Volatility and the Carry Trade

VINEER BHANSALI

VINEER BHANSALI
is an executive vice president, portfolio manager, and head of the Analytics Group at PIMCO in Newport Beach, CA.
bhansali@pimco.com

In the currency carry trade, an investor borrows in a low yielding currency and invests in a high yielding currency. Empirically, it is observed that carry trades do well when currency volatility is low. This makes intuitive sense. To realize the carry in a carry trade, investors are required to hold the position for a while. If the foreign rate is r_f and the domestic rate is r, then the return to the carry trade is proportional, to leading order, to $(r - r_f)T$, where T is the holding period. The risk to the carry trade is an adverse price movement in the level of the exchange rate. We can mitigate this risk by hedging in the currency options markets. If the spot foreign exchange (FX) market and the options markets price risk differently, then one could have potential arbitrage opportunities.

To illustrate this, consider the following strategy: suppose we invest in the higher yielding currency and hedge the exposure by purchasing an at-the-money forward (ATMF) put on the higher yielding currency. Then, since the delta of the out-of-the-money option is approximately 1/2, at inception we would be immune to the movements of the exchange rate. If we adjust the notionals correctly, then by put-call parity this portfolio is the same as an outright ATMF call on the higher yielding currency.

To leading order, we can easily work out the dependence of an at-the-money forward call option to implied volatility in the Black-Scholes framework. The forward foreign exchange rate, F_0, can be written in terms of the domestic interest rate, r, and the foreign interest rate, r_f, as

$$F_0 = S_0 e^{(r-r_f)T} \tag{1}$$

where S_0 is the spot exchange rate and T is the maturity of the option. Then, the Black-Scholes option prices for the calls and puts are simply (e.g., Hull [2006])

$$c = e^{-rT}[F_0 N(d_1) - KN(d_2)] \tag{2}$$

$$p = e^{-rT}[KN(-d_2) - F_0 N(-d_1)] \tag{3}$$

where

$$d_1 = \frac{\ln(F_0/K) + \frac{1}{2}\sigma^2}{\sigma\sqrt{T}} \tag{4}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{5}$$

Here $N(x)$ represents the standardized cumulative normal distribution evaluated at x and K is the strike of the option.

For at-the-money forward options, $F_0 = K$, so, $d_1 = \frac{1}{2}\sigma\sqrt{T} = -d_2$, and the call option price simplifies to

$$e^{-rT} F_0 \left[N\left(\frac{1}{2}\sigma\sqrt{T}\right) - N\left(-\frac{1}{2}\sigma\sqrt{T}\right)\right] \tag{6}$$

But since $N(x) = 1 - N(-x)$, we can write this as

$$c = e^{-rT} F_0 \left[2N\left(\frac{1}{2}\sigma\sqrt{T}\right) - 1 \right] \tag{7}$$

We can expand this about σ to obtain, to leading order

$$e^{-rT} F_0 \sigma \sqrt{\frac{T}{2\pi}} \tag{8}$$

which, in terms of percent premium (percent of forward) is simply

$$e^{-rT} \sigma \sqrt{\frac{T}{2\pi}} \tag{9}$$

Putting the carry relationship together with the leading behavior of an ATMF call option, we conclude that in equilibrium, a carry trade hedged with options would obey, to leading order

$$(r - r_f)T \sim \sigma\sqrt{T} \tag{10}$$

or, in other words

$$\sigma \sim (r - r_f)\sqrt{T} \tag{11}$$

Suppose the actual level of volatility in the market is expected to be lower (i.e., $\sigma^* < \sigma$). Then the fair carry is not $(r - r_f)\sqrt{T}$, but a quantity $(r^* - r_f^*)\sqrt{T}$. If $\sigma^* < \sigma$, then $(r^* - r_f^*) < (r - r_f)$. In the extreme case, where $r_f << r$, $r_f^* << r$ (e.g., when carry trades use the Japanese yen (JPY) as the funding currency), this implies that $r^* << r$, and the carry trade will drive investors to hold bond instruments in the domestic currency by financing them in the lower yielding currency *as a direct bet on lower volatility.* In other words, the carry-volatility equilibrium dictates the availability of profits to investors who are willing to take interest rate risk as foreign exchange volatility falls. Similarly, if we expect volatility to rise, investors will shed bonds in the higher yielding currency. We will see that the simple math is borne out by actual data, especially for emerging market (EM) currencies such as the Mexican peso (MXN) and the Brazilian real (BRL) (see Exhibits 1 and 2).

TRUE OPTION COST

To further explore implementable strategies and their attractiveness, we will define a simple quantity to measure the relative mispricing of FX options which we will call the *true option cost*. The true option cost, O, as a percent of the full premium is

$$O = \frac{P - \Delta C}{P} \tag{12}$$

where P is the option premium, Δ is the delta of the option (approximately 0.48 for at-the-money forward), and C is the carry. This equation states that the true cost of purchasing an ATMF call option on a higher yielding currency should be computed by adjusting the premium

EXHIBIT 1

Relationship between Implied Volatility and Carry for the U.S. Dollar (USD) and Mexican Peso (MXN)

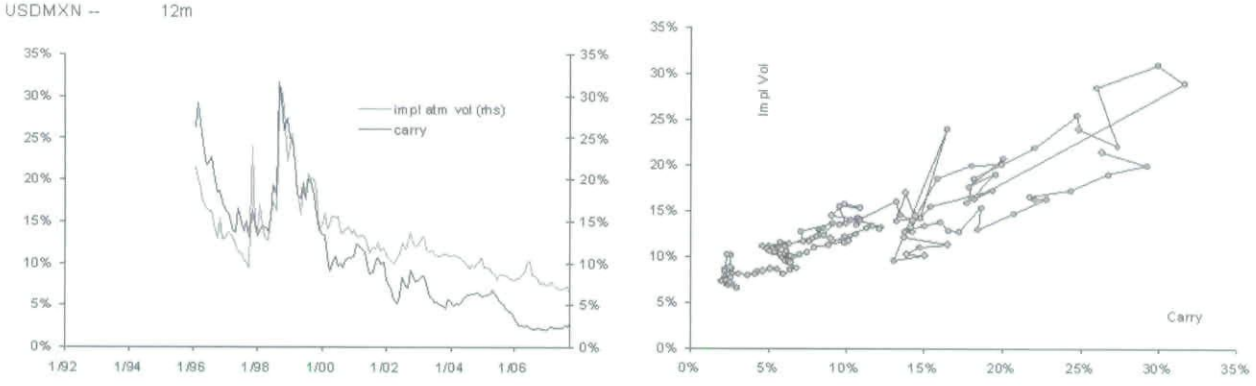
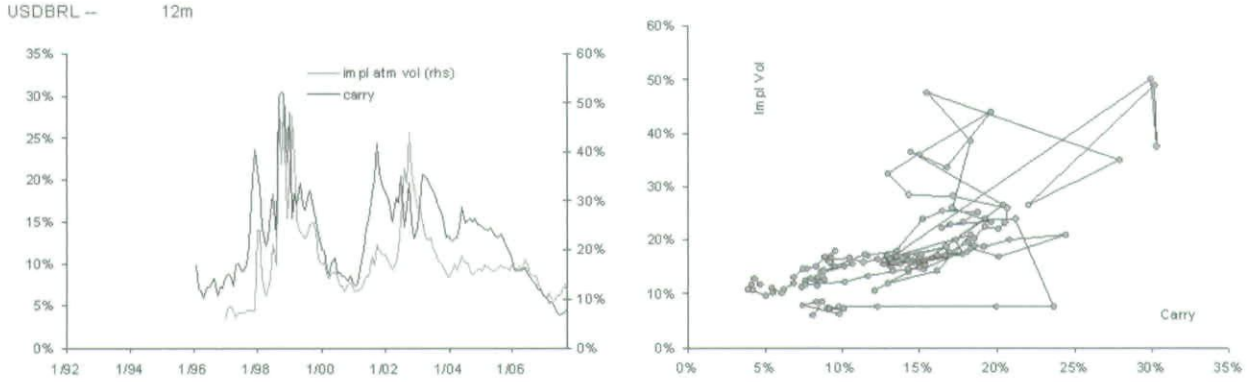


