Optimal Leveraging of Fixed Income Portfolios with Interest Rate Structured Products

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Interest rate structured products are often described as useful leveraging and diversification instruments for fixed income investors. However, the question of which structured products should be used in practice to optimally leverage a fixed income portfolio, as well as the optimal combination of these products, is often left unanswered.

In this article, we tackle this issue for buy-and-hold investors with various investment horizons. For that purpose, we introduce a stochastic curve model based on two correlated Vasicek processes to estimate the joint probability distribution of the average coupons of several interest rate structured products. Neglecting reinvestment of coupon payments, the distribution of the average coupons can be used to approximate the joint distribution of returns over the entire investment period. Assuming an investor whose preferences can be described in terms of expected return and return volatility, we resort to Markowitz [1952] and Tobin's [1958] framework to optimally leverage a portfolio of fixed-coupon bonds with interest rate structured products. We show that an optimal combination of structured products exists in which investors should invest when leveraging a fixed income portfolio.

In *Selection of Interest Rate Structured Products in a Leverage Optimization Context,* we select and describe three instruments within the broad range of interest rate structured products, which together should improve the leverage of fixed income portfolios through diversification effects. In *Derivation of Expected Return and Risk of Interest Rate Structured Products,* we introduce the model used to estimate the joint probability distribution of average coupons. On the basis of this probability distribution, *Leverage Optimization* is devoted to the optimization of the portfolio of structured products and therefore to the optimization of the fixed income portfolio leverage. In *Further Risk Considerations*, we consider further sources of risk and their impact on the optimal portfolio previously identified. Together with a value-at-risk analysis, we perform what we call a "coupon at risk" analysis.

SELECTION OF INTEREST RATE STRUCTURED PRODUCTS IN A LEVERAGE OPTIMIZATION CONTEXT

In purely theoretical terms, any interest rate structure may be used to leverage a fixed income portfolio. In practice, selecting structures that have low correlations among them makes it possible to optimally leverage a bond portfolio using only a limited number of instruments. How to measure these correlations remains an open question, however.

Buy-and-hold investors are primarily interested in the return over the entire investment period. Neglecting reinvestment of coupon payments, this return can be approximated by the average coupon paid over the investment period. Consequently, buy-and-hold investors should be mainly interested in structured products whose average coupons are weakly correlated. For this reason, we have selected three interest rate products with intuitively low-correlated structured coupons: a reverse FRN, a Constant Maturity Swap (CMS) spread FRN, and a volatility bond. The features of these three instruments are described in Exhibit 1 for three investment horizons (5, 10 and 20 years).

Reverse FRNs usually pay a high fixed coupon minus a reference rate—for example, 6-month Euribor, 12-month Euribor or 1-year EUR swap rate. As a result, the coupon payments move conversely to the money market rates; the lower the reference rate, the higher the coupon.

CMS spread FRNs pay a multiple of the spread between two prespecified reference rates, such as the spread between the 10Y and the 2Y EUR swap rate. The coupon payments of CMS Spread FRNs are thus directly linked to the steepness of the swap curve at the fixing dates; the steeper the curve, the higher the corresponding coupon payment.

Volatility bonds typically pay a multiple of the absolute change in a given reference rate. A volatility bond,

EXHIBIT 1

Selected Interest Rate Structured Products for Leverage Optimization over Various Investment Horizons

Structure	Reverse FRN	CMS Spread FRN	Volatility Bond
Maturity	5 years	5 years	5 years
Coupon	Year 1: 3.86%	Year 1: 3.86%	Year 1: 3.86%
	Thereafter:	Thereafter:	Thereafter: 11.00 x (absolute
	7.75% - 2Y EUR swap p.a.	10.05 x (10Y– 2Y-EUR swap) p.a.	change in the 10Y EUR swap rate
	coupon ≥ 0%	coupon ≥ 0%	in the previous coupon period) p.a.
Payment period	Annually	Annually	Semi-annually
Fixing	At the beginning of the period	At the beginning of the period	At the beginning of the period
Issue price	100	100	100
Coupon driver	Money market	Steepness	Volatility
Maturity	10 years	10 years	10 years
Coupon	Year 1: 4.02%	Year 1: 4.02%	Year 1: 4.02%
	Thereafter:	Thereafter:	Thereafter:
	8.05% - 2Y EUR swap p.a.	9.20 x (10Y – 2Y EUR swap)	12.00 x interest rate spread p.a.
	coupon ≥ 0%	p.a.,coupon ≥ 0%	
Interest rate spread, payment frequency, fixing, issue price, and coupon driver as above			
Maturity	20 years	20 years	20 years
Coupon	Year 1: 4.22%	Year 1: 4.22%	Year 1: 4.22%
	Thereafter:	Thereafter:	Thereafter:
	8.40% - 2Y EUR swap p.a.	9.15 x (10Y – 2Y EUR swap)	13.65 x interest rate spread p.a.
	coupon ≥ 0%	p.a.,coupon ≥ 0%	
Interest rate spread, payment frequency, fixing, issue price, and coupon driver as above			

