International Bond Market Cointegration Using Regime Switching Techniques

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This article considers the degree of international bond market integration using daily data over a sample period from 1994 to 2006 for the U.S., German, Japanese, U.K., Swiss and Canadian sovereign bond markets. Using modern cointegration techniques, the analysis demonstrates that these bond markets appear to share a common trend that is subject to structural change.

Interest in market integration issues has until recently focused upon equity market comovements. Hence, while a substantial literature has developed analyzing equity market integration, very few studies have considered the possibility of bond market cointegration. The few exceptions to this are the studies by Clare et al. [1995]; Smith [2002], and Yang [2005]. This article adds to this small literature by highlighting the importance of allowing for structural change using a data set sampled on a daily basis.

Exhibit 1 plots the respective bond index histories for the markets considered in the analysis. The index data exhibit an upward trend through time and are clearly non-stationary, a feature that is formally tested later. One other interesting characteristic is that several series appear to experience sudden breaks through the sample period. Indeed daily price changes of 2% are common throughout the sample period considered here. Exhibit 1 also demonstrates that individual markets are capable of behavior that deviates from that of the aggregate market as a whole. For example the U.S. market appears to under-perform for the early 1990s while the Japanese market out-perfoms until 2001. The U.K. market is seen to closely follow the aggregate market until 2001 but since then has experienced a period of relative underperformance.

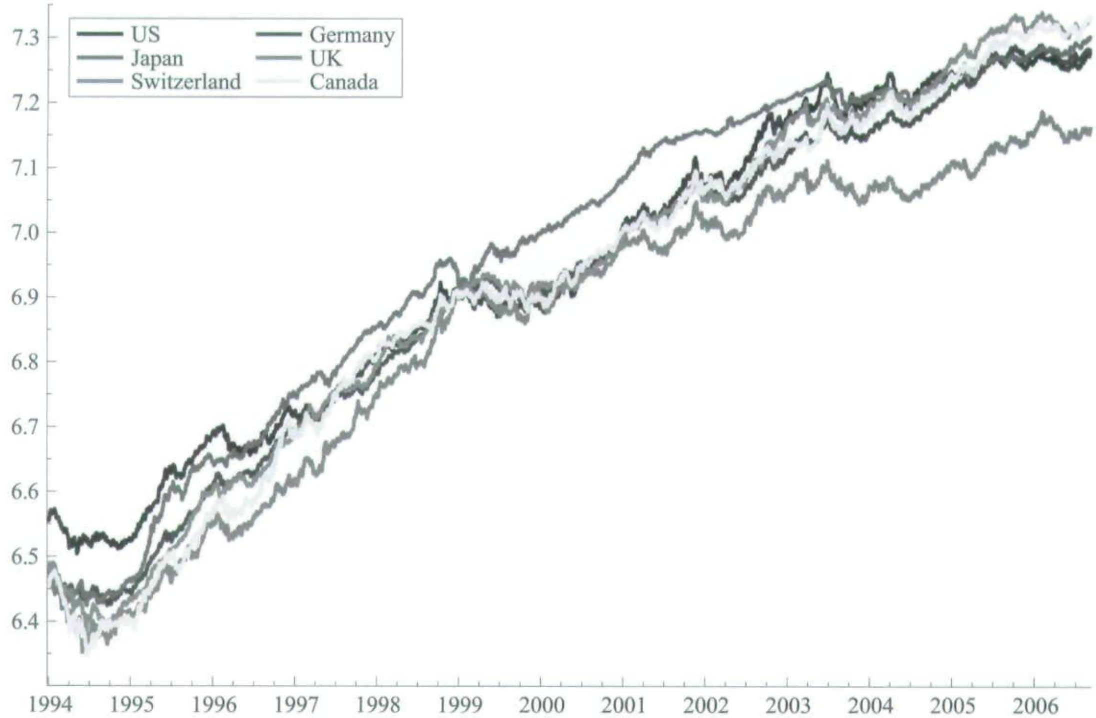
Exhibit 1 demonstrates that while all six markets are clearly trending upwards, it is also the case that individual markets exhibit varying degrees of idiosyncratic behavior for certain subperiods in the data history. The purpose of this article is to assess the importance of allowing for structural change through time in the central long run equilibrium.

MOTIVATION

The issue of market integration using cointegration techniques has a long history in the equity market literature. Studies such as Kasa [1992]; Corhay et al. [1993]; Richards [1995]; Blackman, Holden, and Thomas [1994]; Masih and Masih [2002]; Li [2006], and Davies [2006] all report results that assess market integration issues using cointegration techniques. It is perhaps surprising then to find that relatively little work has been done to address the issue of market integration across international bond markets. One of the earliest attempts to examine bond market cointegration was Clare et al. [1995] who highlighted

EXHIBIT 1

MSCI Bond Indices January 1994 to August 2006



the need to consider the long run comovement of markets in assessing the degree of bond market integration. They considered a long run equilibrium across bond markets for the U.S., U.K., German and Japanese markets using Salomon Brothers total return bond index data from January 1978 to April 1990 at a monthly frequency. Using an Engle Granger two step approach, they find no evidence in favor of bond market integration over the 1980s and conclude that the benefits of international bond market diversification were significant over that period.