EXHIBIT 2

Relationship between Implied Volatility and Carry for the U.S. Dollar (USD) and Brazilian Real (BRL)



for the carry. The reason to multiply by the delta of the option is to equalize the starting notional exposure of the option position (on approximately twice the full notional) to the carry (on the full notional). Using the result from Equation 11, we obtain

$$O \approx \left(1 - \frac{(r - r_f)\sqrt{\frac{1}{2}\pi T}}{\sigma}\right) \tag{13}$$

This simple equation illustrates the influence of volatility on the carry trade in a clear way—as the interest rate differential widens, or volatility embedded in options falls, the true fraction of option cost falls. For options of a fixed maturity, the carry per unit of volatility, $((r - r_f)/\sigma)$, determines the relative attractiveness of entering into the carry trade. When this ratio is large, the carry trade is attractive (true option cost is low), and when the ratio is small, the carry trade is unattractive (true option cost is high).

EMPIRICAL RESULTS

We collected data for a number of major developed market currencies that have been the main beneficiaries of the carry trade.[1] Typically, the JPY and Swiss franc (CHF) have been used as funding currencies, and the Australian dollar (AUD), New Zealand dollar (NZD), and U.S. dollar (USD) have been the higher yielding carry currencies. More recently, the improving fundamentals of the MXN and BRL have enabled them to enter the toolkit of carry traders.

At first blush, Exhibits 1 and 2 show that for the MXN and BRL there is indeed a clear positive link between volatility levels and carry relative to the USD. The relationship is somewhat weaker for GBP–CHF (Exhibit 4) AUD vs. USD and JPY (Exhibits 5 and 7), NZD vs. USD and JPY (Exhibits 6 and 8), and the weakest for USD–JPY (Exhibit 3). A couple of other observations can be made.

The fact that most pairs other than the JPY show a relationship between carry and vol points to an anomaly that is driven by the special relationship between the USD and the JPY. The U.S. has historically been a large debtor of Japan, but now China has joined Japan as the largest creditors of the world. To a large degree, anecdotal evidence suggests that the recycling of surplus by the Japanese and Chinese creates an effective peg for the JPY and Chinese renminbi (RMB) to the USD (for the RMB, the peg is actual), muting the exchange rate volatility of these two countries' currencies vs. the USD. Thus, we see that even though carry of the USD vs. the JPY has varied a fair bit (from a 6% high in 2000 to a 1% low in 2004), the level of implied volatility has remained on a downward trend. We also find a weak relationship of volatility with carry for the CAD vs. the USD (Exhibit 9). However, due to the tight linkage via trade between the USD and the CAD, the CAD is typically not used as a carry trade vehicle. In particular, note that the the interest rate differential for this pair has fluctuated tightly around zero.

When we apply Equation 12 to these pairs, we find that for different pairs the carry reduces the total premium cost of the options from the beneficial effects of the carry. If, as numerous studies have established, it is true that the FX forwards market biases the direction of the

EXHIBIT 3

Relationship between Implied Volatility and Carry for the U.S. Dollar (USD) and Japanese Yen (JPY)

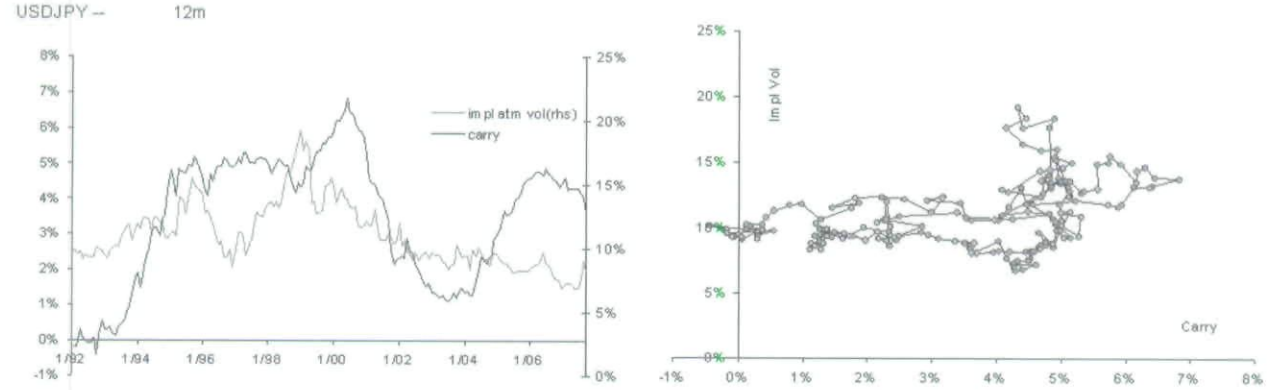


EXHIBIT 4

Relationship between Implied Volatility and Carry for the British Pound (GBP) and Swiss Franc (CHF)

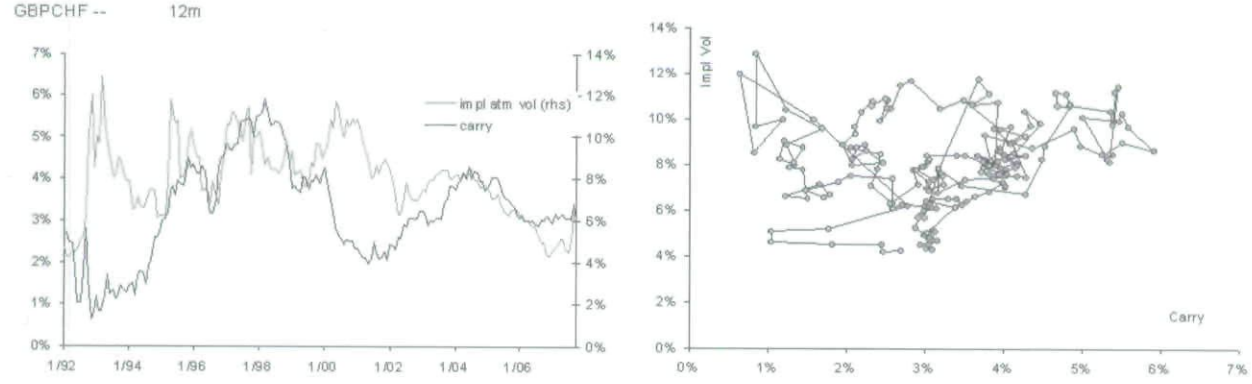


EXHIBIT 5

Relationship between Implied Volatility and Carry for the Australian Dollar (AUD) and U.S. Dollar (USD)

