Analyzing Biases in Valuation Models of Options on Futures

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Trading in options on agricultural futures contracts caused increased study of valuation models for options on futures. Probably the most popular of these models is an extension of the Black-Scholes model (1973) for options on physicals to options on futures by Black (1976). According to Black's option pricing model (OPM), the option's value depends on five factors: (a) the current price of the underlying commodity, (b) the exercise or strike price, (c) the time to option expiration, (d) the interest rate(s) on a risk-free investment realized over the option's life, and (e) the variance rate(s) on the underlying commodity's price return realized during the option's life. Of these factors, three are known and illustrations by Plato and others show that premia are relatively insensitive to the interest rate. Thus, Black's OPM may be used to estimate the market's forecast of variance of the commodity price distribution. Such an estimate is usually standardized and called an implied volatility (IV).

To derive the valuation model, Black made several simplifying assumptions. The extent to which these assumptions cause biases in the resulting premium or IV estimate is an empirical question. Past research addressing these biases have employed one of two general approaches. One approach compares IVs to estimates of future volatility; that is, the variance of price over the remainder of the contract is regressed against IVs, weighted IVs, historical and other variance estimates (see Latane and Rendleman (1976) and Beckers (1981)). Thus, the evidence offered from such studies is with respect to the ability of the IV to forecast the future variance. However, since the future variance is uncertain, it is possible for the realized variance to vary substantially from the IV without indicating anything about biases. The second approach examines option premia or IVs in a structural framework (see Hauser and Neff (1985) and Wilson (1987)). Premia or IVs are regressed on their determinants, including some indicated by the Black model and others designed to capture failures of unrealistic assumptions of the model. The objective of this second type of study is to isolate the factors which are important in the determination of the IVs (or premia), and to allow the importance of various assumptions to be measured.

PURPOSE OF THIS STUDY

Studies of this second type have employed linear models. Since Black's OPM (as well as other OPMs) is nonlinear, the implicit assumption made in these studies is that a lin-

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ear approximation is adequate. One objective of the present analysis is to identify conditions for which the regression framework is appropriate when studying IV biases. This is done through simulation experiments in which coefficients of a first-order Taylor series (TS) expansion of the Black model are compared to regression coefficients. A second and related objective is to investigate whether the simulation results imply alternative approaches for modeling biases. The final objective is to apply general results of the simulation experiments or alternative approaches to a case study in which valuation of biases in the live-cattle option market are investigated.

The following section of this article develops and presents results of simulation experiments designed to analyze the appropriateness of regression in studying IVs. The parameters estimated in the simulation model are used to indicate suitable approaches for the subsequent analysis of live-cattle futures options. The next section of the article discusses potential biases in futures options and examines live-cattle option data for the presence of such biases. Potential biases are then measured using techniques suggested by results of the simulation experiments. The final section summarizes results, reviews their significance, and suggests some directions in which this research could be extended.

THE SIMULATION EXPERIMENT

To assess the biases induced by simplifying assumptions of the Black OPM, it is natural to resort to a regression framework. Using data from the option and futures markets, one may relate IVs (or premia) to determinants and measure significance of biasing factors. The potential problem with such an approach lies in the nonlinear nature of the OPM relationship. A linear regression framework is an approximation to the underlying nonlinear relationship, and thus, regression coefficients are estimates of the first order Taylor series (TS) expansion coefficients for the option pricing formula. This observation has led econometricians to point out that regression estimates of TS coefficients are biased and inconsistent in general (see White (1980) and Byron and Bera (1983)). (A graphical demonstration of the bias is offered in Appendix A.) Thus, the regression framework may be inappropriate for the task of measuring the extent of biases in options data. Again, this is an empirical question.

Thus, there are two sources of biases in the analysis of IVs through linear regression: one arising from simplifying assumptions underlying the Black OPM (model bias); the other is from the potential biases of the regression approximation (coefficient bias). If the regression coefficients are "close" to the TS coefficients at the means of the sample (i.e., coefficient biases are insignificant), then the regression framework is appropriate for detecting the existence of model biases and their magnitudes. Further analysis of actual data can proceed with some confidence that the effects of other variables included in the regression to capture potential model biases will not be adversely affected by the inappropriateness of regression. However, two other general cases are of interest. First, it may be found that the regression framework produces significant coefficient biases. These coefficient biases may be so large that even signs have large probabilities of being wrong. In this case, the regression approximation is unsuitable for analyzing potential sources of model bias, much less their magnitudes. Another, intermediate case is one in which coefficient biases exist, but are small, allowing coefficient signs and the implied directions of impact to be determined with little doubt. Such results would allow the direction of impact of factors influencing model biases to be determined.

A simulation experiment is used here to determine which of these three cases apply to modeling IVs (or premia) using linear regression. The approach involves determining a lognormal distribution from which a time series of futures prices is drawn. Given these futures prices, premium data for the experiment are generated such that the Black model is true. That is, given futures price, strike price, time-to-maturity, interest rate, Black's premium is calculated by using the variance of the underlying futures price distribution. These data are then used to calculate the value of the TS coefficients at the means of the sample, as well as coefficients found through a regression approximation. This process is repeated and the results are summarized using moments of derived empirical distributions of regression and TS coefficients.

Previous application of regression to structural analysis of option markets has followed two approaches. One has been to specify a regression with the premium (or a function of the premium) as the dependent variable. The second approach relates the IV to potential determinants. The simulation framework used here employs the first approach. However, given the other factors, Black's premium is a one-to-one function of the IV. Therefore, the inverse function theorem applies and the approximation of IVs by TS expansion is equivalent to the approximation of the premium. Thus, simulation results for the premium apply to IV-dependent models as well. This is important because direct simulation of the IV requires an assumption about how the market's forecast of futures variance, used in the calculation of equilibrium premia, evolved over time. Results of such experiments would be conditional on the assumption employed, greatly limiting their usefulness.

A number of parameter values must be set to conduct the simulations. These include: the variance of the underlying futures contract, a series of interest rates, the range of strike prices examined, and the length of time over which the option is assumed to be traded. The values were chosen to reflect characteristics of the live-cattle futures and options markets. The cattle market was chosen to provide guides for the experiments' data because the empirical analysis conducted later will involve this market. A full description of the considerations and assumptions employed for the generation of the simulation data is detailed in Appendix B.

