Examining the Validity of a Test of Futures Market Efficiency: A Comment

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In a recent issue of this Journal, Elam and Dixon (1988) reexamine why the empirical testing of futures market efficiency tends to be rejected. They argue that the censoring explanation presented by Maberly (1985) is incorrect and that the correct reason is due to the existence of the unit root in spot price. However, they do not think that the asymptotic distribution of the F-statistic has been derived when spot prices have the unit root. Therefore, they conduct several Monte-Carlo experiments and find that the F-test tends to (wrongly) reject the true model too often. They conclude that the F-test is not a reliable test for price efficiency.

This article agrees with their argument that the random walk of spot price is the main source of rejection. It complements their article by explaining their puzzling Monte-Carlo experiments. Elam and Dixon's testing model is

$$S_{t+i} = a + bS_t + e_{t+i} \tag{1}$$

where S_{t+i} is the spot price at period $t + i$. Pricing is considered efficient if $a = 0$ and $b = 1$. Elam and Dixon are aware that the F-distribution might be invalid in this case but they do not know it has been already derived.

Phillips (1986, 1987) shows that S_t will converge to the Wiener process if S_t has the unit root. The simple intuition of the Wiener process convergence theory can be understood easily by studying the definition of white noise. White noise is defined as the first difference of a univariate Wiener process when sample size goes to infinity. Phillips proves that if the first difference of a univariate time series is white noise, then that series must converge to Wiener process. If S_t converges to Wiener process, then Phillips and Durlauf (1987) show that conventional t- and F- statistics also converge to the Wiener process. In addition, if S_t has the unit root, the distribution of OLS estimate of b skews to the left (Fuller (1976)), then the critical value of the 5 percent significant level moves to the right making the conventional test reject too often.

In fact, Dickey and Fuller (1981) have tabulated the critical vlaues of the F-test if the null hypothesis of (a, b) is (0, 1). They are 0.29 and 6.3 for the 5 percent significance level of the two-sided test when the sample size is equal to 25 ((1981) Table IV). Applying Dickey and Fuller's critical values to Elam and Dixon's Table II, one cannot reject

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The Journal of Futures Markets, Vol. 10, No. 2, 195–196 (1990)

CCC 0270-7314/90/020195-02$04.00

the true model of the market efficiency hypothesis.[1] This can be seen by noting that all the empirical F values estimated by Elam and Dixon are within the range of 0.29 and 6.3. The same case can be applied to the Monte-Carlo experiments of Maberly's Table I. Using the critical values provided by Fuller (1976), one cannot reject the null hypothesis that b is equal to one.

The conventional F-test is invalid when the series has the unit root. The asymptotic distribution of the first autoregression coefficient (here, $\hat{b}$) is discontinuous at the root is equal to one. Using conventional statistics will create the too often rejection problem. In Elam and Dixon's model, the efficiency of the future market can be tested by applying Dickey and Fuller's (1981) critical values and the puzzling Monte-Carlo experimental result is understood. However, if variables on the both sides of (1) are different series, then Dickey and Fuller's test cannot be used. As an alternative test, Engle and Granger (1987) suggest the use of the cointegration technique to test market efficiency.

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