Fixed-Income Portfolio Allocation Including Hedge Fund Strategies: *A Copula Opinion Pooling Approach*

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This article analyzes how the optimal allocation of a traditional bond portfolio is affected by including fixed-income hedge fund strategies, such as convertible arbitrage, fixed-income arbitrage, and distressed securities. A well-known problem with hedge funds is that their returns deviate strongly from normality (Kat and Lu [2002]). Thus traditional mean-variance portfolio optimization is not applicable, because it can lead to suboptimal allocations (e.g., Agarwal and Naik [2005]; Athayde and Flôres [2004]). To capture portfolio risk more adequately, we apply Meucci's [2006a, 2006b] extended version of the Black–Litterman [1990, 1992] framework. This approach is particularly appropriate for hedge fund data because it is independent from any distributional assumptions.

To determine the optimal portfolio allocation and assess whether fixed-income hedge funds enhance the risk-adjusted expected portfolio return, we implement a scenario-based copula opinion pooling (COP) model. In contrast to the original Black–Litterman approach, the approach can incorporate tail risks, which are particularly important for hedge fund investments. We define three scenarios for traditional fixed-income assets: a bearish, a neutral, and a bullish view. No views are expressed for fixed-income hedge fund strategies.

By including these views in the COP framework, we can simulate the posterior distributions of the assets in the investment universe based on the empirical data. We can then use these distributions to compute optimal portfolio allocations. Portfolio optimization is based on an algorithm that maximizes expected portfolio returns without violating two different pre-specified expected shortfall rates.

As expected, our results indicate no *a priori* justification for including fixed-income hedge fund strategies in fixed-income portfolios. We find that optimal portfolio allocation is determined both by an investor's views of expected returns on the fixed-income assets and by his level of risk tolerance. Depending on these parameters, we find that, in our scenarios, the optimal allocation to fixed-income hedge fund strategies ranges from 0% to 85%.

Our results contradict "conventional" asset management wisdom that suggests every portfolio should include hedge funds in order to enhance the risk–return spectrum. We find that investor characteristics and investor expectations are the most important determinants of whether an engagement in fixed-income hedge fund strategies is optimal and of in what proportion these strategies should be included.

ASSET ALLOCATION AND INVESTOR VIEWS

The Markowitz [1952] formulation of modern portfolio theory combines the basic objectives of investing: maximizing expected return while minimizing risk. In a portfolio context, investment risk is measured as the standard deviation (or volatility) of asset returns around their expected value. This traditional portfolio optimization approach leads to a parabolic efficient frontier, which indicates the portfolio compositions with the highest expected return given a certain level of risk.

Although this approach is intuitive and easy to grasp, He and Litterman [1999] note several reasons why mean–variance optimization is difficult to implement in practice. For example, for inputs, Markowitz optimization requires the expected returns for all assets in the investment universe, as well as the complete expected variance–covariance structure.

In contrast, He and Litterman [1999] argue that portfolio managers typically focus on a much smaller segment of the entire investment universe. They look at fundamental ratios, momentum, and different investment styles to identify stocks they regard as undervalued. Return forecasts can be difficult to obtain (Goyal and Welch [2007]; Campbell and Thompson [2008]), and robust estimators for the variance-covariance structure may also be hard to derive (Ledoit and Wolf [1999]). However, Merton [1980] documents that the ex post performance of the resulting weighting scheme depends heavily on the quality and reliability of the input data, especially the vector of expected returns.

Mean–variance optimization implies a trade-off between risk and expected returns along the efficient frontier. Optimal portfolio weights are merely the result of a mathematical optimization. In contrast, portfolio managers often think directly in terms of portfolio weights, and they may deviate from their benchmarks only in the tactical asset allocation (Phillips and Lee [1989]). Therefore, as per He and Litterman [1999], the weights returned by an optimizer are often extreme, suboptimal, and inappropriate for an investor portfolio.

A closely related concern of portfolio managers is that mean–variance optimization leads to extremely unstable portfolio weights when input parameters change. Overall, much of the dissatisfaction with standard portfolio optimization occurs because 1) historical returns are often used instead of expected returns (thereby extrapolating the past into the future), and 2) estimation errors are not taken into account.

Merton [1980] documents that historical returns are poor proxies for future expected returns.[1] He stresses the importance of estimation risk in the implementation of modern portfolio theory. Klein and Bawa [1976], Best and Grauer [1991], and Chopra and Ziemba [1993] also discuss the sensitivity of portfolio weights with respect to uncertainty in the required input data. Optimal portfolio allocations are particularly sensitive to changes in expected returns, and small changes in the input variables can cause dramatic changes in the weighting schemes.

Michaud [1998] goes further and argues that mean–variance optimization is merely estimation error maximization. He notes that optimized portfolios tend to overweight (underweight) assets with high (low) expected returns, negative (positive) correlations, and low (high) variances. This leads to fundamentally flawed portfolio compositions if—as we intuitively expect—assets with extreme returns tend to be most affected by estimation errors.