One factor that is critical to the specification of the simulation data and to understanding of the results is the variance measure employed. Three measures of variance are important. The first measure is the variance of the lognormal distribution from which the futures price is drawn. The second is the market's forecast of the variance over the remainder of the option contract, which is used for option valuation. The third is the researcher's measure, which is used in the regression approximation.

The second measure is referred to as the market's forecast because the participants do not directly observe the variance of the futures' distribution, and thus estimates or forecasts of this quantity must be made when valuing an option. A useful way of thinking about the complex interactions of the market is that the actions of individual participants "sum up" to a market forecast of the variance. Of course, this market forecast does not have to equal the true variance of the futures distribution. To apply the regression approximation to actual option data, researchers have used various forecasts of variance as right-hand-side variables. Again, there is no requirement that such a forecast equal either of the two previous measures. These considerations led to the formulation of the following two basic variance scenarios a "correct variance" scenario and an "incorrect variance" scenario. In both cases, the futures price is assumed to be drawn from a lognormal distribution with a constant daily variance. The variance forecast is considered "correct" if it is used by the market to value the option (calculate the premium) and subsequently to estimate the TS and regression coefficients. Thus in simulations using the "correct" variance, the 30-day historical variance is used to calculate the premium, the market's forecast, and is used in the regression approximation. In this scenario, the market's and researcher's forecast of variance agree.[1] If the variance forecast used to generate the premia and the TS coefficients differs from that used to evaluate regression coefficients it is considered "incorrect." The advantage of the simulation framework is that the performance of the regression approximation can be examined under the "correct" variance scenario, allowing the evaluation under conditions in which all simplifying assumptions are true. These results establish standards with which results from the "incorrect" variance scenario (and/or other cases in which assumptions are violated) may be compared.

General Performance of Regression

Regression coefficients are estimated 500 times for the following specification:

$$C_t = \beta_1 + \beta_2 \mathrm{FUT}_t + \beta_3 \mathrm{STK}_t + \beta_4 \mathrm{TTM}_t + \beta_5 \mathrm{IVM}_t + \beta_6 \mathrm{IR}_t + \epsilon_t \qquad (6)$$

where C_t is the equilibrium call option premium; FUT_t is the price of the underlying futures contract; STK_t is the strike price of the option; TTM_t is the time to maturity in years; IR_t is the real annualized interest rate; and IVM_t represents the volatility measure employed. Depending on the model, IVM_t is either the "correct" or "incorrect" measure. Variables are measured in natural logarithms; the βs are coefficients to be estimated, and ϵ_t is white noise. The TS coefficients depend on the partial derivatives of the Black premium with respect to its five determinants. The relevant derivatives, given by Hauser and Neff ((1985), pp. 572–573), are transformed to elasticities for comparability and are calculated at the sample means of the variables for each simulation sample. The results are then compared.

Table I presents results of two simulation runs. The first simulation is of the "correct" variance scenario; i.e., the variance of the futures price distribution is constant and the 30-day historical variance is used as a forecast to calculate the premia as well as the TS and regression coefficients. The bottom half of Table I gives results of the "incorrect" variance simulation in which a constant variance is used in the calculation of the futures, call premia, and the TS coefficients, but the 30-day historical variance is used in the regressions as an independent variable. This procedure is used because, in practice, the critical aspect of option pricing is in determining the variance level to be used in the option valuation model.

Consider the "correct-variance" results. Comparisons of columns one and four show that the regression coefficients (other than the constant) have the same sign as the TS coefficients. The last two columns give the mean coefficient bias of the 500 simulation samples and its standard error. The mean coefficient bias is the average difference between the regression and TS coefficient, evaluated at the sample means. The coefficient bias divided by its standard error implies "*t* ratios" which range from 3.4 for the interest rate to over 500 for the intercept, indicating that even though the coefficients exhibit expected signs, they are significantly different than those of the TS approximation.

[1]Obviously, the 30-day historical variance is not the only choice possible as a variance for calculation of premia. At any point in the simulation, the variance of the futures over the remainder of the contract is available. Alternatively, the sampling frequency of the futures price could have been increased so as to provide information for a Garman-Klass (1980) type estimator. The realized future variance is employed in several runs with no significant change in the results. However, the 30-day historical variance is used because it can be easily calculated and used by the market.

Table I
FIRST ORDER APPROXIMATION PERFORMANCE OF BLACK'S MODEL

	Regress Coef	STD Error	Prob of Sign Chg[a]	TS Coef	Mean Coef Bias	STD Error
Correct Variance						
INT	0.1256	0.0182	0.0000	−0.1371	0.2627	0.0005
FUT	11.8141	0.8922	0.0000	10.9037	0.9104	0.0299
STK	−10.8061	0.8822	0.0000	−9.8047	−1.0015	0.0288
TTM	0.4912	0.0194	0.0000	0.5687	−0.0775	0.0009
VAR	1.0023	0.0530	0.0000	1.0969	−0.0945	0.0025
IR	−0.0128	0.0373	0.3660	−0.0175	0.0047	0.0017
Incorrect Variance						
INT	0.1264	0.0137	0.0000	−0.1306	0.2571	0.0005
FUT	11.4963	0.4346	0.0000	10.7342	0.7621	0.0237
STK	−10.4903	0.4180	0.0000	−9.6444	−0.8459	0.0225
TTM	0.4896	0.0193	0.0000	0.5638	−0.0741	0.0020
VAR	0.0025	0.0530	0.4810	1.0877	−1.0852	0.0042
IR	−0.0117	0.0378	0.3782	−0.0175	0.0058	0.0017

[a]Calculated assuming the "*t* ratio" is a standard normal.

Of course, testing of coefficient biases requires knowledge of the distribution of the test statistic under the null hypothesis. Normality of the "*t* ratios" is assumed. Empirical sampling distributions of the regression coefficients are in some cases too negatively skewed or kurtotic to be normal. Tests for normality, such as the Chi-square and Jarque-Bera (Judge (1985)), are often significant. The normality assumption is employed for two reasons. First, it is believed that the normal testing framework would be easily understood, and second, the conclusions of tests are the same as those using the actual empirical distributions. Therefore, the normal approximation and resulting "*t*" statistics seem to provide reliable inferences.