More recently Smith [2002] highlighted the fact that the presence of cointegration across markets violates traditional market efficiency arguments since cointegration implies some measure of market return predictability. In effect, cointegration implies that information from one bond market can be used to predict the future direction of another market. Smith also used Salomon bond index data and, in addition to the markets considered by Clare et al. [1995], used bond index data for Canada and France. Using the more advanced Johansen [1988] system-based cointegration technique, Smith employed a monthly data history from February 1985 through to March 1999. Results indicate that for the six markets analysed there are three significant vectors, thereby confirming the presence of cointegration. Smith concluded that there is strong evidence in favor of bond market integration over the late 1980s and 1990s.

Finally, Yang [2005] considered the European experience over 1988 to 2003 using JP Morgan bond index data, again, on a monthly basis. The analysis adopted the Johansen and Juselius [1992] recursive estimation approach for Germany, France, Italy, the U.K., Belgium and the Netherlands. Yang contradicted the findings of Smith [2002] but supported those of Clare et al. [1995] by confirming the absence of cointegration across those six markets. Yang attributed the lack of a common trend across these markets by suggesting the existence of barriers to entry to international bond markets such as differing maturity structures, or perhaps insufficient macroeconomic policy co-ordination during the sample period. Yang therefore concluded that substantial long run diversification possibilities existed across the six markets analysed up to 2003.

With previous work in mind, this article extends the existing literature by using recently developed non-linear cointegration techniques to a daily bond data set that covers the last 12 years. Recently, interest in non-linear cointegration has been sparked by articles such as Breitung and Gourieroux [1997]; Breitung [2001], and Gabriel, Sola, and Psaradakis [2002]. Using equity index data, Li [2006] employs cointegration rank tests developed by Breitung [2001] to test for cointegration across equity markets for Australia, Japan, New Zealand, the U.K., and U.S. Considerable evidence is found in favor of market integration conditional upon a non-linear cointegrating long run relation.

Gabriel, Sola and Psaradakis [2002], in a related equity market application, show that regime switching cointegration is relatively simple to apply using techniques introduced by Dempster, Laird, and Rubin [1977], and extended by Hamilton [1990]. Davies [2006] applies those techniques to an equity index data set to find considerable support in favor of equity market integration conditional upon occasional structural change. The purpose of this article is to adopt a similar approach to determine the extent of bond market integration using data observed at a daily frequency over a 12 year period.

METHODOLOGY

The analysis begins by employing the widely used Engle Granger [1987] two stage technique to test for bond market cointegration. For the application considered here, the first step involves an ordinary least squares (OLS) regression of the log transformation of the level of the bond market index against either the aggregate world index or the five alternate markets analysed in the study. Hence for the U.S., in the case of the parsimonious specification, the first step involves estimating the following relation by ordinary least squares (OLS):

$$\log \text{US Index}_t = \alpha + \beta \cdot \log \text{World ex US Index}_t + \mu_t \quad (1)$$

For the more general specification the first step in the analysis involves estimating the following relation:

$$\begin{aligned}\log \text{US Index}_t = \alpha &+ \beta_1 \cdot \log \text{German Index}_t \\ &+ \beta_2 \cdot \log \text{Japanese Index}_t \\ &+ \beta_3 \cdot \log \text{UK Index}_t \\ &+ \beta_4 \cdot \log \text{Swiss Index}_t \\ &+ \beta_5 \cdot \log \text{Canadian Index}_t + \mu_t\end{aligned} \quad (2)$$

Testing for cointegration across the levels series involves performing an augmented Dickey-Fuller (ADF) test on the residuals $\hat{\mu}_t$ obtained from Equation (1) or (2) as in:

$$\Delta\hat{\mu}_t = \psi_0 \cdot \hat{\mu}_{t-1} + \sum_{i=1}^{n} \psi_i \cdot \Delta\hat{\mu}_{t-i} + \phi + \delta t + \omega_t \quad (3)$$

If the null hypothesis H_0: $\psi_0 = 0$ is rejected in favor of the alternative H_1: $\psi_0 < 0$ then the residual series contain a unit root and are therefore stationary I(0) The levels series are then cointegrated with cointegrating coefficients α and β.

