

DTSSM (Duration Times Spread) for CDS: A New Measure of Spread Sensitivity

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In a recent study, Ben Dor, Dynkin, Hyman, Houweling, Leeuwen, and Penninga [2007] examine the behavior of corporate bond spreads. They find that the volatilities of monthly systematic changes in spreads across sectors tend to increase linearly with the level of spreads.¹ Volatility of the non-systematic component of spread change of a particular bond or issuer is proportional to its spread level as well. Furthermore, the authors show that the linear relationship between spread volatility and spread level implies that excess return volatility is roughly proportional to the product of duration and spread. This new risk measure, termed "DTS"SM (Duration Times Spread), generates better out-of-sample volatility forecasts and lower tracking error for index-replicating portfolios compared with using duration alone.

In this article we extend the analysis of credit spread behavior beyond corporate bonds and look at credit default swaps. Establishing that the conditional volatility of spread change is proportional to the level of spread makes the DTS measure of risk exposure directly applicable to portfolios of CDS. This in turn has important applications in terms of position allocation and risk management.

A priori, we would expect all the previous results to hold for credit default swaps as, in theory, changes in their spreads and those of the underlying bonds should be closely related. In practice, however, this is

not always the case. Some evidence suggests that since CDS are often more liquid than the underlying bonds, their spreads incorporate new information more quickly and may exhibit higher volatility.² In addition, in previous studies corporate bond spreads are computed relative to the Treasury curve, whereas CDS spreads represent spreads over LIBOR. Furthermore, the higher liquidity of CDS contracts as compared with corporate bonds allows us to examine whether the previous findings are still valid when spread changes are analyzed at a weekly frequency.

Another difference between this study and Ben Dor et al. [2007] is the use of quasi-maximum likelihood (QML) to investigate the relation between spread volatility and spread level. This technique addresses the stochastic nature of conditional spread volatility and the fact that it is not directly observable (i.e., latent). Using QML enables us to assess the statistical validity of a pre-specified explicit functional dependence between conditional volatility and spread level, and we discuss the relative merits of such an approach compared with that employed in previous studies.

METHODOLOGY

In studying the relation between spread level and spread volatility we face a problem: volatility is ultimately unobservable. In practice what we observe are spread changes which

represent realizations drawn from an underlying distribution. This distribution is, in turn, a function of the underlying and unobservable volatility of spread movements.

Sample estimates across multiple time periods can serve as a measure of the true underlying spread volatility only if the volatility is fairly stable over time. Yet, if spread volatility is related to the level of spread as was found previously, then it would fluctuate over time in response to changes in the level of spread.

Ben Dor et al. [2007] address the above issue by forming buckets that are populated monthly with bonds trading within a certain spread range. The time-series of average spread changes of all bonds in a given bucket is used to form an estimate of its spread volatility. While a bucket's composition may have changed over time in response to changes in spreads of the underlying bonds, its (average) spread level remains remarkably stable.³ This allows for an analysis of the behavior of spread volatility while holding the level of spread relatively constant.

The advantage of this approach is its flexibility: no assumption is needed regarding the exact nature of the relation between spread volatility and spread level. The finding that spread volatility is linearly related to spread with a proportionality factor of about 10% relies on how the (bucket's) spread volatility reacts in response to changes in the level of spreads. However, the technique does not allow an analysis of spread volatility at the individual security or whole-sector level because of the stochastic nature of volatility, as explained above. It relies on buckets with a homogenous population of bonds trading at similar spreads.

We employ a different technique in this study based on "maximum likelihood." In its purest form, maximum likelihood assumes that the true distribution of the sample data is known. In our analysis we use the normal distribution for spread changes with zero mean and volatility that is not constant over time but rather is a function of the level of spread. However, our results are not dependent on spread changes being normally distributed.⁴ The idea underlying this approach is to identify the shape of the relation between spread volatility and spread level that would maximize the probability (likelihood) of the observed data (e.g., spread changes).

We use the following specification to test the relation between spread volatility in month t and spread level at the beginning of the month:

$$\sigma_t(\Delta s) = \alpha + \beta s_{t-1} + \gamma \hat{s}_{t-1}^2 \quad (1)$$

where $\sigma_t(\Delta s)$ is the volatility of spread during period t and s_{t-1} is the beginning-of-period spread level. The last term in the specification $\hat{s}_{t-1}^2$ controls for a potential non-linear relation between changes in spread level and spread volatility.⁵ The maximum likelihood procedure determines the value of the parameters α , β , and γ such that the likelihood of observing the data in the sample under the assumed (normal) distribution is maximized.