Another issue closely related to estimation risk is that mean–variance optimization does not distinguish strongly held views from vague assumptions. Optimizers generally do not differentiate between the levels of uncertainty associated with input variables. Therefore, the optimal weights often bear no intuitive relationship to an investor's viewpoint. Even worse, the differences in the mean returns are not likely to be statistically significant. Thus, every point on the efficient frontier has a region that includes an infinite number of statistically equivalent portfolios. Although they are equivalently "optimal," these portfolios may have completely different compositions.

One strand of literature suggests using Bayesian methods to account for estimation risk. For example, Jorion [1985, 1986] uses a shrinkage estimator that exploits the properties of the minimum-variance portfolio, and shrinks expected returns toward this common mean. Because the minimum-variance portfolio is independent from expected returns, it is less affected by estimation risk. In this combined portfolio, the relative weights of the tangency portfolio and the minimum-variance portfolio depend on the confidence in the subjective views.

Another remedy often proposed to avoid extreme portfolio weights or weight changes in response to input variable changes is to introduce constraints (e.g., non-negativity constraints). Constraints may come from client-specific and/or legal restrictions. However, with too many restrictions, the "optimized" results will merely reflect the pre-specified views, which are not always economically intuitive. In addition, some constraints may lead to corner solutions and to dropping some assets from the efficient portfolios. These constrained weighting schemes are again hard to implement because they often imply unreasonable weights in only a small subset of assets.

Black and Litterman [1992] proposed a novel Bayesian approach that addresses many of these problems. Their framework allows the subjective market views of a portfolio manager to be blended smoothly with a prior market distribution, under the assumption that all the distributions involved are normal. They start with equilibrium portfolio weights, e.g., the set of asset weights in the market portfolio. Alternatively, a set of strategic or long-term weights can also be used. Both sets of weights are assumed to represent the normal investment behavior of an average investor.

In market equilibrium, these weighting schemes imply a corresponding vector of expected returns for the given asset universe (i.e., the prior). Based on current market states, however, a portfolio manager's expectations about short-term returns may be quite different from those implied by long-term market conditions. The Black–Litterman [1992] approach, however, which uses Bayesian updating rules, is flexible enough to incorporate many different views while consistently combining equilibrium views with an investor's subjective views.

In addition to the level of short-term return expectations, portfolio managers need to specify the degree of confidence they have in these views. All subjective information is translated into symmetric confidence bands around normally distributed returns. In Black and Litterman's [1992] scheme, the higher (lower) the degree of confidence, the more (less) the revised expected returns are tilted toward the investor's views. The vector of revised expected returns (i.e., the posterior) is then handed over to a portfolio optimizer.

As demonstrated by He and Litterman [1999] and Lee [2000], views are spread across all asset classes and imply changes for the entire vector of expected returns. Because asset returns are correlated, views for one asset class also impact the expected returns of other asset classes. These expected return changes imply that estimation risk is diversified across all asset classes, thereby reducing the danger of estimation error maximization. With relative views, the optimal weights change only for asset classes with explicit forecasts. The equilibrium or strategic weights of all asset classes remain unchanged, thereby reflecting the subjective views of the portfolio manager in a consistent way. Overall, the Black–Litterman [1992] model leads to more stable and less extreme portfolio weights.

However, despite its intuitiveness, flexibility, and ease of implementation, the Black–Litterman model has several shortcomings. Most important, it assumes that both the market prior and the portfolio manager's views are normally distributed. This can be an extremely unrealistic assumption because, in many cases, fat tails, skewness, and high dependence among extreme events characterize the joint distribution of asset returns (Füss et al. [2008]).

Although there are no longer closed analytical solutions in this case, Meucci [2006a, 2006b] shows that a copula opinion pooling approach (COP) with Monte Carlo simulation can cope with any distributional assumption. By representing the prior market distribution in terms of a large number of Monte Carlo scenarios, we can simulate an equal number of scenarios from the COP posterior. Similarly to the Black–Litterman approach, the COP posterior blends the market prior with a manager's current views on the market.

Another advantage of the COP approach is that asset interdependence is measured more adequately by a copula than by a linear correlation coefficient. Thus, this approach is better able to capture tail risks in asset returns. As in the Black–Litterman design, the COP posterior distribution can then be handed over to any portfolio optimizer. The consistent twist of the original prior distribution again delivers sensible allocations.

THE COPULA OPINION POOLING APPROACH

This section provides some general insights into the nature of the copula opinion pooling approach and discusses the necessary adjustments in our specific portfolio allocation context. The COP approach is not affected by the shortcomings of the standard portfolio optimization approach. In fact, in addition to incorporating a more adequate measure of asset interdependence, the COP approach allows investors and/or portfolio managers to express their views on all assets contained in the investment universe.

Because of technical problems, a linear correlation coefficient may not be an appropriate concept to measure interdependencies among asset returns. For example, Embrechts et al. [2002] use a two-variable example to demonstrate that a linear correlation coefficient may not capture dependence when the standard deviation of one variable becomes very large, even in the presence of perfect co-monotonicity or counter-monotonicity. In addition, as discussed in Rachev et al. [2005] and Meucci [2006c], the linear correlation coefficient is invariant to non-linear increasing transformations. The copula provides a more reliable measure of interdependence and has been increasingly used in financial applications in recent years.