The assumption of normality also allows the calculation of the probability that the signs of coefficients are wrong. Empirical distributions of coefficients from the simulation always yield probabilities of a sign change which are similar to those of the normal approximation that is reported in Table I. For example, empirical 90% confidence intervals of the regression coefficients for the "correct" variance model are: 0.094 to 0.153 for INT, 10.475 to 13.388 for FUT, −12.392 to −9.497 for STK, 0.463 to 0.523 for TTM, 0.917 to 1.092 for VAR, and −0.076 to 0.047 for IR. Assuming normality the 90% confidence intervals are: 0.096 to 0.156 for INT, 10.346 to 13.282 for FUT, −12.257 to −9.355 for STK, 0.459 to 0.523 for TTM, 0.915 to 1.089 for VAR, and −0.074 to 0.049 for IR.

With the exception of the interest rate coefficient, column three shows that the regression coefficients are unlikely to change sign. These results indicate that the coefficient biases are small in the sense that signs, if not the magnitudes, of the noninterest rate coefficients are reliable when estimated through regressions for the Black premia. The poorly determined sign of the interest rate coefficient further supports the lack of sensitivity of the OPM to the interest rate.

The "incorrect variance" results in the bottom half of Table I reveal two important points. First, the reliability of the variance coefficient's magnitude decreases considerably relative to the known-variance model. The absolute value of the ratio of mean coefficient bias to its standard error increases from 38 to 258, and the probability of a change in sign increases from the minimum, 0.0000, to near the maximum, 0.4810. The second point is that the nonvariance coefficients and statistics are very similar to those of the known-variance model. These findings (along with numerous other simulation scenarios) strongly suggest the directional impacts of futures price, strike price, and time to maturity on the premium can be measured confidently through regression. However, if a variance estimate similar to this one is used, any conclusion about the variance coefficient is suspect. Regardless of what type of variance measure is used, inferences about the interest rate impact must be made cautiously.

It should be noted that the implications drawn from Table I apply to other cases, including: analyses of puts, a wide variety of strike/futures combinations, time to maturity lengths, and interest rate levels. To reiterate, given Black's model, linear regression produces biased estimates of TS coefficients; however, regression and TS coefficients are of the same order of magnitude. Signs of the coefficients are reliably determined, except for the interest rate and "incorrect" variance.

DETECTION OF MODEL BIASES

A model bias in Black's solution, by definition, exists when traders perceive an error in the model and then modify the model or solution to account for this error. Thus, for such a model bias to exist, two conditions must be met: (a) traders must (implicitly or explicitly) recognize the model error and (b) the error must be deemed important enough to correct.

Two sources of model bias will be considered here: (1) biases caused by early exercise and (2) biases caused by transaction costs. These two sources were chosen for analysis because of their obvious departure from reality.[2] In other words, underlying Black's model are the assumptions that the option cannot be exercised before expiration, and that transaction costs are zero. Obviously, neither assumption is true in practice. However, it may or may not be possible to measure model biases associated with them. This measurement is considered below.

Transaction Costs

Contemporary option-valuation models are based on an arbitrage process between the option and underlying commodity that results in a risk-free investment (or loan). This arbitrage process is assumed to occur continuously and without cost. In practice, at least two types of costs are realized: (a) costs associated directly with the riskless hedging process and (b) costs associated with illiquid markets.

When hedging between options and futures, one cost not accounted for under traditional theory is incurred if margins are not in the form of an interest bearing instrument.

[2]Another prominent assumption of Black's model that might be questioned by analysts regards the lognormal diffusion process of the underlying price returns. If it is believed that lognormality is incorrect and that the market makes adjustments for this error, then identification of the bias must rely on a definition of the "better" distributional specification. However, it is not clear that lognormality is incorrect and even less clear what the better specification is. Given a finite variance, the numerous random-walk tests on log-price returns of agricultural futures do not provide clear support for or against the lognormality assumption (Kamara (1982)). The direct tests of distribution have often found leptokurtoticity but there is evidence that these results may be due to a changing variance over time (e.g., Anderson and Danthine (1983), Cootner (1964), Houthakker (1961), and Neftci (1983)). Departures from lognormality are not considered in this study because there is no alternative specification that is clearly superior, leading to a known bias.

Black assumes that margins are placed in the form of Treasury bills and thus there is no investment opportunity cost. This assumption in conjunction with the fact that Black considers the value of holding a futures position only at the end of each day (after the position is marked to market) implies that there is no cost of holding futures.

The initial margin for a futures position must be either in cash or as a U.S. Treasury bill, meaning that a minimum of $10,000 must be deposited if Treasury bills are used (see Tomek (1985)). Thus, small-volume traders and traders whose Treasury bills have been liquidated to meet margin calls face an opportunity cost not considered in Black's model. This cost of holding the underlying commodity causes the futures option to have a positive holding cost.

The effect of holding costs on futures option premia can be seen noting that the only effective difference between the assumptions underlying Black and Scholes' model for valuing options on physicals and Black's model for pricing options on futures is that the cost of holding the physical is positive whereas the cost for futures is zero. While the extent of premium change depends on the cost magnitude, the directional change in Black's solution will be toward Black and Scholes' solution as holding costs become positive. This means that the holding-cost effect causes the premium for calls to increase and the put premium to decrease, implying that the call's IV under Black's model overestimates the variance forecast whereas the put IV is an underestimate.[3] Thus, one potential model bias is that due to holding costs or HCB. Factors which are likely to affect HCB are time to maturity (with longer times increasing the probability of adverse margin calls) and interest rates, reflecting opportunity costs.

One method of detecting the effects of HCB is to estimate the TS coefficients from actual data and compare them to what one would expect under the Black model and the Black-Scholes model. HCB would be reflected in a movement of the coefficients from the Black values toward the Black-Scholes values. Therefore, the Black-Scholes' is simulated here using the "correct" variance scenario. The results are presented in Table II. Comparisons of the simulation results for the Black-Scholes model (Table II) with those of the Black model (Table I, top half) reveals the directions of change for at-the-money calls. The signs and magnitudes of coefficients in Tables I and II agree in all cases except for the interest rate.