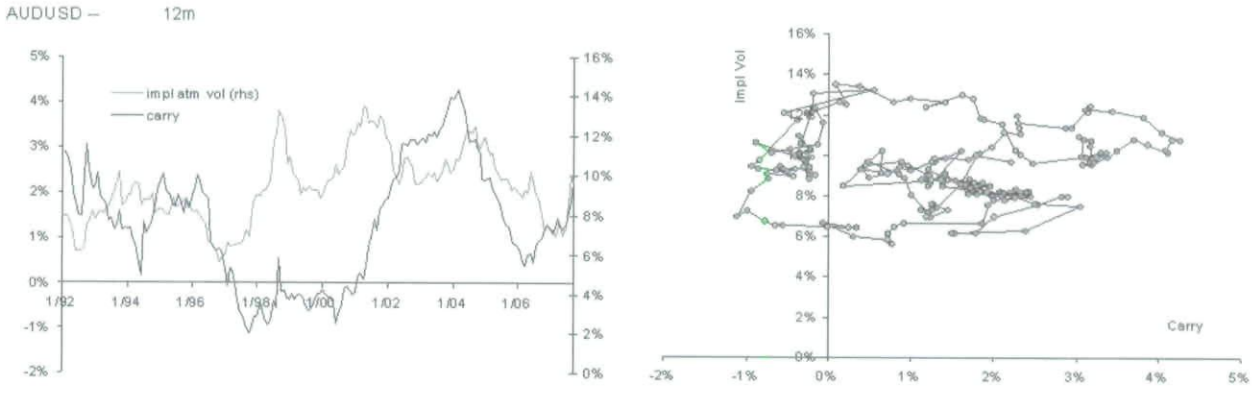


EXHIBIT 6

Relationship between Implied Volatility and Carry for the New Zealand Dollar (NZD) and U.S. Dollar (USD)

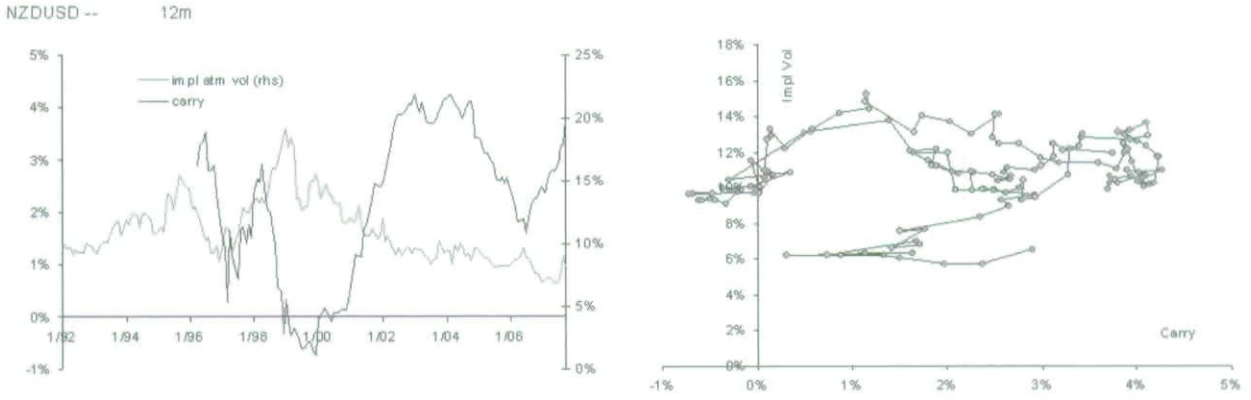


EXHIBIT 7

Relationship between Implied Volatility and Carry for the Australian Dollar (AUD) and Japanese Yen (JPY)

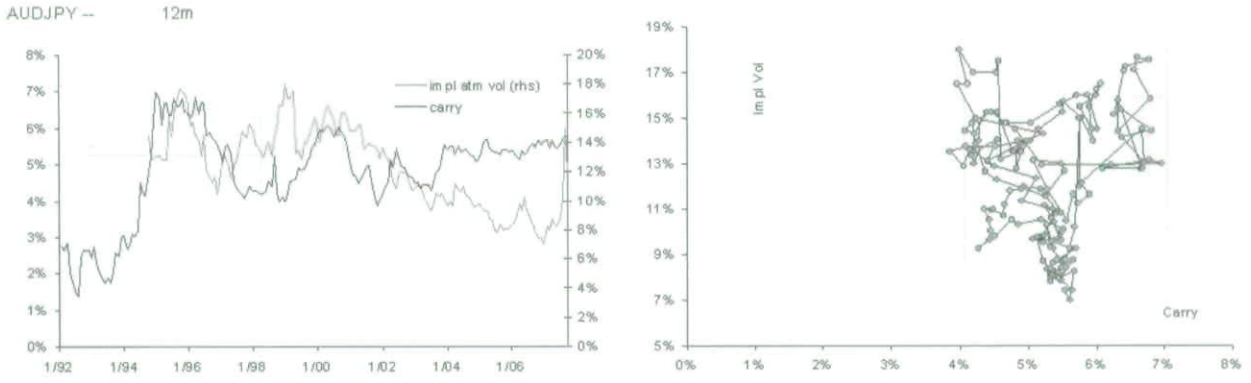


EXHIBIT 8

Relationship between Implied Volatility and Carry for the New Zealand Dollar (NZD) and Japanese Yen (JPY)

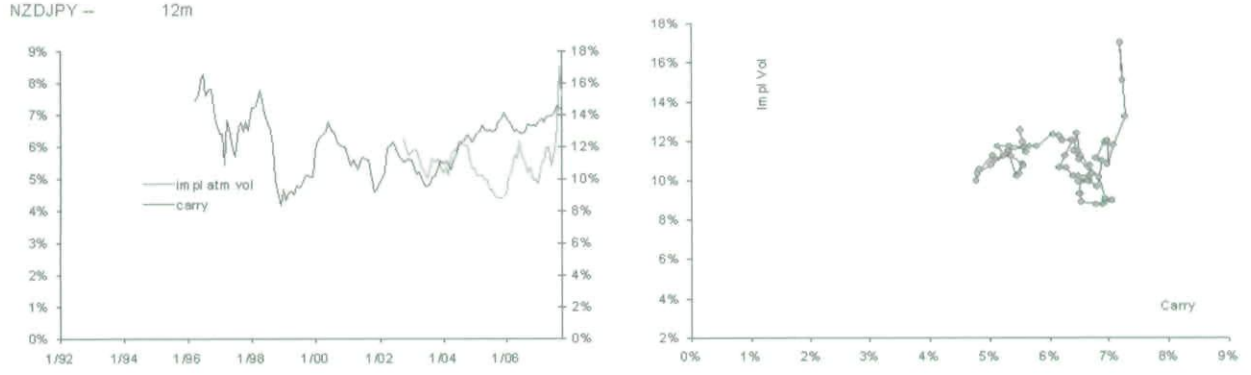


EXHIBIT 9

Relationship between Implied Volatility and Carry for the U.S. Dollar (USD) and Canadian Dollar (CAD)

