Synthetic CDO Equity: *Short or Long Correlation Risk?*

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The purpose of this article is to analyze the widely held belief that the equity tranche in a synthetic collateralized default obligation (CDO) is long (default) correlation risk.[1] That is, the higher the correlation, the lower is the spread to the CDO equity tranche, all else constant. Or alternatively stated, the higher the default correlation, the more valuable is the CDO equity tranche.

This belief is important because it relates to the sign of the equity tranche's hedge. If the sign is wrong, then hedging becomes doubling up instead. Unlike the simpler vanilla interest rate derivatives (e.g., swaps, caps, floors) or credit default swaps (CDS), valuing and risk-managing (hedging) CDOs requires a sophisticated model. This is true for many reasons. First, both the collateral pool underlying the CDO and the CDO's waterfall structure are very complex, sometimes involving various triggers. This complexity makes risk management—hedging the risks of a CDO—too difficult to understand without a model. In some sense, this is also true for the simpler CDS. But, there is a difference. Although one needs a CDS model to determine the exact hedge ratio (e.g., 0.9533), intuition can at least get the sign of the hedge ratio correct. With CDOs, in contrast, this may not be true with respect to the equity tranche. Second, in illiquid markets (such as those experienced since the summer of 2007), valuation can only be done by marking-to-model. In a liquidity crisis, without a good model, financial institutions cannot even determine the asset value of their portfolios. Third, a CDO model enables a financial institution to determine fair value, a useful statistic that is the basis for the construction of profitable quantitative trading strategies.

To study this issue, we first provide two robust examples that provide the relevant intuition. We purposely choose simple examples so that the logic and mathematics are transparent. For both examples, depending upon the model's parameters and the equity's detachment point, equity can be either long or short correlation risk. Since both examples are plausible, this presentation demonstrates that the impact of correlation on CDO equity values is indeterminate and model specific.

After illustrating the intuition in the context of these examples, we next demonstrate the same comparative statics using a realistic, but hypothetical synthetic CDO. Two models are studied: 1) the standard static copula-based approach (see Bluhm and Overbeck [2007] Chapter 2), and 2) a multiperiod and dynamic reduced form model. This section shows that the standard copula-based model implies that equity is long correlation risk, while the reduced form model implies the reverse. Given the well-known shortcomings of the copula-based model (*WSJ* [2005]), this article adds another property to the list.

An outline of this article is as follows. The next section presents a simple synthetic CDO to fix the intuition. The section following presents the realistic synthetic CDO and the last section concludes.

A Simple Synthetic CDO

Let us consider the following simple synthetic CDO. We consider a static single period model (times 0 and 1). The relevant information is contained in the following exhibit:

Assets	*Liabilities*
CDS_1 loss δ_1	Senior tranches
CDS_2 loss δ_2	Equity tranche; loss ξ, detachment point α

- The CDO's asset pool consists of two (standard) credit default swaps (CDS_i) on corporate credits, indexed $i = 1, 2$, both with fair spreads and zero values at time 0.
- Let $\delta_i > 0$ be the losses at time 1 due to a credit event for CDS_i for $i = 1, 2$, measured as a *percentage of the total notional* of the CDO.
- There is a set of senior tranches and an equity tranche. The number of senior tranches is irrelevant to the subsequent discussion. At time 0 the senior and equity tranche spreads are set fairly such that they both have zero value.
- Let the losses to the equity tranche at time 1 be denoted $\xi > 0$.
- We assume that the equity tranche absorbs the first α percentage of losses on the CDO's notional value. That is, α is both the detachment point to equity and the attachment point to the senior tranches.

The percentage losses to the CDO's notional at time 1 are therefore

$$\delta = \delta_1 + \delta_2 \geq 0$$

To keep the logic and analysis simple, we will study the probability that the equity tranche experiences its maximum loss at time 1, that is,

$$\Pr(\xi = \alpha) = \Pr(\delta \geq \alpha)$$

Alternatively stated, this is the probability that the equity tranche is "wiped out." The higher this probability, all else equal, the higher will be the market clearing spread to the equity tranche at time 0. We note, however, that in actuality, the equity spread is determined by the entire distribution function integrated across all possible losses for the equity tranche, and not just the wipe-out probability. Therefore, these simple examples are really most useful for developing one's intuition about the dependence of model structure on equity correlation risk.

Discrete Loss Distributions

The first example assumes a discrete distribution for the losses of both CDS.

Let $p_i = \Pr(default\_i)$ be the probability that CDS_i for $i = 1, 2$, has a credit event by time 1.

Without loss of generality, let us assume that $p_1 \leq p_2$.

As is standard in many CDO models, assume that the loss rates are constant (non-random).