The factorization of a multivariate distribution into the copula (the true interdependence structure between the variables) and the marginal distributions (the randomness of these variables) delivers a standardized measure of the purely joint features of this distribution. The grade of a random variable (uniformly distributed between 0 and 1) is the cumulative distribution function of a one-dimensional random variable, and the copula is the distribution of the grades. Being distributed between 0 and 1, the copula is standardized and can capture all the different kinds of relationships among variables. Specifically, an n-copula $C:[0, 1]^n \rightarrow [0, 1]$ is an n-dimensional distribution function with univariate marginal distributions $U(0, 1)$:

$$C_p(u_1,...,u_n) = P(U_1 \leq u_1,...,U_n \leq u_n) \tag{1}$$

According to Sklar's [1959, 1973] theorem, a copula C is unique if F_i is continuous for all i:

$$H(x_1,...,x_n) = C(F_1(x_1),...,F_n(x_n)) \tag{2}$$

where H is an n-dimensional distribution function with marginal distributions F_i. Different classes and shapes of copulas can be used, such as elliptical (e.g., normal copula or t-copula) or Archimedean copulas (e.g., Gumbel-copula or Clayton-copula) among others.[2] In our analysis, we follow Meucci [2006a, 2006b] and use the symmetrically shaped t-copula.

Meucci [2006a, 2006b] incorporates the concept of copulas into the COP framework. This novel approach has similarities to Bayesian asset allocation methods, particularly to the Black–Litterman [1992] model. However, in contrast to Black–Litterman, the COP approach is not based on a benchmark or equilibrium selection. Moreover, no assumptions are necessary about the return distribution, and hence the well-known problems associated with normality assumptions do not arise.

In the COP framework, an investor can express $K < N$ views for N assets. These views are presented by a $K \times N$ dimensional pick matrix $\overline{P}$. The values in each column of a view range between −1 and +1, depending on the number of assets in a view and on whether the view is in absolute form for a specific asset or in relative terms for comparisons between assets. Lower and upper boundaries reflect the opinion of the investor on a specific asset. Using a univariate distribution, the occurrence of each realization between the lower and upper bound has the same probability. Moreover, the investor must express confidence in each view; stronger confidence leads to higher weights of a view in the following opinion pooling approach.

To influence only those assets on which the investor has expressed a view, the pick matrix is augmented with an orthonormal basis. The resulting pick matrix $\overline{P}$ represents the view-adjusted market coordinates:

$$\overline{P} = \begin{pmatrix} P \\ P^{\perp} \end{pmatrix} \tag{3}$$

Using Monte Carlo methods, the market prior is obtained by first fitting a t-copula to the return matrix, where maximum likelihood estimation is used to choose the appropriate degrees of freedom.

Following the estimation of the t-copula, a kernel smoothing procedure is applied to the empirical cumulative distribution function (*cdf*). In contrast to Meucci [2006b], we do not truncate the empirical *cdf* at points that deliver negative realizations, because we apply the procedure directly to asset returns rather than yields and yield changes. Moreover, there is no need to rule out negative yields, and we can also disregard liquidity premia and habitats.

After the empirical distribution has been smoothed, we generate Monte Carlo simulations of the copula, which are then mapped into the joint distribution of assets. This joint distribution forms the market prior of the assets contained in the investment universe. It can be interpreted as the estimated return distribution based on the true asset interdependence, as measured by the copula.[3]

To obtain the view-adjusted market scenarios, we can combine the market prior with the subjective views by matrix multiplication of the market prior M_{prior} and the pick matrix $\overline{P}$:

$$V_{\text{prior}} = M_{\text{prior}} * \overline{P}' \tag{4}$$

From the scenarios in V_{prior}, we obtain both the marginal prior cumulative distribution functions of the view-adjusted scenarios for all assets in the investment universe (by sorting each row in ascending order) and the market-implied prior copula (by calculating the normalized rank of each entry in the respective row). In the next step, the marginal posterior *cdf* of each view is the weighted average of the *cdf* of the market prior and the *cdf* of the view for a specific asset class, where the weights depend on the views' confidence levels. The joint posterior realizations of all views are then derived by linear interpolation.

Finally, by using simple matrix multiplication of the joint posterior realizations of the views and the inverse of the pick matrix $\overline{P}$, these views are rotated back and deliver the joint posterior realizations of the market distribution:

$$M_{\text{post}} = V_{\text{post}} * \overline{P}^{-1} \tag{5}$$

The posterior matrix, M_{post}, represents a simulated market based on the true interdependence of the assets (as estimated from real data) and an investor's views. No further steps are required when the COP approach is applied to direct asset returns.[4]