Having obtained stationary residuals series $\hat{\mu}_{t-1}$ the second step involves estimating a further regression of the first difference transformations of the bond index series, including the lagged cointegrating relation represented by $\hat{\mu}_{t-1}$ as in:

$$\begin{aligned}\Delta \log \text{US Index}_t = \gamma &\cdot \Delta \log \text{US Index}_{t-1} \\ &+ \delta \cdot \Delta \log \text{World ex US Index}_{t-1} \\ &+ \pi \cdot \hat{\mu}_{t-1} + \varepsilon_t\end{aligned} \quad (4)$$

The size of the coefficient π on the estimated residual term $\hat{\mu}_{t-1}$ obtained from Equation (4) represents the speed of adjustment back towards equilibrium. Furthermore, a negative π coefficient implies that any deviation away from the long run equilibrium can be expected to be reversed over time.

The problems associated with the Engle Granger [1987] approach are well known, in particular the difficulties associated with making valid inferences in the first step of the procedure if the levels series are indeed non-stationary. The purpose of this analysis is to highlight a further problem in that the analysis assumes a single relation to hold over the long run. As already mentioned, financial data are subject to abrupt structural change over time and this seems particularly to be the case with the bond index data considered here. In order to address this issue I propose employing a regime switching approach introduced by Gabriel, Sola, and Psaradakis [2002] that allows for periodic structural change in the long run equilibrium. The approach adopts the Markov-switching estimator developed by Hamilton [1990] and assumes that the regime generating process is governed by an unobservable latent variable s_t such that the long run equilibrium can be expressed as:

$$\log \text{U.S. Index}_t = \begin{cases}\alpha_1 + \beta_1 \cdot \log \text{World ex US}_t + \mu_t & \text{if } s_t = 1 \\ \alpha_2 + \beta_2 \cdot \log \text{World ex US}_t + \mu_t & \text{if } s_t = 2\end{cases} \quad (5)$$

Hence, in the case of the Markov-switching estimator, the residuals forming the error correction term in Equation (4) are computed as:

$$\hat{\mu}_t = \{y_t - [(\hat{\alpha}_1 + \hat{\beta}_1 \cdot \log \text{World ex US}_t) \cdot (Pr(S_t = 1|I_t)) + (\hat{\alpha}_2 + \hat{\beta}_2 \cdot \log \text{World ex U.S.}_t) \cdot (Pr(S_t = 2 \mid I_t))]\} \tag{6}$$

where $Pr(S_t = i \mid I_t)$ denotes the probability for regime i given information available at time t. The estimator employs the expectations maximisation (EM) algorithm developed by Dempster, Laird, and Rubin [1978] by recursively estimating the probability of each regime in each time period. With estimated regime probabilities in place, the α_i and β_i coefficients are then estimated from those observations deemed to apply for each regime. With new estimated coefficients a new set of regime probabilities are obtained with the algorithm repeating itself until convergence. The appendix provides a more detailed explanation of the EM algorithm as applied to regime switching estimation.

DATA

The data used in this article are taken from MSCI total return bond index data archive and relate to a basket of bonds derived from each market of maturities ranging from 1 to 30 years. Returns are converted into U.S. $ returns in an attempt to assess the merits of international bond index diversification from a U.S. $ perspective. In contrast to the existing literature, data are observed on a daily basis beginning in January 1994 and ending in August 2006 to give some 3300 daily observations. The analysis covers the U.S., German, Japanese, U.K., Swiss and Canadian bond markets and therefore considers bond market integration across the major developed fixed income markets. As previously mentioned, I consider market integration in both a parsimonious and general sense, and MSCI data are well suited for this purpose since World index data are constructed excluding a particular market's bonds. Hence, in the parsimonious specification the U.S. market is modeled as a function of the aggregate World index less the U.S. bond universe.

RESULTS

Exhibit 2 begins the analysis by reporting a set of ADF statistics for each of the bond index series used in the subsequent analysis. Evidently each index series is non-stationary I(1), but upon a first transformation becomes I(0) stationary. This result applies regardless of whether a deterministic trend is included within the ADF test specification detailed in Equation (3).

The analysis continues in Exhibits 3 and 4 by considering the parsimonious Engle Granger specification as

EXHIBIT 2
Bond Index ADF Tests

	Level		First Difference	
	Constant	Trend	Constant	Trend
U.S.	-0.787	-1.771	**-55.477****	**-55.474****
Germany	-1.209	-0.769	**-55.769****	**-55.780****
Japan	-2.439	-0.071	**-54.446****	**-54.537****
U.K.	-0.801	-1.127	**-56.077****	**-56.073****
Switzerland	-0.802	-1.180	**-46.166****	**-46.166****
Canada	-0.652	-1.609	**-52.889****	**-52.884****

*Statistical significance is highlighted in bold and is denoted by * and ** at the 5% and 1% levels respectively.*

presented in Equations (1) and (4). Exhibit 2 tests for stationarity of the residual term obtained from Equation (1). With the exception of the U.K., all residual series are I(1) leading to a rejection of the null hypothesis that the individual market and the respective world index are cointegrated and move with a common trend.