Long correlation risk. This example provides the intuition underlying the common market belief that equity is long correlation risk. For this example, we show that as the correlation increases, the probability of equity experiencing its maximum loss decreases.

Let us assume that $\min(\delta_1, \delta_2) \geq \alpha$ (i.e., a credit event in either CDS wipes out the equity tranche).

Then,

$$\begin{aligned}\Pr(\xi = \alpha) &= \Pr(\delta \geq \alpha)\\ &= \Pr(default1 \cup default2)\\ &= \Pr(default1) + \Pr(default2) - \Pr(default\,1 \cap default2)\end{aligned}$$

We next evaluate these probabilities. To understand the impact of correlation, we examine the two extreme situations: i) independent defaults, or zero correlation, and ii) perfectly dependent defaults, or a correlation of one. For independent defaults, we have $\Pr(default\,1 \cap default2) = \Pr(default1)\Pr(default2) = p_1p_2$, and for perfectly correlated defaults, we have $\Pr(default\,1 \cap default2) = \Pr(default1) = p_1$.

Substitution and simplification yields the following exhibit for the equity tranche's wipe-out probability:

$\Pr(\xi = \alpha)$	Correlation
$p_1 + p_2(1 - p_1)$	0
p_1	1

The probability of equity having its maximum loss in the case of zero correlation, $p_1 + p_2(1 - p_1) > p_1$, the probability of equity having its maximum loss in the case of perfect correlation.

Hence, for this example, the equity tranche is long correlation risk. The intuition underlying this example is that increased correlation decreases the equity spread, because it decreases the probability that the loss will be incurred. The magnitude of the losses are irrelevant because a credit event to either CDS causes the entire equity notional to be lost.

Short correlation risk. This example provides the intuition for the contrary situation where equity is short correlation risk.[2] For this example, we show that as the correlation increases, the probability of equity experiencing its maximum loss increases.

Let us assume that $\delta_1 + \delta_2 \geq \alpha > \max(\delta_1, \delta_2)$ (i.e., it takes both credit events to wipe out the equity tranche).

Then,

$$\Pr(\xi = \alpha) = \Pr(\delta \geq \alpha)$$
$$= \Pr(default1 \cap default2)$$

For independent defaults, we have $\Pr(default1 \cap default\,2) = p_1p_2$, and for perfectly correlated defaults, we have $\Pr(default1 \cap default2) = p_1$.

We obtain the following exhibit for the equity tranche's wipe-out probability:

$\Pr(\xi = \alpha)$	Correlation
p_1p_2	0
p_1	1

The probability of equity having its maximum loss in the case of zero correlation, $p_1p_2 < p_1$, the probability of equity having its maximum loss in the case of perfect correlation.

Hence, for this example, the equity tranche is short correlation risk. The intuition underlying this example is that increased correlation increases the equity spread because it increases the probability that the loss will be incurred.

Continuous Loss Distributions

The second example assumes a continuous distribution for the losses of both assets. To keep the mathematics simple, we use normally distributed losses to the CDS. Although this implies losses can be negative, we will ignore this slight difficulty (we assume that the probability of a negative loss is so small that it is inconsequential). It will become clear that the same logic works for alternative loss distributions; as long as the alternative distributions have the property that as the loss correlation increases, the tails of the joint loss distribution increase as well.

Formally, we let the CDS losses δ_i be normally distributed with mean μ_i, variance σ_i^2 and correlation ρ.

Given this assumption, total losses $\delta = \delta_1 + \delta_2$ are also normally distributed with mean $\mu_1 + \mu_2$ and variance $\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2$.

Then,

$$\Pr(\xi = \alpha) = \Pr(\delta \geq \alpha)$$

$$= \Pr\left(\frac{\delta - (\mu_1 + \mu_2)}{\sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2}} \geq \frac{\alpha - (\mu_1 + \mu_2)}{\sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2}}\right)$$

$$= N\left(Z \geq \frac{\alpha - (\mu_1 + \mu_2)}{\sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2}}\right)$$

where $Z = \frac{\delta - (\mu_1 + \mu_2)}{\sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2}}$ and $N(\cdot)$ is the standard normal distribution function.

Long correlation risk. This example provides the intuition for the common belief that equity is long correlation risk. For this example, we show that as the correlation increases, the wipe-out probability of equity decreases.

Assume that $\alpha \leq (\mu_1 + \mu_2)$. Here, the loss level that wipes out equity is less than or equal to the expected loss. Then, as the CDS loss correlation ρ increases, the denominator increases. The numerator, being negative, implies that the probability of equity losing its entire notional value, $\Pr(\xi = \alpha)$, decreases.

Hence, for this example, the equity tranche is long correlation risk (i.e., increased correlation decreases the equity spread, because it decreases the probability that the loss will be incurred).