The COP approach does not include portfolio optimization. Meucci [2006a, 2006b] states that the COP output may be used as an input for any portfolio optimization approach. We opt to take the COP posterior distribution and maximize the expected portfolio return given a predetermined limit of an expected risk measure. Specifically, we maximize the following constrained target function by the choice of the vector of portfolio weights:

$$\begin{aligned}
&\max w * \mathrm{E}[r] && && (6)\\
&\text{subject to: } w = 1 && \text{(full investment)}, &&\\
&\qquad w_i > 0 && \text{(no short selling)}, &&\\
&\qquad esf < esf^* && \text{(maximum expected shortfall), and} &&\\
&\qquad w_i^{\text{low}} < w_i < w_i^{\text{high}} && \text{(asset bounds)}. &&
\end{aligned}$$

where w denotes the vector of portfolio weights, and $E[r]$ is the vector of expected asset returns from the market posterior. esf^* denotes the maximum expected shortfall allowable, defined as the integral of the distribution function below a specified threshold. This expected shortfall risk (the expected tail loss or conditional value at risk, CVaR for continuous distributions) accounts for the concentration in the distribution tails.

In contrast, traditional value at risk (VaR) only indicates the value of the distribution at the threshold. More specifically, CVaR measures the expected loss if a tail event occurs, while the VaR measures the maximum loss not to be exceeded with a certain confidence. Because hedge funds and high-yield products often exhibit severe losses during market downturns, specifying acceptable CVaR values is generally considered an appropriate portfolio optimization approach (Alexander and Baptista [2004], Füss et al. [2007]). The main advantage of this approach is that it allows setting limits according to the level of investor risk aversion.

Note that full investment and no-short-selling are appropriate restrictions, because it is not possible to short sell a hedge fund strategy represented by an index. Moreover, most institutional investors and investment vehicles are restricted to long-only positions. Thus, from a regulatory viewpoint, the no-short-selling constraints are necessary.

CHARACTERISTICS OF FIXED-INCOME HEDGE FUND STRATEGIES

Recent literature has established that hedge funds can serve as effective diversification instruments in conventional portfolios (Brooks and Kat [2002], Morley [2001], Zask [2000], Planta and Banz [2002], Eling [2006]). According to this view, hedge funds can enhance the risk-adjusted performance of traditional portfolios. For example, the advantages of fixed-income arbitrage strategies are low volatility and low correlation with interest rate changes. According to Capocci [2003], investors can also optimize their total portfolio return by adding convertible arbitrage hedge funds. However, when including hedge fund strategies in traditional portfolios, it is important to consider the influence on the skewness and kurtosis of the portfolio return distribution.

Because of their trading strategies, hedge funds are typically non-linear functions of traditional markets. Therefore, linear correlation coefficients as measures of dependence in the context of portfolio optimization are not reliable (Lhabitant [2004]). In general, using correlation coefficients as indicators of dependence among random variables is problematic because 1) only linear dependence is measured, and 2) the results are only meaningful if the multivariate distribution is elliptical (Embrechts et al. [1999, 2002], Kat [2003]).

Because most hedge fund strategies exhibit negative skewness and/or excess kurtosis, the joint distribution is far from being elliptical. Moreover, the correlation coefficient does not exhaust the full interval between [–1, +1]. This can lead to incorrect findings of very low dependence, even when the variables are perfectly correlated.

Convertible arbitrage and fixed-income arbitrage are classified as non-directional or market-neutral strategies. In contrast, because the investments of distressed securities funds range from secured debts to common stock of companies in reorganization and/or bankruptcy, they are classified as event driven. This section provides a more detailed description of these three fixed-income-related hedge fund strategies.

Convertible Arbitrage

Because of the legal complexity of the optional components of convertible bonds, it is possible that the sum of the individually valued components will not equal the market price of the convertible bond (Calamos [2003]). According to Agarwal et al. [2004], equity volatility, creditworthiness, and interest rate risk are the main factors driving convertible arbitrage strategies. These different sources of risk may be hedged individually or specific risks may be actively managed. The rights contained in the option are mainly defined by the contract conditions (such as the strike price or the maturity), and they may differ from those of plain vanilla options. Nevertheless, the option rights are mainly influenced by the price of the underlying stock and its volatility.

The assessment of volatility may lead to the initiation of a convertible arbitrage position. If the implied volatility of the convertible bond differs from the market volatility or the volatility forecast of the hedge fund manager, profit opportunities arise. According to Tran [2006], there are two main ways to generate alpha via convertible arbitrage: 1) valuation errors that occur from an incorrect estimation of creditworthiness, and/or 2) incorrectly priced conversion ratios.

In convertible arbitrage strategies, fund managers' objectives may differ according to the market state. During downward-trending markets, managers need to determine which convertible bonds will decrease more slowly than the underlying equities; during upward-trending markets, they need to determine which convertible bonds will more closely mirror equity prices. As a result, managers will go long in convertible bonds and hedge these positions by shorting the stock of the same company (Nicholas [2000], Tomlinson [1998]).

Fixed-Income Arbitrage

Fixed-income arbitrage managers attempt to take advantage of the inefficiencies in relative valuation among bonds with similar payment characteristics or creditworthiness. It is assumed that, over time, these valuation differences will ultimately narrow or even vanish. Fixed-income arbitrage managers take long and short positions in interest rate–sensitive assets to eliminate interest rate risk (Jaeger [2002]).