The problem here lies in the issues raised by Gregory and Hansen [1996] and Gabriel, Sola, and Psaradakis [2002]. Both articles argue that unit root tests of this sort exhibit a tendency to under reject the null hypothesis of an absence of cointegration with series that are subject to structural breaks. One solution to this problem then is to model those structural breaks explicitly and then use conventional unit root tests to test for cointegration upon the resultant residual time series. This article achieves that later by employing a Markov-switching estimator to replace the first step in the Engle Granger approach.

Exhibit 4 continues with OLS coefficient estimates from the Engle Granger two step procedure. While the aggregate World index appears to enter significantly for several markets, the explanatory power of the error correction equilibrium relation is mixed. The error correction term enters significantly for Canada, Switzerland and the U.K. However, statistical significance is marginal for the largest U.S., German and Japanese markets. Indeed, for Japan the long run equilibrium as proxied by the error correction term actually enters positively, implying that deviations from equilibrium can be expected to persist indefinitely through time. Overall explanatory power as reported in $\bar{R}^2$ statistics is mixed with only the Swiss specification being of reasonable significance.

The contrast of the Engle Granger single regime results with those obtained from the two regime Markov-switching results is marked. Exhibit 5 shows that allowing for periodic structural change within the long run equilibrium, where residuals are obtained from Equations (5) and (6), yields stationary residual series for all six markets. Exhibit 6 further demonstrates that every error correction term enters significantly with a consequent increase in overall explanatory power for every market analyzed in this article. Of particular note are the significant estimated long run equilibria for the largest markets.

In particular, the Japanese error correction term now enters with a significantly negative coefficient implying that deviations from equilibrium are now expected to to correct over the longer term.

Overall, the parsimonious specification highlights the need to allow for periodic structural change within the long run equilibrium. This is found to be particularly important for the largest markets. Furthermore, while a single regime treatment appears to adequately capture the long run equilibrium for smaller markets, it is also clear that allowing for a two regime treatment increases explanatory power for every market considered here.

The analysis is now extended to the more general specification where each market is modeled as a function of the other five markets, as in Equation (2). Exhibit 7 gives ADF test statistics for the residuals obtained from the Engle Granger procedure from Equation (2). With the exception of the U.S. market, all the residual series obtained from the richer general specification appear to be stationary at the 1% level of significance. Exhibit 8 continues with the OLS coefficient estimates from the second step in the Engle Granger procedure as in Equation (4). Here the residuals entering as the error correction term (ECT) are significant for every market except Japan and the U.S. For Japan, as in the parsimonious analysis, the long run ECT enters positively, implying that deviations from equilibrium can be expected to persist indefinitely. Exhibits 7 and 8 demonstrate that the general specification appears to provide some support in favour of a single regime common long run trend within international bond markets. This conclusion cannot, however, be said to hold for the Japanese markets.

As in the parsimonious analysis, I continue by allowing for a two regime Markov-switching specification in modeling the central long run equilibrium. Exhibit 9 provides ADF test statistics for the general Markov-switching equilibrium where residuals are derived from

EXHIBIT 3

Engle Granger OLS Residuals, ADF Tests

	Constant	Trend
U.S.	-1.787	-1.865
Germany	-2.021	-2.073
Japan	-2.344	-2.442
U.K.	**-4.992****	**-4.988****
Switzerland	-2.461	-2.528
Canada	-2.708	-2.731

EXHIBIT 4

Engle Granger Coefficient Estimates

	Δ Log Local Index$_{t-1}$	Δ Log World Index$_{t-1}$	Error Correction Term$_{t-1}$	$\bar{R}^2$	Durbin Watson
Δ Log U.S. Index$_t$	0.027	0.058	-0.004	-0.003	1.999
	[1.251]	[1.534]	[1.930]		
Δ Log Germany Index$_t$	**-0.071****	**0.267****	**-0.010***	0.031	2.021
	[3.205]	**[10.755]**	**[2.178]**		
Δ L n Japan Index$_t$	0.050	**0.133****	0.001	0.008	1.999
	[1.825]	**[7.583]**	[0.626]		
Δ Log U.K. Index$_t$	**-0.094****	**0.371****	**-0.011****	0.017	2.009
	[3.097]	**[6.704]**	**[2.868]**		
Δ Log Swiss Index$_t$	**0.079****	**0.307****	**-0.008****	0.089	2.017
	[2.759]	**[10.811]**	**[2.373]**		
Δ Log Canada Index$_t$	0.031	**0.157****	**-0.009****	0.005	2.001
	[0.813]	**[2.875]**	**[2.445]**		

equations (5) and (6). In every case a stationary residual series is obtained at a 1 % level of significance.