Short correlation risk. This example provides the intuition for the contrasting belief that equity is short correlation risk. For this example, we show that as the correlation increases, the wipe-out probability of equity increases as well.

Assume that $\alpha > (\mu_1 + \alpha_2)$. Here, the loss level that eliminates all equity value exceeds the expected loss. Then, as the CDS loss correlation ρ increases, the denominator increases, and the probability of equity losing its entire notional value, $\Pr(\xi = \alpha)$, increases as well.

Hence, for this example, the equity tranche is short correlation risk (i.e., increased correlation increases the equity spread, because it increases the probability that the loss will be incurred).

Discussion

These examples illustrate that for a simple synthetic CDO, whether equity is long or short correlation depends on the model assumptions regarding the loss distribution and the location of equity's detachment point. Although these examples clarify the intuition, extrapolating from this to realistic CDOs is problematic for three reasons. These examples

1. Have only two CDS in the underlying asset pool,
2. Represent static models, and
3. Demonstrate that the change in the equity tranche's value is equated to the change in the wipe-out probability of the equity tranche, rather than to the value itself.

The next section corrects these omissions by considering a realistic, but hypothetical, synthetic CDO that is calibrated to market data.

A Realistic Synthetic CDO

This section investigates the change in an equity tranche's value when default correlations are increased. We consider two models: 1) the standard copula-based CDO model (see Bluhm and Overbeck [2005], Chapter 2), and 2) a reduced form model. The comparison is done via a hypothetical CDO calibrated to market data.

To calibrate the CDO model to market prices, we obtained the term structure of default probabilities from a hazard rate estimation procedure (the Jarrow-Chava [2004] model[3]) derived from a historical data base with 1.5 million monthly observations from January 1990.[4] The one-month, per year default probabilities from April 27, 2007, were used.

We arbitrarily use a 40% recovery rate in the event of default. This is the standard rate used to price traded CDS.

As a basis for the comparison, we construct a hypothetical synthetic CDO. We assume that the collateral pool consists of CDS on 500 reference names with a notional principal of $1 million each. To obtain the reference names, we selected every company with a BBB+, BBB, or BBB– credit rating as reported by the Standard & Poor's Compustat service on April 7, 2007. This gave us a total of 491 names. For the remaining 9 names, we randomly selected 9 companies rated between BB+ and BB–.

The CDO we construct has a maturity of five years.

Rather than using the typical market tranching conventions for synthetic CDOs, we create tranches in one-percent increments. This is without loss of generality. From this finer structure one can reassemble the tranches to obtain the more typical structures used in practice. In particular, we define the CDO tranches as follows:

- Tranche 1: This tranche takes the first loss up to 1% of notional principal, or $5 million.
- Tranche 2: This tranche takes the second 1% of losses, from 1% of notional principal ($5 million) to 2% of notional principal ($10 million).
- Tranche 3: This tranche takes the third 1% of losses, from 2% of notional principal ($10 million) to 3% of notional principal ($15 million).
- Tranche 4: This tranche takes the fourth 1% of losses, from 3% of notional principal ($15 million) to 4% of notional principal ($20 million).
- Tranche 5: This tranche takes the fifth 1% of losses, from 4% of notional principal ($20 million) to 5% of notional principal ($25 million).
- Tranche 6: This tranche takes the sixth 1% of losses, from 5% of notional principal ($25 million) to 6% of notional principal ($30 million).
- Tranche 7: This tranche takes all of the remaining losses, from 6% of notional principal ($30 million) to a maximum of 100% of notional principal ($500 million).

Of course, Tranche 1 corresponds to the equity tranche and Tranche 7 the super senior tranche.

To provide a slightly richer environment in which to study the impact of the various inputs on the copula CDO valuation, instead of using a static setting we do the analysis on a multi-period basis. That is, we allow our model to experience defaults every month over the CDO's life. By selecting a monthly periodicity, defaults can occur at any one of 60 points in time. This will generate a more realistic cash flow pattern for the CDO.

We price the CDOs using Monte Carlo simulation. The simulation is done using the risk-neutral probabilities,[5] so that the discounted expected (average) value of the coupon payments less the realized losses yields the arbitrage-free price. The simulation consists of 10,000 samples for the first set of analyses.

Since our objective is to analyze the relative differences in CDO tranche valuations under different correlations, the level of coupons used in our simulation is unimportant. Without loss of generality, therefore, we selected as the base case: no default correlation with the annualized one-month default probabilities.

For this base case, we numerically solve for those coupons that are roughly consistent with fair value—a mark-to-market value of zero—for each tranche. Of course, the fair value for each CDS in the reference collateral should also be roughly zero. Exhibit 1 identifies the following coupons as being close to fair value.[6]

Coupons range from 1.53% on Tranche 1 to one basis point on the super senior tranche, Tranche 7. These coupons are fixed for the remainder of the article.

