

Is Jump Risk in iTraxx Sector Indices Diversifiable?

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Research on credit risk has developed quickly over the last decade. Spurred by regulatory developments, different classes of models have been put forward to measure, manage, and price credit risk. Early credit risk models focus on predicting the likelihood of default, but over the last few years the time-series, or dynamic, behavior of credit risk has become increasingly important to academics, practitioners, and regulators.

The credit derivatives market, especially the credit default swap (CDS) market, has grown rapidly since the International Swap and Derivatives Association (ISDA) produced its first version of a standardized contract in 1998. Inferred from the website of the International Index Company (www.indexco.com), notional amounts of credit derivatives were US\$ 34 trillion in 2006 compared with US\$ 700 billion in 2001. These developments make research on credit risk more important and attractive.

While there are many variations of credit derivatives, CDS spreads are among the most intuitive and are commonly cited as a basic building block for more complex structures. A single-name CDS is a contract that protects the buyer, who is the holder of an underlying obligation, from the losses caused by the occurrence of a credit event to the obligation's issuer, referred to as the reference entity. The protection buyer periodically pays—typically a percentage of the notional amount—to the

protection seller. The percentage that gives the contract zero value at initiation is called the spread. When a credit event occurs, the protection buyer is also required to make an accrual payment, the part of the premium payment that has accrued since the last payment date.

Previously, most research on credit risk focused on the corporate bond market or the credit spread, generally defined as the yield of a corporate bond minus the yield of a benchmark risk-free interest rate of the same maturity. Compared with the credit spread, credit default swap spreads are a moderately pure proxy of credit risk, because CDS spreads isolate the credit risk from other possible risks, such as interest rate risk and foreign exchange risk. Furthermore, liquidity in the CDS market, promoted by the use of standardized contracts, makes a short position more easily attainable. Credit default swap spreads are more attractive than credit spreads in credit risk research for several reasons.

First, CDS spreads do not require the specification of a benchmark risk-free yield curve as do credit spreads. The credit spread comprises both a premium for bearing default risk and a premium for bearing interest rate risk. As Longstaff, Mithal, and Neis [2005] pointed out, different benchmark risk-free rates imply different magnitudes for the default component in credit spreads. This component varies from 51% to 95% depending on the risk-free curve chosen and the bond ratings.

Second, short-sale constraints in corporate bond markets mean that credit spreads are not a "pure" measure of credit risk either. Joost [2005] found that the liquidity premium accounts for 17% to 25% of the bond yield over Treasuries. Furthermore, Ericsson and Renault [2006] demonstrated that as default becomes more likely, the components of credit spreads (also called bond yield spreads) attributable to illiquidity increase.

Third, counterparty risk generally does not need to be taken into account in CDS spreads because if a party is unwilling to take on the credit risk of its counterparty, it decides either to cancel the trade or to alleviate the exposure. For example, it is possible to demand that collateral be provided or that the premium be paid upfront instead of periodically (Culp and Neves [1998]). So the information in CDS spreads can be used to directly measure the size of credit risk and to act as the price of credit risk.

With the development of the CDS market, CDS indices have been created in a fashion similar to the way stock indices are created, as a portfolio of individual names. iTraxx, an example of a CDS index, is an equally weighted portfolio of single-name CDS. The iTraxx index family consists of various indices of the most liquid CDS contracts in Europe and Asia (in the U.S., since April 2004, a similar family of indices is Dow Jones CDX). The iTraxx

indices typically trade 5- and 10-year maturities, and a new series is determined by a dealer liquidity poll every 6 months. The Europe and HiVol indices trade 3- and 7-year maturities. The indices are managed and administrated by the International Index Company, owned by a group of the largest global investment banks. According to the International Index Company website (www.indexco.com), index trades have increased rapidly in recent years and represent more than 40% of overall credit derivatives volume.

The benchmark iTraxx Europe index comprises 125 equally weighted European names which are selected by a dealer poll based on CDS volume traded over the previous six months. The European index is further split into several sector indices (nonfinancials, financials, and so on) as well as a corporate index composed of the largest non-financial names (from the 125 names), a crossover index composed of the 25 most liquid subinvestment-grade nonfinancial names, and a HiVol index consisting of the 30 names with the widest CDS spreads. Exhibit 1 gives an overview of these iTraxx index families.

Given the attraction of CDS spread data and the importance of dynamic credit risk modeling, we investigate a multivariate model for its dynamic behavior and possible unobservable common factor behind the iTraxx sector indices data; that is, we chose the iTraxx Non-Financials

EXHIBIT 1

iTraxx Europe Overview

Benchmark Indices	Sector Indices	Derivatives
iTraxx Europe 125 investment - grade entities	Non-Financials 100 entities	Trenched iTraxx Five standardized tranches of iTraxx Europe 0-3% 3-6% 6-9% 9-12% 12-22%
iTraxx Europe HiVol 30 highest spread entities from iTraxx Europe	Financial Sub Same 25 entities Financial Senior	iTraxx Options
iTraxx Europe Crossover 50 subinvestment-grade entities		iTraxx Futures iTraxx Europe exposure traded as a futures contract

Source: www.indexco.com

index and its subindices: iTraxx TMT, iTraxx Industrials, iTraxx Energy, iTraxx Autos, and iTraxx Consumer. All index quotes have been made available to us by the International Index Company. In this article, we only present results for the 5-year maturity iTraxx indices, but similar results can be obtained from the 10-year maturity indices.

In particular, this article focuses on the jump risk in our sample of iTraxx indices. Jump diffusion processes have applied to stock prices and commodity prices for many years. For example, Kim, Oh, and Brooks [1994] recorded nondiversifiable jump risk in stock returns. They presented a multivariate time-series model of the stochastic process that explicitly admits discrete common jumps for an index and its component stocks. Not much work has been done, however, in directly investigating CDS spreads using these models. We follow the Kim, Oh, and Brooks research framework to examine jump risks in the CDS market. We first identify whether jump risks are present individually in each of the component sector subindices that compose the iTraxx Non-Financials index as well as the Non-Financials index itself. If significant jump risk is identified in the iTraxx Non-Financials index, a portfolio of all CDS in the nonfinancial market, we may conclude that such jump risk is nondiversifiable.

The rest of the article is organized as follows. We start by presenting the relevant literature review. Then, we describe the data and the methodology. Finally, we give the empirical results and main conclusions.

