The Slope of Credit Spread Curves

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One widely used approach to modelling credit risk is the so-called structural approach based on Black and Scholes [1973] and Merton [1974]. Recently there have been a number of empirical studies of structural corporate bond pricing models. Whereas the structural approach has been found quite useful in many aspects (e.g., Kealhofer and Kurbat [2001]; Bharath and Shumway [2004]; Leland [2004], and Schaefer and Strebulaev [2004]), the empirical evidence also points out several limitations of existing models. One is that the models are unable to accurately predict yield spreads on corporate bonds (e.g., Jones, Mason and Rosenfeld [1984]; Lyden and Saraniti [2000]; Delianedis and Geske [2001]; Eom, Helwege, and Huang [2004]; Arora, Bohn, and Zhu [2005], and Ericsson and Reneby [2005]). Another is that the models have difficulty explaining both yield spreads and default rates simultaneously, especially for investment-grade bonds (the so-called credit spread puzzle documented in Huang and Huang [2003]).

Another issue is whether the models can correctly predict the slope of the credit yield curve. Fons [1994]; Sarig and Warga [1989], and He, Hu, and Lang [2000] examine the problem and find that spread curves for high yield bonds tend to be downward-sloping. Helwege and Turner [1999] point out that the results of such studies are biased by failing to properly control for issuers' credit quality. Using a matched sample of bonds issued by the same firm, they find that the credit curve is upward-sloping even for speculative-grade bonds.

In addition to precisely controlling for credit quality, an important element of testing the shape of the credit spread curve is to consider the role of the coupon. Some structural models, such as the Merton [1974] model, apply only to zero-coupon bonds. This raises the question as to whether Helwege and Turner's findings, which seem to overturn those of Sarig and Warga, differ because they use a sample of coupon bonds while the latter study is based on zero-coupon bonds.

In this article, we examine the shape of credit spread curves for both investment-grade and speculative-grade bonds using an approach that deals with the presence of coupons in both Treasury and corporate bonds. In particular, we calculate a spot credit spread curve (for zero-coupon bonds) for each set of matched bonds in order to test the predicted slopes of zero-coupon-based structural models. Using a sample of newly issued corporate bonds from Securities Data Corporation (SDC) over a 30-year period, we find that the term structure of credit spreads for both zero-coupon and coupon bonds is usually upward sloping. This is true for both investment-grade and high-yield bonds. This suggests that structural models that predict a downward-sloping credit yield curve for high-yield bonds cannot price corporate bonds well, regardless of coupons.

MODELS OF CREDIT SPREAD CURVES

In this section we describe how to construct spot credit spread curves based on observed corporate bond prices. We introduce two models for the term structure of yield spreads: One is a piecewise constant spot spread model and the other is a piecewise linear spot spread model.

Basic Setup

Let $D(0,T)$ denote the time-0 price of a default-free zero-coupon bond with maturity T and unit par. For simplicity but without loss of generality, we assume continuous compounding. It follows that the T-year (default-free) spot rate is given by

$$r(0,T) = -\ln(D(0,T))/T \tag{1}$$

The time-0 price of a T_n-year default-free coupon bond whose cash flows are $\{CF(t_i)\}_{i=1,\ldots,n}$ is then given by

$$P(0,T_n) = \sum_{i=1}^{n} CF(t_i) \times e^{-r(0,t_i)\times t_i} \tag{2}$$

Consider an n-period defaultable bond with unit face value that matures at T. Denote its cash flows and time-0 price by $\{CF(T_i)\}_{i=1,\ldots,n}$ and $P_c(0,T_n)$, respectively. Assume that in the event of default, the bond recovery is $w < 1$ and it is paid on the first scheduled coupon date after default (discrete-time recovery in the sense of Duffie [1998]). We have (see e.g., Chen and Huang [2001])

$$P_c(0,T) = \sum_{i=1}^{n} D(0,T_i)\cdot CF(T_i)\cdot Q^i(0,T_i) + w\sum_{i=1}^{n} D(0,T_i)\left[Q^{i-1}(0,T_{i-1}) - Q^i(0,T_i)\right] \tag{3}$$

where $Q^i(0,T_i)$ represents the time-0 unconditional survival probability (of the issuer of the bond) by time T_i under the T_i-forward measure, and $T_n = T$. To simplify the implementation of Equation (3), we ignore the correlation between interest rates and the default indicator. It then follows that

$$P_c(0,T) = \sum_{i=1}^{n} D(0,T_i)\cdot CF(T_i)\cdot Q(0,T_i) + w\sum_{i=1}^{n} D(0,T_i)\left[Q(0,T_{i-1}) - Q(0,T_i)\right] \tag{4}$$

where $Q(0,\cdot)$ represents the risk-neutral unconditional survival probability.

Given the observed prices of defaultable bonds, we can first back out survival probabilities $Q(0,\cdot)$ using Equation (4). We can then determine the term structure of spot credit spreads based on $Q(0,\cdot)$. To see this, consider the special case of zero-coupon defaultable bonds with zero recovery. It follows from Equation (4) that

$$P_c(0,T) = D(0,T)Q(0,T) \tag{5}$$

As shown in Litterman and Iben [1991], the (spot) credit spread of this zero-coupon bond is given by:

$$s(0,T) = R(0,T) - r(0,T) = -\ln Q(0,T)/T \tag{6}$$

where $R(0,T)$ is the yield of the zero-coupon defaultable bond. Consequently, once $Q(0,T)$ is known, the T-year spot credit spread can be determined using Equation (6).

In general, using Equation (6), we can rewrite Equation (4) as the following

$$P_c(0,T) = \sum_{i=1}^{n} D(0,T_i)\cdot CF(T_i)\cdot e^{-s(0,T_i)T_i} + w\sum_{i=1}^{n} D(0,T_i)\left[e^{-s(0,T_{i-1})T_{i-1}} - e^{-s(0,T_i)T_i}\right] \tag{7}$$

If there are enough defaultable bonds with observed prices available, then it is straightforward to construct the spot credit spread curve, $s(0,\cdot)$, using Equation (7).

In practice, however, there may not be a sufficient number of defaultable bonds with market prices available on a given day that would allow us to back out $s(0,\cdot)$ so easily. This is particularly true in our analysis where the spot spread curve has to be built based on only a few defaultable bonds. Below we introduce two approximations that enable us to get around this problem.

