Entry-Deterring Contract Specification on Futures Markets

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In a recent article, Lien (1989(b)) shows that within a mean-variance framework the optimal futures contract for a commodity with various grades is a conventional cash settlement contract that dictates the futures price as a linear combination of various cash prices (at maturity) such that the weights are independent of both the variance-covariance matrix of cash prices and the risk aversion coefficients of hedgers. The criteria for optimality are based upon the assumption that the exchange attempts to maximize either the hedgers' expected utility levels or the total futures transaction volume in the absence of competitive pressure.

The two objective functions seem reasonable. In particular, transaction volume maximization is well accepted in the literature because brokers' revenues, market liquidity, and the seat price for floor traders all respond positively to increasing transaction volume (see Silber (1981) and Grossman (1986)). The assumed absence of competitive pressure may, however, be an unrealistic assumption. Based upon empirical evidence, Fischel (1986) argues that the current Commodity Futures Trading Commission (CFTC) regulations appear to favor new contracts that are very similar to existing contracts instead of those that are more innovative.

Moreover, it is argued that the existence of contracts for various grades of the underlying commodity improves cross-hedging effectiveness and, in turn, helps gain CFTC approval. Examples of competing futures contracts are: the Treasury note futures contract traded at the New York Cotton Exchange (NYCE) and those traded at the Chicago Board of Trade (CBOT). Despite differences between the two contracts (they differ in deliverable notes and in the conversion formula used to evaluate the cheapest deliverable notes) trading statistics suggest that these contracts compete directly (Angrist (1988)).

Since there is always a possibility of competing futures contracts, an innovative exchange tries to prevent entry of rival exchanges and thus maintain a monopoly. This article investigates the impact of entry deterrence objectives upon the choice of contract settlement specifications of a new futures contract.

The approach adopted here is a normative analysis. That is, this article asks the question how the exchange "ought to" behave in certain circumstances upon imposing

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specific objective functions. The generation of the actual behavior of the exchanges which would require a positive analysis is not explored.[1]

BASIC MODEL

The model ignores several institutional characteristics such as margins, market liquidity concerns, and to what extent reliable cash prices may be cheaply collected. Therefore, one should not expect the results to succeed in explaining actual behavior.

Consider a two-period world such that $t = 0$ is the present time and $t = 1$ is the common maturity date of the futures contracts. There are three agents: a representative hedger, an innovative exchange (Exchange A), and a rival exchange (Exchange B). The hedger (an elevator operator) buys N grades of a commodity at $t = 0$, and then stores them to sell at $t = 1$. Let P_{it} denote the cash price of the i^{th} grade at time t; $i = 1, 2, \ldots, N$, $t = 0, 1$. Cash prices are assumed to be exogenously determined such that P_{i0} is known to the hedger, but P_{i1} is a random variable at $t = 0$. Following the empirical findings of Labys and Granger (1970), assume that cash prices follow a martingale process with current cash prices being unbiased predictors of future cash prices: $E(P_{i1}) = P_{i0}$, $i = 1, 2, \ldots, N$, where $E(\cdot)$ is the expectation operator conditional on information available at $t = 0$.

The variance-covariance matrix of cash prices is denoted by Σ, i.e., $\Sigma = E[(P_1 - E(P_1))(P_1 - E(P_1))']$, where $P_1 = (P_{11} P_{21} \ldots P_{N1})'$ is an $(N \times 1)$ column vector. The storage cost for each grade is specified as $(c_i/2)S_i^2$ where $c_i > 0$ is the cost factor and S_i is the cash position of grade i (see Bond and Thompson (1985)). The return from storing and processing a unit of grade i is $k_i > 0$. $(S_1 S_2 \ldots S_N)'$ and $(k_1 k_2 \ldots k_N)'$ are denoted by S and k, respectively; S and k are both $(N \times 1)$ column vectors. Also, let C be a $(N \times N)$ diagonal matrix such that $C_{ii} = c_i$ and $C_{ij} = 0$, $\forall\, i \neq j$. The total storage cost $\Sigma_{i=1}^{N} c_i S_i^2/2$ may be written as $S'CS/2$ whereas the total return from cash market operations is $S'k$.