Source: WestLB Research – September 8, 2006.

EXHIBIT 2
Model Calibration

	10 Y Swap Rate	2 Y Swap Rate
Long-term equilibrium	$\theta = 4.34$	$\theta' = 3.51$
Mean reversion parameter	$\varphi = 0.22$	$\varphi' = 0.29$
Volatility parameter	$\sigma = 50\%$	$\sigma' = 64\%$
Correlation	$\rho = 78\%$	

Source: WestLB Research – September 8, 2006

for instance, may pay several times the absolute change in the 10Y EUR swap rate over a predefined period. The greater the volatility of the reference rate, the greater the probability of high coupon payments.

DERIVATION OF EXPECTED RETURN AND RISK OF INTEREST RATE STRUCTURED PRODUCTS

As previously mentioned, the return on investment for buy-and-hold investors can be approximated by the average coupon paid over the entire investment period. It follows that the joint probability distribution of the average coupon of the various structured products is a suitable approximation of the joint probability distribution of returns.

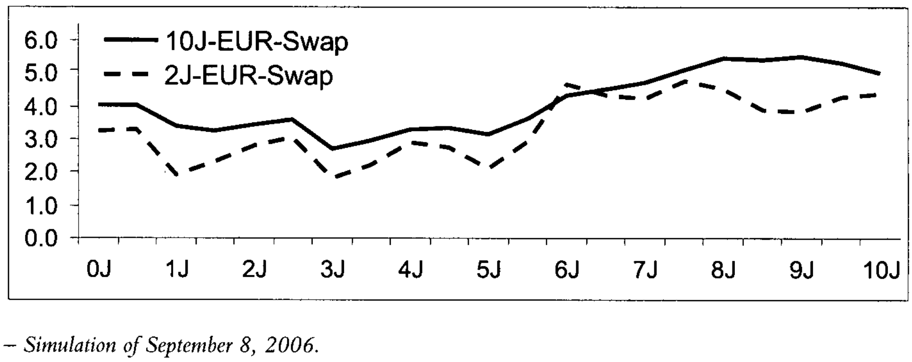
Since the coupon payments of the three aforementioned structured products depend on the behavior of the 2Y and 10Y EUR swap rates over the investment period, there is a clear case for a curve model. In order to reflect reality adequately, this model should take account of the following aspects: 1) Continuous increases (or decreases) in the swap rates should be rather unlikely in the long term. Over long periods, swap rates are more likely to move around a certain level (so-called mean reversion characteristic). 2) The volatility of the swap rates should be foregrounded in the model as it—among other things—directly influences the coupon payments and hence the performance of the Volatility Bond. 3) The model also should focus on the correlation between the 2Y and 10Y EUR swap rates, because of the coupon payments of the CMS Spread FRN, which are directly related to the 2Y as well as the 10Y swap rate.