LITERATURE REVIEW

Generally there are two basic approaches to modeling credit risks: structural models and reduced-form models. Structural models, pioneered by Black and Scholes [1973], and Merton [1974], have been extended by Black and Cox [1976], and Longstaff and Schwartz [1995], among others. In this approach, default occurrence is modeled when the value of the firm hits a given boundary. Reduced-form models, developed by Litterman and Iben [1991], Jarrow and Turnbull [1995], and Jarrow, Lando, and Turnbull [1997], assume that credit risk is determined by the occurrence of default and its recovered amount. Default is often represented by a random stopping time with stochastic or deterministic arrival intensity (hazard rate). In these models, the recovery rate is most often assumed to be constant.

Classical structural models are typically based on diffusion processes where default occurs only when the firm's

value reaches some exogenous pre-specified barrier that can be time-dependent. In contrast, credit markets, even in the short term, do not rule out the possibility that a firm can default instantaneously because of a sudden drop in its value. Such a phenomenon has been identified in empirical work as well. For example, Jones, Mason, and Rosenfeld [1984] found that the credit spreads on corporate bonds are too high to be matched by the model based on the diffusion process.

In the literature, considerable effort has been devoted to empirically modeling the underlying factors describing the dynamic behavior of asset prices. A majority of the articles in this field, however, address that issue for common stocks and commodities. For example, Kim, Oh, and Brooks [1994] studied the diversifiability of jumps in stock returns. They presented a multivariate time-series model of the stochastic process which includes a jump component in the underlying factor. They included both an index and its component stocks. They found that the Poisson-distributed jumps observed from both the index and its component stocks constitute nondiversifiable risk.

Our study of the CDS market follows the framework adopted in Kim, Oh, and Brooks [1994] and explores the existence of jump risk in the CDS sector index and its components.

DATA

The International Index Company provides both bid and ask for each iTraxx index series. The data used in this study consist of daily closing mid-quotes for six iTraxx indices: iTraxx Non-Financials, iTraxx TMT, iTraxx Industrials, iTraxx Energy, iTraxx Autos, and iTraxx Consumer. The iTraxx Non-Financials index encompasses the other five subindices and each subindex accounts for a nonfinancial sector. All of these indices are traded with a 5- or 10-year maturity and are denominated in euros. We focus on the 5-year maturity, which is the most popular in the CDS market. The time period covered is March 22, 2004, to October 3, 2006. The total number of mid-point CDS spread quotes in the panel is 3,900; that is, six iTraxx index series each with 650 time points. All these data have been made available by the International Index Company.

This section also provides some descriptive statistics of these indices which gives readers a rough idea of the dynamic behavior of a CDS index. Exhibit 2 shows these

iTraxx indices for the sample period and Exhibit 4 gives the descriptive statistics. In order to determine if these time series are stationary, we perform both the Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test on our data. For robustness consideration, we try different

model specifications and different lags. The results are quite similar and for simplicity, we present only the results for the unit root test without trend and with four lags.

The unit root test shows that all iTraxx indices are nonstationary at the 5% significance level and all daily

