elton *et al.* ((1976) p. 1352) derived a similar equation. Equation (6) can result from theirs if all returns are assumed equal, correlations are assumed to be zero, and standard deviations are substituted for variances. Many funds likely allocate subjectively rather than the precise manner defined by equation (3). The x_i's from (3) are then used to calculate new percent margins which yield the desired allocation. The new margins are obtained by adjusting the old margins by the ratio of the old allocation $(1/n)$ and the new allocation (x_i):

$$\text{New Margin} = (1/nx_i) \times \text{Old Margin}. \tag{7}$$

This information will be useful to those who choose to ignore the information in historical correlations. Which of these approaches is the best depends on what assumptions one is willing to make about the usefulness of historical means and covariances. An out-of-sample test is conducted to determine the relative usefulness of the approaches.

TRADING MODEL

A computer program simulated the trading of each system and recorded profits, losses, number of trades, and average profit per trade. The model also generated the optimal parameter[4] for each system and commodity based on all previous data prior to the year being traded.

Trading of twenty-three technical trading systems was simulated for eleven years from 1976 through 1986 on thirty futures markets. The optimal parameters were selected by a cumulative reoptimization method. Each year's parameter was selected from all available data from 1975 up to the current year. The criteria for selecting parameters was highest profit. For example, the optimal parameter selected for 1985 was the parameter that yielded the greatest profit from 1975 through 1984. At the end of 1985, the systems were reoptimized with 1975 through 1985 data to select the optimal parameters for 1986. New commodities were added as soon as one year of data became available. This process was used for all markets tested, and assures all results are out-of-sample. Lukac and Brorsen (1988(a)) found no statistical difference between various reoptimization strategies, so the results are likely not dependent upon this choice.

Equal amounts are assumed invested in "margin" for each commodity. Initial margins were assumed to be 5% of contract value for all contracts except Treasury Bills and Eurodollars which are 0.25%.[5] Returns were reported as a percent of total equity invested. Twenty percent of total equity was assumed to be committed to margins, with the rest held in reserve. These margins are one-half those used by Lukac and Brorsen (1988).

[4]The parameter is, for example, the number of days in the moving average, (e.g., 4 days).

[5]These are the exact numbers formerly in use by one public futures fund, Brorsen and Irwin (1987) considered this fund as one that adjusted for relative volatility. The fund now uses the values in Table II with subjective adjustments for mean return and correlation. The allocations in Table III are too uneven for this fund's manager.

The relative weights from the optimization are not affected by the initial assumptions about margins.

Twenty-three technical trading systems were selected to represent the major groups of popularly traded systems. Table I lists the 23 trading systems used in this study. The systems are identified by system type. The systems studied include six breakouts, two oscillators, two moving averages of price, two moving averages of momentum, seven combination systems, one volatility system, one price filter system, one exponential moving average of price, and one point and figure system. The number of parameters for each system varied from one to three.

The thirty futures markets analyzed are: corn, soybeans, soybean meal, soybean oil, wheat, feeder cattle, pork bellies, live cattle, live hogs, silver, platinum, lumber, copper, cotton, cocoa, sugar, frozen concentrated orange juice, gold, British Pound, Deutsche Mark, Japanese Yen, Swiss Franc, Treasury Bills, Treasury Bonds, coffee, heating oil, Treasury Notes, Eurodollar, S&P 500 Index, and crude oil. The daily prices were obtained from the Dunn & Hargitt Commodity Data Bank (Dunn & Hargitt, Inc.). Commodities were added to the portfolio when one year of past data became available. Transaction costs, which include commissions and slippage, were assumed to be $100.00 per trade,