We therefore suggest resorting to two correlated Vasicek processes in order to model the behavior of the two swap rates. The semiannual changes in the 10Y EUR swap rate $Y_{10}(t)$ are modeled as follows:

$$Y_{10}(t+1) - Y_{10}(t) = \varphi\,(\theta - Y_{10}(t)\,) + \sigma\,\varepsilon\,(t)$$

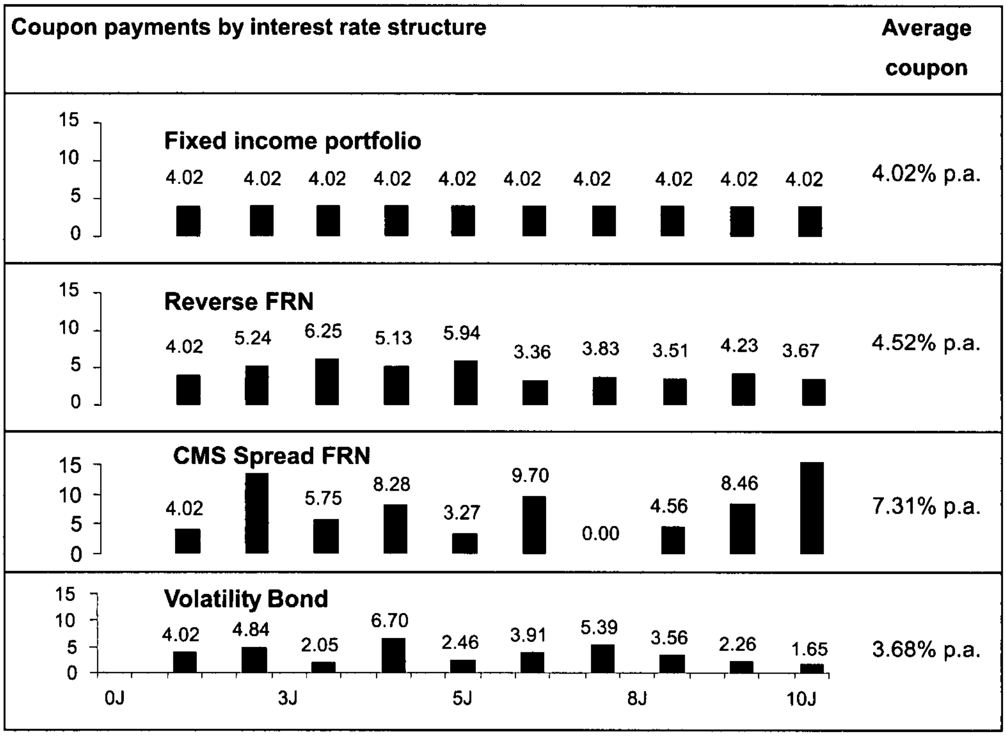
where θ is the long-term level of the 10 Y EUR swap rate, φ a mean reversion parameter, σ a volatility parameter, and

EXHIBIT 3
Example of a Simulation: 10Y and 2Y EUR Swap Rates over 10 Years



Source: WestLB Research – Simulation of September 8, 2006.

EXHIBIT 4
Example of Simulation: Coupon Payments



Source: WestLB Research – Simulation carried out on September 8, 2006.
By way of comparison, the coupon payments of the Volatility Bond are aggregated on a yearly basis.

$\varepsilon(t)$ a normally distributed shock with variance 1 and expectation 0.

The semiannual changes in the 2 Y EUR swap rate $Y_2(t)$ are modeled similarly:

$$Y_2(t+1) - Y_2(t) = \varphi' \left(\theta' - Y_2(t) \right) + \sigma' \varepsilon'(t)$$

where θ' is the long-term level of the 2Y EUR-swap rate, φ' a mean reversion parameter, σ' a volatility parameter, and $\varepsilon'(t)$ a normally distributed shock with variance 1 and expectation 0, such that:

$$\mathrm{Corr}(\varepsilon(t), \varepsilon'(t)) = \rho$$

The model is calibrated with the help of the swap history of the last 10 years. The results of the calibration are summarized in Exhibit 2.

After calibrating the model, we simulate the behavior of the swap curve (2Y and 10Y rates) over the investment period. Exhibit 3 shows an example of such a simulation for an investment horizon of 10 years. The coupon profiles of the three stated structured products are plotted in Exhibit 4 for the curve scenario simulated in Exhibit 3. For the sake of comparison, the payments from a risk-free investment also are shown. In the case of an investment horizon of 10 years, a fixed income portfolio consisting of 10Y fixed-coupon bonds with a negligible default risk is deemed to be risk-free. An alternative risk-free investment is a fixed receiver position in a 10Y swap contract. The average coupon of such a position is simply the 10Y swap rate, i.e., 4.02% at the time this analysis was carried out (September 8, 2006).