For example, maximum likelihood estimates for α and γ that are not significantly different from zero are consistent with spread volatility being linearly proportional to the level of spread. The proportionality factor is given by the β estimate and can be compared with the 9%–10% documented by Ben Dor et al. [2007]. If γ , the coefficient of the quadratic term, is positive (and significant), this would indicate that spread volatility increases with spread in a non-linear manner. Volatility would be lower than in the linear case for tight spreads and higher for wide spreads. Alternatively, if volatility is fairly unchanged over time (which forms the basis for using spread duration in the case of bonds and PV01 for CDS as a risk measure), this would result in a significant α with the estimates of both β and γ not being significantly different from zero.

EMPIRICAL ANALYSIS

Data

The analysis of CDS spread behavior is based on weekly data collected by Mark-it Partners. The list of individual credit default swaps is compiled from the constituents of main 5-year CDX.IG and CDX.XO for the U.S.; and main 5-year iTraxx.IG and iTraxx.XO for Europe. This allows for a comparison of the results across different markets and a wide range of spread levels.

To accurately capture systematic spread changes, only sectors that are represented by at least eight contracts are included in the analysis.⁶ In addition, several index constituents are excluded due to multiple missing observations or spread blow-ups, namely, a spread widening highly unusual for constituents of an index.⁷ To ensure spread changes are not affected by insufficient liquidity, the time period analyzed varies across indices. Exhibit 1 displays exact details on the CDS data population.

To allow for a direct comparison of our results with those reported in Ben Dor et al. [2007], we complement the analysis with monthly spread data of corporate bonds (computed relative to the local Treasury curve). The data

EXHIBIT 1

Description of CDS Dataset Composition

Index	Constituents	Starting Date	Universe population	
			Initial	After exclusions
CDX.IG	Series 1 - 6	4/17/2003	104	92 [*]
iTraxx.IG	Series 1 - 6	10/7/2003	107	99 ^{**}
CDX.XO	Series 6	1/7/2004	35	30 [†]
iTraxx.XO	Series 5	1/7/2004	45	32 [§]

^{*}Three contracts with a substantial number of missing observations (over 30) and four contracts with weekly spread changes exceeding 100 bp are excluded. The Government and Technology sectors are both excluded since they include only two issuers. We also exclude Arrow Electronics Inc., which became an investment grade company only in the recent past. Prior to December 2003, its CDS spreads were well above 200 bp.

^{**}Two contracts with weekly spread changes exceeding 100 bp as well as one contract with time average spread level exceeding 100 bp are excluded. One contract from the Government sector and one from the Energy sector is excluded. Three contracts with substantial numbers of missing observations are eliminated.

[†]One contract from the Utilities sector is excluded. We also eliminate four contracts with substantial numbers of missing points (more than ten weeks).

[§]Eleven contracts with a large number of missing observations (more than 20 weeks) as well as one contract with weekly spread changes exceeding 250 bp are eliminated. The Utilities sector is excluded since it includes only a single contract.

span the period from October 1990 to June 2006 for the Lehman Brothers U.S. Corporate Index, and from June 2000 to June 2006 for the Lehman Brothers U.S. High Yield and Euro Corporate Indices.

Examining the returns of corporate indices and the underlying sectors reveals clear evidence of the effect of large credit events at the index level. Incidents such as Enron and Parmalat influenced sector spread changes in a significant way. Since such events are probably not anticipated by the market even in terms of (higher) volatility, we exclude their effects. In order to do so we use the fact that Lehman Brothers maintains two universes of bond indices: 'Returns' and 'Statistics'.⁸ The former reflects the composition of the index in the current period, whereas the latter includes all the securities that would be part of the index in the subsequent period (and would then form the 'Returns' universe). The different composition of the two universes reflects issuance of new securities, downgrades, index requirements regarding minimum remaining maturity and amount outstanding, etc.

Spread changes for the 'Returns' universe reflect only spread movement of the initial index constituents (at the beginning of the period). In contrast, changes in spreads of the statistics universe also reflect changes in the index composition (due to securities entering and leaving the index at the end of the period). We analyze the time-series of spread changes of the 'Statistics' universe in order

to filter the effect of individual issuers that experience extreme events and comment when the respective results for the 'Returns' universe differ substantially.