We now compute the CDO tranche values for the base case using one-month PDs (probability of defaults) and no default correlations across credits. Under these assumptions, Exhibit 2 graphs the percentile distribution of losses on the reference collateral.

Exhibit 2 shows the cumulative loss quantiles at each of the 60 months over the five-year life of the CDO. The black line corresponds to the worst losses (100%). The first loss occurs in month 3, the second loss occurs in month 12. By the end of the five years there have been five defaults and a total loss (given our 40% recovery rate assumption) of $3 million. This means, of course, that the only tranche that bears losses is Tranche 1, since it takes the first $5 million of losses. At the 95th percentile level, the grey line, losses are $1.2 million. Losses at the 75th percentile are only $600,000 and no losses are observed for percentiles less than the 75th.

The distribution of losses to the equity tranche is given in Exhibit 3. We see here that there are no losses in more than 50% of the scenarios. The equity Tranche 1 has a value of $3,509.

Standard Copula-based Model

The standard copula-based CDO valuation model is static—a single period—model. This implies that default probabilities and correlations are constants over the life of a CDO. For simplicity, the copula-based model usually assumes that all pairs of credits have identical correlations. We follow this approach herein.

To study the sensitivity of the equity tranche value to increased correlation, we fix the one-month PD and we simulate the various tranche values for correlations from 0 to 0.99 in 0.25 increments. Unlike the prior simulations,

EXHIBIT 1
CDO Tranche Coupons

CDO Definition

CDO Type	SYNTHETIC	Portfolio Notional Amount	500,000,000.00
Reference Portfolio	DEMO CDO	Reference Entity Credit Position (%)	0.200000
Origination Date	4/27/2007	Reference Entity Notional Amount	1,000,000.00
Termination Date	4/27/2012	Coupon Frequency	Semi-Annual

Tranche Description	Coupon %	Attachment %	Detachment %	Tranche Size %	Loss Treshold	Recovery Treshold
Tranche 1	1.53	0.00	1.00	1.00	0.00	5,000,000.00
Tranche 2	0.05	1.00	2.00	1.00	5,000,000.00	10,000,000.00
Tranche 3	0.04	2.00	3.00	1.00	10,000,000.0	15,000,000.00
Tranche 4	0.03	3.00	4.00	1.00	15,000,000.0	20,000,000.00
Tranche 5	0.03	4.00	5.00	1.00	20,000,000.0	25,000,000.00
Tranche 6	0.02	5.00	6.00	1.00	25,000,000.0	30,000,000.00
Tranche 7	0.01	6.00	100.00	94.00	30,000,000.0	500,000,000.00