The Piecewise Constant Spot Credit Spread Model

For convenience, given a set of N bonds, we can sort them on maturity such that $T_i < T_j$, if $i < j$. Thus, bond 1 has the shortest time to maturity and bond N has the longest.

Following Li [1998], we assume that the spot credit spread is a piecewise constant function. That is

$$s(0,t) = s_i,\ t \in (T_{i-1}, T_i),\ i = 1, 2, \ldots, N \tag{8}$$

where parameters $(s_i)_{i=1,\ldots,N}$ are constant and $T_0 = 0$. Once the spot spread of the first bond is known, we can recursively compute other spreads, $s_2, s_3, \ldots, s_k$, based on the information from the other $(k-1)$ bonds.

The piecewise constant spot spread model is easy to implement. However, one shortcoming of this model is that the interpolated spot spread curve is not continuous.

The Piecewise Linear Spot Credit Spread Model

In this model, we apply the piecewise linear interpolation to the spot rate spread over the Treasury. Again suppose that we have N bonds in one case and the time to maturity of bond i is T_i, where $T_i < T_j$, if $i < j$. We assume that the spot spread curve is linear between 0 to T_2, i.e., $s(0,t) = s_0 + s_1 * t$, $t \in (0, T_2)$. The information from bond 1 and bond 2 jointly determines the intercept s_0 and the slope s_1. Information contained in the third bond helps us to determine the second slope s_2. The spread curve in domain $[T_2, T_3]$ is then given by: $s(0, t) = s_0 + s_1 * T_2 + s_2 * (t - T_2)$.

In general, the spot credit spread curve for a case of N bonds under the piecewise linear model can be written as follows:

$$s(0,t) = \begin{cases} s_0 + s_1 \times t, & 0 \le t \le T_2; \\ s_0 + \sum_{i=1}^{k-1} s_i \cdot (\tilde{T}_{i+1} - \tilde{T}_i) + s_k & \\ \qquad \cdot (t - T_k), & T_k \le t \le T_{k+1}, k \ge 2 \end{cases} \tag{9}$$

where $\tilde{T}_1 = 0, \tilde{T}_j = T_j, j = 2, 3, \ldots, N$. The intuition of Equation (9) is straightforward. We first determine s_0 (the intercept) and s_1 (the first slope) using the first two bonds (that have the two shortest maturities). We can then recursively compute the slopes, $s_2, \ldots, s_k$, if the information about (k + 1) bonds (where $k \ge 2$) is available.

The Merrill Lynch Exponential Spline Model

The default-free term structure is constructed using the Merrill Lynch Exponential Spline Model, which we will briefly review. See Li, DeWetering, Lucas, Brenner, and Shapiro [2001] for more details about the model.

The basic idea of the model is to choose the following set of exponential functions as the basis for a series expansion of the discount function:

$$F^{(\alpha)}\{f_m(t) = e^{-m\alpha t}, \quad m = 0, 1, 2, \ldots, N\}$$

where α is a decay parameter and the functional inner product of functions $f_m(t)$ and $f_n(t)$ is defined as follows:

$$f_m \otimes f_n = \begin{cases} 1 & m = n = 0 \\ \int_0^\infty f_m(t) f_n(t) dt & \text{otherwise} \end{cases}$$

and the norm is defined by: $\| f(t) \| = (f \otimes f)^{1/2}$.

Given the exponential family $F^{(\alpha)}$, we can construct an orthonormal set of basis functions, $\{e_n(t), n = 1, 2, \ldots, N\}$, by applying the Gram-Schmidt procedure. Namely, we have

$$\| e_n(t) \| = 1 \text{ and } e_n(t) \otimes e_m(t) = 0\ \forall n \ne m$$

An order-N estimator of the discount function $D(0,t)$ is then given by

$$\hat{D}(0,t) = \sum_{i=1}^{N} \lambda_i e_i(t)$$

where

$$\lambda_i = D(0,t) \otimes e_i(t)$$

It then follows that

$$\hat{D}(0,t) = \sum_{m=1}^{N} \xi_m e^{-m\alpha t}$$

where $\{\xi_m\}$ are parameters to be estimated. The intuition here is that the discount function $\hat{D}(0,t)$ is a weighted average of pure discount factors.

Given a default-free bond whose cash flows are $(CF(t_i))_{i=1,\ldots,n}$, its price at time 0 is then given by

$$P_n = \sum_{i=1}^{n} CF(t_i) \times \hat{D}(0,t_i) \tag{10}$$

Let $\{P_n^{obs}, n = 1, 2, \ldots, N_B\}$ denote the market prices of N_B Treasury instruments on a given day. We can determine the coefficients ξ_i, $i = 1, 2, \ldots, N$ by minimizing the following pricing error function:

$$Err^{N_B} = \sum_{n=1}^{N_B} w_n (P_n - P_n^{obs})^2 \tag{11}$$

where w_n is the weight on bond n. In our analysis, we use $w_n = 1 \forall n$.

DATA

Ideally we want to construct credit spread curves (for a given rating category) based on multiple bonds issued by the same company that are identical in every respect except for maturity. In our analysis, we follow Helwege and Turner [1999] and consider only bonds issued on the same day and by the same company.

We first identify all new bond issues in the SDC database for the period January 2, 1970, to April 30, 2001. We then proceed to obtain the final sample of bonds used in our analysis. Exhibit 1 describes the key filters used in our sample selection and the sample size after each major filter applied.

We restrict the sample to noncallable and nonconvertible public bonds sold in the United States with a zero or fixed coupon rate. We also exclude bonds with an S&P rating of "NA," "NR," or "D" (default). This leads to an initial sample of 25,227 investment-grade bonds and 737 non-investment-grade bonds. Within the former group, 24,509 bonds are senior and the remaining 718 bonds are junior. Of those non-investment bonds, 693 are senior ones and 44 are junior.

As we need at least two bonds in order to build a credit spread curve, the next step is to identify sets of bonds issued on the same day by the same company. This leads to 9,137 investment-grade bonds, including 9,094 senior and 43 junior ones, and 181 non-investment grade bonds, with 179 senior and only 2 junior ones.