This simulation is reiterated 3,000 times in order to determine the expected average coupon and its volatility. Exhibit 5 summarizes the results. A higher risk (measured in terms of the volatility of the average coupon) is rewarded with a higher return, i.e., with a larger expected average coupon. This confirms that interest rate structures are suitable for leveraging fixed income portfolios, as illustrated in Exhibit 6.

EXHIBIT 5
Return-Risk Profile

	Fixed income portfolio	Reverse FRN	CMS Spread FRN	Volatility Bond
Return: Expected average coupon	4.02	4.47	7.26	4.91
Risk: Volatility of average coupon	0.00	0.43	2.29	0.83

Source: WestLB Research – Calculations of September 8, 2006.

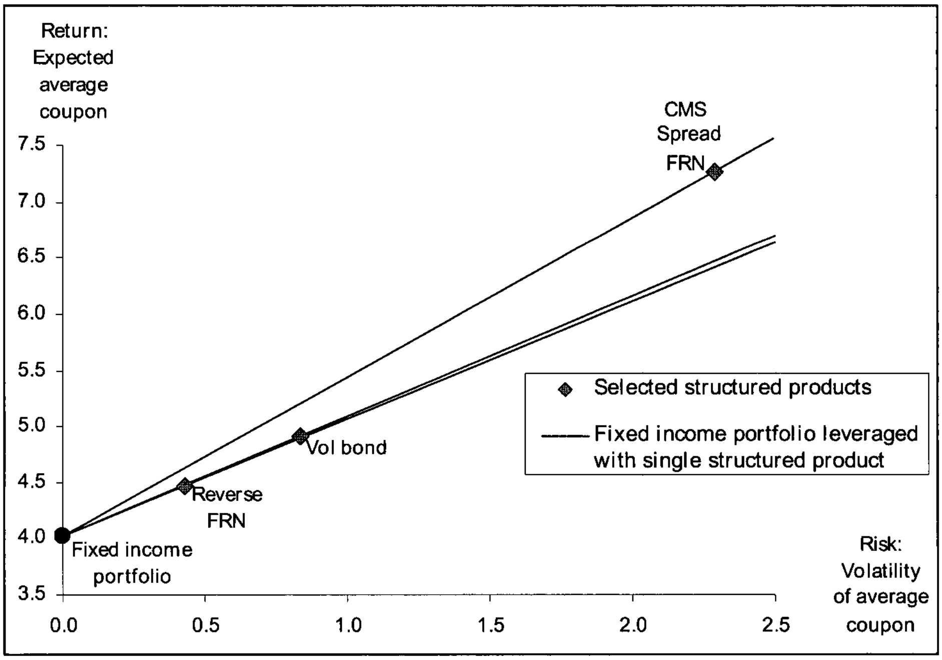
The correlation analysis is even more interesting within the portfolio diversification context (see Exhibit 7). As expected when selecting the interest rate structured products, the instruments chosen have low correlations; the correlations between the average coupons of the corresponding structures do not exceed 42%. In addition, there is almost no correlation between the average coupon of the reverse FRN and the Volatility bond, or between the CMS Spread FRN and the Volatility bond.

LEVERAGE OPTIMIZATION

Based on Tobin's separation theorem [1958], the optimization process as used in this study is divided into two stages. First, we use Markowitz's approach [1952] to derive an optimal portfolio of interest rate structured products. This is then optimally leveraged with the existing fixed income portfolio, which can be regarded as risk-free.

Computing a minimum-risk portfolio of structured products for each level of expected return, we first identify a so-called efficiency curve. In other words, for every level of the expected average coupon, the efficiency curve

EXHIBIT 6
Leveraging of a Fixed Income Portfolio with Single Interest Rate Structured Products



Source: WestLB Research – Calculations of September 8, 2006.

EXHIBIT 7
Correlation Analysis

	Reverse FRN	CMS Spread FRN	Volatility Bond
Reverse FRN	100%	42%	-4%
CMS Spread FRN	42%	100%	2%
Volatility Bond	-4%	2%	100%

Source: WestLB Research – Calculations of September 8, 2006.

shows the portfolio of structured products that has the least volatility as regards the average coupon. In a second step, the fixed income portfolio (•), still considered as risk-free, is leveraged with the tangent portfolio of structured products (♦). Both the efficiency curve and the leverage of the fixed income portfolio with the tangent portfolio are illustrated in Exhibit 8.