Spread Volatility of Credit Default Swaps

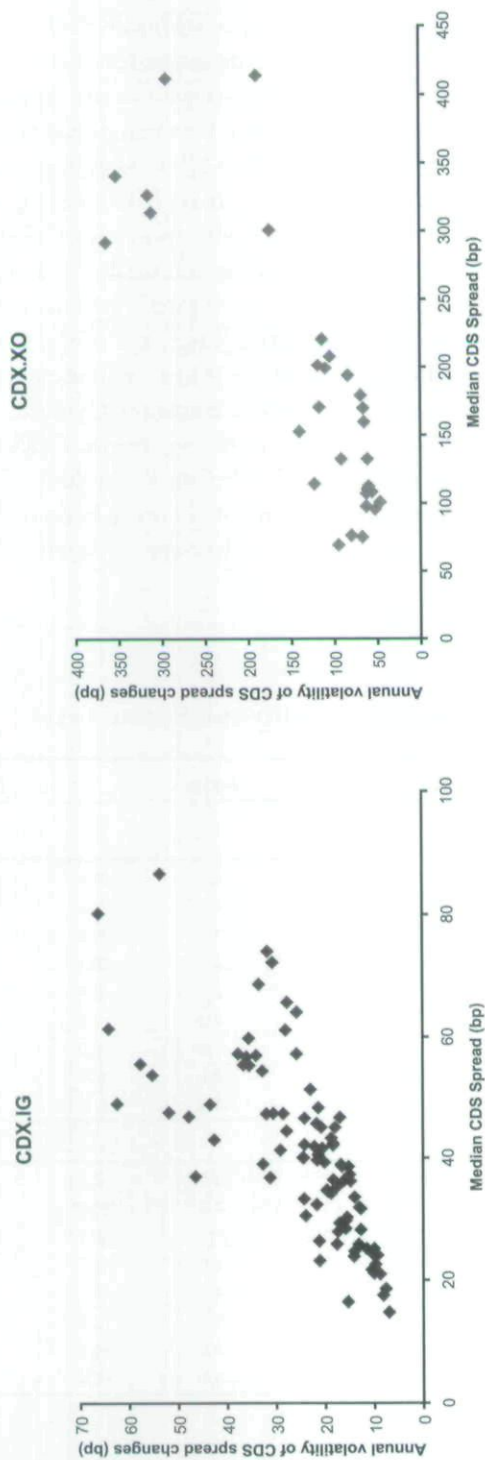
Panel A of Exhibit 2 displays volatilities and corresponding median spreads for all constituents of the CDX.IG and CDX.XO indices between July 2004 and May 2006. It clearly illustrates that the standard deviation of spread changes is increasing with the median level of spread and the plot for CDX.IG indicates a fairly smooth and linear relationship. While the volatility of contracts comprising the CDX.XO is not as well behaved and observations are more scattered, the same general pattern is still evident. This is consistent with Ben Dor et al. [2007] which, using U.S. high yield bonds rated Ba and B, found the linear relation extends well into spreads of 400 bp.

The results for European names included in the iTraxx.IG and iTraxx.XO are very similar (Panel B). Investment grade names are clustered along a line passing through the origin, while contracts trading at higher spreads exhibit more dispersion due to a larger idiosyncratic risk component.⁹ Another phenomenon illustrated in Exhibit 2 is that for very low spreads, the decline in volatility seems to decelerate and converge to some "lower