We next exclude bonds with unusual features such as floating rate coupons or various types of collateral. These types of bonds are reported in Exhibit 2. After applying these filters, we have 922 cases that include 2,403 investment-grade senior bonds, 15 cases with 31 investment-grade junior bonds, 25 cases with 63 high-yield senior bonds and one pair of high-yield junior bonds.

EXHIBIT 1

Creation of the Final Sample of Corporate Bond New Offerings

This exhibit describes how the final sample of sets of matched bonds is created. The initial sample includes all straight corporate bonds issued in the U.S. from January 1970 through April 2001. Data on bond new offerings are provided by Securities Data Corporation (SDC). Both the number of bonds and the number of cases (in parenthesis) are reported here. Each case consists of at least two matched bonds that are issued by the same company on the same day and that have the same seniority.

		Number of Bonds and Number of Cases (in parenthesis)				
		Investment Grade		Non-investment Grade		
		Senior	Junior	Senior	Junior	Total
(1)	All straight public corporate bonds offered for sale in the U.S. from 01/02/1970 through 04/30/2001 (except for those rated "NA", "NR", "D"). All bonds have fixed or zero coupon rate.	24,509	718	693	44	25,964
(2)	All bonds in (1) that were sold on the same day as another bond of the same company	9,094	43	179	2	9,318
(3)	Excluding some specific types of bonds and bonds with "market" and "float" yield or coupon	2,403 (922)	31 (15)	63 (25)	2 (1)	2,499 (963)
(4)	Excluding bonds in (3) whose difference in maturity is too small and with obvious typos in the reported prices	1,445 (643)	31 (15)	45 (20)	2 (1)	1,523 (679)

EXHIBIT 2
Bond Types Excluded from the Sample

This exhibit shows those types of bonds that are excluded from the final sample of corporate bond offerings from the SDC database.

1st Inst Ser Bd	Global Bk Nts	Pass-Thru Certs
Amortizing Nts	Global Bonds	Pass-Thrus, Debt
Bank Notes	GLobLS	Pfanbriefe
CD Notes	GlobalMTNs	PINES
CDs	Global Notes	PATS
Certificates	Gov Trust Certs	Power Bonds
Coll Lease Bds	Gtd ESOP Debs	Power Notes
Coll Trust Bds	Gtd ESOP Notes	QUIBS
Coll Trust Debs	Gtd Mdm-Trm Nts	Remark Notes
Collateral Debs	Gtd Part Certs	Remktd Notes
Cur Cpn Gtd Nts	Gtd Secured Nts	ROARS
Deposit Notes	Gtd Senior Debs	Sec Equip Certs
Drs. SM	Gtd Sr Notes	Sec Fac Bonds
Eq Nt Pass Cert	Guar ETC	Sec Lease Ob Bd
Equip Tr Certs	Guaranteed Bds	Secured Bonds
Equip Tr Notes	Guaranteed Debs	Secured MTNs
Equty Part Secs	Guaranteed Nts	Secured Notes
Exchangeable Bd	Income Securities	Senior Bank Nts
F & Coll Mtg Bd	Ind Amort MTNs	Senior Secured Nts
F & Ref Mtg Bds	Mdm-Trm Bk Nts	Serial Bonds
Flt Rate Bk Nts	Mdm-Trm CD Nts	Shrt-Tm Bank Nt
Flt Rate Bonds	Mdm-Trm Dep Nts	Sink Fd Certs
Flt Rte CDs	Mdm-Trm Fl Nts	Sink Fund Debs
FR MT Bank Nts	MITTS	Sr Flt MT Bk Nt
Fst Mtg Bonds	Money Mult Nts	Sr MT Bank Nts
G & R Bonds	Mtg & Col Tst Bds	Zero Coupon CD
Gen'l Mtg Bds	New Mtg Bonds	Zero Cpn Gtd Nt

Finally, we delete bonds with obvious typos in the reported prices and cases whose bonds share the same maturity or where the difference in their maturities is too small.

The final sample of corporate bond offerings contains 1,445 bonds belonging to 643 cases for the investment-grade senior group; 31 bonds belonging to 15 cases for the investment-grade junior group; 45 bonds and 20 cases for the high-yield senior group and only 2 bonds (in one case) for the high-yield junior group.

Exhibit 3 reports the distribution of the final sample of bonds by ratings. As can be seen from Panel A, most bonds are issued at par or a slight discount. This implies that matched bonds in a given set differ not only in maturity and yield but also in coupon. Out of 643 cases, 36 have bonds with mixed ratings. Namely, these bonds are issued by the same company and at the same day but have different ratings. However, the difference in ratings of these bonds is small. Cases with mixed ratings are only about 5% of the final sample. Thus we treat those 36 cases the same as those with uniform ratings.

Panel C shows that of 21 high-yield cases in the sample, 6 are "BB+," 9 are "BB," 2 are "BB–" and there is only one each from "B+" and "B." Furthermore, there are no bonds rated below "B" in the final sample. In Helwege and Turner [1999], who consider only high yield bonds, the final sample consists of 42 sets of BB-rated bonds (those rated "BB–" or higher), and 21 sets of B-rated bonds, and only one set of bonds rated below "B-." Notice that our final sample of high-yield bonds is smaller than Helwege and Turner's even though our sample period is longer than theirs because we exclude all callable bonds.

The Treasury yield data used in this study are 10 daily constant maturity Treasury (CMT) rate series from the Federal Reserve from January 2, 1970, to April 30, 2001. However, not all 10 instruments are available over the entire sample period. Given the daily CMT series, we construct daily Treasury curves using the Merrill Lynch Exponential Spline Model described previously.

EMPIRICAL RESULTS

Our final sample consists of sets of matched bonds, where each set is composed of two to nine noncallable bonds with the same seniority and issued on the same day by the same company. Bonds in such a set differ only in their maturity, yield, and coupon. As noted in Helwege and Turner [1999], an analysis of the shape of credit spread curves based on such sets of matched bonds is much less likely to suffer from sample selection bias than one based only on ratings.

In this section we present empirical results from our analysis of the slope of the term structure of credit spreads. We provide evidence on the slopes of spot credit spread curves first, then we report empirical results on the slopes of credit spread curves for coupon bonds.

Based on Models with Zero Recovery Rates

We implement our two specifications of the model of spot credit spreads with zero recovery rates. One is the piecewise constant spot spread model and the other the piecewise linear spot spread model.