Since no steeper line crosses both the fixed income portfolio and one of the portfolios of structured products, we have identified the portfolio of interest rate structured products enabling the optimal leverage of the fixed income portfolio. We refer to this portfolio as the optimum portfolio of (interest rate) structured products. Exhibit 9 illustrates the improvement obtained in the leverage using the three selected structured products compared to the leverages involving only one of the structured instruments. Exhibit 10 shows the results for an investment horizon of 10 years, as well as the weights of the individual interest rate structured products for a 5-year and a 20-year investment horizon.

The only issue that remains is which combination of this optimum portfolio of structured products and the fixed income portfolio is optimal from an investor's point of view. Extremely risk-shy investors would invest exclusively in fixed income securities. If, on the other hand, investors are prepared to take a certain risk, then they should invest a portion of their fixed income portfolio in the optimum portfolio of interest rate structured products. Therefore, it is clear that the allocation in interest rate structured products within a fixed income portfolio depends on investors' risk appetite. Many investors define their risk appetite by setting a so-called risk limit that is simply their response to the question, "How much may the portfolio's average coupon differ from the expected average coupon?"

As a concrete example, let us take an investor with a risk limit of 0.15. This risk limit is shown in Exhibit 11 by the broken black line (— —). The optimum portfolio for this particular investor is located at the point of intersection between the risk limit and the solid line showing the optimum leverage (—). It indicates an optimum allocation in interest rate structured products of about 24% (see Exhibit 11) and 76% in fixed income securities. It follows that an investor with a risk limit of 0.15 who invests 24% in structured products can expect an average coupon of 4.31% over 10 years, which is 29 bp more than the return of the fixed income portfolio (4.02%). It can further be shown that the same investor in 96.5% of the cases would get a larger return than investing in fixed income securities only. In the longer term, a higher allocation in interest rate structured products may be recommended. Exhibit 12 summarizes these results.

FURTHER RISK CONSIDERATIONS

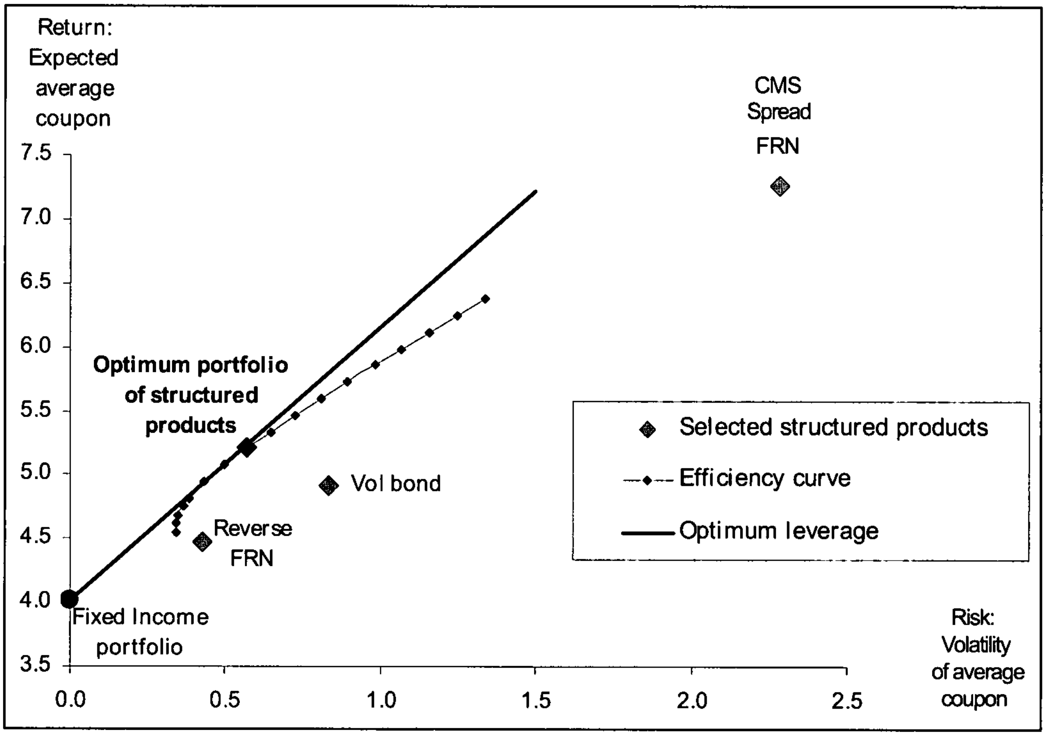
In our analysis, the optimization of the fixed income portfolio of buy-and-hold investors is based on expected returns and return volatilities, i.e., on expectations and volatilities of average coupons. This does not mean, however, that the volatility of the individual coupon payments or the market risk of those investments can be entirely ignored.