EXHIBIT 2

iTraxx Non-Financials 5Y and Its Subindices

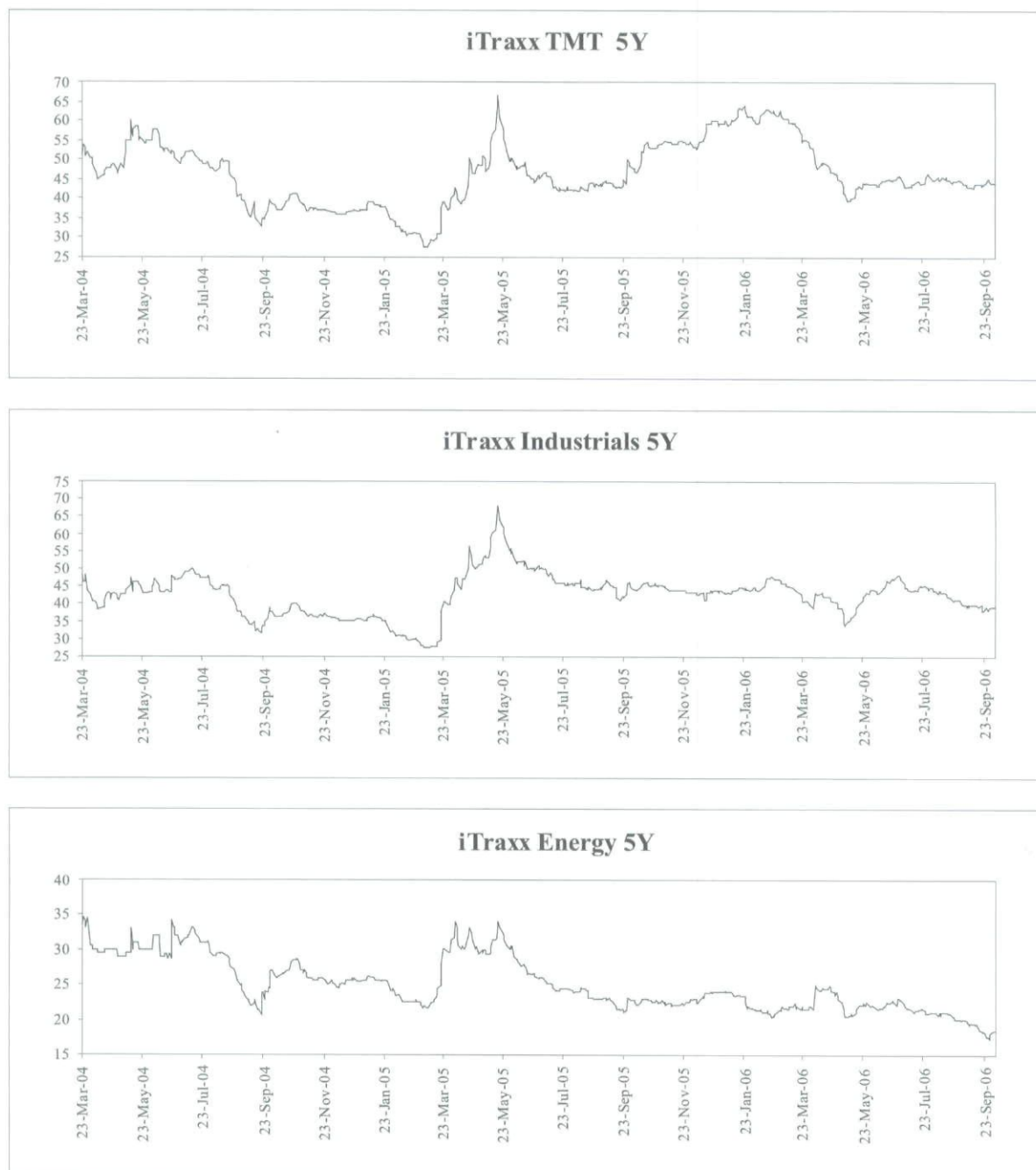
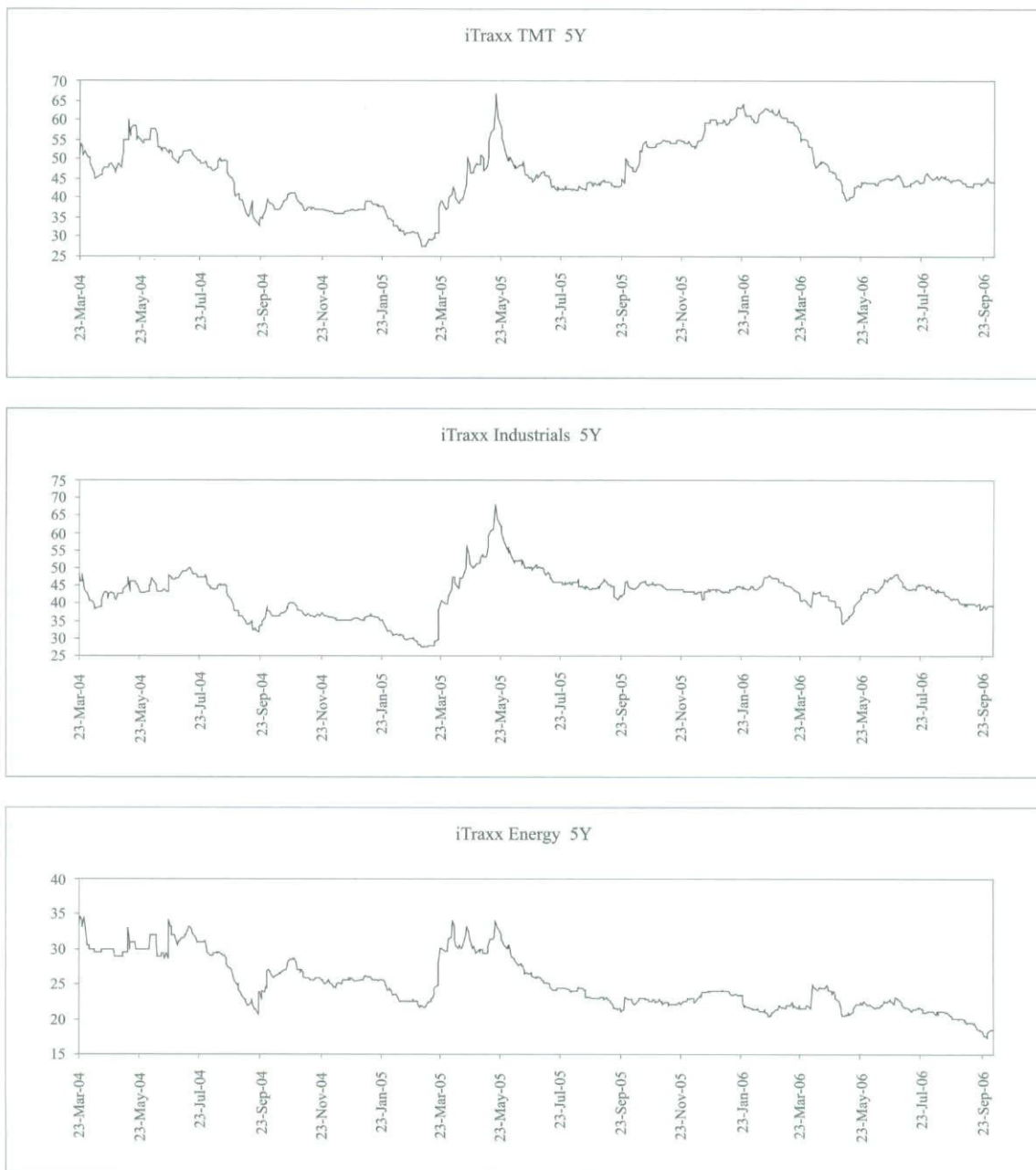


EXHIBIT 2 (Continued)