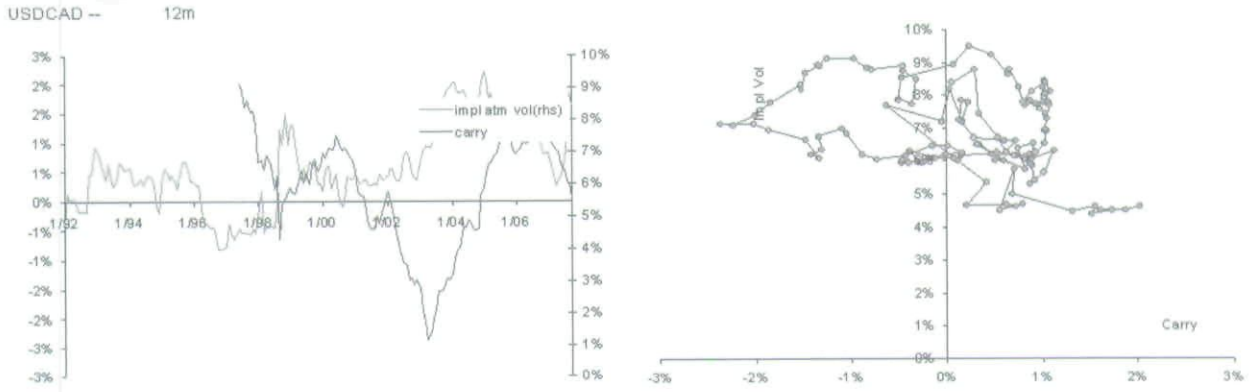


EXHIBIT 10

True Option Cost in October 2007 and January 2007

Oct 20 2007:

Pair	Spot	Forward (=ATM Fwd Strike)	Vol (1y)	Carry (1y)	Premium	Option Premium Net of Delta Weighted Carry	"True" Option Cost
USD/JPY	114.68	110.53	8.33%	3.75%	3.17%	1.4%	43.2%
GBP/CHF	2.4025	2.3307	6.44%	3.08%	2.42%	0.9%	39.0%
AUD/USD	0.8937	0.8713	11.12%	2.57%	4.14%	2.9%	70.2%
AUD/JPY	102.5	96.31	13.86%	6.43%	5.14%	2.1%	40.0%
NZD/USD	0.7541	0.7245	13.46%	4.09%	4.92%	3.0%	60.2%
NZD/JPY	86.47	80.07	16.53%	7.99%	6.04%	2.2%	36.5%
MXN/USD	10.83	11.164	6.65%	2.99%	2.53%	1.1%	43.3%
BRL/USD	1.8013	1.889	13.54%	4.64%	5.15%	2.9%	56.8%

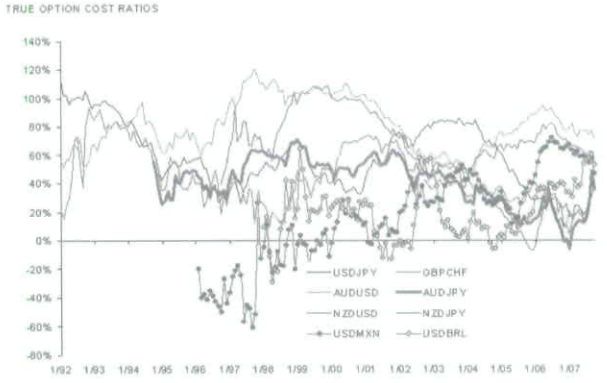
Jan24 2007:

Pair	Spot	Forward (=ATM Fwd Strike)	Vol (1y)	Carry (1y)	Premium	Option Premium Net of Delta Weighted Carry	"True" Option Cost
USD/JPY	121.41	115.89	6.93%	4.76%	2.62%	0.3%	12.7%
GBP/CHF	2.457	2.379	4.85%	3.30%	1.83%	0.2%	13.5%
AUD/USD	0.7803	0.7719	7.16%	1.08%	2.08%	1.6%	75.0%
AUD/JPY	94.72	89.47	7.53%	5.87%	2.82%	0.0%	0.0%
NZD/USD	0.697	0.6809	9.87%	2.39%	2.54%	1.0%	54.8%
NZD/JPY	84.69	78.94	10.69%	7.28%	3.95%	0.5%	11.5%
MXN/USD	10.9237	11.1737	7.65%	2.24%	2.86%	1.8%	62.4%
BRL/USD	2.128	2.2555	10.31%	5.65%	3.90%	1.2%	30.4%

spot exchange rate, then a carry trade implemented through options when volatilies are low, should have a better risk-reward profile than a carry trade done outright. We explore this in the next section. Exhibit 11 shows the history of the true option cost for the pairs. In particular, note that the latest period, which includes August and September of 2007, has witnessed sharp increases in true option costs, primarily because of a rise in currency volatility.

EXHIBIT 11

History of True Option Cost



INVESTMENT STRATEGY

In this section, we present the results of backtesting a carry strategy that uses both information from the