To reduce risk exposure, the hedger may hedge his cash positions by purchasing or selling futures contracts if they are available. Assume Exchange A first recognizes the potential for such a futures market. Consequently, a futures contract is provided (called A-Futures). The contract dictates the futures price as a function of various cash prices at maturity: $f_{A1} = f_A(P_{11}, P_{21}, \ldots, P_{N1})$ where f_{At} is the price of A-Futures at time t, $t = 0,1$. The exchange has the choice of the functional form $f_A(\cdot)$. Given that A-Futures is available, Exchange B must decide whether or not to compete in the market. Since the success/failure of a futures contract depends upon the transaction volume it generates, Exchange B will only consider whether or not to provide the optimal contract that leads to maximum transaction volume among all possible contract specifications. We assume B-Futures (the contract provided by Exchange B) dictates the futures price as a function of cash prices at maturity: $f_{B1} = f_B(P_{11} P_{21} \ldots P_{N1})$ where f_{Bt} is the price of B-Futures at time t, $t = 0,1$. Moreover, assume that there is a critical value, Θ, such that no futures contract can survive if its transaction volume is below Θ. Thus, Exchange B will provide the optimal B-Futures if it generates sufficiently large transaction volume, otherwise the exchange should not compete in the market.

[1]A referee provides examples that the actual behavior of exchanges differs from the normative predictions of this article: (1) While NYCE T-note futures contract (the innovative contract) requires physical delivery, the imitative CBOT contract applies the same settlement method. This occurrence differs from this article's predictio of a conventional cash settlement method and (2) Although CBOT T-bond futures contract requires physical delivery, it seems to succeed in entry deterrence.

Conceivably, the optimal B-Futures takes different forms across various contract specifications of A-Futures. To prevent the entry of Exchange B, with the knowledge of B's subsequent response, Exchange A will choose a contract specification for A-Futures such that the corresponding optimal B-Futures cannot survive. Meanwhile, the success/failure of A-Futures also depends upon its transaction volume. Exchange A therefore chooses the optimal A-Futures to maximize the transaction volume among all possible "entry-preventive" contract specifications. This article's interests center on the form of the optimal A-Futures if it exists.

As described above, this model consists of a sequence of actions by different agents. To derive the optimal entry-deterring contract, the problem must be solved backwards. First, consider the hedging decision of a representative hedger. Assume that the innovative contract does not possess the advantage of higher market liquidity, and hence only hedging effectiveness concerns the hedgers. First, one can derive the optimal hedging for arbitrary pairs of A- and B-Futures (characterized by arbitrary settlement functions $f_A(\cdot)$ and $f_B(\cdot)$). The transaction volumes for both contracts are determined therein. Second, if Exchange B ever decides to compete, the optimal B-Futures contract specification is chosen to maximize the transaction volume of the contract. The resulting solution characterizes the optimal settlement function $f_B(\cdot)$ as a function of $f_A(\cdot)$ since the latter is *a priori* determined by Exchange A when Exchange B is making its decision. Finally, Exchange A notices that its choice of A-Futures contract specification will affect both the optimal B-Futures contract specification and the transaction volumes for both contracts. The optimal entry-deterring contract is chosen with appropriate settlement function $f_A(\cdot)$. These steps are described in the following three sections.

OPTIMAL HEDGING BEHAVIOR

As stated above, the hedger may reduce his risk exposure upon purchasing or selling futures contracts. Let V_A and V_B be the numbers of contracts the hedger purchases/sells. (It is a purchase if $V_i > 0$ and a sale if $V_i < 0$; $i = A, B$). His profit at $t = 1$ is:

$$\tilde{\pi} = S'(P_1 - P_0) + S'k - S'CS/2 + (f_{A1} - f_{A0})V_A + (f_{B1} - f_{B0})V_B.$$

where the first three terms summarize the profits generated from cash market operations; the latter two terms correspond to profits generated from transactions in the two futures contracts.