Table II
FIRST ORDER APPROXIMATION PERFORMANCE OF BLACK-SCHOLES' MODEL

	Regress Coef	STD Error	Prob of Sign Chg[a]	TS Coef	Mean Bias	STD Error
INT	0.0249	0.0062	0.0000	−0.2632	0.2881	0.0005
FUT	11.3118	0.8838	0.0000	9.3460	1.9657	0.0241
STK	−10.3095	0.8664	0.0000	−8.3460	−1.9635	0.0230
TTM	0.5559	0.0227	0.0000	0.5890	−0.0331	0.0010
VAR	0.8612	0.0613	0.0000	0.8291	0.0322	0.0027
IR	0.1298	0.0448	0.0019	0.1460	−0.0162	0.0019

[a]Calculated assuming the "t ratio" is a standard normal.

[3]Hauser and Neff illustrate this cost of holding concept in their binomial examples of pricing options on stocks vs. futures. It is reflected also by Merton's (1973) derivation of a model for pricing a stock paying continuous dividends, which offset the cost of holding. This continuous-dividend model is the same as Black's futures option model.

The Black-Scholes' call premium is positively related to changes in interest rates, while the Black call premium is negatively related. Therefore, an indication of HCB is the sign of the interest rate impact on the call premium. The interest rate coefficient has a 37% chance of being positive even if the Black model is true. However, the Black-Scholes interest rate coefficient is significantly positive, with only a 0.19% chance of being negative. Thus, if HCB has moved the Black solution toward Black-Scholes', this would be revealed by a significant increase in the probability of a positive interest rate coefficient.

For this model bias as well as others discussed below, an empirical investigation of actual option data is conducted. Daily data for live-cattle options are obtained from the Chicago Mercantile Exchange. The Exchange maintains records of option premia, trading volume, open interest, futures prices and strike prices, for all contracts traded.

Six 1986 contracts and six 1987 contracts are considered. The option data includes settle premia, strike prices, trading volumes, and open interest levels. Closing futures prices as well as 91-day Treasury Bill yields (representing the risk-free interest rate) are from the *Wall Street Journal*.[4]

The trading data is used to regress premia on the nonvariance determinants as in eq. (6). Variance is not included since simulations excluding variance has no effect on signs of remaining coefficients.[5]

Six sets of regressions are run. Each set has 12 regressions, including six 1986 contracts and six 1987 contracts. The sets are distinguished by whether puts or calls are examined and by the relationship between the futures and strike prices (i.e., the degree to which the option is in or out of the money). "At-the-money" (ATM) options are defined here as having strike prices within one dollar of the futures prices. A call (put) which is in the money (ITM) has a strike which is less (more) than the futures by three dollars. A call (put) which is out of the money (OTM) has a strike which is more (less) than three dollars than the futures. These categories were defined such that a reasonable number of observations exist for each regression. Thus, 72 regressions (12 contracts and six sets) are considered.

Table III reports the coefficients and standard errors of the interest rate variable. The HCB effect is presumed to be revealed by the movement of the Black solution toward the Black-Scholes' solution. However, it is not clear how far toward BS solution this movement should be. Therefore, even though the BS solution for the interest rate is positive and significant, it is still not clear that HCB in Black's solution would be large enough to cause the expected sign of the interest rate coefficient to be positive. Thus, in interpreting the results of Table III, neither the magnitude nor the sign of any one coefficient is critical. The important result is what the entire group of coefficients implies about the probability of observing a positive interest rate coefficient.

[4]In the simulations, the "real" rate is examined for impact by adjusting the nominal rate down from around 8% to 4%, to capture inflation. This produces no qualitative changes in any of the results, as one might expect given the evidence on insensitivity of Black's premium to the interest rate.

[5]This is not to say that variance is not important in the determination of the option premium. Quite the contrary. However, if a researcher's interest is in determining the influence of nonvariance factors, omitting the variance does not have a significant effect in the simulations. As a peripheral point of study, the effect of excluding other variables from the simulations is investigated also. It is found that the exclusion of TTM or IR, like VAR, has no effect on the remaining coefficients signs and little effect on their magnitudes. However, the exclusion of FUT (STK) causes erratic behavior in both sign and magnitude of the STK (FUT) coefficient. The inclusion of FUT and STK through alternative specifications (e.g., FUT/STK) does not cause the coefficient results to vary much. Thus, an important implication is that both FUT and STK be included in regressions when explaining biases. The specification of these variables does not seem critical.

Table III
HOLDING COST BIAS: INTEREST RATE COEFFICIENTS FROM REGRESSIONS[a]

	Calls			Puts		
	ITM	ATM	OTM	ITM	ATM	OTM
Feb 86	−0.008	0.029	−1.918	−0.168	−0.432	−0.609
	(0.313)	(0.543)	(1.230)	(0.246)	(0.475)	(1.498)
Apr 86	0.105	0.257	0.706	1.195*	1.033*	3.356*
	(0.883)	(0.385)	(1.541)	(0.273)	(0.458)	(1.082)
Jun 86	0.000	0.830*	3.073*	0.064	0.448*	0.804
	(0.000)	(0.321)	(0.564)	(0.110)	(0.223)	(0.644)
Aug 86	0.194	0.140	0.460	−0.050	0.228	3.336*
	(0.311)	(0.408)	(0.780)	(0.085)	(0.319)	(1.564)
Oct 86	−0.157	0.995*	−0.494	0.372	−0.015	−1.105
	(0.304)	(0.496)	(1.218)	(0.291)	(0.424)	(2.032)
Dec 86	0.495*	0.094	0.133	0.689*	−0.010	−1.015
	(0.230)	(0.294)	(0.757)	(0.239)	(0.323)	(1.047)
Feb 87	0.089	0.927*	0.107	−0.138	0.896*	1.513*
	(0.230)	(0.369)	(0.460)	(0.232)	(0.297)	(0.702)
Apr 87	0.318	0.724*	1.219	0.214	0.951*	1.627*
	(0.209)	(0.282)	(0.660)	(0.359)	(0.313)	(0.787)
Jun 87	0.333	0.267	0.716	−0.069	−0.323	−2.070
	(0.179)	(0.410)	(0.803)	(0.149)	(0.385)	(1.616)
Aug 87	−0.230	0.308	0.664	0.096	0.598	1.775*
	(0.109)	(0.356)	(0.698)	(0.130)	(0.349)	(0.745)
Oct 87	0.254	0.068	1.651	0.525*	0.443	2.610*
	(0.165)	(0.300)	(0.928)	(0.217)	(0.315)	(0.845)
Dec 87	0.008	0.126	−0.115	0.048	0.027	0.004
	(0.035)	(0.137)	(0.371)	(0.177)	(0.183)	(0.401)

*Indicates the coefficient is significantly different from zero at a 0.05 level.
[a]In most cases regression equations were corrected for first-order autocorrelation.