A typical fixed-income arbitrage strategy consists of long and short positions of related or similar interest rate assets and their derivatives, which neutralize each other. Parallel shifts in the interest rate term structure curve thus will not affect the portfolio. Decisions are based primarily on mathematical and statistical valuation models. Managers try to detect and profit from valuation anomalies that originate from changes in interest rate curves, creditworthiness ratings, or volatility curves (Lhabitant [2002]).

Fixed-income arbitrage managers need to identify interest rate assets that display a high level of mathematical, fundamental, or historical correlation and that are also characterized by a present or expected future price inefficiency.[5] Such price inefficiencies may occur, for example, because of structural changes in the interest rate market, shifts in investor preferences, or exogenous shocks that impact supply and demand. A fund manager will profit if the current price ratio of these interest rate assets converges toward a historical "standard ratio." In the field of interest rate arbitrage, a high leverage level of 10 or more is common due to the very small price differences.[6]

To generate profits, fixed-income arbitrage strategies exploit interest rate changes among government and corporate bonds of high creditworthiness, bonds with differing creditworthiness, bonds of a debtor with different guarantees, or bonds of the same issuer with different maturities. According to Duarte et al. [2006], almost all excess returns generated by fixed-income arbitrage stem from interest rate risk and market risk. As a result, the alphas are significant, even after deducting the usual hedge fund fees.

Fung and Hsieh [2002] and Jaeger and Wagner [2005] documented, however, that heavy losses can occur under certain conditions, such as investors choosing flight over quality, a sudden expansion in credit spreads, a lack of liquidity, or an emerging market downturn. Nevertheless, the events of summer 1998, particularly the case of LTCM, indicate that fixed-income arbitrage can resemble a short option, incorporating the risk of significant losses during periods of constant positive performance (Jaeger and Wagner [2005]).

Distressed Securities

Funds using the distressed securities strategy invest in securities of companies that are economically, financially, or organizationally distressed (Füss et al. [2007, 2008]). This strategy is thus referred to as "results oriented."[7] A distressed situation may arise if 1) the financial situation of a company significantly deteriorates, 2) a company is unable to meet its financial obligations and/or its debt obligations, or 3) insolvency has occurred due to other factors. Distressed securities normally trade at prices far below the notional amount or former prices. The goal is thus to buy these assets at a deep discount and hold them until the company has overcome its difficulties.

Distressed securities managers attempt to invest in companies they expect will recover, where the return potential neutralizes the obvious risk. A typical company of interest might have temporary financial problems but excellent potential future prospects due to, for example, a specific area of expertise. However, hedge fund managers must have excellent sector knowledge in these cases, as well as close management contacts within the company in question. Distressed securities investments are mostly long-term and are thus quite sensitive to liquidity pressure (this is why most require a long capital lock-up period).

Owners of distressed securities are often willing to close their positions even with significant losses. Large price cuts may result from false valuations of the distressed securities by market participants. Selling pressure may also result from the fact that some investor groups (e.g., insurance companies) are unable to invest in distressed securities.[8] Hedge funds, however, have no restrictions on allocation, whether from insolvency, liquidation, or ratings downgrades.

PORTFOLIO ALLOCATION USING THE COPULA OPINION POOLING APPROACH

In our empirical analysis, we use monthly return data from February 1996 through August 2007. To capture fixed-income securities as an asset class, we use the government bond indexes of Germany, the U.K., Japan, and the U.S. According to their risk–return profile, these four indexes represent the most conservative fixed-income investments.

To fully capture the bond universe, we also use three Lehman indexes, consisting of an aggregate corporate bond index, a high-yield credit bond index, and a mortgage-backed securities index, as well as the Exane convertible bond index. To examine whether fixed-income hedge fund strategies should be included in a portfolio that consists of these fixed-income asset classes, we include the composite hedge fund index and the strategy subindexes for convertible arbitrage, fixed-income arbitrage, and event-driven/distressed securities from the CS/Tremont hedge fund database.

Exhibit 1 gives the descriptive statistics for our fixed-income investment universe. Fixed-income hedge fund strategies exhibit pronounced non-normality, with negatively skewed and leptokurtotic returns. Therefore, the classical models based on normality assumptions are not suitable to model bond portfolios that include these strategies. In contrast, Meucci's [2006a, 2006b] COP approach would be an appropriate alternative.

In the first step of the COP approach, we estimate the prior market distribution using copula-based Monte Carlo simulations. Exhibit 2 provides the descriptive statistics. Comparing Exhibits 1 and 2, we infer that the copula-based Monte Carlo simulations track the empirical data very well, although almost all the simulated series exhibit slight deviations from the empirical series. However, this is not surprising, given that we use a maximum likelihood function to determine the degrees of freedom for the copula, which is the input for the Monte Carlo simulations of the market realizations.

Furthermore, because a continuous distribution is approximated, the series' cumulative distribution functions are smoothed with a kernel procedure. Comparing the distribution of the market prior with the assets' empirical distributions using QQ plots (not shown here), we infer that the simulation procedure is very efficient.