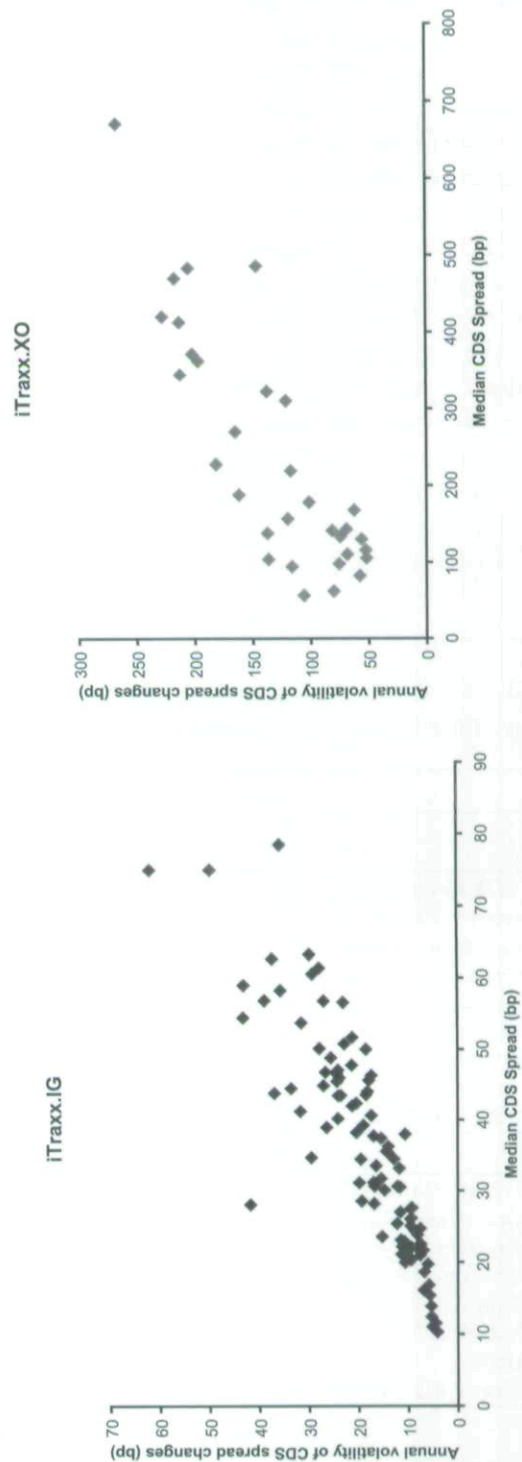
EXHIBIT 2

Volatility of Spread Changes Versus Median Spread Levels

Panel A



Panel B



Volatility of spread changes of individual CDS contracts versus median spread level; weekly observations from July 1, 2004 to June 1, 2006.

bound.” A similar effect has been documented in Ben Dor et al. [2007] for agency spreads.

Systematic volatility To analyze the time series of systematic spread changes of CDS we first compute the average spread change in each sector of the CDX.IG and iTraxx.IG indices with at least eight contracts.¹⁰ We then estimate the parameters in Equation (1) separately for each sector using (quasi) maximum likelihood. For CDX.XO and iTraxx.XO we calculate a single weekly aggregate figure, since the idiosyncratic component of spread change for crossover names is relatively high, therefore requiring a high degree of diversification in order to isolate systematic movements in spread.

Exhibit 3 reports the parameter estimates for α and β and their associated t -statistics. For investment grade CDS, the estimates for the linear spread term (β) are significant across all sectors except Consumer Cyclical and Materials for CDX.IG, and Consumer Stable for iTraxx.IG. Comparing estimates for CDX sectors with

those of equivalent sectors in iTraxx shows them to be fairly similar. In addition, most of the coefficients lie in the interval 0.04–0.06, which is in line with the 0.09–0.10 estimated using corporate bonds data over a monthly frequency.¹¹ None of the estimates of α are found to be significant, also lending support to the previous results which found that the relation between spread volatility and spread level is best described by a linear function which intersects the origin (i.e., α is equal to zero).

The last two columns in the exhibit show estimates of γ which control for potential non-linear aspects of the relationship between spread level and volatility and their associated t -statistics.¹² Review of the figures reveals that such effects are evident in several sectors of the CDX.IG (Communications, Consumer Cyclical, and Consumer Stable) as well as in the aggregate CDX index at a 10% significance level. The positive values of the coefficients imply that conditional volatility becomes less sensitive to changes in spread at low spread levels. This is consistent

EXHIBIT 3

Estimation of the Conditional Relation between Systematic Spread Volatility and Spread Level

	NO CDS	NO OBS	α	t -stat	β	t -stat	γ	t -stat
CDX								
CDX.IG (Index*)	92	161	3.7E-05	0.8	3.4%	2.9	9.5	1.73
Communications	8	161	1.5E-05	0.2	5.1%	4.5	9.0	2.28
Consumer Cyclical	17	161	6.5E-05	0.7	3.1%	1.4	31.0	2.38
Consumer Stable	11	161	4.2E-05	0.7	3.6%	2.5	18.1	2.18
Financial	19	161	-1.7E-06	0.0	6.1%	2.7	-0.2	-0.01
Industrial	14	161	-2.3E-07	0.0	4.3%	4.0	-1.4	-0.31
Materials	9	161	1.1E-04	0.9	3.2%	1.2	35.0	1.54
Utilities	8	161	-4.8E-05	-0.7	5.8%	3.8	4.8	0.65
CDX.XO (Index)	30	103	-1.2E-04	-0.3	6.6%	2.6	6.7	1.1
iTraxx								
iTraxx.IG (Index**)	99	149	1.1E-05	0.1	4.1%	2.0	22.5	1.0
Communications	14	149	1.0E-05	0.1	5.1%	2.1	26.9	1.6
Consumer Cyclical	13	149	-1.2E-04	-0.7	7.9%	2.3	36.1	1.3
Consumer Stable	14	149	1.3E-05	0.1	4.0%	1.5	34.9	1.1
Financial	21	149	-1.3E-05	-0.5	5.3%	4.1	17.7	1.0
Industrial	9	149	1.9E-05	0.2	4.3%	2.1	12.9	0.7
Utilities	14	149	2.7E-05	0.8	3.2%	3.0	10.1	1.4
iTraxx.XO (Index)	32	103	3.4E-04	0.4	3.6%	0.9	19.6	2.6

*Excluding the energy sector.