The Piecewise Constant Spot Credit Spread Model. Given a set of matched bonds along with their prices and other characteristics such as coupon and maturity, we can obtain the spot credit spread curve using Equation (7) and the piecewise constant spot credit spread model specified in Equation (8). As such, spot credit spreads

EXHIBIT 3

Summary Description of the Final Sample of Bonds

This exhibit reports the distribution of the final sample of cases (sets) by rating and issue type. The number reported is the number of cases. A case of matched bonds is comprised of all bonds issued on the same day by the same firm that all have the same characteristics except maturity, yield and possibly coupon. Entries shown on column "Mix" refer to those cases where matched bonds have mixed S&P ratings. Entries shown on row "Mix" refer to those cases which have different bond issue types. For example, a two-bond case with one bond issued at par and the other at discount will be put in this category. The sample period is January 1970 – April 2001.

Panel A: Sets of Investment Grade Senior Bonds

	Mix	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	Total
Mix	5	4	1	6	12	11	15	13	10	11	4	92
ZeroCpn	0	0	0	0	0	0	0	0	0	1	0	1
Par	25	25	6	8	22	41	48	43	24	22	9	273
Premium	0	0	0	0	0	0	0	0	0	0	0	0
Discount	6	6	5	10	27	29	52	35	35	42	30	277
Total	36	35	12	24	61	81	115	91	69	76	43	643

Panel B: Sets of Investment Grade Junior Bonds

	Mix	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	Total
Mix	0	0	0	0	0	0	0	1	1	0	1	3
ZeroCpn	0	0	0	0	0	0	0	0	0	0	0	0
Par	0	0	0	0	0	0	2	0	0	1	0	3
Premium	0	0	0	0	0	0	0	0	0	0	0	0
Discount	0	0	0	3	0	3	1	1	0	1	0	9
Total	0	0	0	3	0	3	3	2	1	2	1	15

Panel C: Sets of Non-investment Grade Bonds

	BB+	BB	BB-	B+	B	Total
Mix	2	4	0	0	0	6
ZeroCpn	0	0	0	0	0	0
Par	0	5	2	1	1	9
Premium	0	0	0	0	0	0
Discount	4	2	0	0	0	6
Total	6	11	2	1	1	21

control for difference in coupons of matched bonds whereas this is not the case for credit spreads based on matched bonds (that are examined later in this article).

For a given set of matched bonds and corresponding spot credit spreads with same maturities, if the yield spreads increase with the maturity, we label the set of bonds as a case of "All Up" (a case of the upward-sloping spot credit spread curve). Otherwise, the set of bonds is labeled as "Non-Up" (a case whose spot credit spread curve is not upward sloping).

Exhibit 4 reports the number and percentage of the "All Up" cases based on the model of piecewise constant spot spread curves with zero recovery. Panel A shows that of 643 sets of investment-grade senior bonds, 357 or 55.5% of the group are "All Up" cases. Notice that these are mainly driven by the two-bond cases (331 out of 357). In addition, out of those 532 two-bond cases, 62% are "All Up" outcomes, which is notably higher than 55.5%. In comparison, in the 78 three-bond cases the percentage of "All Up" cases drops to 33%. Among those four-bond or higher cases, none is All Up. Panel B shows that of 15 sets of investment-grade junior bonds, the "All Up" cases number 9 (60%) and belong entirely to the two-bond category. Panel C reports results for high-yield bonds. The percentage of All Up cases is 52.3%, which is lower than its counterpart for investment-grade bonds (55.5%) and is entirely due to the two-bond cases, which have a much higher percentage (64.7%) than the average.

EXHIBIT 4

Results from the Piecewise Constant Spot Credit Spread Model with Zero Recovery

The sample includes all cases of matched bonds issued from 01/02/1970 through 04/30/2001 in the SDC database. A case of matched bonds is comprised of all bonds issued on the same day by the same firm that all have the same characteristics except maturity and yield. "All Up" includes those cases where the spot credit spread increases as the bond maturity increases. "Non-UP" includes all other cases. Spot credit spreads are calculated using the piecewise constant spot spread model with zero recovery.

Panel A: Cases of Investment Grade Senior Bonds

	Number of Bonds in One Case							
	2	3	4	5	6	7	9	ALL
# of All UP	331	26	0	0	0	0	0	357
# of Non-UP	201	52	25	5	1	1	1	286
% of All UP	62%	33%	0%	0%	0%	0%	0%	55.50%

Panel B: Cases of Investment Grade Junior Bonds

	Number of Bonds in One Case		
	2	3	ALL
# of All UP	9	0	9
# of Non-UP	5	1	6
% of All UP	64%	0%	60%

Panel C: Cases of High Yield Bonds

	Number of Bonds in One Case			
	2	3	4	ALL
# of All UP	11	0	0	11
# of Non-UP	6	3	1	10
% of All UP	64.70%	0%	0%	52.30%

Overall, results in Exhibit 4 provide some evidence that spot spread curves for both investment-grade and high-yield bonds are upward sloping, but the evidence is not that strong. To further examine the shape of spot spread curves, we follow Helwege and Turner [1999] and apply *t*-tests and rank tests to the set of bonds in the sample. More specifically, we apply the two tests to a sample of matched pairs. If a set of bonds includes more than two, then we obtain matched pairs of bonds using only adjacent bonds. That is, with a set of N bonds, we can have $(N-1)$ matched pairs. We can then calculate the difference between the two spreads for each matched pair. This completes the construction of a sample of spread differences. The first three rows of Exhibit 5 report the number of matched pairs, the mean and median spread differences, respectively.

We can do a *t*-test to see if the mean spread difference is significantly different from zero. Exhibit 5 shows results from such a test. The mean difference is significantly positive at the 10% level for the investment-grade (IG) senior group but is not significant for the other two groups—the IG junior and high-yield (HY) bonds. The results from the Wilcoxon signed rank test are similar but are much stronger for the IG senior group. Results from Exhibit 5 indicate that spot spread curves are upward sloping for investment-grade senior bonds and are not upward sloping for the other two groups.