Exhibit 2
Distribution of Losses to the Collateral Pool

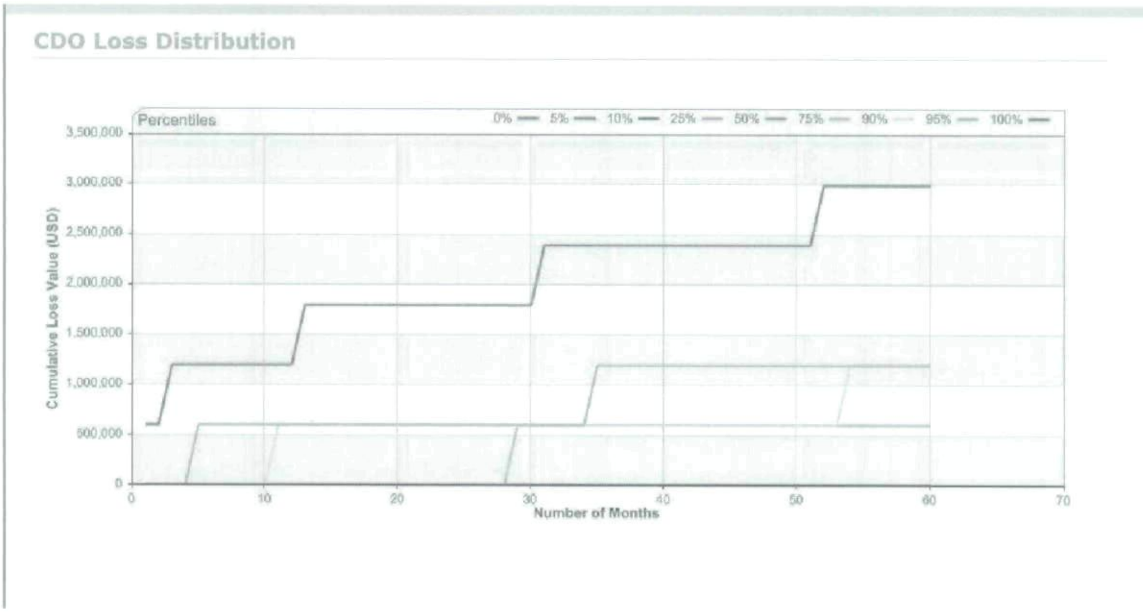
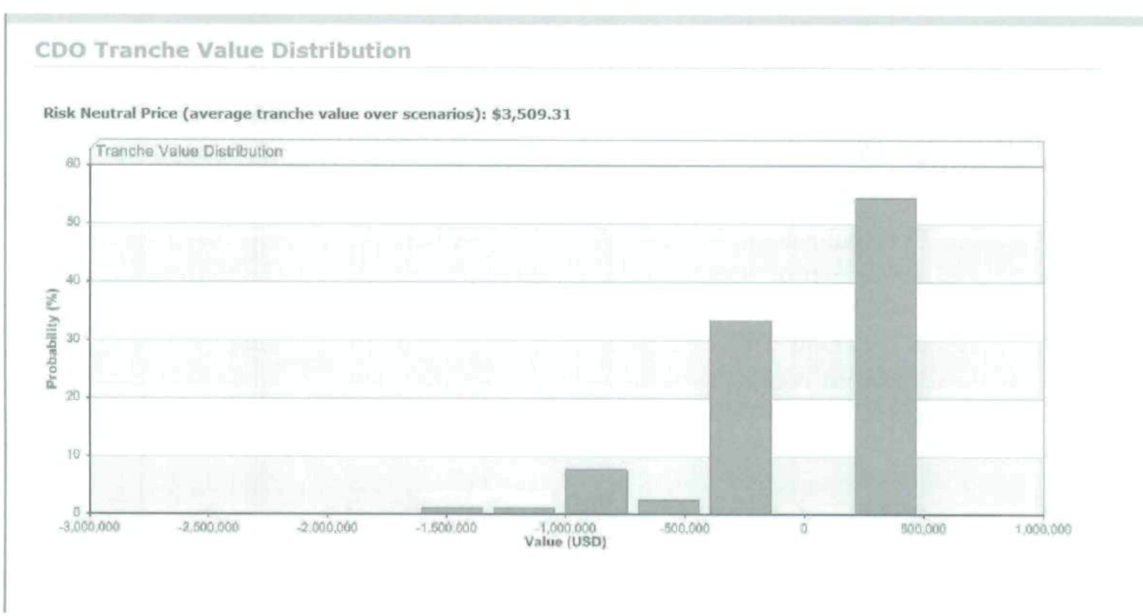


Exhibit 3
Distribution of Losses to the Equity Tranche



we increased the scenario count to 500,000 scenarios to reduce the sampling error. The results are reported in Exhibit 4.

The base case for the 500,000 scenario simulation differs from the value obtained in the 10,000 scenario simulation due to sampling error. Of course, the 500,000 scenario simulation is more precise.

As indicated, the equity tranche increases in value as the correlation increases. In contrast, the values for the other tranches decline as the correlation increases. Hence,

EXHIBIT 4
Tranche Values as a Function of Correlation

copula model	*base case*				
correlation	0	0.25	0.50	0.75	0.99
equity tranche	– $64,035	– $33,522	$9,798	$68,951	$133,260
tranche 2	$11,060	$7,879	– $10,614	– $29,337	– $42,066
tranche 3	$8,848	$8,599	$3,475	– $7,132	– $19,478
tranche 4	$6,636	$6,594	$4,634	– $1,088	– $8,824
tranche 5	$6,636	$6,635	$5,856	$2,409	– $2,118
tranche 6	$4,424	$4,424	$4,057	$1,915	– $808
tranche 7	$207,940	$207,940	$207,531	$201,827	$191,076

Note: The base case for the 500,000 scenario simulation differs from the value obtained in the 10,000 scenario simulation due to sampling error, Of course, the 500,000 scenario simulation is more precise.

for the standard copula-based model, the equity tranche is long correlation.

Reduced Form Model

An alternative to the standard static copula model is a dynamic, multi-period model that allows both the default probabilities and correlations to vary with the business cycle. In this regard, we use a reduced form model (see Jarrow and Turnbull [1995]) where default correlations are implicitly captured in the default intensities through their dependence on common state variables (see Jarrow and van Deventer [2005]).

Given that we estimate these default intensities using market data, formulating a comparative static analysis for changes in the CDO equity tranche value via a change in the correlation structure is more difficult for the reduced form model. To obtain such a comparison, however, we can fix the dynamic default intensities at their initial values on April 27, 2007. Fixing the default intensities will remove the default correlation from the reduced form model in the Monte Carlo simulation. Fortunately, this zero correlation modification is just the base case CDO model presented earlier. Hence, to contrast the impact of a change in default correlations on the reduced form model, we need only compare the base case CDO model presented earlier with a properly estimated reduced form model. This is the purpose of this section.