For many investors, it's not only the long-term performance that counts, but also the annual coupon payments. Since the lowest possible coupon for the three structures considered is zero each year, it is not possible to differentiate among these structured products in terms of coupon risk. For this reason, we introduce the concept of "coupon at risk" or CaR. The 95% CaR of an investment is the lowest coupon paid in 95% of the most favorable cases. For example, how great is the uncertainty regarding the second coupon payment of the Reverse FRN? In 95% of the cases, it is more than 3.14%. Exhibit 13 summarizes the 95% CaRs for all coupon dates and structured products considered.

A glance at Exhibit 13 reveals that the risk of low coupon payments for individual interest rate structured

EXHIBIT 8

Optimal Portfolio of Interest Rate Structured Products for an Investment Horizon of 10 Years



Source: WestLB Research – Calculations of September 8, 2006.

products should not be underestimated. For example, each year the coupon of the CMS Spread FRN is zero in over 5% of the simulations. In the case of the Volatility Bond, the second coupon may be below 1.20% in 5% of the cases, while for the Reverse FRN, the second coupon is above 3.14% in 95% of the simulations.

Because of the positive diversification effects, the minimum coupon of the optimum portfolio of interest rate structured products is between 2.92% and 2.97% in 95% of the cases (depending on the year). For an investor with a risk limit of 0.15, the minimum coupon is about 3.74%, with a probability of 95%. The risk of lower coupon payments thus remains extremely limited for the optimum portfolio of structures, as well as for the optimized investor portfolio. The underperformance of the investor optimum portfolio compared to an investment in fixed income securities is at most around 28 bp in 95% of the cases.

To account for the market risk of such a strategy, we also perform a value-at-risk analysis. The results presented here are based on the historical method using one year of market data. The horizon for the computation is set to one day, and the confidence level is set to 95%. Bp volatilities are assumed to have remained constant over time. The results are presented in Exhibit 14.