Another way to investigate the slope of spread curves is to regress the bond spread against the bond maturity and then examine the sign of the regression coefficient on maturity. However, Helwege and Turner [1999] point out that regressions using pooled bonds with the same

EXHIBIT 5

Matched-Pair Statistical Tests of the Slopes of Piecewise Constant Spot Credit Spread Curves with Zero Recovery

Bonds issued on the same day by the same firm that have the same characteristics except maturity, yield and coupon constitute a matched set. A such set of N bonds yields (N – 1) matched pairs of spot credit spreads that are then used in the t-tests and ranked tests here. Difference in spread is calculated for each pair of spreads. Spot credit spreads are calculated using the piecewise constant spot spread model with zero recovery. The sample period is January 1970 – April 2001.

	IG Senior	IG Junior	High Yield	Total
Number of matched pairs	802	16	26	844
Median difference in spread (bp)	4.9	3.9	22.5	5
Mean difference in spread (bp)	4.2	18.5	21.7	5
t-statistic for mean difference in spread equal to zero	1.8	1.55	1.2	2.18
P-value (2-tailed t-test)	0.0725	0.14	0.24	0.03
P-value (Wilcoxon signed rank test)	<0.0001	0.45	0.56	<0.0001

whole-letter rating categories tend to generate a negative slope coefficient for high-yield bonds and thus a bias toward finding a downward-sloping spread curve for high-yield bonds.

Exhibit 6 reports results from pooled regressions of spot credit spreads against the corresponding bond maturities. As can be seen from the exhibit, the coefficient on maturity (the slope coefficient) is significantly positive at the 10% level for the IG senior group. Also, the slope coefficient is indeed negative for the other two groups although it is not statistically different from zero. This result appears to indicate that the bias pointed out by Helwege and Turner [1999] may also exist in pooled regressions of spot credit spreads.

To summarize, results from the piecewise constant spot spread model with zero recovery provide some evidence that spot spread curves for investment-grade senior bonds are upward sloping and provide no evidence that spread curves for high-yield bonds are upward sloping.

The Piecewise Linear Spot Credit Spread Model. One unattractive feature of the piecewise constant spot spread model is that spread curves are not continuous. In this subsection we implement the piecewise linear spot spread model and present the corresponding empirical results.

Given a set of matched bonds along with their prices and other characteristics such as coupon and maturity, we can obtain the spot credit spread of these bonds using

EXHIBIT 6

Regression Estimates of Piecewise Constant Spot Credit Spread Curves with Zero Recovery

This exhibit presents regression coefficients for each of three groups of bonds. The regression runs the spot credit spread against the bond maturity. Spot spreads used in the regression are calculated using the piecewise constant spot spread model with zero recovery. The sample period is January 1970 – April 2001. *t*-statistics are in parentheses. ***, **, * indicate significance at the 0.01, 0.05, and 0.10 levels, respectively.

	IG Senior	IG Junior	HighYield	All
Number of obs.	1,445	31	47	1,523
Intercept	$4{,}19 \cdot 10^{-3}$	$5{,}91 \cdot 10^{-3}$	$1{,}30 \cdot 10^{-2}$	$4{,}46 \cdot 10^{-3}$
	(23.70)***	(4.82)***	(5.37)***	(24.07)***
Maturity	$1{,}09 \cdot 10^{-5}$	$-3{,}51 \cdot 10^{-5}$	$-3{,}75 \cdot 10^{-5}$	$1{,}04 \cdot 10^{-5}$
	(1.57)*	(−0.84)	(−0.34)	(1.43)
R^2	0.017	0.024	0.0026	0.013

Equation (7) along with the piecewise linear spot credit spread model specified in Equation (9).

As indicated in Equation (9), to implement the model in the case of a set of N bonds, we need to determine the intercept s_0 and the slopes s_i, $i - 1, \ldots, N - 1$. If all the slopes are positive, we label the case as "All Up." Otherwise, we label it as a "Non-Up" case.

Exhibit 7 reports the number and percentage of the "All Up" cases based on the piecewise linear spread model with zero recovery. Compared with results shown in Exhibit 4, both the number and percentage of "All Up" cases is significantly higher in all three rating and seniority groups. For instance, Panel A shows that in the investment-grade senior group, the number of "All Up" cases is 467 (73% of the entire group) whereas its counterpart in Panel A of Exhibit 4 is 357 (55.5%).

As can be seen from Exhibit 7, among the "All Up" cases, the two-bond case accounts for about 91% (424 out of 467), 92% (12 out of 13), and 93% (14 out of 15), in the IG senior, IG junior, and high-yield groups, respectively. That is, the high percentage of "All Up" cases is still mainly driven by the two-bond cases. However, it is less so than in the case of the piecewise constant spot spread model. Recall from Exhibit 4 that the two-bond cases there contribute about 93% (331 out of 357), 100% (9 out of 9), and 100% (11 out of 11), in the three rating and seniority groups, respectively.

Nonetheless, as with Exhibit 4, Exhibit 7 provides some evidence that spot spread curves for both investment-grade and high-yield bonds are upward sloping, but the evidence is mainly based on cases with only two bonds.

EXHIBIT 7

Results from the Piecewise Linear Spot Credit Spread Model with Zero Recovery Rates

The sample includes all matched cases of bonds issued between 01/01/1970 and 04/30/2001 in the SDC. A matched cases of bonds is comprised of all bonds issued on the same day by the same firm that all have the same characteristics except maturity, yield and coupon. "All Up" includes those cases where the spot credit spread increases as the bond maturity increases. "Non-UP" includes all other cases. Spot credit spreads are calculated using the piecewise linear spot spread model with zero recovery.

Panel A: Investment Grade Senior bond

	No. of Bonds in One Case							
	2	3	4	5	6	7	9	ALL
# of All UP	424	39	4	0	0	0	0	467
# of Non-UP	108	39	21	5	1	1	1	176
% of All UP	80%	50%	16%	0%	0%	0%	0%	73%

Panel B: Investment Grade Sub-Senior

	No. of Bonds in One Case		
	2	3	ALL
# of All UP	12	1	13
# of Non-UP	2	0	2
% of All UP	86%	100%	86.7%

Panel C: Non-investment Grade

	No. of Bonds in One Case			
	2	3	4	ALL
# of All UP	14	1	0	15
# of Non-UP	3	2	1	6
% of All UP	82%	33%	0%	71.4%

As in the preceding subsection, we examine further the slope of spot spread curves by applying t-tests and Wilcoxon signed rank tests to a sample of matched pairs. Recall that a given set of N (matched) bonds can generate $(N - 1)$ matched pairs and that we compute the difference between the two spreads for each matched pair to obtain a sample of spread differences.