Table I
TRADING SYSTEMS IDENTIFIED BY SYSTEM TYPE AND NUMBER OF PARAMETERS

System Number	System Type	Number of Parameters
1	Price Filter	1
2	Volatility	2
3	Oscillator	2
4	Oscillator	2
5	Breakout	1
6	Breakout	3
7	Breakout	3
8	Comb. of EMA[a] & Breakout	3
9	Comb. of EMA & Breakout	3
10	Comb. of Systems 5 & 15	3
11	Comb. of Systems 5 & 19	2
12	Comb. of Systems 5 & 17	3
13	Comb. of Systems 5 & 3	3
14	Comb. of Systems 5 & 2	3
15	Simple Avg. of Momentum	2
16	Simple Avg. of Price	2
17	Simple Avg. of Price	2
18	Breakout	1
19	Simple Avg. of Momentum	1
20	EMA of Price	3
21	Point & Figure	2
22	Breakout	3
23	Breakout	3

[a]EMA: Exponential Moving Average.

including rollover trades. No trading was allowed on days where markets opened at the limit and stayed there all day. Brorsen and Irwin (1987) found futures funds trade the nearby contract (the one closest to expiration) 80% of the time. All trading in this simulation is in the nearby contract month, except that several lightly traded contract months such as September corn are not traded. This was done to keep trading in the contract month with the highest open interest. Rollovers into new months occur on the third Monday of the month prior to expiration. Only data on the contract month being traded were used to determine buy/sell signals.

This study demonstrates its results with the channel system, and the aggregate of all 23 technical systems. The channel system was chosen because Lukac *et al.* (1988) suggest it as the standard system to which others should be compared. The mean return and standard deviation using all 23 systems are calculated as the mean and standard deviation of all observations and thus should be interpreted as the mean and standard deviation when the trading system for each month is randomly selected.

RESULTS FOR 1976–1986

The mean returns for each commodity for the aggregate of twenty-three technical trading systems and for the channel trading system are presented in Table II. Eighteen of the thirty commodities yielded positive net returns. The commodities with higher returns for all twenty-three systems tended to be the same ones with higher returns for the channel system. Returns in the livestock commodities were poor and returns in the currencies were good. Table I also shows the standard deviation of returns varies widely across commodities. This suggests the potential to greatly improve the Sharpe's ratio of the portfolio over the ones from the initial allocation.

The results show a small positive return which is consistent with past research using trend-following trading systems (Lukac (1988)). The Sharpe's ratio is .04 for all trading systems and 0.13 for the channel system.

To compare the portfolio returns in Table II to returns from public futures funds it is necessary to add the risk-free rate and subtract some additional costs. Public pools averaged costs of 8.5% a year for administrative, managerial, and incentive fees (Irwin and Brorsen (1985)). U.S. Treasury Bill returns were 9.8% a year during Murphy's (1986) observation period and Irwin and Brorsen (1985) estimated interest earnings for 1978–83 at 8.47% a year. Thus, returns in Table I are approximately comparable to returns from futures funds. Murphy (1986) found mean returns for eleven technically traded public futures funds was 0.64% a month. Irwin and Brorsen (1985) estimated returns for technically traded public futures funds at 0.63% a month over 1978–83. Murphy (1986) found monthly returns ranged from −0.76% to 1.77%. The monthly returns of 0.31% for all 23 systems and 0.95% for the channel are well within this range. Murphy (1986) found standard deviations ranged from 5.92–9.93. Thus, the standard deviations of 7.20 for the 23 systems and 7.36 for the channel are also well within the range found for public futures funds.

The portfolio allocation using equal variances is presented in Table III. This procedure as defined in eq. (3) allocates more to the less variable commodities and less to the more variable ones. The initial allocation for each commodity was $1/n$ or $3.\overline{33}$ since $n = 30$. Thus, any commodity in Table III that has an allocation greater than $3.\overline{33}$ needs more funds allocated to it. The initial allocation was most over committed to sugar (new allocation = 1.20) and least committed to Treasury Notes (new allocation = 8.82). The new adjusted margins are quite similar to historical average margins. This is expected since margins are presumably based on volatility. They are a performance margin de-