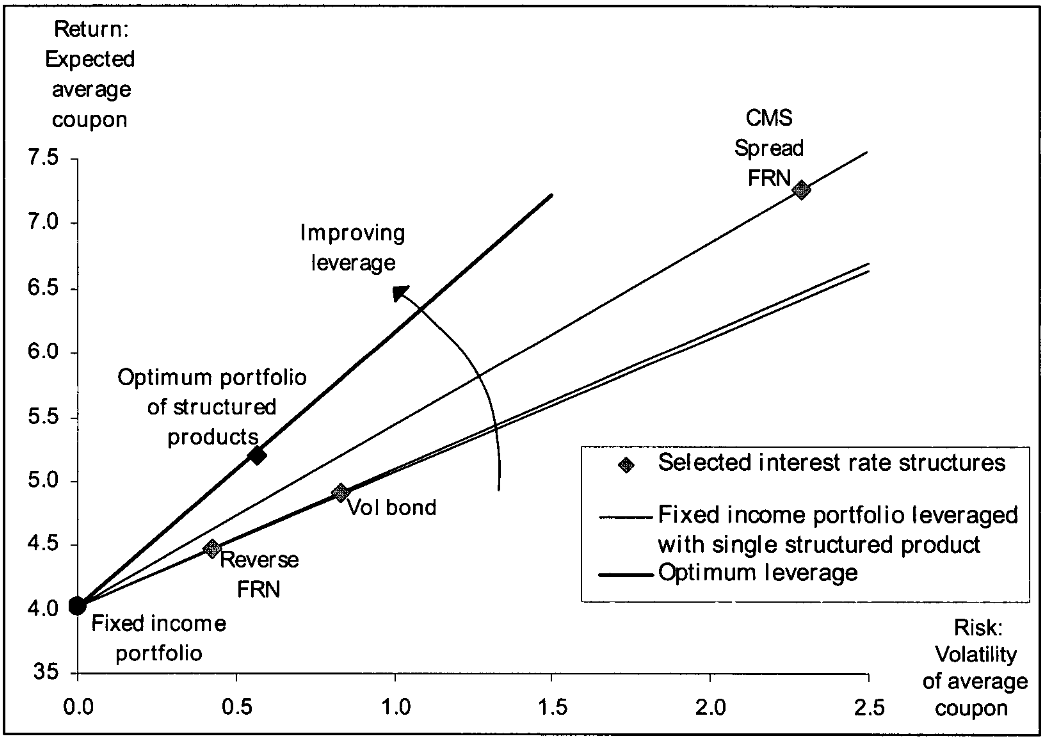
Over a 10-year horizon, for example, the VaR of the individual structures is between 0.53% and 0.94%. A VaR of 0.94% for the Reverse FRN, for example, means that in only 5% of the simulations the loss of value within a trading day is above 0.94%. The VaR of the optimum portfolio of structures is 0.76%. The fixed income portfolio, which is risk-free as regards the coupon payments only and is subject to market fluctuations, has a VaR of 0.48%. Finally, the VaR of the optimum portfolio of an investor with a risk limit of 0.15 is just slightly higher than the VaR of the fixed income portfolio and amounts to 0.56%.

CONCLUSION

In this study we have suggested a quantitative approach to optimally leverage fixed income portfolios using interest rate structures. We have shown that a significant increase

EXHIBIT 9

Leverage Improvement Using a Portfolio of Interest Rate Structured Products



Source: WestLB Research – Calculations of September 8, 2006.

in the expected return of a fixed income portfolio can be achieved when investing in structured products, with only a minor increase in risk. For instance, on a 10-year horizon, investors can expect 29 bp additional return when investing 24% of their portfolio in structured products, with a risk of underperforming the risk-free benchmark in only 3.5% of the cases. Furthermore, we have shown that the optimal portion that investors should invest in interest rate structured products is an increasing function of their risk appetite and investment horizon.

We have deliberately limited the scope of our study to simple interest rate structures. Fixed income portfolios would certainly benefit from diversification in additional interest rate structured products. For instance, callable

EXHIBIT 10

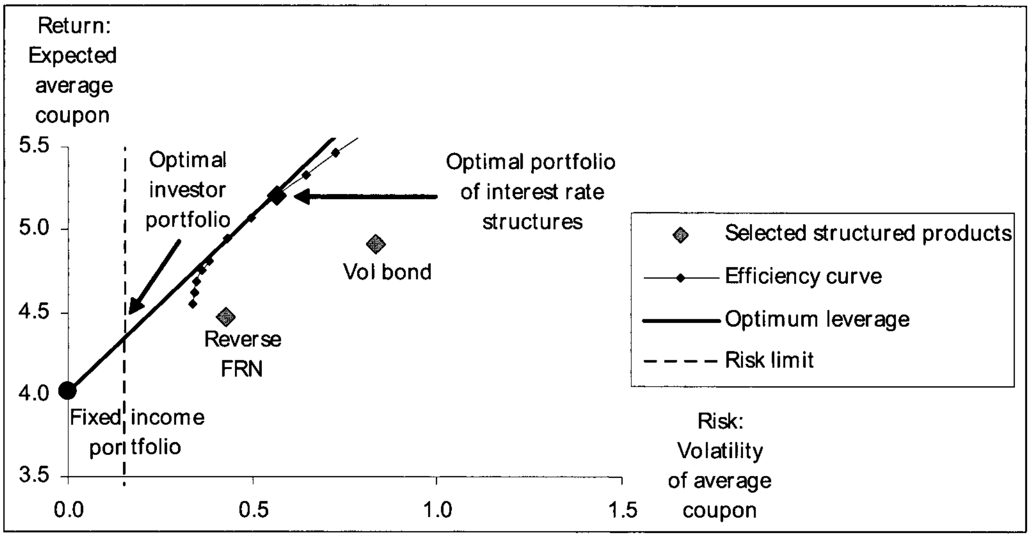
Optimal Portfolio of Interest Rate Structured Products for Various Investment Horizons

Investment horizon	5 years	10 years	20 years
Reverse FRN	31%	43%	45%
CMS Spread FRN	26%	18%	12%
Volatilit y Bond	43%	39%	43%
Optimum portfolio of structures	100%	100%	100%

Source: WestLB Research – Calculations of September 8, 2006.