**Excluding the Materials sector and unclassified contracts.

Figures in bold are significant at the 5% level. Estimates are based on weekly observations. Sample periods vary by index (see Exhibit 1).

with Ben Dor et al. [2007], who find that spread volatility is roughly constant for spreads below 20 bp.¹³

Regarding crossover names, β is found to be significant for CDX.XO at a 5% level. The magnitude of the estimate (0.132 when converted to a monthly frequency) is generally higher than those for investment grade names and is consistent with previous results for high yield bonds.¹⁴ In contrast, β is not significant for iTraxx.XO but γ is significant at the 5% level.

The effect of the non-linear term on systematic volatility can be substantial. As an illustration, we consider predicted volatility of the CDX.IG Communications sector where both coefficients β and γ are significant. Exhibit 4 shows predicted volatilities with and without the non-linear term. As we see, the difference between predicted volatilities, especially for high spread levels, can be substantial.

Idiosyncratic volatility We also analyze the idiosyncratic spread volatility of constituents of the four CDS indices. Idiosyncratic spread changes are calculated by subtracting the average sector or index spread change from that of the individual contract. As before, we specify conditional volatility of idiosyncratic spread changes as a simple function of spread level (Equation 1) and use maximum likelihood to estimate the parameters.

Exhibit 5 shows the distribution of the β estimates separately for constituents of CDX and ITRAXX. The two distributions are fairly similar, with the majority of estimates falling in the interval 0.034–0.064. Only a few estimates (three and ten for CDX and iTraxx, respectively) have negative values.

Exhibit 6 reports the results of a “pooled estimation.” The time-series of idiosyncratic spread changes of individual contracts are combined within a sector (for investment grade names) or the entire index (for crossover names), and the parameters in Equation (1) are estimated as before. The result is a single set of estimates of the parameters representative of the relationship between idiosyncratic spread volatility and spread level in each sector (or aggregate index for crossover contracts).

Overall, the results in Exhibit 6 lend further support to the linear relationship between spread volatility and spread level. The estimates for the linear term are always significant at the 1% significance level. In addition, the magnitudes of the estimates are quite similar and consistent with the estimates in Ben Dor et al. [2007].

The figures in the last column indicate that a non-linear effect is detected in a single sector (Utilities of iTraxx.IG), similar to the case of systematic spread volatility (Exhibit 3).

EXHIBIT 4

Fitted Spread Volatility of CDX.IG Communication Sector—Linear Versus Non-Linear Model Specification