The 72 strike-contract configurations regressions result in 54 positive interest rate coefficients, of which 19 are significant. Eighteen coefficients are negative; with none significant. Given the simulation results, one would have expected more negative coefficients than positive, and such intuition leads to a method for determining whether these results suggest the presence of HCB. If the simulation probability of a sign change for the interest rate coefficient is accepted as true, then the distribution of the observed number of negative coefficients is binomial. It is possible to calculate the probability of observing 18 negative draws using a normal approximation to a binomial with parameters $p = 0.63$ (the probability that the interest rate coefficient has a negative sign for the "correct" variance results in Table I) and $n = 72$. If the number of negative coefficients is approximated by a normal with mean, $np = 45.36$ and variance, $np(1 - p) = 16.78$, then the observed value of only 18 negative coefficients is over six standard deviations from the mean, implying that HCBs are present in Black's IVs. Alternatively, taking the coefficients in Table III as random draws from a normal population implies the probability of observing a negative interest rate coefficient is approximately 0.33. This represents a 50% movement of the Black toward the BS solution (probability of negative coefficient is 0.02 in Table II). Though the size of the HCB is inconclusive, this is strong evidence that an HCB exists.

A second type of transaction cost is related to the degree of uncertainty associated with the ability to enter or exit the market quickly at a "fair" price. As trading volume or liquidity decreases, this uncertainty presumably increases, causing trading costs to increase. Therefore, it is expected that trading volume and premium are inversely related. An alternative measure of liquidity, suggested by Ward and Dasse (1977), is the relationship between trading volume and commitments. A thin market is one in which changes in commitments are large relative to trading volume. This measure is employed in a manner similar to volume. It produces qualitatively similar results in simulations and regressions using live-cattle data.

The model bias considered here is related to a variable which is not included in Black's valuation model. To help indicate whether the bias is easily detectable, an experiment is conducted in which a proxy for liquidity cost (LC) is added to equation (6). This is done by first generating a random volume variable as a function of time to maturity by the following specification:

$$\mathrm{VOL}_t = \exp(-1.56\ \ln(\mathrm{TTM}_t) + \epsilon_t)\,; \qquad \epsilon_t \sim N(0, 0.5)$$

where VOL is volume, ranging from 0 to 2000 contracts with higher volumes being more likely as the option contract nears expiration, and ϵ_t is normally distributed about zero with a variance of 0.5. The coefficient, -1.56, and the error structure are based on the 1986–87 empirical data on cattle options. LC is then defined as a logistic function of volume, expressed as:

$$\mathrm{LC}_t = \frac{1.0}{1.0 + e^{.25\mathrm{VOL}_t}}$$

This LC function is chosen (somewhat arbitrarily) so that it asymptotes to zero fairly quickly. The maximum LC is .50 when VOL = 0; LC = .05 when VOL = 10; and LC = .01 when VOL = 20. Black's premia biased by liquidity costs are generated as $C_t + LC_t$.

The regression approximation results for both "correct" and "incorrect" variances are presented in Table IV. The VOL regression coefficient is of the expected sign (negative) and significant for both. As important, however, are the coefficient levels and statistics of the other variables relative to those in Table I. Note that the levels of standard errors and probabilities of a change in sign in Table IV are very close to those found in Table I. Therefore, these simulation results indicate that, if the LC function is reasonable, any model bias associated with liquidity should be easy to detect and that its impact is independent of the other variables' impacts.

Regressions using the 1986–87 live-cattle data are conducted in which premia were regressed on futures price, strike price, time to maturity, interest rate, and volume. Table V presents the volume coefficients and standard errors for six categories of each of the 12 option contracts. Only two of the 72 coefficients are significantly negative. The OTM call and put results are particularly counter to expectations. All but two of the OTM coefficients are positive, and 16 of the 24 are significant. When the liquidity measure of Ward and Dasse is employed for the live-cattle data instead of volume, five of the 72 coefficients are significantly negative; and only two of the 24 OTM coefficients are negative. Thus, based on measures employed in this study there is not a detectable liquidity bias.

Table IV
FIRST ORDER APPROXIMATION PERFORMANCE OF BLACK'S MODEL PLUS VOLUME

	Regress Coef	STD Error	Prob of Sign Chg[a]	TS Coef	Mean Bias	STD Error
Correct Variance						
INT	0.0287	0.0096	0.0014	−0.2047	0.2334	0.0006
FUT	11.6042	0.9736	0.0000	9.8825	1.7217	0.0265
STK	−10.6288	0.9737	0.0000	−8.8825	−1.7463	0.0258
TTM	0.4737	0.0246	0.0000	0.5161	−0.0424	0.0011
VAR	0.9841	0.0628	0.0000	0.9985	−0.0144	0.0028
IR	−0.0157	0.0486	0.3734	−0.0175	0.0018	0.0022
VOL	−0.0271	0.0074	0.0001			
Incorrect Variance						
INT	0.0293	0.0092	0.0007	−0.1916	0.2210	0.0005
FUT	11.3277	0.4861	0.0000	9.7617	1.5659	0.0216
STK	−10.3488	0.4597	0.0000	−8.7617	−1.5871	0.0205
TTM	0.4740	0.0225	0.0000	0.5161	−0.0421	0.0010
VAR	0.0023	0.0552	0.4837	0.0000	0.0023	0.0025
IR	−0.0147	0.0435	0.3673	−0.0175	0.0028	0.0019
VOL	−0.0265	0.0066	0.0000			

[a]Calculated assuming the "*t* ratio" is a standard normal.