Now, a mean-variance approach is adopted. Let $(\lambda/2)$ be the risk aversion coefficient of the hedger; $\lambda > 0$ such that a more risk averse hedger has a larger λ. The expected utility for the hedger is then $E(\tilde{\pi}) - (\lambda/2)\mathrm{var}(\tilde{\pi})$ where $\mathrm{var}(\cdot)$ is the variance operator conditional on information available at $t = 0$. We further assume that a martingale equilibrium characterizes the futures market: $E(f_{i1}) = f_{i0}$, $i = A, B$. As a result, the hedger's expected utility may be written as follows:

$$u = S'k - S'CS/2 - \lambda(\tilde{S}'V_B)\tilde{\sum}(\tilde{S}'V_B)'_2,$$

where $\tilde{S} = (S'V_A)'$ is an $(N + 1) \times 1$ column vector, $\tilde{\Sigma}$ is the variance-covariance matrix of cash and futures prices.[2] We adopt the following partition, it is adopted to simplify the derivation of optimal V_B:

[2]The notations are adopted to simplify mathematical expositions. Essentially, the $(N + 2)$ cash and futures positions are grouped into two terms: a portfolio of cash positions and A-Futures position. B-Futures position.

$$\tilde{\Sigma} = \begin{bmatrix} \tilde{\Sigma}_A \beta_2 \\ \beta_2' \ \gamma_2 \end{bmatrix}$$

such that $\tilde{\Sigma} = \text{var}(P_1 f_{A1})$, $\beta_2' = (\text{cov}(P_1', f_{B1}), \text{cov}(f_{A1}, f_{B1}))$, and $\gamma_2 = \text{var}(f_{B1})$. Upon taking partial derivatives of u with respect to $\tilde{S}$ and V_B, after algebraic manipulation, the optimal cash and futures positions for the hedger are derived as follows;

$$\tilde{S}^* = \left(\tilde{\sum}_A + \tilde{E} - \gamma_2^{-1}\beta_2\beta_2'\right)^{-1} \tilde{h}; \tag{1}$$

$$V_B^* = -\gamma_2^{-1}\beta_2'\left(\tilde{\sum}_A + \tilde{E} - \gamma_2^{-1}\beta_2\beta_2'\right)^{-1} \tilde{h} \tag{2}$$

where $\tilde{h} = (k'0)'/\lambda$ and $\tilde{E} = \tilde{C}/\lambda$ with $\tilde{C}$ being the augmented matrix of C by additions of zero elements to form an $(N + 1) \times (N + 1)$ diagonal matrix. If one further applies the formula for the inverse of a partitioned matrix, Equation (2) may be rewritten as:

$$V_B^* = -\beta_2'\left(\tilde{\sum}_A + \tilde{E}\right)^{-1} \tilde{h} \Big/ \left[\gamma_2 - \beta_2'\left(\tilde{\sum}_A + \tilde{E}\right)^{-1} \beta_2\right]. \tag{2'}$$

In general, one cannot ensure that each of the optimal cash positions is nonnegative and/or either of the optimal futures positions is nonpositive. To satisfy "no short sell" conditions in each of the cash markets, the i^{th} grade from the framework to be dropped if $S_i^* < 0$. The solution to the resulting new maximization problem takes the same form as (1)–(2). Without loss of generality, assume $S_i^* \geq 0$, $\forall\, i = 1, \ldots, N$. On the other hand, since the problem is considered from the viewpoint of a short hedger, one would expect $V_A^* \leq 0$ or $V_B^* \leq 0$, otherwise, the roles of short hedgers and long hedgers would be reversed. If V_A^* and V_B^* have opposite signs, the two futures are complementary.[3] There is no concern of entry deterrence. Thus, the problem is meaningful only when both V_A^* and V_B^* are nonpositive. These assumptions are maintained throughout this article.