Exhibit 10 provides OLS coefficient estimates for the first difference estimation step where the ECT is derived as the Markov-switching residuals from Equation (5). A comparison of the coefficient estimates across Exhibits 8 and 10 shows that the Markov-switching residuals now enter with a negative coefficient for every market including Japan. Furthermore, explanatory power is improved in every market, except Switzerland, by allowing for a long run equilibrium that is allowed to switch periodically across two different states.

As a final diagnostic check regarding the Markov-switching approach adopted here, Exhibit 11 provides details regarding the characteristics of the regimes themselves for the general system. The motivation here is to recognise that the regimes appear to be relatively persistent through time. This is important since the Markov-switching

EXHIBIT 5

Markov Switching Residualsm ADF Tests

	Constant	Trend
U.S.	**-3.669****	**-3.667****
Germany	**-4.085****	**-4.848****
Japan	**-4.992****	**-4.998****
U.K.	**-6.011****	**-5.980****
Switzerland	**-5.085****	**-5.086****
Canada	**-5.360****	**-5.359****

EXHIBIT 6

Markov Switching Coefficient Estimates

	Δ Log Local Index_{t-1}	Δ Log World Index_{t-1}	Error Correction Term_{t-1}	$\bar{R}^2$	Durbin Watson
Δ Log U.S. Index_t	0.035	0.065	**-0.018****	0.002	1.999
	[1.629]	[1.737]	**[4.086]**		
Δ Log Germany Index_t	**-0.067****	**0.267****	**-0.028****	0.034	2.021
	[3.013]	**[10.781]**	**[3.829]**		
Δ Log Japan Index_t	**0.057****	**0.131****	**-0.014****	0.013	1.998
	[2.081]	**[7.431]**	**[3.258]**		
Δ Log U.K. Index_t	**-0.079****	**0.365****	**-0.044****	0.025	2.006
	[2.657]	**[6.654]**	**[5.367]**		
Δ Log Swiss Index_t	**0.080****	**0.307****	**-0.012***	0.089	2.017
	[2.805]	**[10.782]**	**[2.028]**		
Δ Log Canada Index_t	0.047	**0.146****	**-0.041****	0.014	2.001
	[1.251]	**[2.699]**	**[5.849]**		

approach has been used to identify two alternative equilibria. Persistence is therefore a desirable property for any long run equilibrium since its absence would indicate an unstable long run equilibrium relation. Exhibit 10 shows that regime persistence is a characteristic of the bond markets data analyzed here.

The first two columns in Exhibit 11 give the transition matrix from one regime to another. Column 3 gives the overall probability of observing a specific regime and column 4 gives the average regime duration. Hence for the U.S. the probability of moving from regime 1 to 2 is 0.002 while the overall probability of being in regime 1 is roughly one third. The average duration of regime 1 is 454 days. Overall the probability of moving from one regime to another is low and the average regime duration is at least 80 days. Typical regime durations are around 200 to 300 days or roughly one year.

EXHIBIT 7

General Specification: Engle Granger Residuals, ADF Tests

	Constant	Trend
U.S.	**-3.235***	-3.251
Germany	**-4.430****	**-4.413****
Japan	**-4.603****	**-4.648****
U.K.	**-4.117****	**-4.083****
Switzerland	**-5.393****	**-5.390****
Canada	**-4.585****	**-4.585****

EXHIBIT 8

General Specification: Engle Granger Coefficient Estimates

	Δ U.S.$_{t-1}$	Δ Ger$_{t-1}$	Δ Jap$_{t-1}$	Δ U.K.$_{t-1}$	Δ Swis$_{t-1}$	Δ Can$_{t-1}$	ECT$_{t-1}$ $\hat{\mu}_{t-1}$	$\bar{R}^2$	Durbin W
Δ U.S.$_t$	0.007	0.061	0.004	0.010	-0.040	0.021	**-0.004****	-0.003	1.999
	[0.215]	[1.302]	[0.145]	[0.433]	[-1.104]	[0.685]	**[-2.089]**		
Δ Ger$_t$	**0.145****	**-0.085****	**0.061****	0.004	-0.019	0.038	**-0.009***	0.036	2.023
	[6.549]	**[-2.473]**	**[3.527]**	[0.213]	[-0.728]	[1.853]	**[-1.968]**		
Δ Jap$_t$	**0.082****	**0.089****	0.051	-0.017	-0.003	-0.014	0.001	0.009	2.000
	[3.881]	**[2.654]**	[1.813]	[-0.862]	[-0.126]	[-0.616]	[0.297]		
Δ U.K.$_t$	**0.204****	-0.035	0.051	-0.036	-0.077	0.036	**-0.012****	0.034	2.019
	[5.391]	[-0.546]	[1.539]	[-1.047]	[-1.658]	[0.965]	**[-4.509]**		
Δ Swiss$_t$	**0.111****	0.060	0.033	0.009	**0.099****	**0.047***	**-0.007***	0.099	2.018
	[5.647]	[1.986]	[1.903]	[0.538]	**[3.253]**	**[2.360]**	**[-2.193]**		
Δ Can$_t$	0.024	0.054	0.037	0.021	-0.033	0.052	**-0.011****	0.004	2.002
	[0.615]	[1.058]	[1.431]	[0.797]	[-0.806]	[1.133]	**[-3.433]**		