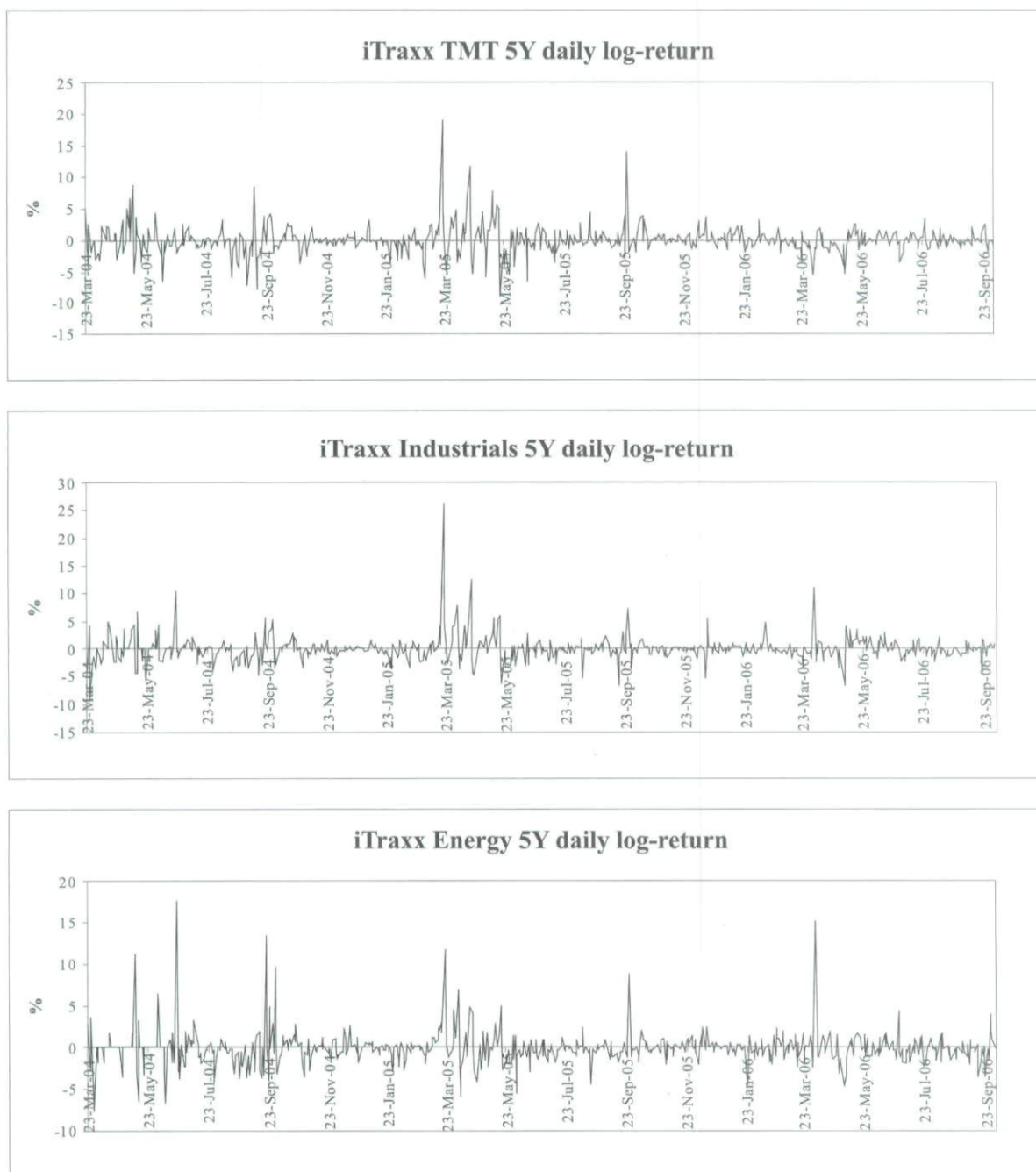
log-returns on iTraxx indices are stationary. Except for the energy sector, the size of the CDS spread (in basis points) varies somewhat more over the time period than across the sectors.

Byström [2006], using the similar dataset for the period from June 21, 2004, to April 18, 2005, found the distribution of the CDS index spread changes to be much more skewed and leptokurtic than the stock index return

distribution. The iTraxx index is also at least two to three times as volatile (with a standard deviation of 30% to 40% on an annual basis) as the corresponding stock portfolio (with a standard deviation of 10% to 15% on an annual basis). Such a finding may also provide a hint about jump risk in the CDS market.

EXHIBIT 3

Daily Log-Return on iTraxx Non-Financials 5Y and Its Subindices



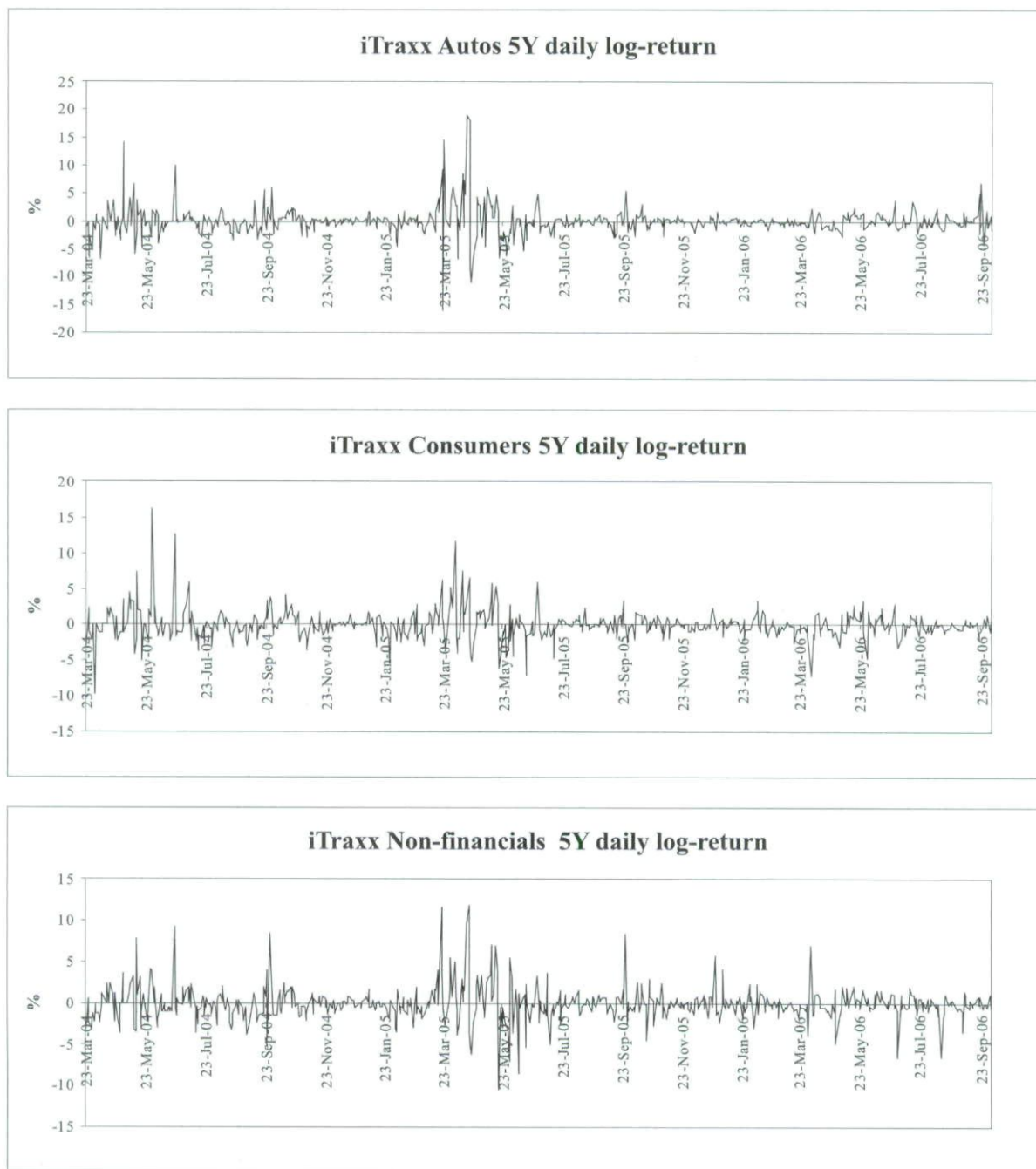
METHODOLOGY

Normality Test for CDS Index Distribution

A nonparametric investigation quantifies what the visual inspection suggests—that neither levels nor log-returns are

normally distributed. Exhibit 4 also lists excess kurtosis and skewness for the iTraxx index level and its daily log-return, respectively. The Jarque-Bera test rejects the null hypothesis of normality for all iTraxx indices in our sample (p -values do not exceed even 0.01%). All CDS indices and their log-returns exhibit significant excess kurtosis, indicating that a pure

EXHIBIT 3 (Continued)



diffusion probably did not generate the data. Also, positive skewness for the iTraxx levels together with the excess kurtosis values confirms that large jumps are mostly positive.

