A Note on Hedging Performance and Portfolio Effects

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Under current regulations, any proposal for a new futures contract must demonstrate the underlying hedging demand. The most popular measure of hedging effectiveness is that of Ederington (1979) where a representative hedger minimizes his risk upon hedging the position in a single cash commodity through transactions in a single futures contract.[1] Thus, a simple linear regression with the cash price level (or difference) being the dependent variable and with the futures price level (or difference) and a constant term being the independent variable generates the coefficient of determination, R^2, corresponding to the Ederington measure of hedging effectiveness. The method is popular in the literature.

Nonetheless, as pointed out in Stein ((1986), p. 59–60), the main users/hedgers of futures contracts are the intermediaries which simultaneously hold various cash commodities. Any futures contract serves both direct and cross hedging purposes. A more reasonable and realistic measure of hedging effectiveness should, therefore, consider a portfolio of various cash commodities. On the other hand, a hedger may simultaneously transact in several futures markets to reduce the overall risk of his cash portfolio. Examples are CBOT and KCBT wheat futures for a soft wheat grower (Lien (1989)) and Deutsche mark and British pound sterling futures for a holder of European currencies (Saunder and Sienkiewicz, (1988)). Anderson and Danthine (1981) present a framework for hedging a futures portfolio against a cash portfolio. An immediate implication is that the hedging effectiveness of any specific futures contract cannot be determined by itself unless it is the sole component of the futures portfolio, otherwise the composition of the portfolio will affect the hedging effectiveness (in which case the hedging effectiveness of a futures contract may be defined as its "marginal" contribution to the reduction of the overall risk). Moreover, the cash portfolio affects the hedging effectiveness in both cases.[2] The impact of cash and futures portfolios is termed "portfolio effects."

To illustrate, assume there are h futures contracts and consider a risk minimizing hedger with the cash portfolio $S = (S_1 S_2 \cdots S_k)'$ where S_i is his cash position in the i^{th}

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[1]Upon taking into account the risk-return characteristics and the transaction cost, alternative measures are proposed in Howard and D'Antonio (1984) and Gjerde (1987). Both measures, however, also pertain to the case of a single cash commodity and a single futures contract.

[2]A similar concept is provided in Lypny (1988). Nevertheless, Lypny assumed a specific cash portfolio with equal cash positions on the two cash commodities.

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commodity, $i = 1, 2, \cdots, k$. Upon adopting the risk minimizing strategy, his risk (in terms of the variance of his profit) is

$$V^* = S'[\text{var}(p) - \text{cov}(p,f)\ \text{var}^{-1}(f)\ \text{cov}(f,p)]S,$$

where var(p) is the variance-covariance matrix of the k cash prices, cov(p,f) is the covariance matrix of the k cash prices and the h futures prices, var(f) is the variance-covariance matrix of the h futures prices. Finally, cov(f,p) is the transpose of cov(p,f). If the hedger places no hedge, his risk is $V^0 = S'\ \text{var}(p)S$. The hedging effectiveness of the futures portfolio may be written as: $E = (V^0 - V^*)/V^0$, or more precisely,

$$E = S'\ \text{cov}(p,f)\ \text{var}^{-1}(f)\ \text{cov}(f,p)S/S'\ \text{var}(p)S.$$

Obviously the cash portfolio (S) affects the hedging effectiveness (E) in a nonlinear fashion. While a single cash commodity is a special case of the cash portfolio, the hedging effectiveness thus generated provides limited information. Specifically, a futures portfolio may be a poor hedging instrument for certain cash portfolios despite its great hedging performance against each cash commodity individually. Thus, the Ederington measure may not be representative of the true hedging effectiveness (which should take into account different cash portfolios across hedgers).

Given that E varies with the cash portfolios, an attempt to characterize an upper and a lower bound upon considering all possible nonzero portfolios is attempted here. Let $A = \text{cov}(p,f)\ \text{var}^{-1}(f)\ \text{cov}(f,p)$ and let $B = \text{var}(p)$. Consider the solutions $\lambda_1, \ldots, \lambda_k$ for the following equation:

$$\det(B^{-1}A - \lambda I) = 0,$$

where det($\cdot$) is the determinant operator and I is an identity matrix. Then $0 < \lambda_{\min} \leq E \leq \lambda_{\max} < 1$, where $\lambda_{\min} = \min(\lambda_1, \ldots, \lambda_k), \lambda_{\max} = \max(\lambda_1, \ldots, \lambda_k)$. [For details, see the Appendix]. The two numbers, $\lambda_{\min}$ and $\lambda_{\max}$, provide better information regarding the hedging effectiveness: whatever the cash portfolio may be. The highest hedging effectiveness the futures portfolio can provide is $\lambda_{\max}$ and the lowest is $\lambda_{\min}$.