EXHIBIT 9

General Specification: Markov Switching Residuals, ADF Tests

	Constant	Trend
U.S.	**-5.215****	**-5.223****
Germany	**-9.242****	**-9.235****
Japan	**-8.013****	**-8.105****
U.K.	**-7.256****	**-7.255****
Switzerland	**-10.005****	**-10.003****
Canada	**-7.834****	**-7.833****

Finally, Exhibit 12 plots the first regime probability through time. An interesting finding, and one that encourages future research, is the fact that regime 1 appears to apply for the early part of the data history whereas regime 2 appears to apply during the latter period. Since this result appears to be relatively uniform across all six markets, an interesting avenue for future research will be to search for a common factor explaining the regime generation process itself. Possible candidate variables might include inflation expectations or perhaps financial market volatility.

CONCLUSION

This article has considered the extent of bond market integration over the past 12 years on a daily basis using bond index data series that are widely used by the professional investment community. The analysis adds to the relatively sparse literature in this area by suggesting a long run equilibrium that is subject to occasional structural change. While a single regime treatment yields mixed results regarding bond market integration, a regime switching treatment provides compelling evidence in favor of a common bond market trend.

EXHIBIT 10

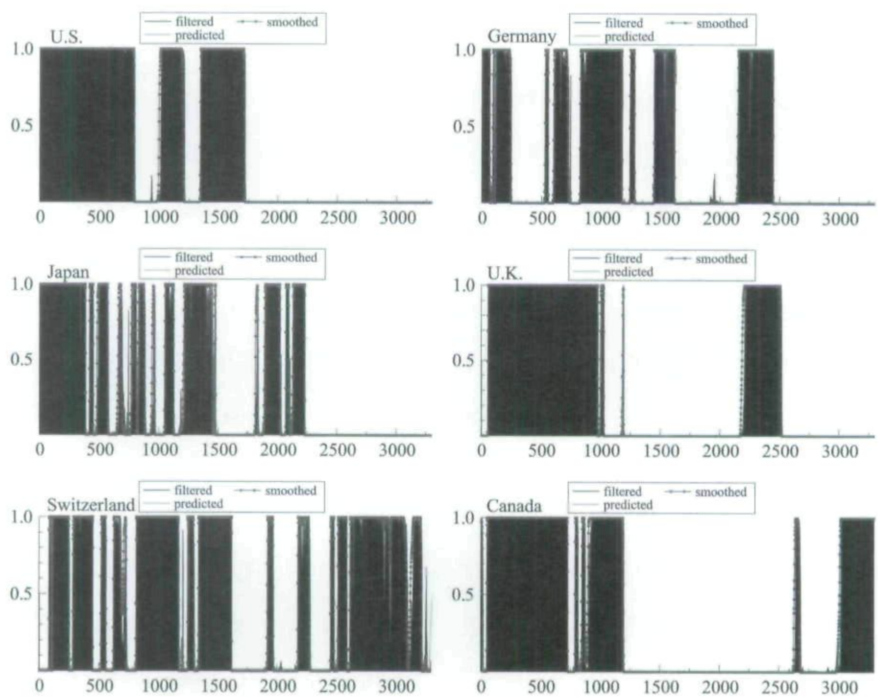
General Specification: Markov Switching Coefficient Estimates

	Δ U.S.$_{t-1}$	Δ Ger$_{t-1}$	Δ Jap$_{t-1}$	Δ U.K.$_{t-1}$	Δ Swis$_{t-1}$	Δ Can$_{t-1}$	ECT$_{t-1}$ $\hat{\mu}_{t-1}$	$\bar{R}^2$	Durbin W
Δ U.S.$_t$	0.009	0.065	0.001	0.009	-0.039	0.022	**-0.013****	0.001	1.999
	[0.298]	[1.378]	[0.044]	[0.405]	[-1.082]	[0.710]	**[-4.054]**		
Δ Ger$_t$	**0.145****	**-0.074***	**0.059****	0.003	-0.022	0.037	**-0.037****	0.041	2.022
	[6.571]	**[-2.150]**	**[3.424]**	[0.196]	[-0.834]	[1.796]	**[-4.277]**		
Δ Jap$_t$	**0.082****	**0.087****	0.055	-0.016	-0.004	-0.014	-0.005	0.010	2.000
	[3.875]	**[2.609]**	[1.953]	[-0.855]	[-0.152]	[-0.626]	[-1.337]		
Δ U.K.$_t$	**0.199****	-0.044	0.060	-0.029	-0.069	0.041	**-0.024****	0.034	2.018
	[5.295]	[-0.704]	[1.826]	[-0.841]	[-1.484]	[1.082]	**[-4.655]**		
Δ Swis$_t$	**0.110****	0.059	0.033	0.009	**0.100****	**0.048****	-0.010	0.099	2.018
	[5.648]	[1.937]	[1.933]	[0.550]	**[3.299]**	**[2.403]**	[-1.715]		
Δ Can$_t$	0.024	0.046	0.038	0.017	-0.031	0.059	**-0.028****	0.009	2.002
	[0.601]	[0.908]	[1.477]	[0.637]	[-0.760]	[1.306]	**[-4.925]**		