The dynamic CDO valuation model we employ uses the reduced form probabilities obtained from a hazard rate estimation procedure similar to that employed by Jarrow and Chava [2004]. We estimate the business-cycle-dependent annualized one-month default probabilities for each reference name using a logistic regression that links each company's default probability to 27 potential macroeconomic variables.[7] These macroeconomic variables perform two functions. First, they enable the one-month default probabilities to vary across the business cycle. Second, they induce business cycle variations in the default correlations as well. This business cycle dependence allows for the inclusion of default contagion. Indeed, in a downturn, all pro-cyclic companies will have their default probabilities increase. This will increase the likelihood of multiple defaults (i.e., contagion). This inclusion of business-cycle-dependent default correlation and contagion is not present in the standard copula-based CDO valuation model.

The macroeconomic variables used in the regression are provided in Exhibit 5. These macro-variables are the usual candidates included to understand the business cycle: oil prices, stock indices (level and volatility), interest rates, and exchange rates.

To simulate the monthly changes to the annualized one-month default probabilities over the CDO's five-year life, we first need to simulate the realizations of these macroeconomic variables. To do this, we assume that these macroeconomic variables follow either a geometric (lognormal distribution) or standard (normal distribution) Brownian motion process. The historical volatilities and correlations for each of these macroeconomic factors are computed and used in the subsequent simulations. Exhibit 6 lists the volatilities and the distribution applied to each macro-variable.

Another issue arises when simulating the business-cycle-dependent default probabilities. To simulate the changes in the default probabilities, we need to simulate

EXHIBIT 5
Macro-Variables Used in the Reduced Form Model

Number	Variable Name	Description
1	EUR_USD	EUR/USD exchange rate
2	EUR_USD1	1 year change of EUR/USD exchange rate
3	EUR_USD2	2 year change of EUR/USD exchange rate
4	EUR_USD3	3 year change of EUR/USD exchange rate
5	JPY_USD	JPY/USD exchange rate
6	JPY_USD1	1 year change of JPY/USD exchange rate
7	JPY_USD2	2 year change of JPY/USD exchange rate
8	JPY_USD3	3 year change of JPY/USD exchange rate
9	OILPRICE	Oil price
10	OILCHG1	1 year change of oil price
11	OILCHG2	2 year change of oil price
12	OILCHG3	3 year change of oil price
13	TR10YR_U	U.S. government 10 year treasury yield
14	TR10YR_C	Canada government 10 year treasury yield
15	TR10YR_E	Euro government 10 year treasury yield
16	TR10YR_J	Japan government 10 year treasury yield
17	TR10YR_G	U.K. government 10 year treasury yield
18	IDXCHG_U	U.S. stock maket index 2 year change
19	IDXCHG_C	Canada stock maket index 2 year change
20	IDXCHG_E	Euro stock maket index 2 year change
21	IDXCHG_J	Japan stock maket index 2 year change
22	IDXCHG_G	U.K. stock maket index 2 year change
23	IDXSTD_U	Standard deviation of U.S. stock market index 2 year change
24	IDXSTD_C	Standard deviation of Canada stock market index 2 year change
25	IDXSTD_E	Standard deviation of Euro stock market index 2 year change
26	IDXSTD_J	Standard deviation of Japan stock market index 2 year change
27	IDXSTD_G	Standard deviation of U.K. stock market index 2 year change

both the macro-variables and the firm-specific variables. Rather than simulating the firm-specific variables, we consider adding them as idiosyncratic risk, and we simulate the idiosyncratic component instead. This reduces the dimensionality of the simulation considerably.

To model the idiosyncratic component we follow this process:

- First, we identify a time series for Company A's default probabilities that we regard as "true," denoted PD_t. Here we use the Jarrow-Chava reduced form model as the "true" time series of default probabilities. This model includes macro and company-specific variables.
- Second, we use these default probabilities PD_t as the dependent variable in a second logistic regression where the independent variables correspond to the macro-variables alone. The residual from this second regression is the idiosyncratic risk component. By construction, this idiosyncratic risk will be uncorrelated across companies and across time, consistent with the conclusions of Jarrow, Lando and Yu [2005].
- Third, in the simulation, we include the idiosyncratic risk via independent draws from a standard normal distribution. The parameters of this normal distribution are obtained from the second logistic regression.