Similar approaches may be applied to determine the hedging effectiveness of a specific futures contract. For example, assume one adds a new contract to the futures portfolio. The new minimum risk is

$$\overline{V} = S[\text{var}(p) - \text{cov}(p,\bar{f})\ \text{var}^{-1}(\bar{f})\ \text{cov}(\bar{f},p)]S,$$

where $\bar{f}$ is the vector of the $(h + 1)$ futures prices. The hedging effectiveness for the new contract is then

$$U = (V^* - \overline{V})/V^* = S'CS/S'DS.$$

where $D = B - A$ and $C = \text{cov}(p,\bar{f})\ \text{var}^{-1}(\bar{f})\ \text{cov}(\bar{f},p) - A$; A and B are defined above. Let $\theta_{\max}$(resp. $\theta_{\min}$) be the maximum (resp. minimum) of the solutions for the equation: $\det(D^{-1}C - \theta I) = 0$. Then $0 < \theta_{\min} \leq U \leq \theta_{\max} < 1$. The highest hedging effectiveness for the new futures contract is $\theta_{\max}$, the lowest is $\theta_{\min}$. To derive the hedging effectiveness for a futures contract that is already a component of the futures portfolio, consider the portfolio of the remaining $(h - 1)$ contracts. The resulting minimum risk is

$$\tilde{V} = S'[var(p) - \text{cov}(p,\tilde{f})\ \text{var}^{-1}(\tilde{f})\ \text{cov}(\tilde{f},p)]S,$$

where $\tilde{f}$ is the vector of the $(h - 1)$ futures prices. The hedging effectiveness of the component futures contract is

$$W = (\tilde{V} - V^*)/\tilde{V}.$$

Following similar procedures, an upper and a lower bound for W can be ascertained.

The above results have important implications for determining the success/failure of a new futures contract. Specifically, Black (1986) argued that the success/failure of a new futures contract is determined by its hedging effectiveness as compared to the hedging effectiveness of existing futures contracts. Thus Saunders and Sienkiewicz (1988) compared ECU futures to a portfolio of Deutsche mark and British pound sterling futures and concluded that the ECU futures contract is doomed to fail. The approach assumes that the new futures contract is competing against existing futures. However, one could argue that the new contract is more likely to supplement the existing futures, at least during earlier periods. The usefulness of the contract must be based upon the comparison between the original futures portfolio and the new portfolio in which the new contract is also a component. This approach provides the range of the hedging effectiveness of the new contract and its success/failure can be determined therein.

Finally, note that when deriving the upper and lower bounds, it is assumed that the structure of cash and futures portfolio is well specified such that there is no ambiguity about which cash commodity and which futures contract should be a component of the portfolios. While the assumption on the cash portfolio may be acceptable, the hedger needs to construct his futures portfolio. Obviously any additional futures contract always helps to reduce his risk. The determination of the futures portfolio structure therefore must rely upon other factors such as the transaction cost induced by adding a futures contract into the portfolio. Alternatively, a futures contract could be added to the portfolio only when its contribution to risk reduction is statistically significant.

Appendix

To find the extreme points of $S'AS/S'BS$, take partial derivatives of the objective function with respect to S. The resulting first order condition is

$$(S'BS)AS - (S'AS)BS = 0.$$

Let $\lambda = S'AS/S'BS$, (λ is a scalar), then:

$$(A - \lambda B)S = 0 \quad \text{or} \quad (B^{-1}A - \lambda I)S = 0.$$

In other words, λ is an eigenvalue of the matrix $B^{-1}A$. Since both A and B are positive definite, and since $B - A$ is positive definite, then $0 < \lambda < 1$. (See Dhrymes, (1980), p. 581–583.) Moreover, the maximum of $S'AS/S'BS$ over S equals to λ_{max} and the minimum of $S'AS/S'BS$ equals to λ_{min}.

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