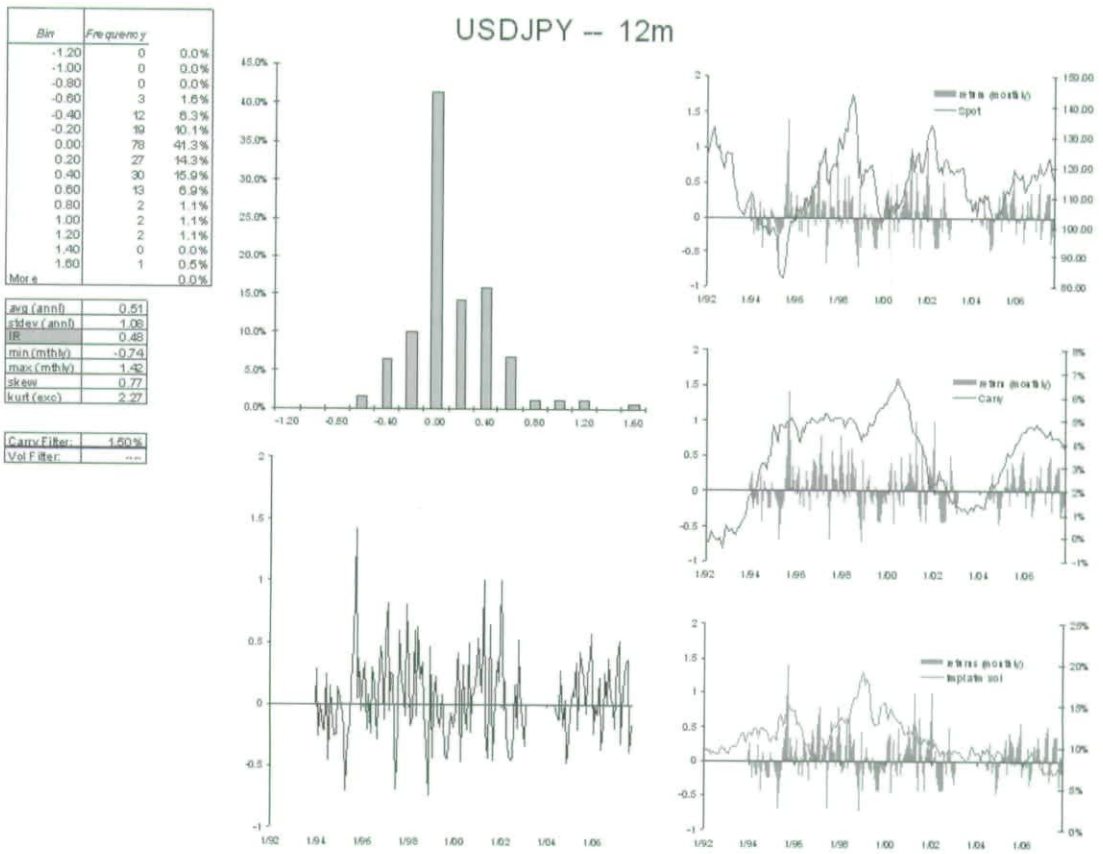
options markets and from the interest rate differentials between currencies. The basic idea is to purchase at-the-money forward calls on a higher yielding currency, and hold them to maturity. If there is indeed a forward-exchange-rate bias in the market, then simple carry strategies should result in positive returns; implementing the strategy through options when volatility is low would further improve the risk profile of the strategy.

Another way of thinking about this is as with options purchases, we think of the daily time decay as the cost of truncating the possibility of negative outcomes. However, if we note that as time passes the forward will rollup toward the current spot level, we can reduce the cost of the option by the embedded carry because the ATMF option is going into-the-money as time passes.

The strategy we explore invests in the higher yielding currency via 1 year at-the-money forward options, and rolls them every month; the results are largely similar for both shorter-dated and longer-dated options. We explicitly include transaction costs by taking the average transaction costs over the previous year. Additionally, we refined the strategies by putting optional filters on both volatility and carry (i.e., enter the trade only if the carry is higher than a threshold and/or if volatility is lower than a threshold).

In Exhibits 12, 13, 14, 15, 16, 17, and 18, we show the statistics of these strategies for a variety of pairs from many different perspectives. In each of these exhibits, the first table on the top left shows the returns per month binned by the returns. The histogram immediately to the

EXHIBIT 12
Statistics of USD–JPY Carry Trade with Options



Bin	Frequency	
-1.20	0	0.0%
-1.00	0	0.0%
-0.80	0	0.0%
-0.60	3	1.6%
-0.40	12	6.3%
-0.20	19	10.1%
0.00	78	41.3%
0.20	27	14.3%
0.40	30	15.9%
0.60	13	6.9%
0.80	2	1.1%
1.00	2	1.1%
1.20	2	1.1%
1.40	0	0.0%
1.60	1	0.5%
More		0.0%

avg (annl)	0.51
stdev (annl)	1.08
IR	0.48
min (mthly)	-0.74
max (mthly)	1.42
skew	0.77
kurt (exc)	2.27

Carry Filter:	1.50%
Vol Filter:	---

The first table on the top left shows the returns per month binned by the returns. The histogram immediately to the right and the statistics below the table show the positive skew of the option-based implementation, along with Sharpe ratios and annualized expected returns. We also display at the bottom left the carry and vol filters (if any) used in the back test. In the example above, the carry trade is only entered if the carry exceeds 150 bps at all implied vol levels. The charts below the histograms how the monthly performance time series of the strategy. The three charts on the right show the option-based carry strategy return vs. spot levels for the pair, returns vs. the carry for the pair, and returns vs. the implied volatility for the pair.