The simulation algorithm using the full 27 macroeconomic variables can now be described. It consists of the following steps:

EXHIBIT 6
Volatilities and Distribution for the Macro-Variables

Number	Variable Name	Description	Volatility	Distribution
1	EUR_USD	EUR/USD exchange rate	9.48%	LOGNORMAL
2	EUR_USD1	1 year change of EUR/USD exchange rate	11.30%	NORMAL
3	EUR_USD2	2 year change of EUR/USD exchange rate	18.83%	NORMAL
4	EUR_USD3	3 year change of EUR/USD exchange rate	24.80%	NORMAL
5	JPY_USD	JPY/USD exchange rate	9.53%	LOGNORMAL
6	JPY_USD1	1 year change of JPY/USD exchange rate	10.19%	NORMAL
7	JPY_USD2	2 year change of JPY/USD exchange rate	13.58%	NORMAL
8	JPY_USD3	3 year change of JPY/USD exchange rate	11.44%	NORMAL
9	OILPRICE	Oil price	33.35%	LOGNORMAL
10	OILCHG1	1 year change of oil price	34.80%	NORMAL
11	OILCHG2	2 year change of oil price	49.62%	NORMAL
12	OILCHG3	3 year change of oil price	50.22%	NORMAL
13	TR10YR_U	U.S. government 10 year treasury yield	21.44%	LOGNORMAL
14	TR10YR_C	Canada government 10 year treasury yield	14.09%	LOGNORMAL
15	TR10YR_E	Euro government 10 year treasury yield	15.79%	LOGNORMAL
16	TR10YR_J	Japan government 10 year treasury yield	35.22%	LOGNORMAL
17	TR10YR_G	U.K. government 10 year treasury yield	15.02%	LOGNORMAL
18	IDXCHG_U	U.S. stock maket index 2 year change	28.68%	NORMAL
19	IDXCHG_C	Canada stock maket index 2 year change	27.11%	NORMAL
20	IDXCHG_E	Euro stock maket index 2 year change	33.81%	NORMAL
21	IDXCHG_J	Japan stock maket index 2 year change	29.19%	NORMAL
22	IDXCHG_G	U.K. stock maket index 2 year change	25.58%	NORMAL
23	IDXSTD_U	Standard deviation of U.S. stock market index 2 year change	38.82%	LOGNORMAL
24	IDXSTD_C	Standard deviation of Canada stock market index 2 year change	46.56%	LOGNORMAL
25	IDXSTD_E	Standard deviation of Euro stock market index 2 year change	44.54%	LOGNORMAL
26	IDXSTD_J	Standard deviation of Japan stock market index 2 year change	48.51%	LOGNORMAL
27	IDXSTD_G	Standard deviation of U.K. stock market index 2 year change	47.37%	LOGNORMAL

1. In month 1, random values of the 27 correlated macroeconomic variables were generated.
2. The idiosyncratic term of the default probability function was also generated for the 500 reference names by making 500 independent draws from the standard normal distribution, scaling by the standard deviation appropriate for each of the 500 companies.
3. The month 1 annualized one-month default probabilities for each name were then computed.
4. Default/No default was then simulated for each name based on these default probabilities.
5. If default occurred, losses were realized with a 40% recovery rate.
6. For those companies who survived month 1, a similar simulation was done for month 2, given the month 2 values of the simulated macro-variables.
7. This simulation ran for 60 months, and then it was repeated for a total of 10,000 scenarios.

The loss distribution for the full amount of the reference collateral is shown in Exhibit 7. The median cumulative losses after 60 months are approximately $15 million or 3% of the original notional principal. The worst-case (100%) quantile's cumulative losses were in excess of $50 million. Even in the best case scenario, losses after five years totaled about $5 million.

The distribution of losses to the equity Tranche 1 is provided in Exhibit 8. As indicated, all of the 10,000 scenarios show losses, and Tranche 1's CDO value is –$4,709,461, almost the entire notional principal of the tranche.

The values of the other tranches are provided in Exhibit 9.

Relative to the base case, we see that the value of the equity tranche declines significantly as the default correlation increases. The remaining tranches also decline. For the reduced form model, the equity tranche is short correlation risk.