The second step of the COP procedure requires the specification of an investor's market views. To examine how investor views may influence the outcome of the copula opinion pooling and the subsequent portfolio optimization, we define three scenarios: 1) a fixed-income bullish scenario, where the assets perform well with respect to the structure implied by the market data and the prior; 2) a neutral scenario, where all asset classes perform as indicated by the prior (i.e., no opinion pooling is required, and the resulting portfolio is only shown for the sake of comparison); and 3) a fixed-income bearish scenario,

EXHIBIT 1
Descriptive Data Statistics

Fixed Income Asset/Strategy	Mean	Standard Deviation	Min	Max	Skewness	Kurtosis	Jarque–Bera
10-Year Bund Yield	–0.14%	0.65%	–4.78%	5.90%	0.41	2.77	4.02*
10-Year Gilts Yield	–0.21%	0.72%	–7.44%	6.96%	0.22	3.03	1.21
10-Year Japan Yield	–0.14%	0.63%	–6.24%	13.56%	1.79	13.59	790.32***
10-Year Treasury Yield	–0.09%	0.89%	–6.48%	10.87%	0.46	3.55	7.29**
Lehman Aggregate Bond Index	5.81%	3.58%	–40.32%	32.40%	–0.55	3.75	11.48**
Lehman High Yield Credit Bond Index	6.60%	7.00%	–88.44%	89.88%	–0.68	6.51	91.20***
Lehman Mortgage-Backed Securities Index	5.84%	2.65%	–22.44%	25.68%	–0.35	3.04	3.16
Exane Europe Convertible Bond Index	10.21%	9.59%	–118.38%	103.24%	0.13	5.08	29.07***
CS/Tremont Convertible Arbitrage	9.32%	4.54%	–56.11%	42.82%	–1.53	7.22	169.41***
CS/Tremont Fixed-Income Arbitrage	6.02%	3.72%	–83.58%	24.63%	–3.22	20.07	2,048.09***
CS/Tremont Event Driven—Distressed	12.74%	6.09%	–149.46%	46.88%	–3.55	27.15	3,971.28***
CS/Tremont Hedge Fund Composite	10.65%	7.27%	–90.59%	102.34%	0.02	6.16	64.99***

*Notes: Annualized (linear) returns and standard deviation.***, **, and *denote significance at the 1%, 5%, and 10% levels, respectively (rejection of the normal distribution).*

Source: Thomson Financial Datastream.

where the assets perform poorly with respect to the structure implied by the market data and the prior.

We test these three scenarios to determine whether fixed-income hedge fund strategies should be included in a fixed-income portfolio, and to what extent investor views influence the portfolio weights when the COP output is handed over to the portfolio optimizer.

The third step of the COP approach requires setting up the pick and view matrices. We set each view separately, i.e., we formulate one view for each of the eight fixed-income indexes. We define a return span of zero to half the maximum return implied by the prior for the fixed-income bullish view and zero to half the minimum return implied by the prior for the bearish view. For all views, we use a confidence level of 50%. Note that we assume no subjective views for the fixed-income hedge fund strategies—they remain neutral as specified in the prior.

Exhibit 3 shows the results of the copula opinion pooling for all scenarios.[9] As expected, the structure of the posterior distribution of the fixed-income indexes changes considerably and consistently reflects the investor's views. The minimum and maximum returns are not affected when the ranges are set between these values. According to the views, we find that the skewness of the return distributions is affected in a way that is implied by a shift in the mean.

In the final step, we calculate six optimal portfolios. To specify the constraints in Equation (6), we optimized the scenario portfolios given a maximum acceptable expected shortfall, denoted as esf^*, of –1% and –3% at a

EXHIBIT 2
Descriptive Statistics of the Simulated Prior

Fixed Income Asset/Strategy	Mean	Standard Deviation	Min	Max	Skewness	Kurtosis	Jarque–Bera
10-Year Bund Yield	–0.08%	0.69%	–5.63%	7.19%	0.47	2.99	3,674.10***
10-Year Gilts Yield	–0.14%	0.78%	–8.24%	8.42%	0.37	3.23	2,464.30***
10-Year Japan Yield	–0.03%	0.75%	–6.93%	16.31%	2.51	16.34	84,5340***
10-Year Treasury Yield	0.01%	0.97%	–7.76%	13.05%	0.67	4.23	13,758***
Lehman Aggregate Bond Index	6.02%	3.74%	–44.31%	37.51%	–0.38	3.47	3,392.20***
Lehman High Yield Credit Bond Index	7.31%	7.57%	–96.67%	105.85%	–0.21	6.18	42,924***
Lehman Mortgage-Backed Securities Index	6.00%	2.76%	–26.01%	29.45%	–0.23	2.98	888.86***
Exane Europe Convertible Bond Index	11.03%	10.19%	–129.00%	122.45%	0.30	4.71	13,643***
CS/Tremont Convertible Arbitrage	9.63%	4.76%	–62.84%	48.90%	–1.20	6.27	68,437***
CS/Tremont Fixed-Income Arbitrage	6.28%	3.74%	–87.71%	27.83%	–2.61	16.10	828,070***
CS/Tremont Event Driven—Distressed	13.17%	6.15%	–156.25%	53.34%	–2.83	21.69	1,586,500***
CS/Tremont Hedge Fund Composite	11.43%	7.92%	–98.73%	120.74%	0.40	6.10	42,759***

*Notes: Annualized (linear) returns and standard deviations.***, **, and *denote significance at the 1%, 5%, and 10% levels, respectively (rejection of the normal distribution).*

monthly horizon. This accounts for the defensive nature of fixed-income portfolios.