Table II
MEAN MONTHLY NET RETURNS OF SIMULATED TRADING OF TECHNICAL TRADING SYSTEMS FROM 1976 THROUGH 1986

	23 Trading Systems			Channel System		
Commodity	Mean % Return	Standard Deviation	Sharpe's Ratio	Mean % Return	Standard Deviation	Sharpe's Ratio
Corn	1.2239	19.61	0.062	2.3700	21.80	0.108
Soybeans	0.5474	24.77	0.022	0.7010	26.78	0.026
Soybean Meal	0.6978	27.78	0.025	1.9347	29.12	0.066
Soybean Oil	0.0277	34.69	0.001	1.2905	35.44	0.035
Wheat	−2.1625	22.93	−0.094	−3.8639	24.10	−0.158
Feeder Cattle	−0.5181	18.22	−0.028	−0.0325	20.51	−0.001
Pork Bellies	−2.7711	36.36	−0.076	−1.9780	36.41	−0.054
Live Cattle	−0.9896	22.51	−0.044	−2.3778	24.00	−0.098
Live Hogs	−0.1719	29.19	−0.006	0.8251	29.69	0.027
Silver	3.5718	37.87	0.094	6.6041	39.53	0.166
Platinum	1.1658	41.00	0.028	3.4812	39.94	0.087
Lumber	−1.9807	30.13	−0.066	−1.3931	29.53	−0.047
Copper	−2.5070	25.94	−0.097	−1.9613	26.10	−0.075
Cotton	1.8682	22.05	0.085	3.2365	23.57	0.137
Cocoa	1.6641	31.56	0.053	−0.0063	32.38	0.000
Sugar	5.3718	56.06	0.097	7.2429	60.20	0.120
FCOJ	−1.1493	37.71	−0.030	−0.6499	38.59	−0.017
Gold	1.8016	26.50	0.068	3.3000	26.37	0.125
British Pound	−0.1126	12.98	−0.009	−0.1909	13.71	−0.014
Deutsche Mark	1.4433	11.71	0.123	1.8900	11.54	0.164
Japanese Yen	2.2409	11.54	0.194	2.1944	12.76	0.172
Swiss Franc	1.2696	13.34	0.095	2.9260	12.84	0.228
Treasury Bills	0.7616	16.75	0.045	3.2763	15.61	0.210
Treasury Bonds	0.5476	14.18	0.039	1.2039	14.38	0.084
Coffee	0.4459	33.96	0.013	2.3270	40.01	0.058
Heating Oil	−0.4215	28.83	0.015	1.8100	25.47	0.071
Treasury Notes	0.6667	7.62	0.087	1.8050	6.74	0.268
Eurodollar	0.5732	10.98	0.052	−1.8580	15.19	−0.122
S & P 500	−1.2403	12.13	−0.102	−2.0160	12.17	−0.166
Crude Oil	−2.5041	44.47	−0.056	−3.4587	44.39	−0.178
Portfolio	0.3121	7.20	0.043	0.9547	7.36	0.130

Note: Returns are calculated assuming transaction costs are $100 per trade for all trades including rollover trades.

signed to protect the exchange against a default. The new allocation in Table III produced a substantially larger Sharpe's ratio (63% higher for the 23 systems and 31% higher for the channel system). This is mostly due to a reduction in the standard deviation by 26% since the changes in the means were not large.

Table IV presents the allocations from maximizing the Sharpe's ratio. Again, the allocations from the aggregate and the Channel are similar. Based on historical means, very little is allocated to the agricultural commodities and large amounts are allocated to the currency and interest rate futures. Using historical means produces Sharpe's ratios sev-