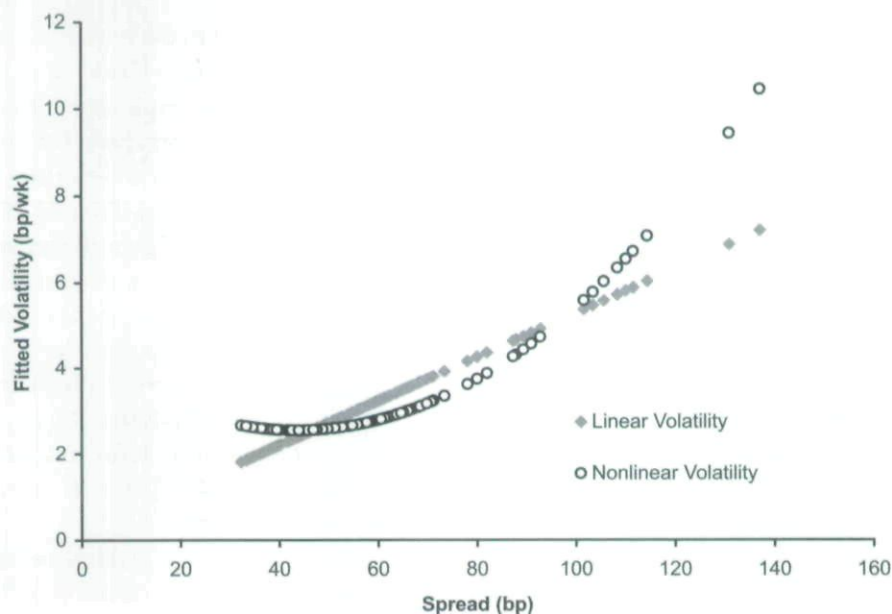
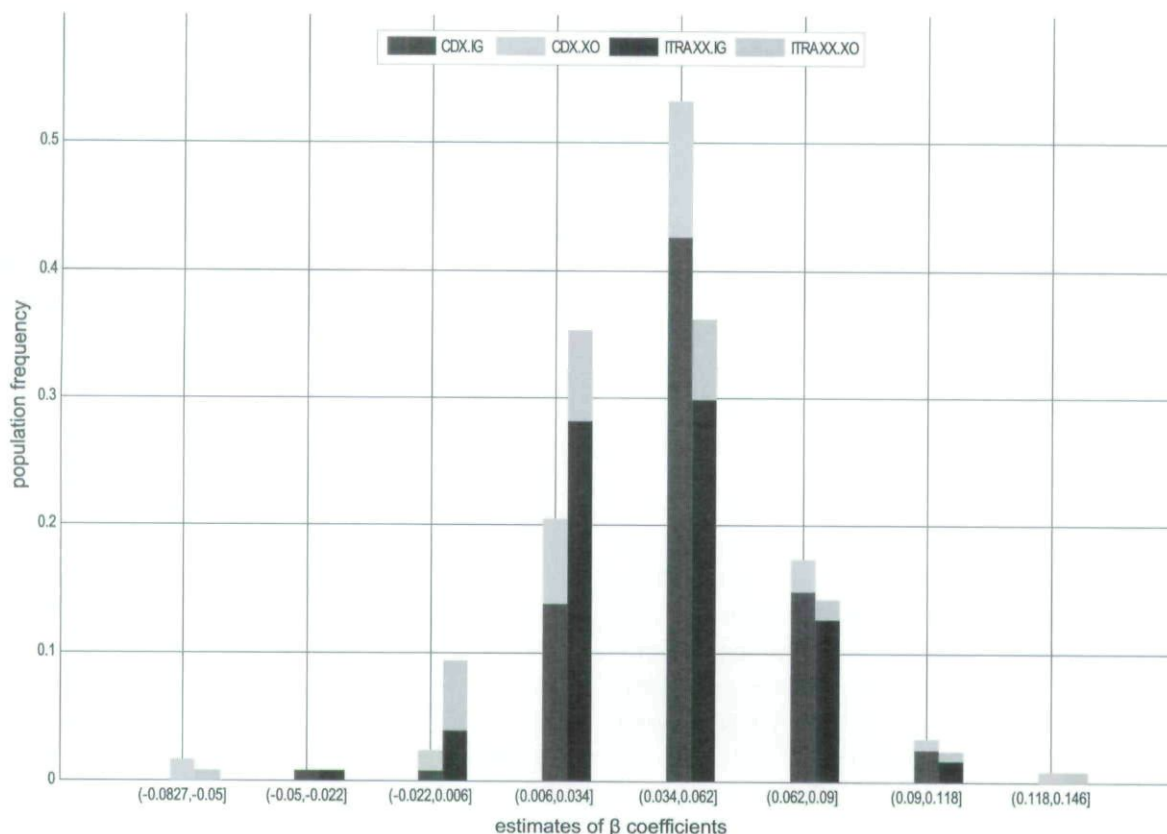


EXHIBIT 5

Distributions of Linear Coefficients for Idiosyncratic Spread Changes



Beta coefficients are estimated separately for each contract based on weekly observations. Sample periods vary by index (see Exhibit 1).

Spread Volatility of Corporate Bonds

We complete our study with an analysis of systematic spread volatility using monthly bond data. Unlike Ben Dor et al. [2007], which use individual bond data, our analysis is based on (aggregated) spread changes of several Lehman Brothers indices (U.S. and Euro Corporate Indices and U.S. High Yield Index). We report results for three major sectors (Financials, Industrials, and Utilities) as well as for various underlying sub-sectors. To ensure proper measurement of systematic spread changes, we examine only sectors with at least ten issuers for investment grade bonds and 20 issuers for high yield bonds (monthly).

Exhibit 7 presents the results for the U.S. Corporate Index. The table reports four main statistics: estimates of beta, gamma, and the associated *t*-statistics. Many investors

believe that credit markets changed fundamentally in 1998 following the "Russian Crisis" and the downfall of LTCM. To ensure that our results are not an artifact of a specific time-period, we report results for the entire sample period and separately for two sub-periods: October 1990–June 1998 and October 1998–January 2006.¹⁵

The estimates of β for the broader Financials (12.7%) and Industrials (12.5%) sectors are in agreement with the earlier results of 10% despite the different statistical techniques and sample periods. The estimates are also significant for all sub-sectors. Comparing the results for the two sub-periods reveals that the estimates are often larger in the second period. This contrasts with the findings of Ben Dor et al. [2007] that the proportionality factor is fairly stable over time.¹⁶

The reported results are based on the 'Statistics' universe with monthly re-balancing of the index. As explained