EXHIBIT 11

Optimal Fixed Income Portfolio after Addition of Interest Rate Structured Products for an Investment Horizon of 10 Years



Source: WestLB Research – Calculations of September 8, 2006.

instruments could be added to the scope of this research but that would require pricing the callable structures at each call date, so that not only the dynamic of the swap curve but also the dynamic of the volatility cube would have to be modeled.

Further studies also could include credit, inflation, equity, or hybrid structures. This requires a stochastic model accounting for the dynamic of interest rates, credit spreads, inflation, and equity. A challenging issue!

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EXHIBIT 12

Optimal Fixed Income Portfolio for Various Investment Horizons Investment Horizons

Investment horizon	5 years	10 years	20 years
Optimal portfolio of interest rate structured products	81%	76%	55%
Risk-free bonds	19%	24%	45%
Optimal investor portfolio (risk limit = 0.15)	100%	100%	100%

Source: WestLB Research – Calculations of September 8, 2006.

Exhibit 13
Coupon at Risk (95%) for an Investment Horizon of 10 Years

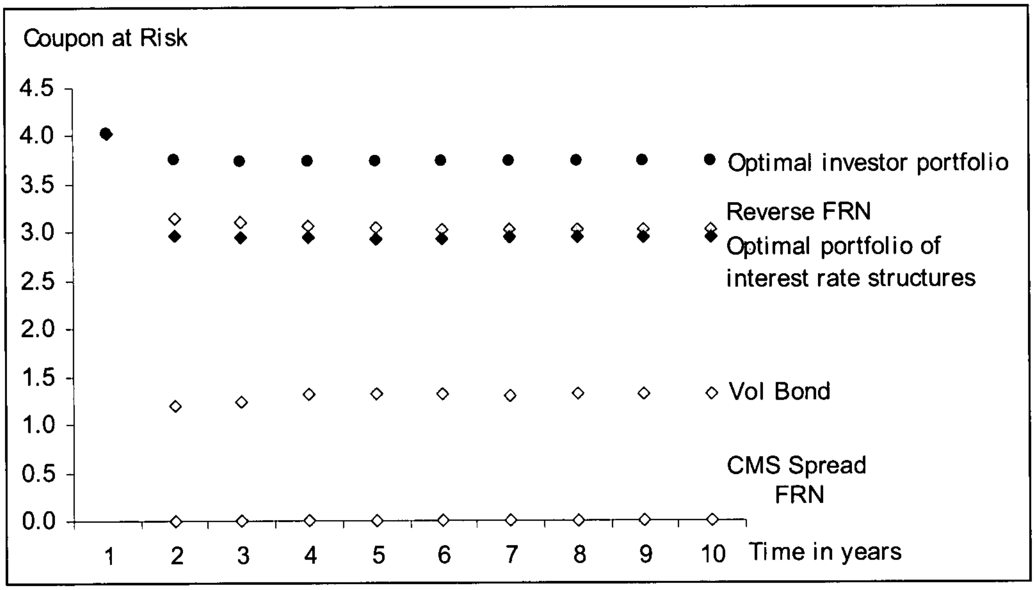

Exhibit 14
Value at Risk

Maturity	5 years	10 years	20 years
Reverse FRN	0.53%	0.94%	1.57%
CMS Spread FRN	0.81%	0.90%	1.04%
Volatility Bond	0.23%	0.53%	0.84%
Optimum portfolio of structures	0.41%	0.76%	1.15%
Fixed income portfolio	0.25%	0.48%	0.81%
Optimum investor portfolio (risk limit = 0.15)	0.28%	0.56%	0.96%

Source: WestLB Research – Calculations September 8, 2006.