The tail section is chosen at the lower 5%, in line with the 95% confidence level commonly used for market risk measurement. To avoid extreme portfolio allocations, we set the upper bound for each asset weight at 25%. Presumably, this will ensure that we could obtain either an all-fixed-income hedge fund portfolio or an all-fixed-income portfolio from the portfolio optimization algorithm (as long as its expected shortfall does not exceed the acceptable value).

Exhibit 4 shows the resulting portfolio allocations. Note that the allocation to fixed-income assets is extremely large. This finding is not surprising given that we have restricted the allocations not to exceed the –1% or –3% expected shortfall. Even in the bearish scenario, where all fixed-income indexes exhibit a negative mean, the total fraction allocated to these indexes is very high for both risk levels because of their lower concentration in the left-hand tail of the distribution.

Note further that the high allocation to the fixed-income indexes is particularly robust when the restrictive –1% expected shortfall is applied. The aggregate weights are 79%, 92%, and 100% for the bearish, neutral, and bullish scenarios, respectively. The rest of the portfolio is allocated to fixed-income hedge funds. This low allocation to hedge funds also implies that the bearish scenario produces a very low expected portfolio return.

In contrast, when we apply the less restrictive –3% expected shortfall, we find that the investors' subjective views strongly influence the portfolio weights. The aggregate weights of the fixed-income assets for the bearish, neutral, and bullish scenarios, respectively, are 15%, 49%, and 77%. These lower weights imply higher allocations

EXHIBIT 3

Expected Return and Standard Deviation of Fixed-Income Bullish and Bearish Scenarios (Posterior) and Neutral Scenario (Prior)

Fixed Income Asset/Strategy	Mean FI Bullish	Std. Dev. FI Bullish	Mean Neutral (Prior)	Std. Dev. Neutral (Prior)	Mean FI Bearish	Std. Dev. FI Bearish
10-Year Bund Yield	0.86%	0.60%	–0.08%	0.69%	–0.73%	0.55%
10-Year Gilts Yield	0.98%	0.68%	–0.14%	0.78%	–1.09%	0.66%
10-Year Japan Yield	2.03%	0.93%	–0.03%	0.75%	–0.88%	0.62%
10-Year Treasury Yield	1.63%	0.91%	0.01%	0.97%	–0.95%	0.77%
Lehman Aggregate Bond Index	7.69%	2.90%	6.02%	3.74%	–2.54%	3.85%
Lehman High Yield Credit Bond Index	16.89%	6.79%	7.31%	7.57%	–8.44%	7.58%
Lehman Mortgage-Backed Securities Index	6.69%	2.15%	6.00%	2.76%	–0.24%	2.76%
Exane Europe Convertible Bond Index	20.82%	8.54%	11.03%	10.19%	–10.62%	10.27%

Note: Annualized (linear) returns and standard deviations.

EXHIBIT 4

Descriptive Statistics of the Optimized Portfolios

Portfolio Results	*esf**	Mean	Std. Dev.	Skewness	Kurtosis	FI Allocation	Non Govt. FI Allocation Share	FI Hedge Fund Allocation
FI Bullish Scenario	–1%	5.47%	2.16%	–0.25	4.04	100.00%	48.38%	0.00%
FI Bullish Scenario	–3%	14.27%	5.83%	–1.00	8.93	77.29%	100.00%	22.71%
Neutral Scenario	–1%	3.04%	1.97%	0.30	6.72	92.33%	37.50%	7.67%
Neutral Scenario	–3%	8.76%	4.41%	–1.14	8.62	49.25%	100.00%	50.75%
FI Bearish Scenario	–1%	1.81%	1.70%	0.83	11.54	79.45%	0.00%	20.55%
FI Bearish Scenario	–3%	8.97%	5.08%	–0.97	12.17	14.63%	0.00%	85.37%

Note: Annualized (linear) returns and standard deviations.

to fixed-income hedge funds. In the bearish state, for example, the allocation is as high as 85%. With this less stringent shortfall restriction, therefore, investor views more strongly affect the allocation to fixed-income hedge funds.

Our results show that both investor views and risk attitudes strongly drive optimal portfolio allocations. Overall, our findings shed doubt on overoptimistic hedge fund proposals, such as the often-heard recommendation that hedge funds must be included in any portfolio for diversification benefits. On the one hand, we find that very risk-averse investors should only include fixed-income hedge fund strategies in their traditional fixed-income portfolios if they are extremely bearish on the assets. On the other hand, we find that more risk-tolerant investors, e.g., those as indicated by a –3% expected shortfall risk within one month, would be well advised to allocate more to fixed-income hedge fund strategies.