EXHIBIT 7
Distribution of Losses to the Collateral Pool

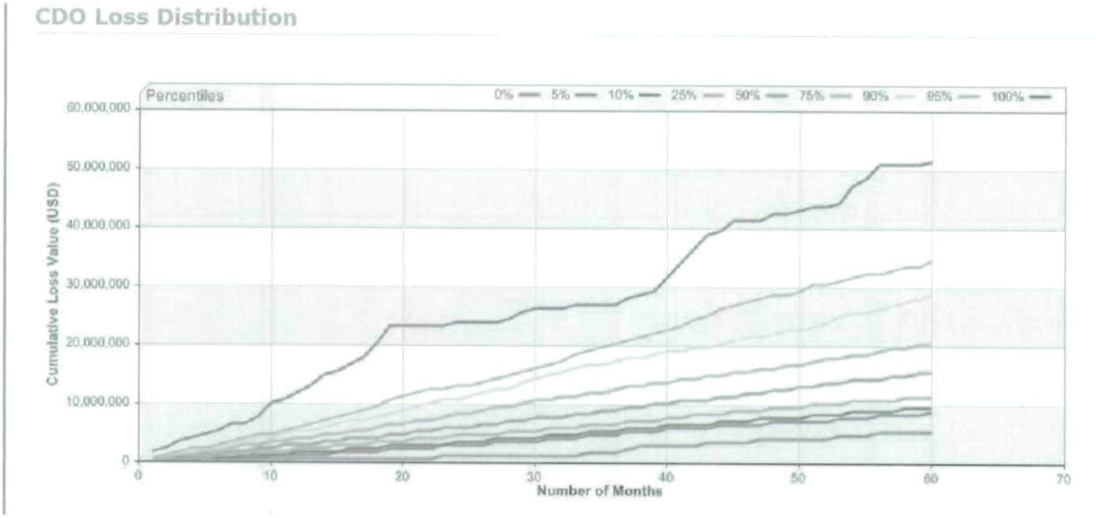
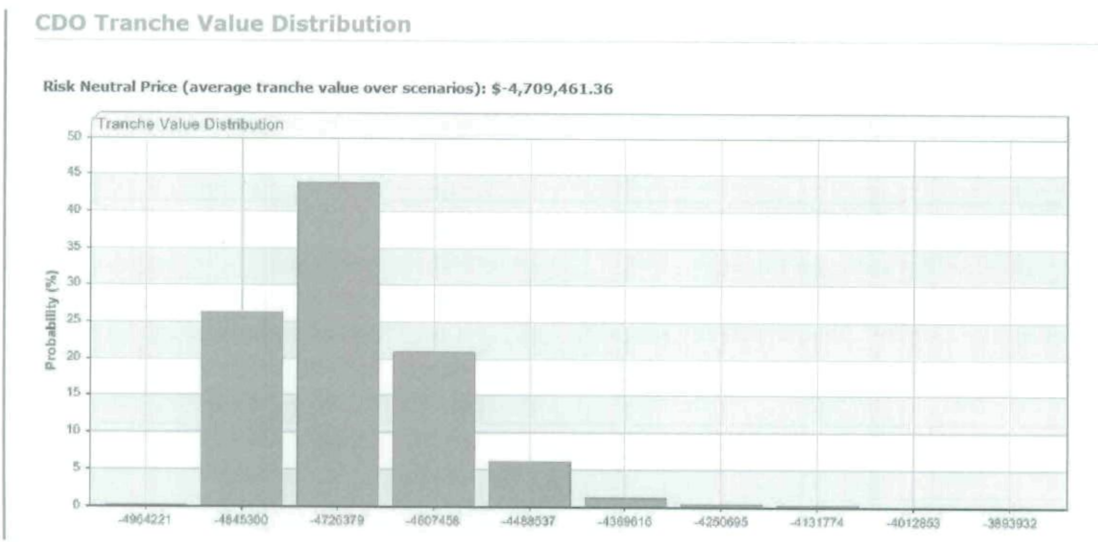


EXHIBIT 8
Distribution of Losses to the Equity Tranche



CONCLUSION

This article analyzes the commonly held belief that a synthetic CDO's equity tranche is long correlation risk. We show, through a series of simple examples and a realistic simulation, that this belief need not be true. In fact, whether equity is long or short correlation risk depends on the CDO model selected.

ENDNOTES

Helpful comments from Gunter Meissner are gratefully acknowledged.

[1]This article is similar in spirit to a recent working paper by Agca and Islam [2007].

[2]This case was pointed out by Gunter Meissner.

[3]Related models are Bharath and Shumway [2004], and Campbell, Hilscher, and Szilagyi [2007].

[4]Default probabilities were provided by the Kamakura Corporation KRIS default probability service (see www.

Exhibit 9
Tranche Values Comparison

	base case	reduced form
equity tranche	−$64,035	− $4,709,461
tranche 2	$11,060	− $4,277,124
tranche 3	$8,848	− $3,016,776
tranche 4	$6,636	− $1,631,544
tranche 5	$6,636	− $709,304
tranche 6	$4,424	− $379,104
tranche 7	$207,940	− $376,965

kamakuraco.com for a more complete description of the KRIS methodologies).

[5]Using the hazard rate estimation procedure generates the statistical default probabilities. However, the insights of Jarrow, Lando, and Yu [2005] show that these default probability estimates are also the risk-neutral default probabilities.

[6]In the simulation, we use the web-based synthetic CDO valuation of KRIS-CDO provided by Kamakura Corporation.

[7]Coefficients and statistical significance are those embedded in the Kamakura Risk Information Services KRIS-CDO web service.

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