OPTIMAL FUTURES CONTRACT FOR EXCHANGE B

Given the contract specification of A-Futures, Exchange B attempts to choose β_2 and γ_2 to maximize $(-V_B^*)$. [Note that every functional form of $f_B(\cdot)$ determine a unique vector $(\beta_2'\gamma_2)$, but not vice versa.] Two different cases are distinguished below.

Case I: A-Futures is not a Conventional Cash Settlement Contract

In this case, $\tilde{\Sigma}_A$ is a nonsingular matrix. To ensure that $\tilde{\Sigma}$ is a variance-covariance matrix, a necessary condition would be: $\gamma_2 \geq \beta_2' \tilde{\Sigma}_A^{-1} \beta_2$. From equation (2), clearly $(-V_B^*)$ increases as γ_2 decreases. The optimum solution satisfies $\gamma_2 = \beta_2' \tilde{\Sigma}_A^{-1} \beta_2$, which implies $f_{B1} = \tilde{b}_1'(P_1 f_{A1})$ where $\tilde{b}_1$ is an $(N + 1) \times 1$ column vector. Upon substituting the relationship into (2′) and then upon solving the maximization problem, one determines

$$\tilde{b}_1 = \delta_b \tilde{E}^- \tilde{k} = \delta_b (k'C^{-1}0)',$$

where the superscript "−" denotes the generalized inverse, and δ_b is a constant term. The resulting maximum transaction volume is $(-V_B^*) = 1/\delta_b$. To determine δ_b, assume

[3] Futures contracts traded at various exchanges may be complementary to each other. For example, Gray (1961) argued that the survival of both KCBT (Kansas City Board of Trade) and MGE (Minneapolis Grain Exchange) wheat futures is actually supported by CBOT wheat futures. The argument is supported in Lien (1989(a)).

that the exchange has to choose a basis grade such that the futures price converges to the cash price of the basis grade (in expectation forms) at maturity. Thus, the optimal B-Futures is a conventional cash settlement contract with $f_{B1} = b_1' P_1$ where $b_1' = \delta_b k' C^{-1}$, and

$$\delta_b = \min\{E(P_{i1}) / \sum_{j=1}^{N} (k_j / C_j) E(P_{j1})\} \tag{3}$$

Some properties are in order: (i) Exchange B will adopt a specific conventional cash settlement contract, independent of the contract specification of A-Futures, to maximize its transaction volume. Moreover, the resulting maximum is a constant and is independent of A-Futures, (ii) The optimal B-Futures chooses the cheapest grade to be the basis grade [see eq. (3)]; and (iii) Compared with the result from Lien (1989(b)), the optimal B-Futures derived here is exactly the optimal contract Exchange A will adopt in the absence of competitive pressure. In other words, if A can successfully devise a futures contract to prevent the entry of B, the resulting transaction volume of the contract is less than or equal to $1/\delta_b$.

Case II: A-Futures is a Conventional Cash Settlement Contract

Let $f_{A1} = a' P_1$ where a is an $(N \times 1)$ column vector. Consequently, $\tilde{\Sigma}_A$ is a singular matrix. To ensure $\tilde{\Sigma}$ is a variance-covariance matrix, a necessary condition is $\gamma_2 \geq \beta_{21}' \Sigma^{-1} \beta_{21}$; β_{21} is the first N components of β_2, i.e., $\beta_{21} = \mathrm{cov}(P_1, f_{B1})$. Upon applying the above reasoning, one sees that the optimal B-Futures must satisfy $\gamma = \beta_{21}' \Sigma^{-1} \beta_{21}$. Thus, it is a conventional cash settlement contract with $f_{B1} = b_2' P_1$ for some b_2. Substituting the above two relationships (concerning f_{A1} and f_{B1}) into (2′) and then solving for the maximization problem, one yields[4]