EXHIBIT 11
Regime Characteristics

		Reg1	Reg2	Prob	Duration
U.S.	Reg1	0.998	0.002	0.321	454.500
	Reg2	0.001	0.999	0.679	961.540
Germany	Reg1	0.992	0.008	0.342	132.140
	Reg2	0.004	0.996	0.658	254.120
Japan	Reg1	0.989	0.011	0.371	89.060
	Reg2	0.007	0.993	0.629	151.030
U.K.	Reg1	0.997	0.003	0.392	323.990
	Reg2	0.002	0.998	0.608	503.190
Swiss	Reg1	0.992	0.008	0.603	125.810
	Reg2	0.012	0.988	0.397	82.970
Canada	Reg1	0.996	0.004	0.418	230.240
	Reg2	0.003	0.997	0.582	320.380

EXHIBIT 12



APPENDIX

The EM Algorithm

Initially consider the following two regime lagged relation:

$$y_t = \begin{cases} \phi_{0,1} + \phi_{1,1} \cdot x_{t-1} + \varepsilon_t & if\ s_t = 1 \\ \phi_{0,2} + \phi_{1,2} \cdot x_{t-1} + \varepsilon_t & if\ s_t = 2 \end{cases} \tag{7}$$

or:

$$y_t = \phi_{0,1} + \phi_{1,s} \cdot x_{t-1} + \varepsilon_t \tag{8}$$

Assuming that the ε_t are normally distributed, the density of y_t conditional on the state s_t and history Ω_{t-1} is given by:

$$f(y_t | s_t = j, \Omega_{t-1}; \theta) = \frac{1}{\sqrt{2\pi\sigma}} exp\left\{\frac{-(y_t - \phi_j' x_t)^2}{2\sigma^2}\right\} \tag{9}$$

where $x_t = (1, x_{t-1})'$, $\phi_j = (\phi_{0,j}, \phi_{1,j})'$ for j = 1, 2 and $\theta = (\phi_1', \phi_2', \rho_{11}, \rho_{22}, \sigma^2)$ with ρ_{11} as the transition probability of remaining in regime 1.

Next consider the forecast, inference and smoothed regime probability vectors:

$$\text{Forecast: } \hat{\varepsilon}_{t+1|t} = P \cdot \hat{\varepsilon}_{t|t} \tag{10}$$

$$\text{Inference: } \hat{\varepsilon}_{t|t} = \frac{\hat{\varepsilon}_{t|t-1} \otimes f_t}{1'(\hat{\varepsilon}_{t|t-1} \otimes f_t)} \tag{11}$$

$$\text{Smoothed: } \hat{\varepsilon}_{t|n} = \hat{\varepsilon}_{t|n} \otimes (P'[\hat{\varepsilon}_{t+1|n} \div \hat{\varepsilon}_{t+1|t}]) \tag{12}$$

where for example $\varepsilon_{t|n}$ is the 2×1 vector containing the smoothed conditional regime probabilities as:

$$\hat{\varepsilon}_{t|n} = \begin{matrix} P(s_t = 1 | \Omega_n; \theta) \\ P(s_t = 2 | \Omega_n; \theta) \end{matrix} \tag{13}$$

and P is the matrix of regime transition probabilities.

Estimation begins with some initial guess of θ, call it $\hat{\theta}^{(0)}$ and $\hat{\varepsilon}_{1|0}$. Using the inference regime probabilities from Equation (11) we obtain $\hat{\varepsilon}_{1|1}$ and can then obtain $\hat{\varepsilon}_{2|1}$ from the forecast regime probabilities from Equation (10). Repeating this routine we obtain forecast and inference regime probabilities for all t up to n.

Next, using the final $\hat{\varepsilon}_{n|n}$ from Eqation(11) we can obtain $\hat{\varepsilon}_{n-1|n}$ from the smoothed regime probability in Equation (12). This is then repeated backwards for all t from n–1 to 1.