Early Exercise Bias

Black's solution is an equilibrium premium for "European" options (options that can be exercised only at expiration) but is not an equilibrium solution for "American" options (which can be exercised before expiration). All agricultural options traded in the U.S. can be exercised any time during the option's life. Because Black's model is easy to use, practitioners often use the model to guide pricing decisions for these options. However, since Black's solution is for a European option, the resulting IV may overstate the market's variance forecast, since the correction for early exercise increases the premium. The extent of adjustment needed is illustrated by Hauser and Neff under various levels of interest rate, volatility, time to maturity, and difference between strike and futures prices. Relatively large adjustments are needed when the option has a long time to maturity and is deep in the money. Under most other conditions, the adjustment is quite small and of little practical importance. Note, however, that in the case of an early exercise bias (EEB), the factors which are likely to determine the size of the model bias also influence the determination of the premium directly. For instance, the difference between the futures and strike prices determines the amount the option is in the money. The deep in the money options are more likely to be exercised early, but this means the coefficient of futures and strike in the regression should be larger in absolute value. Yet, the simulation results of Table I indicate that even if the model is exactly true, the coefficient biases are in that direction. Therefore, to detect the EEB, another approach must be considered.

One pricing model which explicitly incorporates the possibility of early exercise is the binomial option pricing model of Cox, Ross, and Rubinstein (1979). Rather than assuming the continuous diffusion process, Cox, Ross, and Rubinstein assume a discrete diffu-

Table V
LIQUIDITY COST BIAS: VOLUME COEFFICIENTS FROM REGRESSIONS[a]

	Calls			Puts		
	ITM	**ATM**	**OTM**	**ITM**	**ATM**	**OTM**
Feb 86	0.384	−1.011	2.005	−0.370	−0.061	5.906*
	(0.357)	(0.542)	(1.128)	(0.379)	(0.816)	(2.333)
Apr 86	−0.625	−0.451	3.757*	0.924*	−0.579	3.244*
	(0.421)	(0.367)	(1.066)	(0.374)	(0.593)	(1.425)
Jun 86	0.000[b]	0.386	4.863*	0.051	−0.374	1.380
	(0.000)	(0.538)	(0.979)	(0.448)	(0.571)	(1.022)
Aug 86	0.291	1.605	−0.803	−0.340	−0.187	8.258*
	(0.368)	(1.057)	(1.488)	(0.337)	(0.800)	(2.566)
Oct 86	−0.313	−0.308	4.220*	1.103	−0.646	9.501*
	(0.581)	(1.096)	(1.114)	(1.225)	(0.648)	(2.645)
Dec 86	−0.574	0.035	1.640	−0.818	0.092	6.817*
	(0.713)	(0.653)	(1.398)	(0.525)	(0.553)	(1.932)
Feb 87	0.413	−0.240	5.019*	−0.680	−0.625	2.899
	(0.435)	(1.017)	(1.485)	(0.635)	(0.874)	(1.593)
Apr 87	0.393	1.503	3.040*	1.631*	−1.905*	5.668*
	(0.543)	(0.889)	(0.955)	(0.776)	(0.710)	(1.481)
Jun 87	0.078	−0.525	5.126*	2.241*	−2.086*	5.207*
	(0.299)	(0.974)	(1.041)	(0.320)	(0.629)	(1.770)
Aug 87	0.134	−0.120	3.865*	−0.116	0.089	8.710*
	(0.410)	(0.798)	(1.012)	(0.369)	(0.734)	(1.734)
Oct 87	0.025	0.301	0.791	0.455	−0.762	3.689*
	(0.304)	(0.606)	(1.217)	(0.303)	(0.606)	(1.763)
Dec 87	0.323	−0.112	−0.915	−0.851	−1.034	1.098
	(0.240)	(0.488)	(0.928)	(1.294)	(0.658)	(1.113)

*Indicates the coefficient is significantly different from zero at a 0.05 level.

[a]All coefficients and standard errors have been multiplied by 100 for ease of presentation. In most cases regression equations were corrected for first-order autocorrelation.

[b]Insufficient observations were available for the regression.

sion process, the multiplicative binomial. Intuitively, the life of the option is divided into stages. At each stage the option price is assumed to increase or decrease by a determined amount, with probabilities p and $(1 - p)$, respectively. This formulation of the option price has both advantages and disadvantages over Black's model. Since the process is viewed as happening in discrete stages, it is possible at each stage to evaluate whether it is better to hold the option or exercise early. Thus, the binomial model is not subject to the EEB. The solution technique for the binomial process is to treat it as a stochastic dynamic programming problem.[6] One starts at expiration, moving backward in time, picking the strategy in each time period which maximizes the expected present discounted value of the option. As the number of stages is increased the binomial process

[6]The binomial model, as developed by Cox, Ross, and Rubinstein, is designed to model physicals, with or without dividends. To apply it to futures, the dividend in each period is adjusted so as to make the holding cost of the futures zero, i.e., the dividend rate is equal to the risk-free rate (see Hauser and Neff for details).

converges to the lognormal process (for properly chosen parameters), while it maintains the possibility of early exercise. The main disadvantage of the binomial model is that the calculations involved are cumbersome relative to the "closed-form solution" of the Black model.

To determine the extent of the EEB in Black's premium relative to the binomial model, a comparison between the two models' implied volatilities for observed market premia can be made. Observed differences should be related to the characteristics which promote early exercise; i.e., how far the option is in the money and the length of time to the maturity date.

To illustrate, futures, strikes, time-to-maturities, Black's IVs, and interest rates are generated under the "correct variance" simulations. Next, binomial IVs are generated and the percentage difference between Black and binomial IVs are calculated. This procedure differs from that of earlier simulations in two ways. First, regression and TS coefficients are not calculated because of the biases noted above. Instead, the means and ranges of percentage differences between Black and binomial IVs are calculated. Second, replications are not performed because of the time consuming requirement in calculating binomial IVs. Results of the correct-variance simulation are presented in Table VI indicate the effects of early exercise opportunities.

For the simulated data, the mean percentage difference between Black and binomial IVs was .51%, with a range of .00% to 1.00% for at-the-money calls. Two things stand out in these simulated results. First, as expected, the EEB declines as the option moves from in- to at- to out-of-the-money. Second, puts are generally less biased than calls for similar strike-futures relationships.

Calculations are made also for the 12 live-cattle options traded in 1986 and 1987. The means and ranges for each is calculated for ITM, ATM, and OTM options. Only those options with 30–60 days to maturity are included in the calculations to minimize the time-to-maturity effects. These empirical results are presented in Table VII. The means and ranges are calculated also for each contract separately, resulting in the same implications as those of Table VII. The 72 contract specific means and ranges are available from the authors upon request.