$$b_2 = \phi_b M \tilde{h};$$

$$M = \left[\Sigma - \left(\Sigma\, \Sigma a\right)\left(\tilde{\Sigma}_A + \tilde{E}\right)^{-1}\left(\Sigma\, \Sigma a\right)'\right]^{-}\left(\Sigma\, \Sigma a\right)\left(\tilde{\Sigma}_A + \tilde{E}\right)^{-1};$$

$$\phi_b = \min\{E(P_{i1}) / \tilde{h} M' E(P_1);\ i \neq \text{basis grade of } A\text{-Futures}\}. \tag{4}$$

The resulting transaction volume is $(-V_B^*) = 1/\phi_b$. In this case, the optimal B-Futures and the transaction volume it generates both depend upon the contract specification of A-Futures.

OPTIMAL FUTURES CONTRACT FOR EXCHANGE A

The above results characterize the optimal B-Futures as a response to the contract specification of A-Futures. Knowing the response, Exchange A attempts to initiate a futures contract such that B will not enter and compete in the market. That is, A must reduce the transaction volume of B-Futures, $-V_B^*$, to a level below Θ. In Case I, if $(-V_B^*) \leq \Theta$, then A-Futures cannot survive since $(-V_B^*)$ is greater than the transaction volume of A-Futures. To successfully prevent the entry, Exchange A therefore must rely upon conventional cash settlement contracts.

[4]Let $H = (\Sigma - \Sigma a)(\tilde{\Sigma}_A + \tilde{E})^{-1}(\Sigma\, \Sigma a)'$, then it can easily be shown that $a'Ha = 0$. Thus, H is a singular matrix; H^{-1} does not exist. Instead, the generalized inverse of H must be applied.

Assume that the A-Futures satisfies $f_{A1} = a'P_1$. The problem for Exchange A may be written as follows:

$$\text{Maximize } -\tilde{V}_A = a'\sum\left(\sum + E\right)^{-1} h/a'\left(\sum - \sum\left(\sum - E\right)^{-1}\sum\right)a$$

$$\text{Subject to: } \phi_b \geq 1/\Theta,$$

where $(-\tilde{V}_A)$ is the transaction volume of A-Futures given that it is the only futures available in the market. If the constraint is not binding, then the optimal A-Futures for the above maximization problem is derived in Lien (1989(b)) such that $a = \delta_b C^{-1}k, \delta_b$ is given in eq. (3). That is, if a large transaction volume is required for the survival of a futures contract (i.e., Θ is large), Exchange A chooses the same conventional cash settlement contract whether or not there is competitive pressure expected (assuming $\phi_b \geq 1/\Theta > \delta_b$). In most cases, the constraint is expected to be binding, and thus the optimal entry-deterring contract differs from the optimal contract in the absence of competition. Because the latter contract provides the maximum expected utility level (Lien, 1989(b)), Exchange A and hedgers both incur losses to deter possible entry of B. Nonetheless, the optimal contract still takes the form of a conventional cash settlement contract.

CONCLUDING REMARKS

This article investigates the impact of entry deterrence objectives upon the choice of contract settlement specifications of a new futures contract. The main results of this investigation are as follows: (1) Whatever the specification of the innovative contract, if the rival exchange decides to compete, it will employ a conventional cash settlement contract. In the settlement method, the assignment of weights to each grade depends upon the innovative contract's settlement specification and (2) In an effort to prevent competing contracts, a conventional settlement contract must be adopted by the innovative exchange. This contract differs from the optimal contract when competitive pressure is absent. Thus, the futures market transaction volume and the hedger's utility are both lower than the optimal levels. The exchanges and hedgers all incur losses when contracts are specified to deter the entrance of competing contracts.

This article represents a first attempt to model the entry-deterring factors employed in new contract specifications. It is certainly not a complete resolution of the problem.

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