Model for Individual iTraxx Indices

We follow the Kim, Oh, and Brooks [1994] framework to model the iTraxx index, a proxy for credit risk

price which is denoted by S_t , and assume this price evolves as follows:

$$\frac{dS_t}{S_t} = \gamma dt + \sigma dW_t + (\tilde{Y} - 1)dq_t \quad (1)$$

EXHIBIT 4

Descriptive Statistics for iTraxx Index Level and Its Daily Log-Return

	Non-Financials	TMT	Industrials	Energy	Autos	Consumer
Panel A : Descriptive Statistics for iTraxx Index						
Min	31.650	27.400	27.335	17.330	32.200	29.990
Mean	41.734	46.468	42.674	25.048	45.846	46.760
Media	41.103	45.305	43.695	24.050	44.683	47.053
Max	66.035	66.679	68.210	34.665	87.175	75.180
Stdev	5.298	8.186	6.089	3.763	8.000	8.531
Skewness	0.859	0.128	0.166	0.572	1.811	0.046
Kurtosis	4.892	2.478	4.548	2.340	7.944	2.989
ADF Test	0.189	0.236	0.098	0.113	0.164	0.571
PP test	0.163	0.315	0.134	0.172	0.097	0.602
Panel B: Descriptive Statistics for Daily Log-Return on iTraxx Index						
Min*10 ²	-10.522	-9.135	-9.221	-7.522	-15.810	-9.737
Mean*10 ²	-0.061	-0.025	-0.031	-0.093	-0.041	-0.078
Median*10 ²	-0.103	-0.089	-0.069	-0.042	-0.090	-0.100
Max*10 ²	11.925	18.930	26.092	17.505	18.979	16.149
Stdev*10 ²	1.989	2.162	2.169	2.031	2.404	1.930
Skewness	0.967	1.648	3.320	2.870	1.680	1.632
Kurtosis	11.531	17.309	38.882	23.820	22.067	17.246
ADF Test	0.000	0.000	0.000	0.000	0.000	0.000
PP Test	0.000	0.000	0.000	0.000	0.000	0.000

MacKinnon approximate *p*-value for $Z(t)$ is listed in row ADF test and PP test.

where dW_t is a standard Brownian motion, dq_t represents a jump process, and dW_t and dq_t are assumed to be orthogonal (Kim, Oh, and Brooks [1994]):

$$dq \text{ is the jump term} = \begin{cases} 0 & \text{if no jump occurs} \\ 1 & \text{if a jump occurs} \end{cases} \quad (2)$$

The jump process is assumed to be a Poisson process with intensity λ , and $\tilde{Y} - 1$ is the percentage change in the asset price if the Poisson event occurs. $\tilde{Y}$ is assumed to have a log normal distribution with mean θ and variance v^2 .

The sample path for S_t will be continuous most of the time with finite occasional jumps of different sizes and signs occurring at discrete points in time. If a jump occurs within a small time interval $(t, t + dt)$, the stochastic process is

$$\frac{dS_t}{S_t} = \gamma dt + \sigma dW_t + \tilde{Y} - 1 \quad (3)$$

If a jump does not occur within that time interval, the stochastic process is

$$\frac{dS_t}{S_t} = \gamma dt + \sigma dW_t \quad (4)$$

By Ito's lemma, the jump-diffusion process in Equation (1) can be expressed as

$$\frac{S_{t+dt}}{S_t} = e^{(\gamma - \frac{\sigma^2}{2})dt + \sigma dW_t} \tilde{Y}(J) = e^{\mu dt + \sigma dW_t} \cdot \tilde{Y}(J) \quad (5)$$

where J is the number of occurrences of Poisson-distributed events during the interval of dt , which is characterized by single parameter λdt ; $\tilde{Y}(J) = 1$ if $J = 0$, and $\tilde{Y}(J) = \prod_{j=1}^J \tilde{Y}_j$, if $J \geq 1$. Its discrete time version is

$$\frac{S_{t+\Delta t}}{S_t} = e^{\mu\Delta t + \sigma Z_t} \tilde{Y}(J) \quad (6)$$

We obtain Equation (7) by taking the natural logarithm of both sides of Equation (6)

$$Ln\left(\frac{S_{t+\Delta t}}{S_t}\right) = \mu\Delta t + \sigma Z_t + \ln(\tilde{Y}(J)) \quad (7)$$

where Z_t follows a normal distribution with mean 0 and variance Δt

$$Ln\left(\frac{S_{t+\Delta t}}{S_t}\right) \sim \sum_{j=0}^J \frac{e^{-\lambda\Delta t} (\lambda\Delta t)^j}{j!} N(\mu\Delta t + j\theta, \sigma^2\Delta t + j\nu^2)$$

In this article, $Ln\left(\frac{S_{t+\Delta t}}{S_t}\right)$ is treated as the return on the iTraxx index.

The log-likelihood function is

$$LnL(R_{iTraxx,t} : \mu, \sigma^2, \lambda, \theta, \nu^2) = \sum_{t=1}^T Ln\left(\sum_{j=0}^{\infty} \frac{e^{-\lambda\Delta t} (\lambda\Delta t)^j}{j!} \times (2\pi(\sigma^2\Delta t + j\nu^2))^{-1/2} \exp\left[-\frac{(R_{iTraxx,t} - (\mu\Delta t + j\theta))^2}{2(\sigma^2\Delta t + j\nu^2)}\right]\right) \quad (8)$$

Basically, J can be any positive number, but for simplicity and ease of computation, we set J equal to 1. This simplification is possible because the sample path for the iTraxx index will be continuous most of the time with finite occasional jumps of different sizes and signs. Based on the daily data we use, there is a small chance that the CDS rate may jump twice within a day. The estimation results are listed in Exhibit 5.