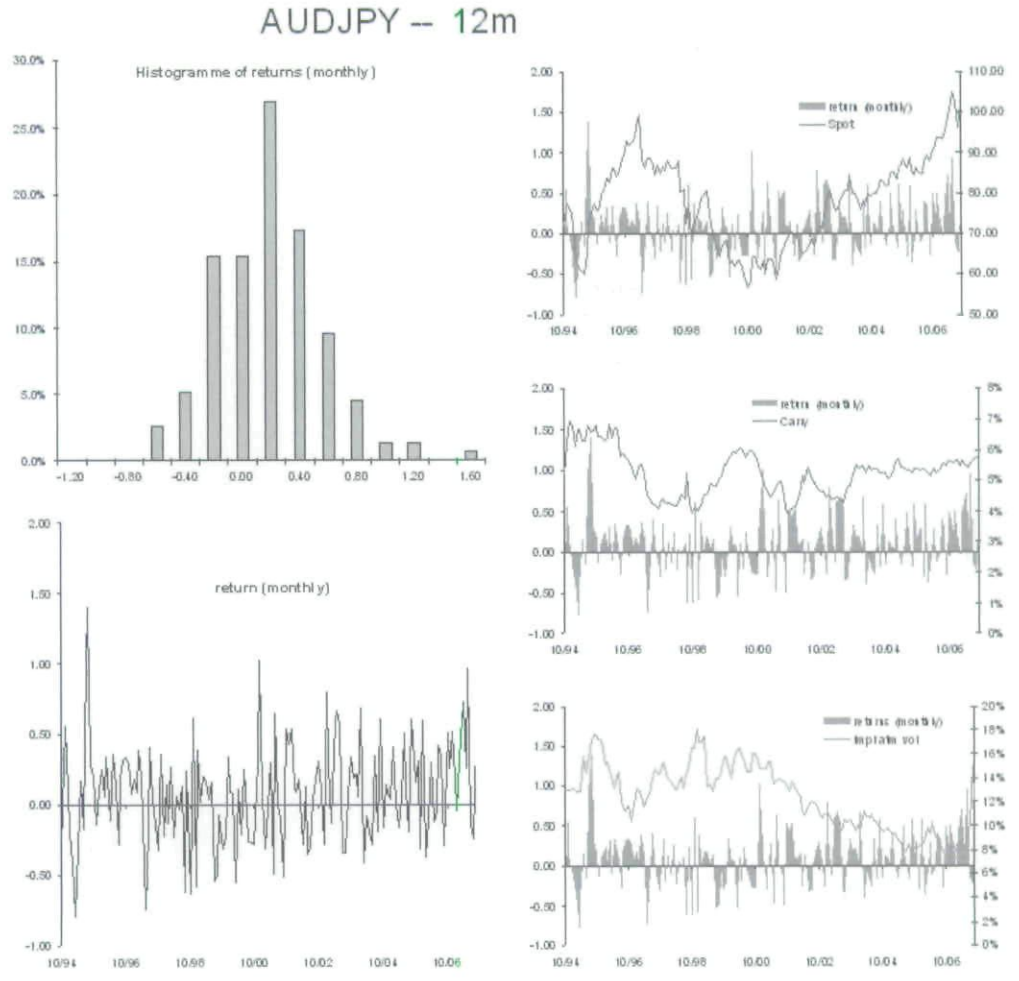
EXHIBIT 13

Statistics of AUD–JPY Carry Trade with Options

Bin	Frequency	
-1.20	0	0.0%
-1.00	0	0.0%
-0.80	0	0.0%
-0.60	4	2.6%
-0.40	8	5.1%
-0.20	24	15.4%
0.00	24	15.4%
0.20	42	26.9%
0.40	27	17.3%
0.60	15	9.6%
0.80	7	4.5%
1.00	2	1.3%
1.20	2	1.3%
1.40	0	0.0%
1.60	1	0.6%
More		0.0%

avg (annl)	1.02
stdev (annl)	1.27
IR	0.80
min (mthly)	-0.79
max (mthly)	1.41
skew	0.35
kurt (exc)	0.60

Carry Filter:	1.50%
Vol Filter:	----



right and the statistics below the table show the positive skew of the options-based implementation along with Sharpe ratios and annualized expected returns. We also display at the bottom left the carry and vol filters (if any) used in the backtest. The charts below the histogram show the monthly performance time series of the strategy. The three charts on the right show the options-based carry strategy return vs. spot levels for the pair, returns vs. the carry for the pair, and returns vs. the implied volatility for the pair. The AUD–JPY and NZD–JPY pairs over the period observed (NZD–JPY data is only available from 2002) show the highest information ratio, presumably because the carry over this period has been high. The NZD, in particular, has had very high short-term rates due to the explicit inflation-targeting nature of the central bank. It is obvious why in recent episodes of credit driven risk unwinds the AUD and NZD have been the first to sell off relative to the JPY and USD.

CONCLUSION

In this article we explored the connection between carry and volatility levels, and find both theoretical and empirical evidence supporting a positive relationship between volatility and carry. We explore whether the breakdown of the relationship, even loosely, can be used to implement options-based carry strategies that yield a high information ratio and favorable distributions of returns. We find that this is indeed the case.

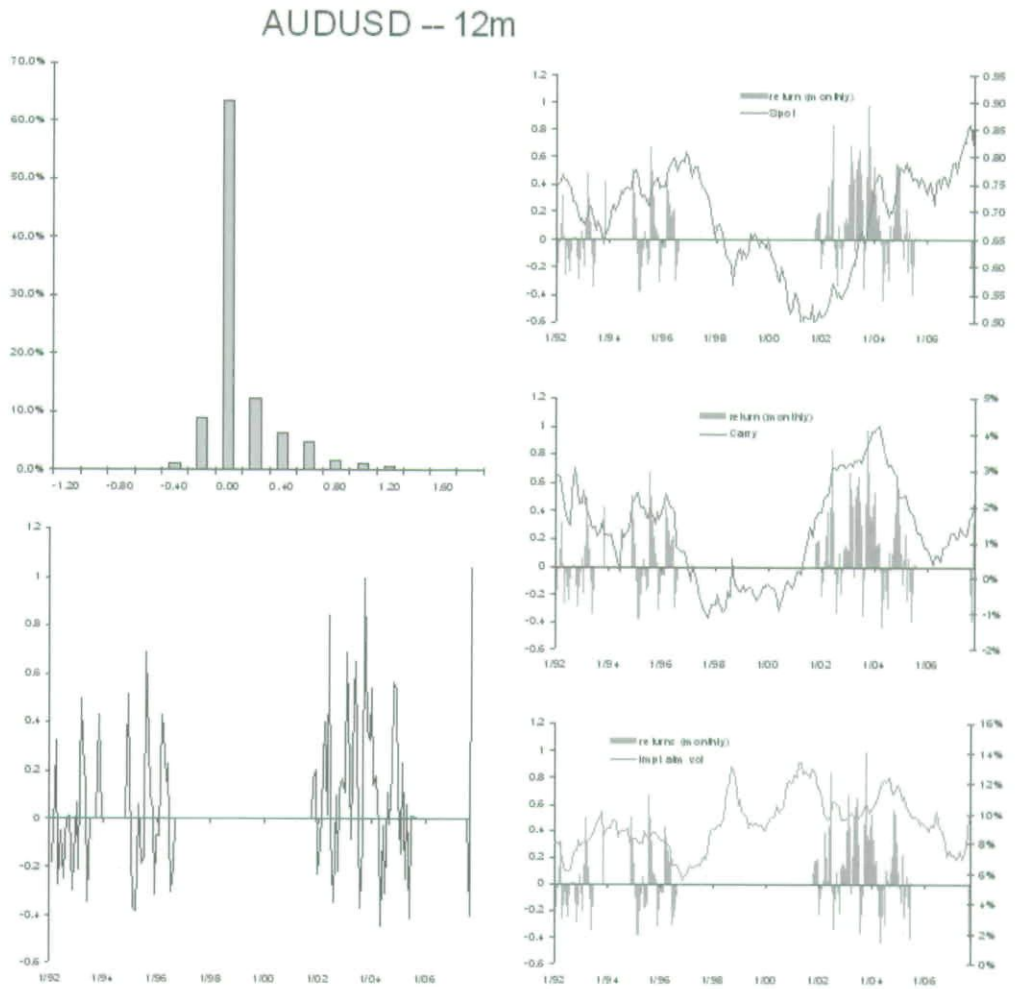
EXHIBIT 14

Statistics of AUD–USD Carry Trade with Options

Bin	Frequency	
-1.20	0	0.0%
-1.00	0	0.0%
-0.80	0	0.0%
-0.60	0	0.0%
-0.40	2	1.1%
-0.20	17	9.0%
0.00	120	63.5%
0.20	23	12.2%
0.40	12	6.3%
0.60	9	4.8%
0.80	3	1.6%
1.00	2	1.1%
1.20	1	0.5%
1.40	0	0.0%
1.60	0	0.0%
1.8	0	0.0%

avg (annl)	0.48
stdev (annl)	0.79
IR	0.61
min (mthly)	-0.45
max (mthly)	1.04
skew	1.52
kurt (exc)	4.44

Carry Filter:	1.50%
Vol Filter:	----



Note the period in the middle of the chart at the bottom where the carry filter of 1.5% kept investors out of the carry trade.