The first three rows of Exhibit 8 report the number of matched pairs and the mean and median differences in spreads. The mean spread difference for the IG senior group is much higher than the mean for the other two groups. The fourth row of the exhibit shows the results from a t-test that tests if the mean spread difference is significantly different from zero. The entries on the fourth row indicate that the mean spread difference (a measure of the slope) is significantly positive at the 10% level for the two IG groups but is not so for the HY group. Results from the Wilcoxon signed rank test (a nonparametric test) reported in the last row of the exhibit provide evidence that the slope of spot spread curves is positive for all three rating and seniority groups.

As before, another way to examine the slope of spot spread curves is to regress spot spreads against maturities and look at the sign of the regression coefficient. A positive sign would indicate an upward sloping spread curve. To implement this idea, for each set of N bonds whose maturities are T_i, $i = 1, \ldots, N$, we first determine the corresponding set of spot spreads using the piecewise linear spread model. We then pool all the bonds together and regress the spot spread against the maturity in each of the three rating and seniority groups.

Regression results, reported in Exhibit 9, show that the coefficient on maturity is significantly positive only for the IG Senior subsample (at the 1% level). As in the case of the piecewise constant spot spread model with zero recovery, the slope coefficient is negative, albeit not significantly so, for the other two subsamples.

Based on Models with Positive Recovery Rates

We obtain the empirical results reported so far by assuming that the recovery rate is zero. In this section, we present results from models with positive recovery rates. For brevity, we implement only the piecewise linear spot spread model, which is more likely to generate an upward-sloping spot spread curve than the piecewise constant spot spread model is (as shown in the previous section).

Empirical evidence has documented that bond seniority is the main factor affecting recovery rates. Nonetheless, to take into account the variation of recovery rates across ratings in our analysis, we use the estimates of recovery rates from Altman and Kishore [1998] that, for completeness, are given in Exhibit 10. Given the recovery rate, we can obtain spot spread curves using the piecewise linear spread model.

Once we have a sample of matched cases of spot spreads, we examine the slope of spot spread curves as

EXHIBIT 8

Statistical Tests of the Slopes of Piecewise Linear Spot Spread Curves with Zero Recovery Rates

Bonds issued on the same day by the same firm that have the same characteristics except maturity, yield and coupon constitute a matched set. A such set of N bonds yields (N – 1) matched pairs of spot credit spreads that are then used in the t-tests and ranked tests here. Difference in spread is calculated for each pair of spreads. Spot credit spreads are calculated using the piecewise linear spot spread model with zero recovery. The sample period is January 1970 – April 2001.

	IG Senior	IG Junior	High Yield	All
Number of matched pairs	802	16	26	844
Median difference in spread (bp)	1.1	0.67	1.69	1.11
Mean difference in spread (bp)	3.29	1.33	1.16	3.19
t-statistic for mean difference in spread equal to zero	1.96	2.08	0.721	1.99
P-value (2-tailed t-test)	0.0506	0.0546	0.478	0.0464
P-value (Wilcoxon signed rank test)	<0.0001	0.0042	0.0094	<0.0001

EXHIBIT 9

Regression Estimates of the Piecewise Linear Spot Spread Model with Zero Recovery Rates

This exhibit presents regression coefficients for each of three groups of bonds. The regression runs the spot credit spread against the bond maturity. Spot credit spreads used in the regression are calculated using the piecewise linear spot spread model with zero recovery. The sample period is January 1970 – April 2001. t-statistics are in parentheses. ***, **, * indicate significance at the 0.01, 0.05, and 0.10 levels, respectively.

	IG Senior	IG Junior	High Yield	All
Number of obs.	802	16	26	844
Intercept	$-4{,}90 \cdot 10^{-3}$	$6{,}51 \cdot 10^{-3}$	$1{,}30 \cdot 10^{-2}$	$-4{,}47 \cdot 10^{-3}$
	(−3.40)***	(3.78)***	(4.84)***	(−3.22)***
Maturity	$4{,}98 \cdot 10^{-4}$	$-1{,}72 \cdot 10^{-5}$	$-2{,}19 \cdot 10^{-5}$	$4{,}83 \cdot 10^{-5}$
	(11.16)***	(−0.38)	(−0.22)	(11.22)***
R^2	0.135	0.01	0.0020	0.130

before in each of the three rating and seniority groups (subsamples). First, we look at the percentage of "All Up" cases. We then apply t-tests and Wilcoxon signed rank tests to the sample of spot spread differences. Finally, we run pooled regressions and see if they tend to generate biased results in favor of downward-sloping spot spread curves for high-yield bonds.

Exhibit 11 reports the number and percentage of the "All Up" cases based on the piecewise linear spread model with non-zero recovery rates. Compared with results shown in Exhibit 7, the percentage of "All Up" here is higher in both IG senior (75% vs. 73%) and HY groups (81% vs. 71%) and is the same in the IG junior group (87%). Furthermore, this percentage increase comes mainly from three- and four-bond cases. As a result, incorporating recovery rate information in the piecewise linear spread curve model may more likely generate upward-sloping spot spread curves.

EXHIBIT 10

Recovery Rates by Ratings

This exhibit shows the percentage of par that a bond is worth one month after bankruptcy by original rating of the bond and is from Altman and Kishore [1998].

Original Rating	Recovery Rate (%)
AAA	68.34
AA	59.59
A	60.63
BBB	49.42
BB	39.05
B	37.54
CCC	38.02
Default	0

Exhibit 12 shows results from statistical tests of the slope of piecewise linear spot spread curves under the assumption of positive recovery rates. The first three rows of the exhibit show the number of matched pairs of spot spreads and the mean and median differences in spreads. We can see that both the mean and median are significantly higher than their counterparts under the piecewise linear spread model with zero recovery rates in each of the three groups (in Exhibit 8). Results from the t-tests (shown in rows 4 and 5) and the Wilcoxon signed rank tests (shown in the last row) indicate further that the slopes of spot spread curves are positive for both IG senior and HY groups.