EXHIBIT 6

Estimation of Idiosyncratic Conditional Volatility of Spread Changes

	NO CDS	NO OBS	α	<i>t-stat</i>	β	<i>t-stat</i>	γ	<i>t-stat</i>
CDX								
CDX.IG (Index*)	92	14,652	3.1E-05	2.0	5.1%	16.8	0.9	1.1
Communications	8	1,288	9.4E-05	2.4	3.8%	6.9	2.3	1.6
Consumer Cyclical	17	2,737	3.6E-05	1.6	4.3%	7.1	0.7	0.4
Consumer Stable	11	1,771	-3.6E-06	-0.2	6.2%	11.9	0.6	0.5
Financial	6	3,059	5.4E-05	1.7	5.1%	8.8	1.2	0.9
Industrial	19	2,254	4.5E-05	2.7	3.5%	8.2	0.6	0.7
Materials	14	1,449	6.6E-05	2.5	3.4%	5.6	0.8	0.4
Utilities	8	1,288	2.9E-05	0.6	4.0%	4.1	1.7	0.8
CDX.XO (Index)	30	3,058	3.6E-04	4.0	4.8%	10.9	0.3	1.019
iTraxx								
iTraxx.IG (Index**)	99	14,759	2.6E-06	0.2	4.7%	17.5	0.5	0.5
Communications	14	2,128	-8.9E-06	-0.4	4.4%	10.0	1.4	1.2
Consumer Cyclical	13	1,976	-5.3E-05	-1.9	5.9%	9.8	0.0	0.0
Consumer Stable	14	2,128	-1.0E-05	-0.6	5.2%	11.1	1.1	0.8
Financial	21	3,192	9.9E-06	1.0	3.6%	9.0	2.5	1.3
Industrial	9	1,368	4.8E-05	1.3	3.4%	6.5	3.7	0.6
Utilities	14	2,128	1.1E-05	1.0	2.9%	8.2	2.4	2.6
iTraxx.XO (Index)	32	3,230	7.7E-04	5.7	2.4%	5.4	0.0	0.2

*Excluding the energy sector.

**Excluding the Materials sector and unclassified contracts.

Figures in bold are significant at the 5% level. Estimates are based on weekly observations. Sample periods vary by index (see Exhibit 1).

earlier, we focus on the 'Statistics' universe because it excludes, to a large extent, extreme spread changes that result from defaults or downgrades of the underlying issuers. Such bonds are excluded from the universe in the beginning of the month following the event. The degree to which the results are affected by extreme issuer-specific events can be seen if we perform a similar analysis using the 'Returns' universe. Consider, for example, the Electrical sub-sector of the U.S. Corporate Index. Enron and NRG (an Illinois power company) are both constituent issuers in this sub-sector. In the case of Enron, the company's bonds experienced extreme spread widening over the course of a few days in November 2001, trading at spreads in excess of 1000 bp, and were subsequently excluded from the index at the end of that month. Similarly, bonds issued by NRG suffered huge losses during July 2002 and left the index at the next month-end. These two companies have a large effect on returns of the Electrical sector in the 'Returns' universe.

Exhibit 8 plots the time-series of spread changes of the two universes for the Electrical sector. It clearly illustrates that issuer-specific events are less pronounced in spread changes of the 'Returns' universe. If we re-estimate the parameters for the Electrical sector using the 'Returns' universe rather than the 'Statistics' universe, as in Exhibit 7, the estimate of β rises substantially from 15.4% to 19.3% and is less in line with estimates for the Financials and Industrials sectors.

Comparing parameters estimated based on the 'Returns' and 'Statistics' universes for the U.S. High Yield and Euro Corporate Indices further highlights the importance of accounting for issuer-specific events. For example, the result for the Communications sector of the U.S. High Yield Index (Exhibit 9—Panel A) supports the proportionality of spread changes with a significant *t*-statistic. However, when we re-estimate the coefficients using the Returns universe, the *t*-statistic value drops to 0.5. Similarly, the Communications, Utilities, and

EXHIBIT 7

Estimation of Systematic Conditional Volatility of Spread Changes for the Lehman Brothers U.S. Corporate Index