Exhibit 5 shows that both the sector iTraxx indices and the iTraxx Non-Financials 5Y (which acts as an index for the sector iTraxx indices) experienced jumps during the sample period. The null hypothesis test that there is no jump ($\lambda = 0$) in the iTraxx sector index and iTraxx Non-Financials based on the underlying stochastic process is rejected at the 1% level. Even though the iTraxx Non-Financials 5Y acts as a portfolio of all CDS in the nonfinancial sector and should benefit from diversification, the results in Exhibit 5 reject the hypothesis $\lambda = 0$, even

EXHIBIT 5

ML Estimates of the Jump-Diffusion Process

	μ	σ^2	λ	θ	ν^2	t
1. TMT 5Y	-0.115 (0.057)	1.124 (0.045)	0.169 (0.030)	0.534 (0.527)	3.485 (0.279)	5.62
2. Industrials 5Y	-0.148 (0.050)	0.978 (0.037)	0.176 (0.027)	0.669 (0.560)	3.696 (0.187)	6.65
3. Energy 5Y	-0.199 (0.049)	1.038 (0.036)	0.108 (0.022)	0.979 (0.876)	4.286 (0.542)	4.84
4. Autos 5Y	-0.163 (0.049)	1.026 (0.036)	0.145 (0.022)	0.836 (0.639)	4.709 (0.362)	6.50
5. Consumer 5Y	-0.162 (0.055)	1.165 (0.042)	0.106 (0.025)	0.792 (0.680)	3.638 (0.447)	4.33
6. Non-Financials 5Y	-0.162 (0.051)	1.041 (0.041)	0.154 (0.028)	0.651 (0.483)	3.352 (0.392)	5.46

Asymptotic standard error is reported below each estimated parameter and asymptotic t-statistic of λ is reported in the last column. We adopt the BHHH estimator as variance estimators for an MLE. More detailed information may be found in Greene [2002].

at the 1% significance level. Since diversification does not remove the jump risk, then they may be driven by factor(s) other than firm- or industry-specific abnormal information. In other words, the results in Exhibit 5 show that the jumps observed in the iTraxx index may be systematic or nondiversifiable risk.

Exhibit 5 also shows that the estimated lognormally distributed mean jump sizes θ are all positive, which may provide some evidence that most jumps happening in the CDS market are positive, although they are mostly insignificant. Comparing the variance of jump sizes and the instantaneous volatility conditional on no jumps confirms that the jump risk matters and the average variance of the jump size is around 3.5 times the variance of instantaneous variance. Because the mean jump sizes are almost insignificant, for simplicity in later estimation, we can impose the restriction that θ equals zero for computational ease.

Multivariate Analysis of jumps in the iTraxx Index

Multivariate Model with One Common Factor. We adopt a vector autoregression framework with common trends (Kim, Oh, and Brooks [1994]) to model the dynamic behavior of CDS spreads. The common factor is assumed to behave as a random walk with a deterministic trend.

Let ΔX_t denote the daily log-return on each of the sector iTraxx indices: iTraxx TMT 5Y, iTraxx Industrials 5Y, iTraxx Energy 5Y, iTraxx Autos 5Y, and iTraxx Consumer 5Y. Let G_t be an unobserved single index which corresponds to the log of the iTraxx Non-Financials 5Y, so that ΔG_t will be interpreted as the log-return of the estimated common driver of the CDS indices. The model can be written mathematically as

$$\Delta X_t = \mu_x + \beta \Delta G_t + v_t, v_t \sim N(0, \Sigma) \quad (9)$$

where, for simplicity, Σ is restricted to be a diagonal matrix.

$$\Sigma = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5^2 \end{bmatrix} \quad (10)$$

$$\Delta G_t = \mu_G + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_G^2) \quad (11)$$

The model is in classical state space form and may be estimated using a Kalman filter framework. To clearly identify the measurement and to state the equation of the state space form we restate the model in matrix difference notation. The measurement equation is given by

$$\begin{bmatrix} \Delta X_{1t} \\ \Delta X_{2t} \\ \Delta X_{3t} \\ \Delta X_{4t} \\ \Delta X_{5t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} + \begin{bmatrix} \beta_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & 1 & 0 & 0 & 0 & 0 \\ \beta_3 & 0 & 0 & 1 & 0 & 0 & 0 \\ \beta_4 & 0 & 0 & 0 & 1 & 0 & 0 \\ \beta_5 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} \Delta G_t \\ v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \\ v_{5,t} \\ G_{t-1} \end{bmatrix} \quad (12)$$

$$\text{or} \quad \Delta X_t = \mu_x + Z_t \alpha_t \quad (13)$$

The state vector α_t has the following dynamic form:

$$\begin{bmatrix} \Delta G_t \\ v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \\ v_{5,t} \\ G_{t-1} \end{bmatrix} = \begin{bmatrix} \mu_G \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \Delta G_{t-1} \\ v_{1,t-1} \\ v_{2,t-1} \\ v_{3,t-1} \\ v_{4,t-1} \\ v_{5,t-1} \\ G_{t-2} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \zeta_t \\ v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \\ v_{5,t} \end{bmatrix} \quad (14)$$

$$\text{or} \quad \alpha_t = \mu_\alpha + T_t \alpha_{t-1} + R_t \eta_t \quad (15)$$

where $E(\eta_t \eta_t') = Q_t = \text{diag}\{\sigma_G^2, \sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2\}$ (16)

The parameter set is given by the vector

$$\Theta \equiv \{\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_G, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2\} \quad (17)$$

The variance of the unobservable common component is set equal to 1, since the single factor in the model is assumed to be the market portfolio.