Table III
PORTFOLIO ALLOCATION BASED ON
HOLDING POSITIONS WITH EQUAL VARIANCES, 1976–1986

	23 Trading Systems		Channel System		
Commodity	Allocation (%)	Adjusted Margin (%)	Allocation (%)	Adjusted Margin (%)	Historical[a] Margins(%)
Corn	3.43	4.9	3.17	5.3	4.7
Soybeans	2.71	6.2	2.58	6.5	6.0
Soybean Meal	2.42	6.9	2.37	7.0	--
Soybean Oil	1.94	8.6	1.95	8.5	--
Wheat	2.93	5.7	2.87	5.8	5.0
Feeder Cattle	3.69	4.5	3.37	4.9	--
Pork Bellies	1.85	9.0	1.90	8.8	--
Live Cattle	2.99	5.6	2.88	5.8	4.0
Live Hogs	2.30	7.2	2.33	7.2	--
Silver	1.77	9.4	1.75	9.5	--
Platinum	1.64	10.2	1.73	9.6	--
Lumber	2.23	7.5	2.34	7.1	--
Copper	2.59	6.4	2.65	6.3	5.5
Cotton	3.05	5.5	2.93	5.7	--
Cocoa	2.13	7.8	2.13	7.8	11.5
Sugar	1.20	13.9	1.15	14.5	14.2
FCOJ	1.78	9.4	1.79	9.3	--
Gold	2.54	6.6	2.62	6.4	--
British Pound	5.18	3.2	5.04	3.3	--
Deutsche Mark	5.74	2.9	5.99	2.8	2.8
Japanese Yen	5.82	2.9	5.41	3.1	--
Swiss Franc	5.04	3.3	5.38	3.1	--
Treasury Bills	4.01	0.21	4.43	0.17	0.16
Treasury Bonds	4.74	3.5	4.80	3.5	2.5
Coffee	1.98	8.4	1.73	9.6	--
Heating Oil	2.33	7.2	2.71	6.2	--
Treasury Notes	8.82	2.9	10.25	1.6	--
Eurodollar	6.12	0.14	4.55	0.18	--
S & P 500	5.54	3.0	5.68	2.9	--
Crude Oil	1.51	11.0	1.56	10.7	--
Portfolio					
Mean	0.38		0.91		
Std. Dev.	5.36		5.43		
Sharpe's Ratio	0.07		0.17		

[a]Historical margins are from Brorsen and Irwin (1987).

eral times larger than the initial allocation. These are not achievable results since they use information that would not be available to an investor. But, they are the type of pretested results that appear in many advertisements. They also illustrate the potential influence of the portfolio allocation on the results of academic studies.

The results from assuming equal Sharpe's ratios are reported also in Table IV. The objective function in this case considers the correlations between commodity returns. The

Table IV
OPTIMAL CAPITAL ALLOCATION WITHIN
A FUTURES FUND PORTFOLIO, 1976–1986

Commodity	23 Trading Systems: Historical Means	23 Trading Systems: Equal Sharpe's Ratio	Channel System: Historical Means	Channel System: Equal Sharpe's Ratio
Corn	5.98	3.40	5.37	3.62
Soybeans	0.0	0.0	0.0	0.0
Soybean Meal	0.0	0.0	0.0	1.26
Soybean Oil	0.0	0.0	0.0	0.0
Wheat	0.0	1.051	0.0	2.47
Feeder Cattle	0.0	3.31	0.0	4.79
Pork Bellies	0.0	0.65	0.0	1.57
Live Cattle	0.0	2.17	0.0	3.11
Live Hogs	1.51	2.64	2.59	3.04
Silver	2.31	0.0	2.49	0.87
Platinum	0.0	0.0	0.0	0.22
Lumber	0.0	2.15	0.0	3.45
Copper	0.0	1.29	0.0	1.71
Cotton	10.01	8.12	10.34	8.61
Cocoa	1.18	0.0	0.0	1.66
Sugar	2.48	0.0	1.92	0.88
FCOJ	0.0	0.96	0.0	2.38
Gold	4.34	3.66	4.14	3.64
British Pound	0.0	6.52	0.0	5.10
Deutsche Mark	14.83	5.94	7.41	2.70
Japanese Yen	28.47	11.98	13.39	11.20
Swiss Franc	0.0	0.0	8.08	0.41
Treasury Bills	2.36	3.96	9.33	6.30
Treasury Bonds	0.0	0.0	0.0	0.0
Coffee	0.0	0.14	0.65	2.34
Heating Oil	0.0	1.86	2.72	4.08
Treasury Notes	20.71	23.64	31.59	11.18
Eurodollar	5.83	7.32	0.0	6.79
S & P 500	0.0	9.25	0.0	6.64
Crude Oil	0.0	0.0	0.0	0.0
Portfolio				
Mean	1.61	0.55	2.53	0.94
Std. Dev.	5.23	3.93	4.49	4.39
Sharpe's Ratio	0.31	0.14	0.56	0.21