Results from pooled regressions of spot credit spreads against maturity, reported in Exhibit 13, show that the regression coefficient on maturity is still significantly positive (at the 1% level) for the IG Senior group but is negative (albeit not significantly so) for the other two groups. This result indicates again that pooled regressions using broad rating categories tend to generate a negative slope coefficient for high yield bonds. Thus our findings using spot credit spread curves complement those of Helwege and Turner [1999].

To summarize, empirical results obtained using the piecewise linear spread model with non-zero recovery rates indicate that spot spread curves are upward-sloping for both investment-grade senior bonds and high-yield bonds in our sample.

EXHIBIT 11

Results from the Piecewise Linear Spot Spread Model with Positive Recovery Rates

The sample includes all matched cases of bonds issued between 01/01/1970 and 04/30/2001. A matched case of bonds is comprised of all bonds issued on the same day by the same firm that all have the same characteristics except maturity, yield and coupon. "All Up" includes those cases where the spot credit spread increases as the bond maturity increases. "Non-UP" includes all other cases. Spot credit spreads are calculated using the piecewise linear spot spread model with the recovery rates given in Exhibit 12.

Panel A: Investment Grade Senior bond

	No. of Bonds in One Case							
	2	3	4	5	6	7	9	ALL
# of All UP	423	54	6	0	0	0	0	483
# of Non-UP	109	24	19	5	1	1	1	160
% of All UP	80%	69.2%	24%	0%	0%	0%	0%	75.1%

Panel B: Investment Grade Sub-Senior

	No. of Bonds in One Case		
	2	3	ALL
# of All UP	12	1	13
# of Non-UP	2	0	2
% of All UP	86%	100%	86.7%

Panel C: Non-investment Grade

	No. of Bonds in One Case			
	2	3	4	ALL
# of All UP	14	3	0	17
# of Non-UP	3	0	1	4
% of All UP	82.4%	100%	0%	81%

EXHIBIT 12

Statistical Tests of the Slopes of Piecewise Linear Spot Spread Curves with Positive Recovery Rates

Bonds issued on the same day by the same firm that have the same characteristics except maturity, yield and coupon constitute a matched set. A such set of N bonds yields (N – 1) matched pairs of spot credit spreads that are then used in the t-tests and ranked tests here. Difference in spread is calculated for each pair of spreads. Spot credit spreads are calculated using the piecewise linear spot spread model with the recovery rates given in Exhibit 12. The sample period is January 1970 – April 2001.

	IG Senior	IG Junior	High Yield	Total
Number of matched pairs	802	16	26	844
Median difference in spread (bp)	2.93	1.34	3.52	2.93
Mean difference in spread (bp)	8.14	2.94	3.86	7.91
t-statistic for mean difference in spread equal to zero	3.19	1.92	3.23	3.26
P-value (2-tailed *t*-test)	0.0015	0.074	0.0034	0.0011
P-value (Wilcoxon signed rank test)	<0.0001	0.0042	0.0025	<0.0001

EXHIBIT 13

Regression Estimates of Piecewise Linear Spot Spread Curves with Positive Recovery Rates

This exhibit presents regression coefficients for each of three groups of bonds. The regression runs the spot credit spread over the corresponding bond maturity. The *t*-statistics are in parentheses. Spot credit yields are calculated using the piecewise linear spot spread model with the recovery rates given in Exhibit 12. The sample period is January 1970 – April 2001. ***, **, * indicate significance at the 0.01, 0.05, and 0.10 levels, respectively.

	IG Senior	IG Junior	High Yield	All
Number of obs.	802	16	26	844
Intercept	$3{,}65 \cdot 10^{-4}$	$1{,}41 \cdot 10^{-2}$	$2{,}16 \cdot 10^{-2}$	$8{,}03 \cdot 10^{-4}$
	(0.11)	(3.84)***	(5.53)***	(0.26)
Maturity	$7{,}67 \cdot 10^{-4}$	$-3{,}15 \cdot 10^{-5}$	$-1{,}33 \cdot 10^{-5}$	$7{,}43 \cdot 10^{-4}$
	(7.58)***	(–0.32)	(–0.09)	(7.62)***
R^2	0.0669	0.0075	0.0003	0.0645

Term Structure of Credit Spreads for Coupon Bonds

Empirical results presented so far are based on spot credit spread curves (for zero-coupon bonds). In order to make our analysis more comprehensive, we examine the slope of the term structure of coupon bond credit spreads in this section. As such, we provide evidence on the slope of spread curves for both zero-coupon and coupon bonds.

Essentially we repeat the analysis of Helwege and Turner [1999], who consider high-yield bonds only, but we extend their study to include investment-grade bonds as well. In addition, as mentioned earlier, our sample of matched bonds is different from theirs as our sample period is longer and we exclude all callable bonds.

To proceed, we first calculate the credit spreads of those matched bonds in each case in the sample by taking the difference between the bond yield and the nearest on-the-run Treasury yield (spot spread models are no longer needed here). Such spreads are actually supplied by SDC. We eliminate the few cases where the spread is missing in SDC. Given the credit spreads, we can then examine the slope of credit spread curves in the same fashion as we do with spot spread curves. However, as mentioned earlier, newly issued bonds in a given matched case in general have different coupons and, as such, credit spreads from SDC do not adjust for coupon.

Exhibit 14 reports the number and percentage of the "All Up" cases based on the term structure of credit spreads for coupon bonds. Specifically, the percentage level is 85% for the IG senior group, 93.3% for the IG junior group, and 81% for the high-yield group. The levels for the first two groups are significantly higher than their counterparts under the piecewise linear spot spread model (see Exhibit 11). Loosely speaking, for investment-grade bonds, credit spread curves for coupon bonds are more likely to be upward sloping than spot spread curves in our sample.

The level of 81% for the high-yield group in our sample is also notably higher than the percentage of "All Up" cases of BB noncallable bonds reported in Helwege and Turner (61% as shown in their Table II). (Recall from Exhibit 3 that our high-yield group includes only two single B cases out of 21.)

Exhibit 15 presents results from statistical tests of the slopes of credit spread curves for coupon bonds. We can see that both the mean and median differences in spreads are significantly higher than their counterparts under the piecewise linear spot spread model with non-zero recovery rates (see Exhibit 12). Results from both *t*-tests and Wilcoxon signed rank tests indicate clearly that the slopes of credit spread curves for coupon bonds are significantly positive for both investment-grade and high-yield bonds. Again, the evidence concering high-yield bonds here appears even stronger than that for BB noncallable bonds reported in Helwege and Turner (see their Table III).

