

Annual Default Rates Are Probably Less Than Long-Run Average Annual Default Rates

NICHOLAS M. KIEFER

NICHOLAS M. KIEFER is Ta-Chung Liu Professor in the Departments of Economics and Statistical Science at Cornell University in Ithaca, NY, and Senior Economist, Office of the Comptroller of the Currency, Risk Analysis Division in Washington, DC.
nicholas.kiefer@cornell.edu

The Basel II (B2) capital requirements—see Basel Committee for Banking Supervision [2006]—are based on a one-factor model due to Gordy [2003] that accommodates systematic temporal variation in asset values and hence in default probabilities. This model can be used as the basis of a model that allows temporal variation in the default probabilities and hence correlated defaults within years. The value of the i th asset in time t is

$$v_{it} = \rho^{1/2} x_t + (1 - \rho)^{1/2} \varepsilon_{it}$$

where ε_{it} is the time and asset-specific shock and x_t is the common time shock, inducing correlation ρ across asset values within a period. The random variables are assumed to be standard normal and independent. A mean of zero is attainable through translation without loss of generality since we are only interested in default probabilities. Suppose default occurs if $v_{it} < d$, a default threshold value. The overall or marginal default rate apparently required by B2 is $\theta = \Phi(d)$. However, in each period the default rate depends on the realization of the systematic factor x_t ; denote this θ_t . The model implies a distribution for θ_t . Specifically, the distribution of v_{it} conditional on x_t is $N(\rho^{1/2} x_t, 1 - \rho)$.

Hence the period t default probability is

$$\theta_t = \Phi[(d - \rho^{1/2} x_t) / (1 - \rho)^{1/2}]$$

Thus for $\rho \neq 0$ there is random variation in the default probability over time. The distribution is given by

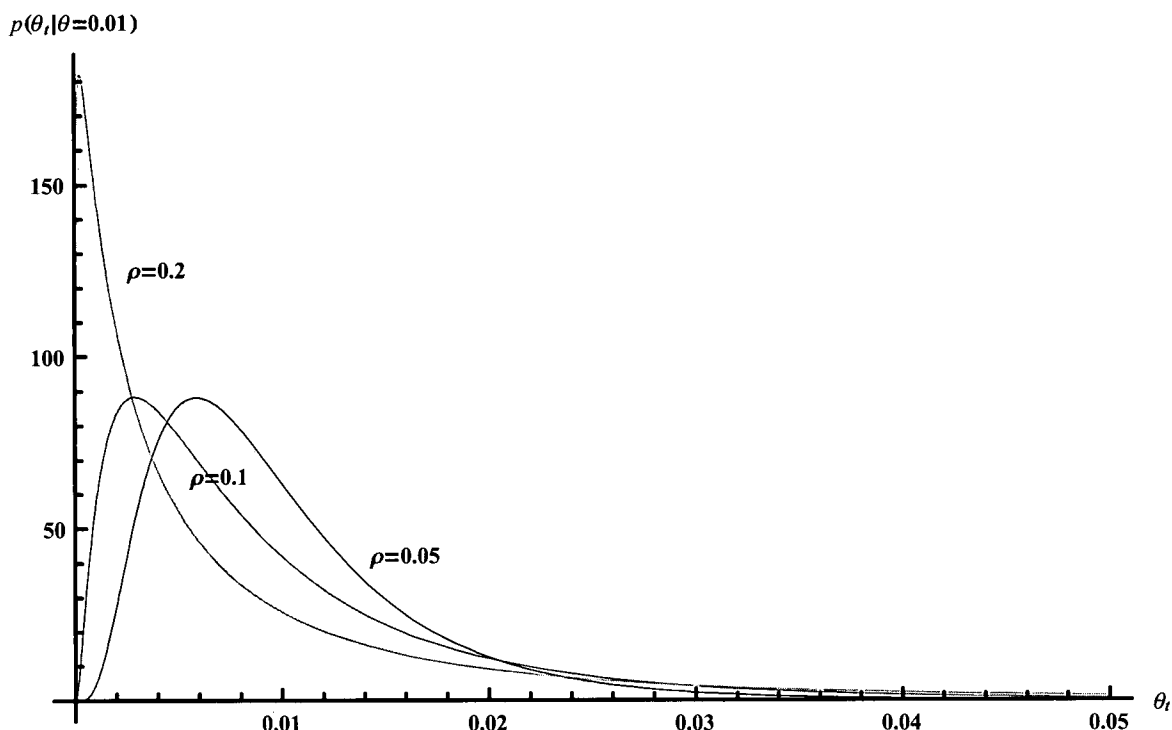
$$\begin{aligned} \Pr(\theta_t \leq A) &= \Pr(\Phi[(d - \rho^{1/2} x_t) / (1 - \rho)^{1/2}] \leq A) \\ &= \Phi[((1 - \rho)^{1/2} \Phi^{-1}[A] - \Phi^{-1}[\theta]) / \rho^{1/2}] \end{aligned}$$

using the standard normal distribution of x_t and $\theta = \Phi(d)$. This is known as the Vasicek distribution. Differentiating gives the density $p(\theta_t | \theta, \rho)$. The parameters are θ , the marginal or long-run mean default probability, and the asset correlation ρ . Values for ρ are in fact prescribed in Basel Committee for Banking Supervision [2006] for different asset classes. There is very little data evidence on ρ so we defer to the B2 formulas and set ρ to a prescribed value (0.20) but give results to show sensitivity to a range of values of the asset correlation. To provide some background, Exhibit 1 plots the density $p(\theta_t | \theta, \rho)$ for $\theta = 0.01$ and $\rho = \{0.05, 0.1, 0.2\}$.