Sharpe's ratios are higher than when positions of equal variance were held in each commodity (doubled for the 23 systems returns and 24% higher for the channel system).

The proportions placed in Treasury Notes and the Japanese Yen look extremely high, but this is partly due to assuming higher than actual margins for these commodities. Overall, the results say more needs to be allocated to currencies and interest rates and less to grains, metals, and soft foods. Versus the equal Sharpe's ratio allocation, the one using historical means suggests allocating less to livestock, frozen concentrated orange juice (FCOJ), heating oil, and the S&P 500 and increasing the share allocated to sugar, silver, Deutsche Mark, and Swiss Franc.

OUT-OF-SAMPLE TEST

The results in the previous section are intended for a fund manager who would want to implement one of these allocation methods and thus uses recent historical data. But, it does not provide a test of these methods. Therefore, an out-of-sample test is performed.

The out-of-sample test uses the 23 commodities which were continuously traded over 1978–1986. The portfolio weights were estimated using returns from 1978–1982. These portfolio weights were then applied to the returns for 1983–1986.

Standard errors for the estimated Sharpe's Ratios are difficult to determine by conventional means, so they are calculated using bootstrapping (see Efron and Gong, (1983)). Bootstrapping is a Monte Carlo resampling method. One hundred samples were randomly selected with replacement from the data. Each of the 100 samples yielded one Sharpe's Ratio. The standard deviation of these 100 Sharpe's Ratios is the bootstrap standard error.

The out-of-sample results which are reported in Table V show a tendency for higher Sharpe's Ratios when more information is considered in selecting the portfolio weights. But, the advantage of the approaches that consider additional information is considerably less than found for in-sample results. The only Sharpe's Ratio which is significantly different from zero is that for the channel system when all historical information is used. The advantages of optimal portfolio allocation are, of course, greater for trading systems that are more profitable. The channel system was selected because it is known to have positive historical returns.

CONCLUSIONS

The research reported in this article sought to define the optimal way for futures funds to determine how many contracts to trade of each commodity. Futures funds keep only a small percentage of their total funds invested in margins and therefore they can easily increase their leverage. Actual funds choose their portfolio of commodities in a variety of ways, but most consider only relative variances. The objective of futures funds is as-

Table V
OUT-OF-SAMPLE SHARPE'S RATIOS FOR 1983–86 BASED ON THE OPTIMAL PORTFOLIOS FOR 1978–82

	23 Trading Systems	Channel System
Portfolio Selection	Sharpe's Ratio	Sharpe's Ratio
Constant % of Contract Value	−0.015	0.063
	(0.15)	(0.14)
No Correlation, Equal Means	0.047	0.120
	(0.14)	(0.14)
Equal Means	0.017	0.1587
	(0.15)	(0.14)
All Historical Information	0.162	0.345
	(0.13)	(0.13)

Note: The numbers in parentheses are the bootstrap standard errors (Efron and Gong (1983)).

sumed to be to maximize the ratio of the mean to the standard deviation (the Sharpe's ratio). Results showed large improvements were possible if information on actual sample means, variances, and covariances were known *ex ante*. The out-of-sample test which is what really matters, did not reveal a statistically significant improvement in performance.