We leave it to further research to explore more thoroughly why such opportunities occasionally present themselves in today's markets. One possible reason is that the currency markets, which are macro in nature, have discounted the desire of central bankers to keep exchange rate volatilities low, while the rate markets respond more to the borrowing and lending needs of participants within their own countries. Whether this is the correct story or not remains to be seen, but it does seem no obvious instruments exist to exploit the differential risk-premium pricing between the rate and FX markets. Currency options, however, do provide a mechanism to weakly arbitrage gross mispricing of risk in these markets.

Indeed, evidence shows that when option volatilities become cheap, levered hedge funds implement carry trades through options. Just the presence of cheap options might drive the carry trade to be a good risk-reward opportunity in most states of the world, and when done through options, creates artificial directional biases due to the residual delta.

Another possibility is the action of funds that follow dynamic option replication strategies. If, instead of purchasing options, a large number of participants set their strategy-scaling rules based on the combined inputs of carry and implied volatility, the effective deltas would impact the direction of the market.

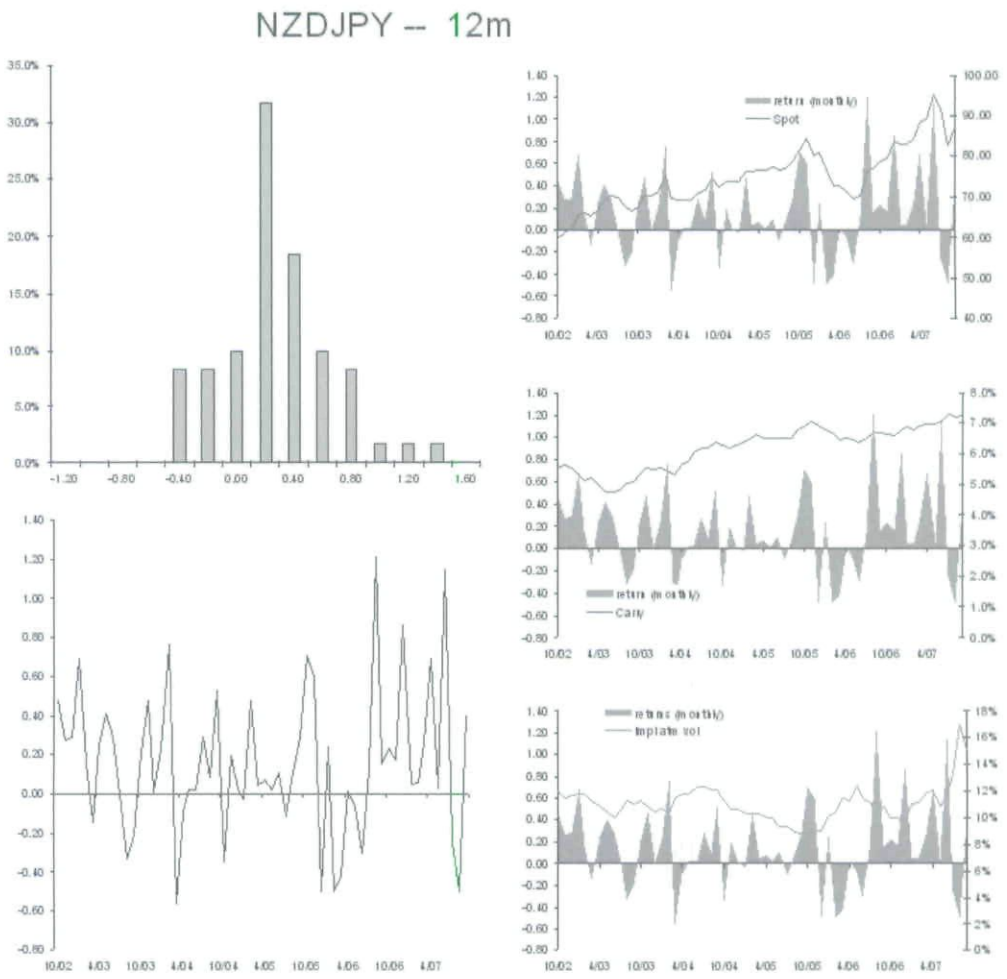
EXHIBIT 15

Statistics of NZD–JPY Carry Trade with Options

Bin	Frequency	
-1.20	0	0.0%
-1.00	0	0.0%
-0.80	0	0.0%
-0.60	0	0.0%
-0.40	5	8.3%
-0.20	5	8.3%
0.00	6	10.0%
0.20	19	31.7%
0.40	11	18.3%
0.60	6	10.0%
0.80	5	8.3%
1.00	1	1.7%
1.20	1	1.7%
1.40	1	1.7%
1.60	0	0.0%
More	0	0.0%

avg (annl)	1.88
std ev (annl)	1.31
IR	1.44
min (mthly)	-0.56
max (mthly)	1.21
skew	0.50
kurt (exc)	0.85

Carry Filter:	1.50%
Vol Filter:	----



Arguably all of these explanations are speculative and deserve more thorough empirical investigation which is challenging due to the proprietary nature of many such strategies. It would also be interesting to explore the impact of volatility skews (as measured by currency risk reversals) and the cross-currency basis swap on the performance of the carry trade. We leave these for future studies.

ENDNOTES

The author would like to thank his colleague Michael Althof for excellent research assistance.

[1]Data was provided by Citibank from January1992 to October 2007.

REFERENCE

Hull John. *Options,Futures, and Other Derivatives*. Upper Saddle River, NJ: Prentice Hall, 2006.

EXHIBIT 16

Statistics of GBP–CHF Carry Trade with Options

Bin	Frequency	
-1.20	0	0.0%
-1.00	0	0.0%
-0.80	0	0.0%
-0.60	2	1.1%
-0.40	14	7.4%
-0.20	28	14.8%
0.00	63	33.3%
0.20	34	18.0%
0.40	21	11.1%
0.60	10	5.3%
0.80	6	3.2%
1.00	5	2.6%
1.20	3	1.6%
1.40	1	0.5%
1.60	2	1.1%
More		0.0%

avg (annl)	0.84
stdev (annl)	1.33
IR	0.48
min (mthly)	-0.79
max (mthly)	1.50
skew	1.23
kurt (exc)	2.25