The recursive algorithm of the Kalman filter (for simplicity all initial values of the filter are set equal to 1) is outlined as follows:

$$\alpha_{t|t-1} = \mu_G + T_t \alpha_{t-1|t-1} \quad (18)$$

$$P_{t|t-1} = T_t P_{t-1|t-1} T_t' + R_t Q_t R_t' \quad (19)$$

$$F_t = Z P_{t|t-1} Z' \quad (20)$$

$$\alpha_{t|t} = \alpha_{t|t-1} + P_{t|t-1} Z_t' F_t^{-1} (X_t - \mu_X - Z_t \alpha_{t|t-1}) \quad (21)$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} Z_t Z_t' P_{t|t-1} F_t^{-1} \quad (22)$$

The log-likelihood function is given by

$$\ln L = -\frac{nT}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T \ln(\det(F_t)) - \frac{1}{2} \sum_{t=1}^T \xi_t' F_t^{-1} \xi_t \quad (23)$$

Multivariate Analysis with One Common Factor and Jump Component. If the jump risk cannot be diversified—that is, the unobservable common factor may include a jump component in its sample path—its process can be modified to include Poisson jumps as follows:

$$\Delta G_t = \mu_G + \varepsilon_t + \bar{y}, \varepsilon_t \sim N(0, \sigma_G^2) \quad (24)$$

where $\bar{y} \sim N(J \cdot \theta, J \cdot \sigma_Y^2)$ for $J = 0, 1$

The probability density function (PDF) of innovation can be expressed as

$$L = \sum_{j=0}^{\infty} \frac{\exp\{-\lambda\} \lambda^j}{j!} (2\pi(\sigma_G^2 + j \cdot \sigma_Y^2))^{-1/2} \times \exp\left[-\frac{(\Delta G_t - \mu_G - j \cdot \theta)^2}{2(\sigma_G^2 + j \cdot \sigma_Y^2)}\right] \quad (25)$$

In terms of the Kalman filter process, this modification amounts to replacing the σ_G^2 term in the covariance matrix, Q_t , in Equation (16) by $\sigma_G^2 + \sigma_Y^2 j$, $j = 0, 1$.

Hence, the parameter set is given by the vector

$$\Theta \equiv \{\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_G, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \lambda, \sigma_Y^2\} \quad (26)$$

Eighteen parameters need to be estimated which is not easy for optimization. A different initial value should not theoretically have an impact on the final optimal result for parameter estimation, but in a practical sense it does. We use a numerical optimization algorithm in Matlab (ML) 7.4 to estimate the parameters. In order to get stable results, we use 20 sets of initial values for estimation work and choose the result with the largest value of log-likelihood function. More detailed results are available upon request. Although the estimators are slightly different based on their different upper and lower bounds (we keep the bound at a small value for a lower computational burden), the key conclusion of this article—that there is nondiversifiable positive jump in the CDS spread market—does not change.

EMPIRICAL RESULTS

Exhibit 6 lists the results for the ML estimation of the one-factor model with jump component. It shows that the jump risk embedded in the iTraxx sector indices is not diversifiable. The variance of jump sizes is much larger than the instantaneous variance conditional on no jumps, namely 36 versus 2. This result confirms again that the jump risk embedded in CDS spreads is significant and undiversified. The estimated value of λ , which is the mean rate of jumps per unit of time is 0.1412 per day, and the asymptotic

EXHIBIT 6

ML Estimates of the One-Factor Jump-Diffusion Model

	μ_X	β	σ^2
1. TMT 5Y	0.168 (137.630)	0.647 (0.050)	2.126 (0.085)
2. Industrials 5Y	0.192 (158.890)	0.747 (0.056)	1.311 (0.063)
3. Energy 5Y	0.075 (119.31)	0.561 (0.045)	2.211 (0.077)
4. Autos 5Y	0.160 (143.335)	0.6742 (0.054)	3.011 (0.060)
5. Consumer 5Y	0.0741 (108.536)	0.511 (0.041)	2.138 (0.047)
Common Factor			
λ	0.1412 (0.028)		
μ_G	-0.439 (212.58)		
σ_Y^2	35.995 (7.181)		

$$\Delta X_t = \mu_X + \beta \Delta G_t + v_t, v_t \sim N(0, \Sigma)$$

$$\Delta G_t = \mu_G + \varepsilon_t + \bar{y}, \varepsilon_t \sim N(0, \sigma_G^2), \bar{y} \sim N(J \cdot \theta, J \cdot \sigma_Y^2) \text{ for } J = 0, 1$$

X_t and G_t , respectively, denote the daily log price of the iTraxx index and an unobserved single common factor which is corresponding to the log of iTraxx Non-Financials 5Y. Asymptotic standard error for each parameter is also reported below each estimated parameter.

standard error is 0.0283. Exhibit 7 illustrates how the common factor of the CDS indices evolves over the sample period and also shows different suitable proxies of the common factor behind the entire sector index. Panel A of Exhibit 7 gives the average log-sector index, which is calculated as

$$EWAIndex_t = \frac{1}{5} (\log(TMT_t) + \log(Industrial_t) + \log(Energy_t) + \log(Autos_t) + \log(consumers_t)) \quad (27)$$

Here TMT_t , $Industrial_t$, $Energy_t$, $Autos_t$, and $Consumers_t$ denote iTraxx TMT 5Y, iTraxx Industrials 5Y, iTraxx Energy 5Y, iTraxx Autos 5Y, and iTraxx Consumer 5Y, respectively.

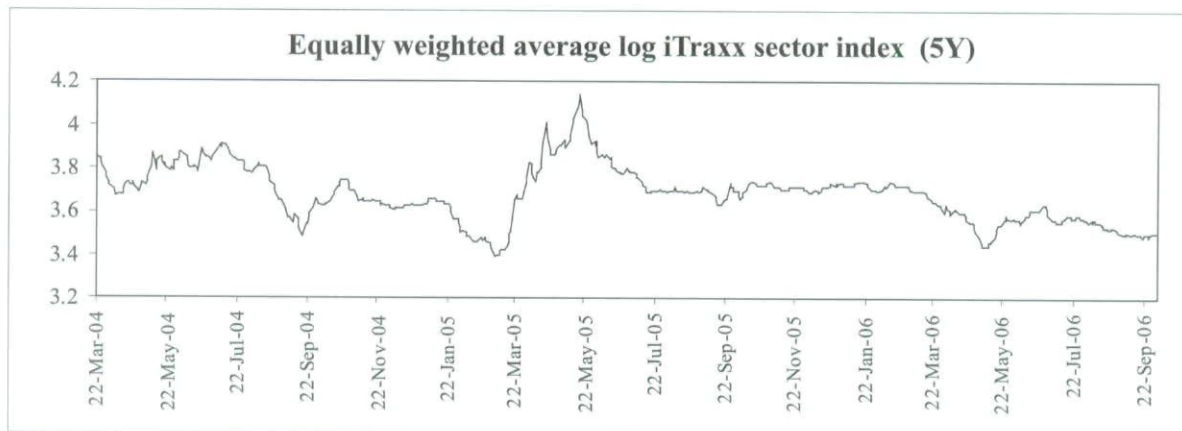
Exhibit 7 Panel B shows the one-period ahead predicted common factor by the one-factor model. The model predicts the pattern of the common factor which underlies the entire sector indices quite well. Exhibit 7 Panel C shows the iTraxx Non-Financials index along the same time axes. Compared to Panel C, time-series plots in Panels A and B are smoother.