Exhibit 16 reports results from pooled regressions of credit spreads against maturity. As in the case of the piecewise linear spot spread model with non-zero recovery rates (see Exhibit 13), the regression coefficient on maturity (the slope coefficient) here is still significantly positive at the 1% level for the IG Senior group and is negative (albeit not significantly so) for the other two groups. This result indicates again that pooled regressions may generate biased results for high yield bonds.

The regression results for high-yield bonds are also consistent with the findings of Helwege and Turner, who find that the slope coefficients of BB and B bonds from

Exhibit 14

Results from Credit Spread Curves for Coupon Bonds

The sample includes all matched cases of bonds issued between 01/02/1970 and 04/30/2001. A matched case of bonds is comprised of all bonds issued on the same day by the same firm that all have the same characteristics except maturity, yield and possibly coupon. Bond credit yield spreads are calculated over the nearest on-the-run Treasury bond and are provided by SDC. "All Up" includes those cases where the bond credit spread increases as the bond maturity increases. "Non-UP" includes all other cases.

Panel A: Cases of Investment Grade Senior Bonds

	Number of Bonds in One Case							
	2	3	4	5	6	7	9	ALL
# of All UP	472	58	17	2	0	0	0	549
# of Non-UP	60	20	8	3	1	1	1	94
% of All UP	89%	74%	68%	40%	0%	0%	0%	85%

Panel B: Cases of Investment Grade Junior Bonds

	Number of Bonds in One Case		
	2	3	ALL
# of All UP	13	1	14
# of Non-UP	1	0	1
% of All UP	92.9%	100%	93.3%

Panel C: Cases of High Yield Bonds

	Number of Bonds in One Case			
	2	3	4	ALL
# of All UP	16	1	0	17
# of Non-UP	1	2	1	4
% of All UP	94%	33.3%	0%	81%

pooled regressions are negative and largely not statistically significant (see their Panel B of Table IV).

To summarize, our analysis of credit spread curves (for coupon bonds) indicates that the curve is upward sloping for both investment-grade and high-yield bonds. In particular, the results for the latter group reinforce the main finding of Helwege and Turner [1999].

CONCLUSION

In this article we present perhaps the first comprehensive empirical analysis of the slope of the credit spread curve, using a sample of newly issued corporate bonds from Securities Data Corporation. Specifically, we examine credit spread curves for both zero-coupon and coupon bonds and for both investment-grade and high-yield bonds. Importantly, we control for credit quality in our analysis by examining sets of bonds issued on the same day by the same company that have the same characteristics except maturity, yield and coupon.

This method of controlling for credit quality is first proposed by Helwege and Turner [1999]. However, they limit their analysis to credit spread curves for high-yield coupon bonds. We contribute to the literature by extending their analysis to include investment-grade bonds and, in addition, to study credit spread curves for zero-coupon bonds.

We find that the term structure of credit spreads for both zero-coupon and coupon bonds is usually upward sloping regardless of whether the bond is rated investment-grade or speculative-grade.

Our results contradict previous evidence on the slope of credit spread curves for both zero-coupon (Sarig

EXHIBIT 15

Statistical Tests of the Slopes of Credit Spread Curves for Coupon Bonds

A matched case of bonds is comprised of all bonds issued on the same day by the same firm that all have the same characteristics except maturity, yield and coupon. A such set of N bonds yields (N – 1) matched pairs of spot credit spreads that are then used in the t-tests and ranked tests here. Difference in spread is calculated for each pair of spreads. Bond credit yield spreads are calculated over the nearest on-the-run Treasury bond and are provided by SDC. The sample period is January 1970 – April 2001. ***, **, * indicate significance at the 0.01, 0.05, and 0.10 levels, respectively.

	IG Senior	IG Junior	High Yield	Total
Number of matched pairs	747	15	25	787
Median difference in spread (bp)	11.0	11.0	15.0	11.0
Mean difference in spread (bp)	14.0	15.7	17.2	14.1
t-statistic for mean difference in spread equal to zero	18.1	3.80	4.97	19.0
P-value (2-tailed t-test)	<0.0001	0.0020	<0.0001	<0.0001
P-value (Wilcoxon signed rank test)	<0.0001	0.0018	0.0009	<0.0001

EXHIBIT 16

Regression Estimates of Credit Spread Curves for Coupon Bonds

A matched case of bonds is comprised of all bonds issued on the same day by the same firm that all have the same characteristics except maturity, yield and coupon. "All Up" includes those cases where the bond credit spread increases as the bond maturity increases. "Non-UP" includes all other cases. Bond credit yield spreads are calculated over the nearest on-the-run Treasury bond and are provided by SDC. The sample period is January 1970 – April 2001. ***, **,* indicate significance at the 0.01, 0.05, and 0.10 levels, respectively.

	IG Senior	IG Junior	High Yield	All
Number of obs.	1,370	29	46	1,445
Intercept	$7{,}22 \cdot 10^{-3}$	$10{,}77 \cdot 10^{-3}$	$2{,}52 \cdot 10^{-2}$	$7{,}76 \cdot 10^{-3}$
	(37.48)***	(6.79)***	(7.98)***	(34.86)***
Maturity	$8{,}48 \cdot 10^{-5}$	$-4{,}92 \cdot 10^{-5}$	$-7{,}62 \cdot 10^{-5}$	$8{,}15 \cdot 10^{-5}$
	(11.40)***	(–0.92)	(–0.54)	(9.48)***
R^2	0.087	0.031	0.0065	0.059

and Warga [1989]) and coupon bonds (Fons [1994]). On the other hand, our results complement the findings of Helwege and Turner [1999] on the slope of credit spread curves for high-yield coupon bonds.

One implication of our results is that a realistic credit risk model should be able to generate an upward-sloping credit spread curve for both zero-coupon and coupon bonds, regardless of the rating of the bond. However, a caveat about the results on high-yield bonds is worth mentioning here. Recall that our final sample of high-yield bonds includes only those rated B or higher. As mentioned in Helwege and Turner [1999], spread curves for sufficiently low credit quality might be hump shaped as predicted by some credit risk models. See Lando and Mortensen [2005] for evidence based on credit default swap data.

ENDNOTE

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