Armed with the smoothed regime probabilities from Equation (12) for all t denoted $P(s_t = j | \Omega_n; \theta)$ we can use the initial transition probabilities $\rho_{i,j}^{(0)}$ from $\theta^{(0)}$ to obtain $\rho_{i,j}^{(0)}$ from:

$$\rho_{i,j} = \frac{\Sigma_{t=2}^{n} P(s_t = j, s_{t-1} = i | \Omega_n; \hat{\theta})}{\Sigma_{t=2}^{n} P(s_{t-1} = i | \Omega_n; \hat{\theta})} \tag{14}$$

Finally new autoregressive coefficients and residual variance are obtained from:

$$\hat{\beta}_j = \left(\sum_{i=1}^{n} x_t(j) x_t(j)'\right)^{-1} \left(\sum_{t=1}^{n} x_t(j) y_t(j)\right) \tag{15}$$

$$\hat{\sigma}^2 = \frac{1}{2} \sum_{t=1}^{n} \sum_{j=1}^{n} (y_t - \hat{\beta}_j' x_t)^2 \cdot P(s_t = j | \Omega_n; \hat{\theta}) \tag{16}$$

with:

$$y_t(j) = y_t \sqrt{P(s_t = j | \Omega_n; \hat{\theta})} \tag{17}$$

$$x_t(j) = x_t \sqrt{P(s_t = j | \Omega_n; \hat{\theta})} \tag{18}$$

Overall we obtain a new set of coefficient estimates $\hat{\theta}^{(1)}$. The algorithm continues until we obtain a set of coefficient estimates that change by less than some pre-specified magnitude from the previous recursion of the algorithm.

REFERENCES

Balvers, R., Wu, Y., and Gilliland, Z. "Mean Reversion Across National Stock Markets and Parametric Contrarian Investment Strategies." *Journal of Finance*, Vol. 55, No. 2 (2000), pp. 745–772.

Blackman, S.C., Holden, K., and W.A. Thomas. "Long-term Relationships Between International Share Prices." *Applied Financial Economics*, 4 (1994), pp. 297–304.

Breitung, J. "Rank Tests for Nonlinear Cointegration." *Journal of Business and Economic Statistics*, 19 (2001), pp. 331–340.

Breitung, J., and C. Gourieroux. "Rank Tests for Unit Roots." *Journal of Econometrics*, 81 (1997), pp. 7–27.

Clare, A.D., M. Maras, and S.H. Thomas. "The Integration and Efficiency of International Bond Markets." *Journal of Business Finance & Accounting*, 22 (1995), pp. 313–322.

Corhay, A., A. Tourani Rad, and J.P. Urbain. "Common Stochastic Trends in European Stock Markets." *Economic Letters*, 42 (1993), pp. 385–390.

Davies, A. "Testing for International Equity Market Integration using Regime Switching Cointegration Techniques." *Review of Financial Economics*, 15, pp. 305–321.

Engle, R.F., and C.W.J. Granger. "Cointegration and Error Correction: Representation, Estimation and Testing." *Econometrica*, 55 (2006), pp. 251–276.

Gabriel, V.J., M. Sola, and Z. Psaradakis. "A Simple Method for Testing for Cointegration Subject to Multiple Regime Changes." *Economics Letters*, 76 (2002), pp. 213–221.

Gregory, A.W., and B.E. Hansen. "Residual-based Tests for Cointegration in Models with Regime Shifts." *Journal of Econometrics*, 71 (1996), pp. 321–341.

Hamilton, J.D. "Analysis of Time Series Subject to Changes in Regime." *Journal of Econometrics*, 45 (1990), pp. 39–70.

Johansen, S. "Identification of the Long Run and Short Run Structure an Application to the ISLM Model." *Journal of Econometrics*, Vol. 63, No. 1 (1994), pp. 7–36.

——. "Statistical Analysis of Cointegrating Vectors." *Journal of Economic Dynamics and Control*, 12 (1988), pp. 231–245.

Johansen, S., and K. Juselius. "Maximum Likelihood Estimation and Inference on Cointegration with Applications to Money Demand." *Oxford Bulletin of Economics and Statistics*, 52 (1990), pp. 169–210.

Kasa, K. "Common Stochastic Trends in International Stock Markets." *Journal of Monetary Economics*, 29 (1992), pp. 95–124.

Li, X-M. "A Revisit of International Stock Market Linkages: New Evidence from Rank Tests for Nonlinear Cointegration Analaysis." *Scottish Journal of Political Economy*, (2005).

Masih, A.M.M. and R. Masih. "Propagative Causal Price Transmission Among International Stock Markets: Evidence from the Pre- and Post-globalisation Period." *Global Finance Journal*, 13 (2002), pp. 63–91.

Richards, A.J. "Comovements in National Stock Market Returns: Evidence of Predictability but not Cointegration." *Journal of Monetary Economics*, 36 (1995), pp. 631–654.

Smith, K.L. "Government Bond Market Seasonality, Diversification and Cointegration: International Evidence." *Journal of Financial Research*, 25 (2002), pp. 203–221.

Yang, J. "Government Bond Market Linkages: Evidence from Europe." *Applied Financial Economics*, 15 (2005), pp. 599–610.