Differences in liquidity costs were not considered.[6] This factor can be important as Brorsen and Irwin (1987) found public funds traded mostly high volume commodities, presumably to reduce liquidity costs. Small volume markets like feeder cattle, lumber, FCOJ, and coffee likely have higher liquidity costs. Allocations to these commodities might need to be reduced, particularly for a large fund which would need to submit large orders. Adjustments for liquidity costs and historical returns could still be handled in this article's framework by adjusting the expected returns. To do this, however, would require additional research on liquidity costs and forecasting of individual commodity returns. The suggested procedures offer some risk reduction beyond what most managers are now doing. But, as the mostly statistically insignificant results show, this risk reduction is of little value unless the trading system is profitable.

Bibliography

Bodie, Z., and Rosansky, V. (1980, May/June): "Risk and Return in Commodity Futures," *Financial Analysts Journal,* 36:27–37.

Brorsen, B. Wade, and Irwin, Scott H. (1987): "Futures Funds and Price Volatility," *The Review of Futures Markets,* 6:118–135.

Cornew, Ronald W. (1986): "Note on Initial Margin to Net Asset Values: Average Values for the Commodity Pool Industry," *Journal of Futures Markets,* 6:495–502.

Dreman, David (1979): *Contrarian Investment Strategy,* New York: Random House.

Dunn & Hargitt, Inc., "Dunn & Hargitt Commodity Data Bank," Lafayette, Indiana.

Efron, B., and Gong, (1983, February): "A Leisurely Look at the Bootstrap, the Jackknife, and Cross-Validation," *The American Statistician,* 37:36–48.

Elton, Edwin J., and Gruber, Martin J. (1973, December): "Estimating the Dependence Structure of Share Prices-Implications for Portfolio Selection," *Journal of Finance,* 28:1203–1232.

Elton, Edwin J., Gruber, Martin J., and Padberg, Manfred W. (1976, December): "Simple Criteria for Optimal Portfolio Selection," *Journal of Finance,* 31:1341–1357.

Elton, Edwin J., Gruber, Martin J. and Rentzler, (1987): "Professionally Managed, Publicly Traded Commodity Funds," *The Journal of Business,* 60:175–200.

Greer, Thomas V., and Brorsen, B. Wade (1989): "Execution Costs for a Public Futures Fund," Paper presented at NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management, Chicago, Illinois.

Irwin, Scott H., and Brorsen, B. Wade (1985): "Public Futures Funds," *The Journal of Futures Markets,* 5:463–485.

Irwin, Scott H., and Landa, Diego (1987, Fall): "Real Estate, Futures, and Gold as Portfolio Assets," *The Journal of Portfolio Management,* 13:29–35.

Irwin, Scott H., and Uhrig, J. W. (1984): "Do Technical Analysts Have Holes in Their Shoes?," *The Review of Futures Markets,* 3:264–277.

Lukac, Louis P., and Brorsen, B. Wade (1988): "Technical Trading Systems," Selected Paper, 1988 AAEA Meetings, Knoxville, Tennessee.

[6]Greer and Brorsen (1989) present estimates of execution costs for a public futures fund, They find execution costs are considerably higher than bid-ask spreads.

Lukac, Louis P., and Brorsen, B. Wade (1989, February): "The Usefulness of Historical Data in Selecting Parameters for Technical Trading Systems," *Journal of Futures Markets,* 9(1):55–65.

Lukac, Louis P., Brorsen, B. Wade, and Irwin, Scott H. (1988): "A Test of Futures Market Disequilibrium Using Twelve Different Technical Trading Systems," *Applied Economics,* 20:623–639.

Murphy, J. Austin (1986): "Futures Fund Performance: A Test of the Effectiveness of Technical Analysis," *The Journal of Futures Markets,* 6:175–185.

Murtagh, B. A., and Saunders, M. A. (1980): *MINOS/AUGMENTED User's Manual.* Systems Optimization Laboratory Tech. Rep. SOL 80-14, Stanford University

Schwager, Jack D. (1984): *A Complete Guide to the Futures Markets,* New York: John Wiley & Sons.

Sharpe, William F. (1970): *Portfolio Theory and Capital Markets,* New York: McGraw-Hill.