Carry Filter:	1.50%
Vol Filter:	----

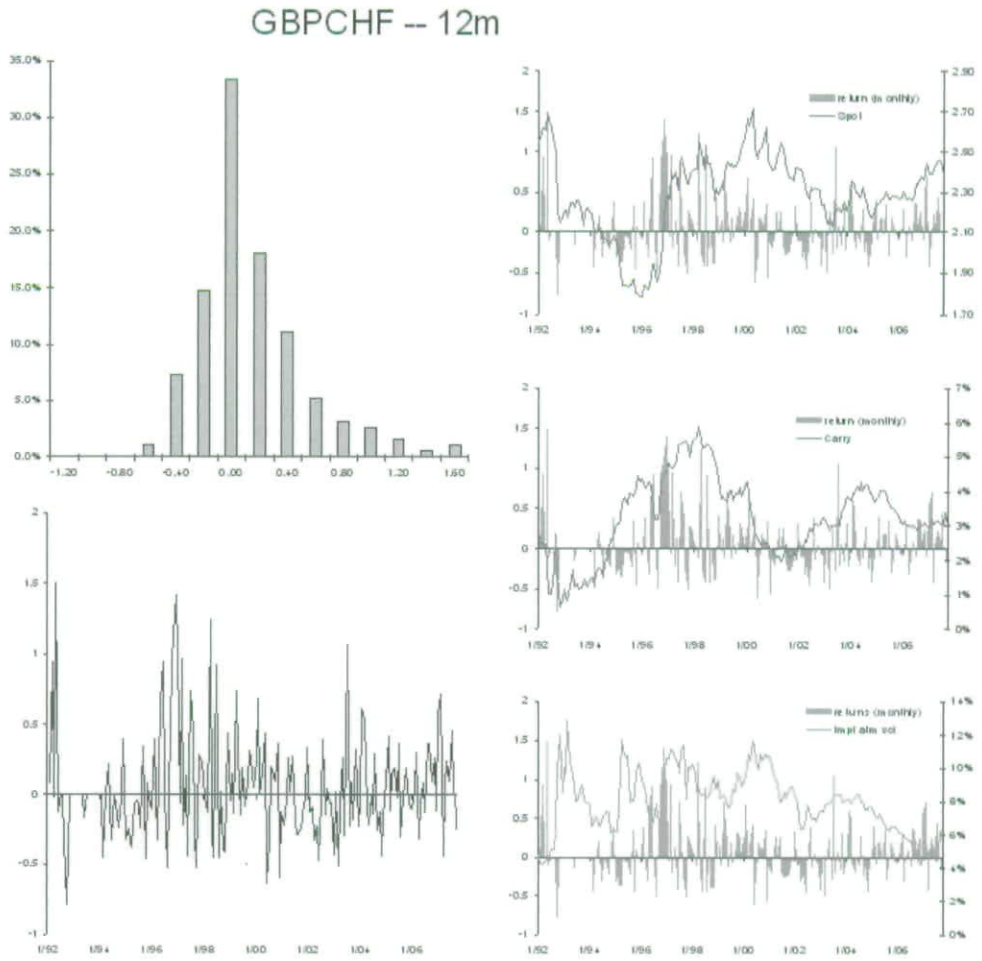


EXHIBIT 17

Statistics of USD–MXN Carry Trade with Options

Bin	Frequency	
-1.20	0	0.0%
-1.00	0	0.0%
-0.80	0	0.0%
-0.60	0	0.0%
-0.40	2	1.4%
-0.20	21	15.0%
0.00	41	29.3%
0.20	46	32.9%
0.40	27	19.3%
0.60	2	1.4%
0.80	1	0.7%
1.00	0	0.0%
1.20	0	0.0%
1.40	0	0.0%
1.60	0	0.0%
More		0.0%

avg (annl)	0.10
stdev (annl)	0.78
IR	0.13
min (mthly)	-0.46
max (mthly)	0.71
skew	0.16
kurt(exc)	-0.24

Carry Filter:	1.50%
Vol Filter:	---

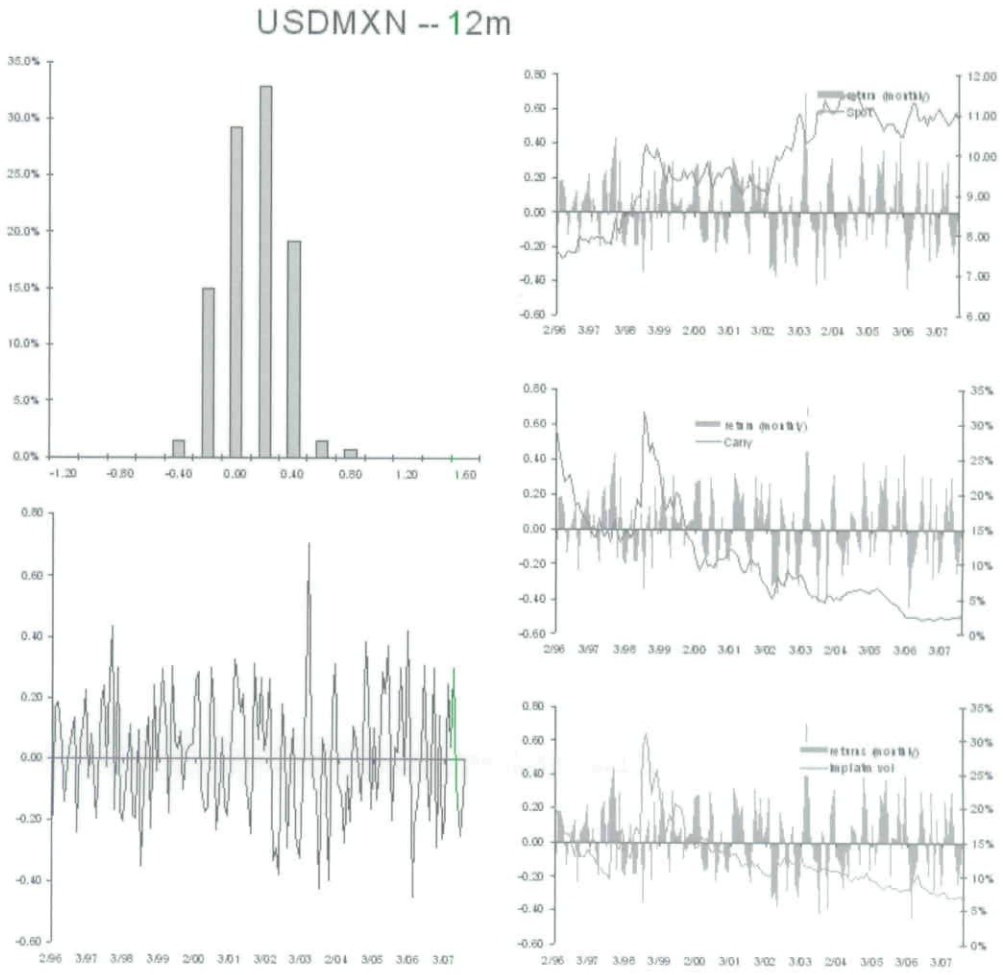


EXHIBIT 18

Statistics of USD–BRL Carry Trade with Options

Bin	Frequency	
-1.20	0	0.0%
-1.00	0	0.0%
-0.80	0	0.0%
-0.60	1	0.9%
-0.40	3	2.6%
-0.20	18	15.4%
0.00	35	29.9%
0.20	31	26.5%
0.40	17	14.5%
0.60	8	6.8%
0.80	1	0.9%
1.00	2	1.7%
1.20	1	0.9%
1.40	0	0.0%
1.60	0	0.0%
More		0.0%

avg (annl)	0.50
stdev (annl)	1.02
IR	0.49
min (mthly)	-0.61
max (mthly)	1.00
skew	0.62
kurt (exc)	0.94

Carry Filter:	0.00%
Vol Filter:	---