The entire CDS market experienced a significant rise beginning at the end of April 2005 and peaking around May 20, 2005, after which the CDS market returned to its average level. The market's movement was strongly affected by Standard & Poor's downgrade to junk status of the debt of two of the world's biggest automakers, Ford and General Motors. At the downgrade, General Motors had \$290 billion and Ford had \$450 billion in outstanding debt¹, a massive amount of debt. The iTraxx Non-Financials 5Y rose from 49.63 bps on May, 5, 2005, to 66.04 bps on May, 28, 2005, while the iTraxx Autos

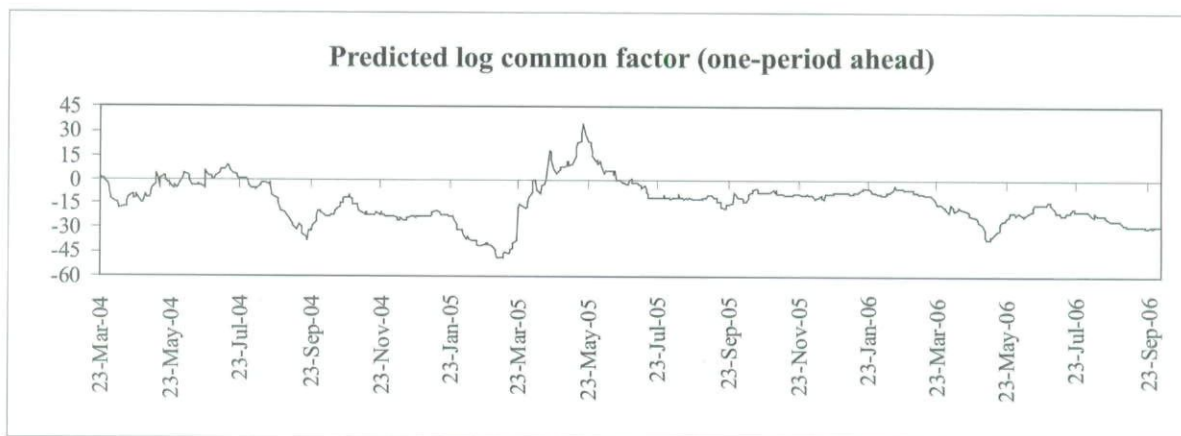
EXHIBIT 7

Different Proxies Log-Sector Index (equally weighted)

Panel A: Average Log-Sector Index (equally weighted)



Panel B: One-Period Ahead Predicted Common Factor



Panel C

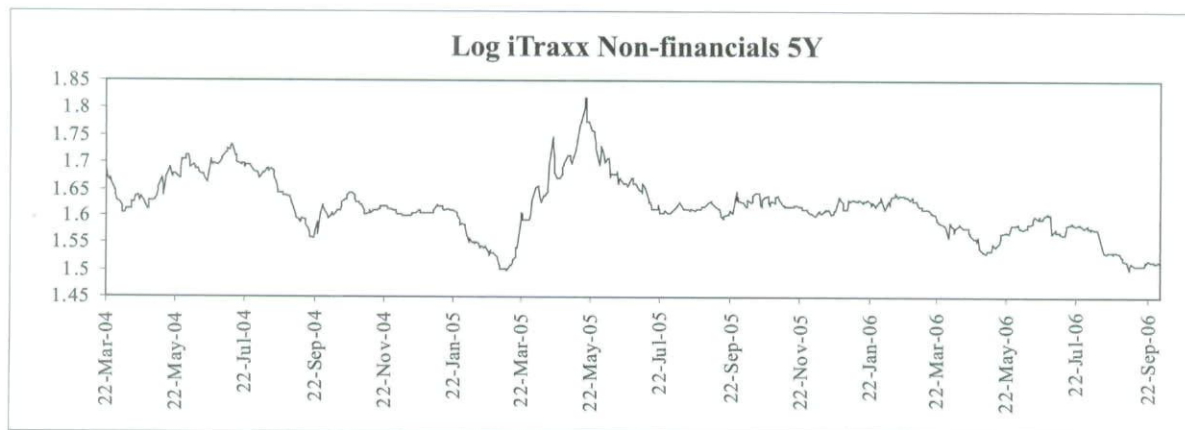


EXHIBIT 8

Correlation between Equally Weighted Average Log iTraxx Sector Index, One-Period-Ahead Predicted Log Common Factor, and Log iTraxx Non-Financials

	Equally weighted average	Predicted log common factor	Log iTraxx Non-financials
Equally weighted average	1		
Predicted log common factor	0.962	1	
Log iTraxx Non-financials	0.970	0.913	1

5Y was up 16.45 bps from 65.655 bps to 82.105 bps over the same period.

Exhibit 8 gives the correlation coefficient for the three series: the equally weighted average log iTraxx sector index, the one-period-ahead predicted log common factor, and the log iTraxx Non-Financials. Not surprisingly, these three series are highly correlated with each other.

Based on the estimated results, we conclude that the jump risks are present in iTraxx sector indices and they are not diversifiable. For robustness consideration, we also use a two-factor model (detailed results are available upon request) to capture the characteristic of the dynamic behavior of the iTraxx sector index. We find a similar result for the jump risk.

CONCLUSION

The credit derivatives market has developed rapidly over the last several years and of the credit default swap (CDS) is the most popular product in the market. This article examines this quickly developing market and investigates the jump-diffusion process for iTraxx (a popular index for the CDS market) in a multivariate structural time-series setting for the stochastic process. Our empirical study detects significant Poisson-distributed jumps both in the iTraxx Non-Financials index and its subindices.

Our finding that nondiversifiable jump risk exists in the CDS market is quite strong and is robust based on the statistical analysis used in the article. Of course, further analysis with an alternative methodology and different dataset should be conducted, but the issue of nondiversifiable jump risk certainly warrants further empirical investigation.

ENDNOTES

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¹According to the website of www.financialpolicy.org.

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