Consistent with the simulated results, empirical means decline as one moves further out of the money. As expected, the differences reflect an increased likelihood of exercise as the option gets further in the money. Also consistent with the simulated results is the fact that, for any one contract and strike-futures relationship, means and ranges are larger for calls than for puts. For example, ATM calls have mean difference of .37%, while ATM puts are .19% and so on. This is true for all strike futures configurations. A similar relationship exists for the ranges. This result is not expected a priori. It seems

Table VI
MEANS AND RANGES OF PERCENTAGE DIFFERENCES BETWEEN BLACK AND BINOMIAL IMPLIED VOLATILITIES FOR SIMULATED DATA

Calls			Puts		
ITM	**ATM**	**OTM**	**ITM**	**ATM**	**OTM**
1.22[a]	0.51	0.22	0.79	0.34	0.10
(.40, 2.40)[b]	(.00, 1.00)	(−.10, .60)	(.00, 1.60)	(−.10, .70)	(−.10, .40)

[a]Mean percentage differences.
[b]Range of percentage differences.

Table VII
MEANS AND RANGES OF PERCENTAGE DIFFERENCES BETWEEN BLACK AND BINOMIAL IMPLIED VOLATILITIES FOR LIVE-CATTLE DATA

Calls			Puts		
ITM	ATM	OTM	ITM	ATM	OTM
1.58[a]	0.37	0.01	0.80	0.19	−0.04
(.39, 4.27)[b]	(.04, .63)	(−.18, .27)	(.06, 3.82)	(−.06, .37)	(−.17, .14)

[a]Mean percentage differences.
[b]Range of percentage differences.

unlikely that EEBs are larger for calls than puts due to other model biases. Neither the Black nor binomial models allow for positive holding costs and thus both Black and binomial IVs are equally affected by any holding cost biases which exist in actual premia. Furthermore, a significant liquidity bias is not found in live cattle options premia. So the evidence in Table VII suggests (1) that there are small EEBs in Black IVs and premia and (2) that calls are more likely to be exercised early than puts.

SUMMARY AND CONCLUSIONS

A simulation framework is developed to examine the validity of using a regression approximation in conducting structural analysis of option premia. This is done for two reasons. First, linear regression has been used in past research to identify and measure model biases in Black's option pricing model. Second, the examination may lead to alternative techniques with which to pursue the issue of option model biases. While there is little doubt that Black premia are biased by some assumptions, the degree to which such biases are measurable, or even detectable, is an open question.

Black's option pricing formula is expanded in a Taylor series (TS), and TS coefficients (evaluated at sample means) are compared to regression coefficients. From this stage of the analysis, two conclusions can be drawn about the general performance of regression results when analyzing model biases. First, regression coefficients are biased estimates of those of the Taylor series, i.e., coefficient biases exist. Second, while these coefficient biases are significant, they are small relative to the magnitude of the coefficients, allowing the direction of impacts to be inferred with confidence. The exceptions to this second conclusion apply to the interest rate coefficient, which has a high probability of the wrong sign in the regressions; and to the impact of the variance if the variance used to calibrate the regressions is not that which is used in the market to determine the premium. The errors-in-variables problem (i.e., using the wrong variance estimate) does not appear to have an adverse impact on the remaining coefficients. Indeed, simulations omitting the variance variable yield results similar to the correct variance model for the remaining variables. It is critical, however, that in a structural implied volatility (IV) analysis, both futures and strike price variables be included when explaining biases.

Simulation results are used then to help analyze empirical data on live cattle options. The objective is to isolate effects of potential model biases in Black's IVs. The model biases arise from two sources. Black's pricing model derives values for European options, which can only be exercised at maturity. Thus, there exists a possible early-exercise bias (EEB) in Black IVs for live-cattle American options. The other source of potential model bias in Black's model emanates from the assumption of zero transaction costs. One bias is in the form of a liquidity cost bias (LCB). The other is a holding cost bias (HCB). All three of the biases are examined using simulations and live-cattle options data. The results of the simulations indicate that all three of the biases should be detectable. Results using live-cattle data demonstrate the presence of only EEB and HCB.

Significant EEB is found in all live-cattle options. EEB is investigated by considering differences between the IV determined by Black's model and that determined by a model accounting for the possibility of early exercise. The magnitude of the EEB is lowest for out-of-the-money (OTM) options, which has a maximum EEB of less than .27%. EEB is greatest for in-the-money (ITM) options. For these options, the EEB is as high as 4.27%. The at-the-money (ATM) options have maximum EEBs of about .63%.

Significant HCBs are found also in live-cattle options. The evidence is based on the observation that if HCB is present, then the Black model solutions move toward the Black-Scholes solution. This movement is revealed by the regression coefficient of the interest rate. For the Black model, simulations demonstrate that one should expect this coefficient to be negative on average and insignificant. On the other hand, the Black-Scholes' simulation indicates that the interest rate coefficient should be significantly positive. The regression using live-cattle options data for various contracts and strike-futures relationships for calls and puts produces far too many positive interest rate coefficients to be the result of chance if the zero holding cost assumption is true. The interpretation of the results in Table III show that they reflect a significant movement of the Black toward the Black-Scholes solution, offering strong evidence of the presence of HCB in live-cattle options. The magnitudes of HCB, however, are not known.

Finally, an LCB is not detected. The simulation results suggest that if an increment to the option premium exists for low liquidity in the market, then a regression of the premium on its determinants and the trading volume would reveal this bias through a volume coefficient which is negative and significant. The live-cattle regressions provide no such evidence for the coefficient of volume or other liquidity measures.

What are the implications of these findings for researchers and practitioners who work with IVs and premia? One general message is that the use of regression as a tool for structural analysis of options is justified when inferring directional impacts of option pricing factors. However, caution should be used when interpreting impacts related to the interest rate and, unless the researcher/practitioner develops a variance forecast model which reflects that actually used by the market, inferences about variance effects are weak. The specification of the regression equation should, at minimum, include variables representing the strike and futures prices.

Another implication is related to the type of option which should be used by the researcher/practitioner when estimating the market's forecast of variance in the form of IVs. In short, if the analyst feels comfortable in adjusting Black's IV to account for time-to-maturity and moneyness effects, then at-the-money options should be used. Otherwise, out-of-the-money options which trade at least five times per day are advised. Finally, while conventional wisdom among traders is that calls are "better understood" than puts, the early-exercise biases of live-cattle options suggest puts are less likely to be exercised early and, therefore, suffer smaller biases than calls.