All of these densities have a mean approximately equal to 0.01, but it is clear that the mass below the mean depends crucially on the correlation ρ .

EXHIBIT 1

Densities $p(\theta_i | \theta = 0.01)$ for Different ρ



IMPLICATIONS

We claim that default rates annually are more likely to be less than the long-run average than to be above when defaults are correlated. To illustrate the claim, let us use a mid-portfolio value for the long-run default rate θ , say $\theta = 0.01$ (100 basis point). With the asset correlation set at 0.2, the probability that θ_i , a draw from the density $p(\theta_i | \theta, \rho)$, is less than 0.01, the long-run default rate, is 0.709, over two-thirds. Thus, we are more than twice as likely to see a realized default rate in any given year that is below the long-run rate than one above the long-run rate. How does this occur? Not because the location is incorrect, the mean of the distribution of θ_i is almost exactly 0.01, the long-run rate. Rather it is because lower than expected rates, occurring about 71% of the time, are expected to be 0.0034, or about 1/3 of the long-run value, while higher than expected rates, when they occur, are expected to be 0.026, 2 1/2 times the long-run value.

Having set the stage with an example, we turn to a systematic description of the relationship between the long-run default rate and the probability that an annual realized default rate will be below the long-run rate.

Exhibit 2 graphs the probability that the realization is less than the mean against the mean (long-run rate).

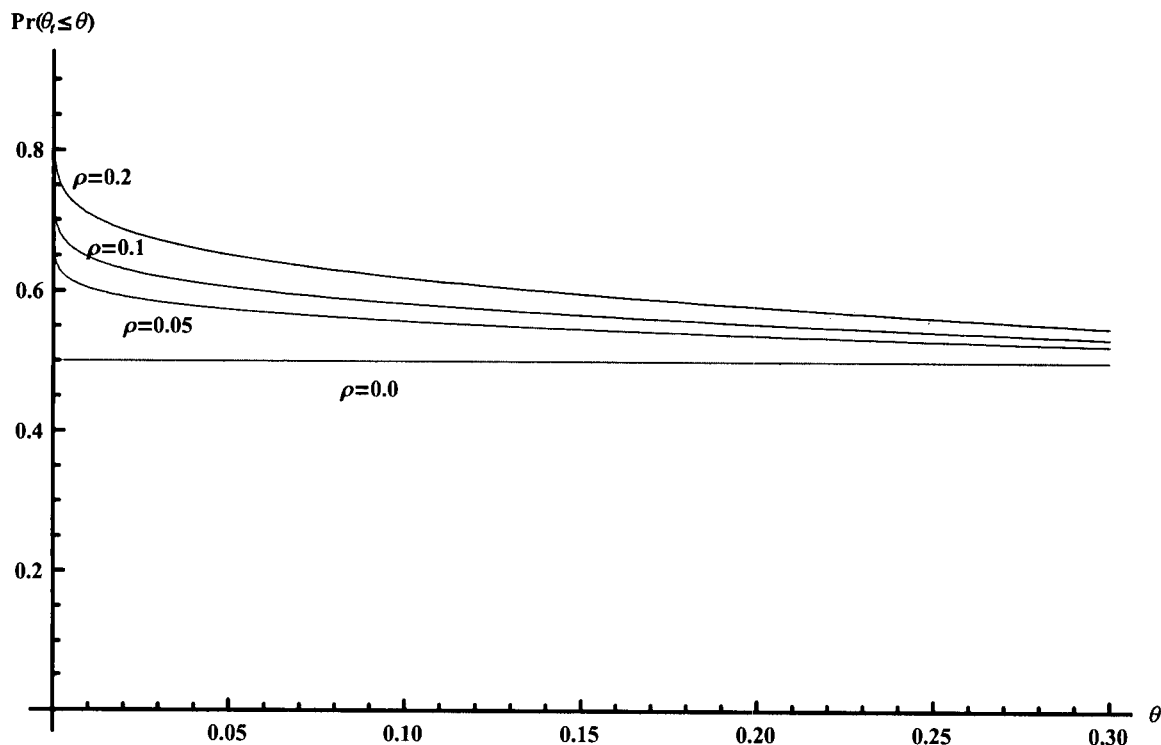
For low values of the default rate, correlation has stronger implications that the observed defaults will be fewer than expected. Thus for very safe, highly correlated assets the realized default rate in a short period is likely to be substantially lower than the true long-run average default rate. The effect is reversed as the long-run rate crosses 0.5 (not a relevant default rate, one hopes). The graph is drawn for $\rho = (0.05, 0.1, 0.2)$, perhaps covering the relevant range of asset correlations. For $\rho = 0$ the annual default rate is equal to the long-run average for any value of the default rate.

CONCLUSION

Average default rates in any year are likely to be less than the long-run average when defaults are correlated. This fact provides substantial support for the B2 insistence on data covering a full business cycle for estimating the long-run average default. It raises special questions for very new assets. Practitioners should be wary of short datasets especially on new, very safe assets. When default rates are

EXHIBIT 2

Probabilities $\Pr(\theta_i \leq \theta)$ for Different ρ



correlated, the default rates experienced over the first year or two are likely to underestimate substantially the long-run average default rate. When the asset correlation is 0.2, a value suggested by B2 when default rates are near 0.01, the understatement in a year of experience might be by a factor of 1/3. Perhaps it would be useful to rely on expert judgment about these assets, at least until adequate data series are observed. See Kiefer [2007]. Is this observation relevant for recent experience with new asset types?

ENDNOTE

I thank Dennis Glennon, Robert Jarrow, Erik Larson, Mark Levonian, and Mitch Stengel for useful comments.

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