Finally, one should not confuse the high fixed-income allocations with high safe-haven investments. With a –3% expected shortfall, the weights of non-governmental fixed-income assets within the fixed-income investments are 0%, 100%, and 100%, respectively, for the bearish, neutral, and bullish scenarios. This suggests that, with a higher acceptable level of risk, high-yielding fixed-income assets can substitute to some extent for fixed-income hedge fund strategies. Since these assets are more risky than government bonds, however, they drop out of the portfolio in the bearish scenario.

CONCLUSION AND OUTLOOK

In this article, we analyze how the allocation of a traditional bond portfolio changes when including fixed-income hedge fund strategies such as convertible arbitrage, fixed income, and distressed securities. The Black–Litterman model [1990, 1992] assumes that both the market prior and the portfolio manager's views are normally distributed. To circumvent the problems associated with non-normality, we apply the copula opinion pooling approach recently developed by Meucci [2006a, 2006b]. This approach also allows for non-linear dependencies between asset returns, i.e., the interdependence between asset returns is measured more adequately by a copula than by a linear correlation coefficient.

Similar to the Black–Litterman approach, the COP posterior blends the market prior with a manager's market views. We first represent the prior market distribution via a large number of Monte Carlo scenarios. We can then simulate an equal number of scenarios from the COP posterior distribution.

To examine how an investor's market views influence the outcome of the copula opinion pooling and the later portfolio compositions, we posit three scenarios: 1) a fixed-income bullish scenario, where the assets perform well with respect to the structure implied by the market data and the prior, 2) a neutral scenario, where the markets perform as indicated by the prior, and 3) a fixed-income bearish scenario, where the assets perform poorly with respect to the structure implied by the market data and the prior.

To account for the defensive nature of fixed-income portfolios, we optimize the scenario portfolios using values of –1% and –3% for the maximum acceptable expected shortfall on a monthly horizon. In line with the 95% confidence level commonly used for market risk measurement, we look at the lower 5% tail region. To avoid extreme results, we further set a 25% upper bound for each asset weight.

The allocation in fixed-income assets is extremely large. Even in the bearish scenario, where all fixed-income assets exhibit a negative mean, their aggregate share in the optimal portfolio is dominant because of the lower concentration in the left-hand tails of the distribution.

We find that an investor's subjective views heavily influence the outcome of the portfolio optimizations when the less stringent –3% expected shortfall is applied. In this setting, the aggregate portfolio weights of the fixed-income assets are 15%, 49%, and 77%, respectively, for the bearish, neutral, and bullish scenarios. Because the rest of the portfolio is allocated to fixed-income hedge fund strategies, these weights can become significant.

In contrast, when the more restrictive –1% expected shortfall is applied, we find that the aggregate portfolio weight of the fixed-income assets is very high, at 79%, 92%, and 100%, respectively, for the three scenarios. Accordingly, the weights of fixed-income hedge fund strategies in the optimal portfolio are much lower. In particular, in the fixed-income bullish scenario, hedge funds are no longer included in the portfolio.

Overall, our results indicate that both an investor's views and his attitude toward risk are crucial for determining optimal hedge fund allocations in fixed-income portfolios. As we note, our findings cast doubt on the proposition that including hedge funds in every portfolio is necessary due to the diversification benefits. On the one hand, very risk-averse investors are advised to abstain from investing in fixed-income hedge fund strategies, except in very bearish fixed-income market scenarios. More risk-tolerant investors, on the other hand, may find that larger allocations to fixed-income hedge fund strategies can enhance their risk–return spectrum.

ENDNOTES

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[1]See Campbell [2001] and Jagannathan et al. [2000] for more recent discussions.

[2]Archimedean copulas are particularly useful in credit risk management applications and other applications that attempt

to capture joint tail occurrences on one side of the distribution. Embrechts et al. [2003] discuss properties of different copulas with financial applications.

[3]If an entry in the prior matrix does not yield a result (due to numerical problems), we erase the respective line and this one scenario.

[4]In applications of the COP approach to risk factors, risk premia, or subfactors of the variables, the factors must be mapped into the variables or the asset returns.

[5]Identifying market price anomalies between similar interest rate securities is quite difficult. Fixed-income arbitrageurs typically use complex analytical computer models to detect these potential anomalies.

[6]Due to the heavy use of outside capital, risk control is of the utmost importance when using this strategy. If interest rate differences do not develop as expected, investors must have sufficient market liquidity to close the position. Hence, it is critical to maintain a permanent comparison between position sizes using average historical and current traded volume.

[7]We can further classify the distressed securities strategy into two components, active and passive, depending on the level of investor influence. In the active strategy class, investors may try to assume control of a company; in the passive strategy class, we may observe non-control-oriented trading positions. Hedge funds that follow an active strategy closely resemble private equity.

[8]Distressed securities hedge funds have a large exposure to default risk. This risk can be most efficiently hedged by the use of credit default swaps (CDS). However, it can be very difficult to find counterparties for a CDS on distressed securities. These funds thus perform well if credit spreads narrow, and vice versa (Stefanini [2006]).

[9]The hedge fund results are not reported because the numbers have not changed from those in Exhibit 2.

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