Appendix A

GRAPHICAL DEMONSTRATION OF OLS BIAS

In Figure 1, which is similar to that of White, the nonlinear function is a plot of $f(x) = x^{-1}$. The scatter of points around this line are given by $f(x) + \epsilon$, with $\epsilon \sim N(0, .025)$. A first-order Taylor series (TS) expansion allows the approximation of a nonlinear function by expanding around a point, usually the sample means. The TS

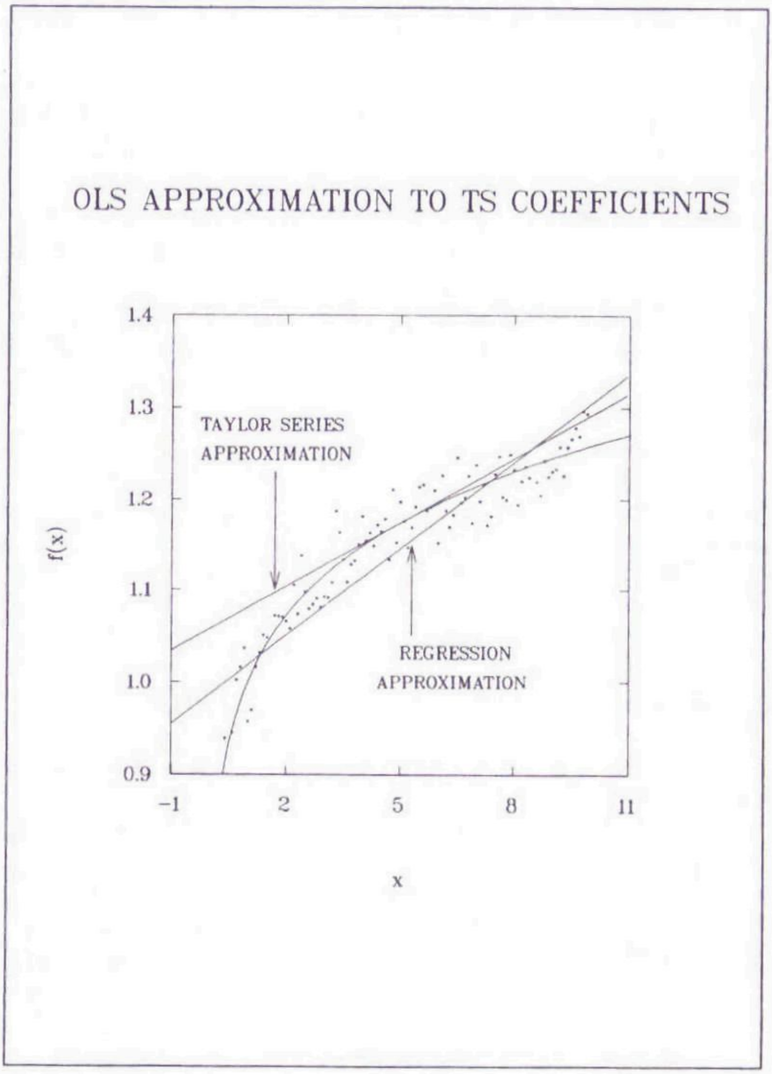

Figure 1.

coefficients are the slopes of the function with respect to its determinants. Thus, the behavior of the function with respect to changes in the determining factors are revealed, locally. The figure demonstrates the bias of OLS approximations (lower straight line) to TS coefficients (upper straight line) for a nonlinear function. Note, that while the OLS slope is different from the TS at the sample mean, the signs are the same. Thus, often the direction of impact of determining factors may be inferred through the use of linear regression.

Appendix B

SIMULATION DATA

The simulation experiment follows Black's assumptions that: (1) the futures price follows a lognormal diffusion process with zero mean and constant variance; (2) the interest rate is known and constant or evolves according to a known function of time;

(3) options may be exercised only at maturity; and (4) a portfolio of options, futures, and a risk-free bond may be continuously adjusted in a manner which generates the risk-free rate of return.

A critical component of the experiment is the distribution from which futures prices are drawn. Given lognormality of the underlying futures price distribution, there are two parameters which completely specify the density: the mean and variance. When defining the price distribution expected at expiration, the assumptions underlying the Black model imply the current log-futures price is the expected log-futures price. This assumption is maintained across all subsequent simulations.

To help determine a "variance rate," daily futures prices for the February live-cattle contract from 1966–86 are used to calculate annualized variances. The historical annualized standard deviation percentages are calculated for each year of the period and range from a low of 10 to a high of 50. Simulations for a number of volatilities in this range are run. For the sake of space, the results which assume a constant IV of 30 (futures price annual variance is 0.09) are presented. These results are representative.

The length of time over which a contract trades is based on the average length of seven option contracts. (While data is available on all twelve options traded in 1986 and 1987, data for only seven of the contracts is complete.) Although cattle options are offered for about one year before expiration, the average time to expiration on the first day of trading is 184 days, or about six months. Trading is always thin for the first two months and thus excluded. As is typical in the other option studies, the last two weeks of trading is excluded also due to unusual trades made to close out positions as the option expires. This leaves an average of about 100 trading days for live-cattle options.

An interest rate distribution is based on the daily yields for 91-day Treasury Bills offered from February through July, 1985. Interest rates are assumed to be lognormally distributed with a mean of log(.08), and an annualized standard deviation of 0.09. Data on 91-day Treasury Bills are from the *Wall Street Journal*.

A variety of strike prices are used for in-the-money (ITM) and out-of-the-money (OTM) options. The results of these simulations are similar to those for at-the-money (ATM) options and so ATM results are presented. Finally, simulations for both puts and calls produce similar results, and thus only those for calls are presented.[7]

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[7]Three final considerations are important in the experimental design of the simulations. First, since the underlying model is multiplicative, i.e., $\log(F_{t+1}/F_t)$, the logarithmic form of the TS expansion is chosen. Second, in the subsequent analysis of live-cattle options, many of the regressions do not have enough observations to estimate a second order TS model and because of the collinearity one would expect between the second order terms involving the futures price and the strike price, all simulations are on first order TS expansions. This is not what is recommended by Byron and Bera. It is, however, in keeping with what has been done in past IV studies. Third, results presented in the text are based on 500 replications. They are similar to results based on 1000 and 2000 replications.

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