	Avg. No. Bonds	October 1990 - June 2006				October 1990- June 1998				October 1998 - June 2006			
		β	t-stat	γ	t-stat	β	t-stat	γ	t-stat	β	t-stat	γ	t-stat
Financials	886	12.7%	5.4	1.6	0.3	9.5%	4.1	6.0	1.2	13.9%	5.6	2.5	0.5
Banking	359	12.6%	4.7	1.8	0.5	11.8%	4.9	3.0	0.7	11.8%	3.4	5.5	0.6
Brokerage	105	15.2%	2.6	2.7	0.5	16.1%	3.4	1.4	0.2	12.6%	3.6	5.4	0.6
Finance Comp'	277	16.8%	7.5	0.4	0.2	11.5%	4.6	7.6	0.9	19.3%	7.0	0.4	0.2
Insurance	54	16.2%	3.4	-0.2	0.0	NA	NA	NA	NA	16.2%	3.4	-0.2	0.0
REITS	41	5.4%	3.0	2.3	0.7	NA	NA	NA	NA	5.4%	3.0	2.3	0.7
Industrials	1166	12.5%	5.2	0.2	0.0	10.3%	2.7	9.2	1.0	11.8%	3.7	2.5	0.4
Basic Industrials	144	8.8%	5.8	-1.5	-0.3	8.0%	2.8	4.8	0.6	5.9%	2.2	7.8	1.6
Capital Goods	146	11.5%	5.8	1.5	0.3	11.3%	4.4	4.8	0.9	9.9%	3.0	4.6	0.6
Communications	124	18.8%	4.7	1.0	0.3	NA	NA	NA	NA	18.8%	4.7	1.0	0.3
Consumer Cyclicals	216	20.1%	5.4	0.9	0.2	15.5%	3.4	5.0	0.9	17.2%	1.8	16.3	0.9
Energy	137	10.9%	2.1	-3.3	-0.3	13.5%	2.2	7.3	0.4	6.8%	1.7	1.4	0.2
Consumer NonCyc	228	10.5%	2.7	-1.5	-0.2	11.4%	2.0	17.4	0.5	8.3%	2.3	1.4	0.2
Technology	49	17.2%	3.0	0.2	0.0	16.8%	3.6	7.9	1.0	17.8%	3.4	0.1	0.0
Transportation	99	10.2%	4.2	-0.3	-0.1	8.8%	3.5	3.3	0.4	15.5%	3.8	-2.3	-0.7
Utilities	359	13.6%	3.7	1.0	0.3	8.3%	2.3	9.9	0.5	16.3%	3.2	1.1	0.4
Electric	229	15.4%	2.9	0.9	0.2	11.8%	2.9	7.4	0.2	17.8%	2.4	0.7	0.2
Natural Gas	80	13.5%	5.5	1.8	0.7	5.4%	3.4	1.2	0.2	17.3%	4.9	2.8	0.9

Figures in bold are significant at the 5% level.

EXHIBIT 8

Spread Changes of the Lehman Brothers U.S. Corporate Index Electric Sector Based on the 'Returns' and 'Statistics' Universes

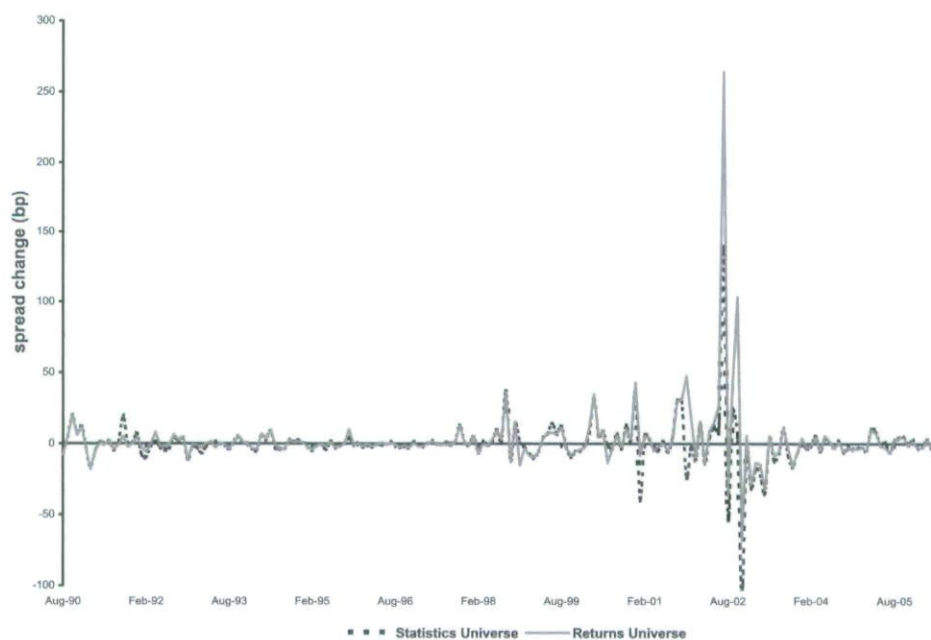


EXHIBIT 9

Estimation of Systematic Conditional Volatility of Spread Changes for the Lehman Brothers
U.S. High Yield and Euro Corporate Indices

	Avg. NO of Bonds	β	Jun 2000 - Jun 2006		
			t -stat	γ	t -stat
Panel A: U.S. HY					
Financials	27	14.1%	2.1	-2.5	-0.9
Industrials	418	10.3%	3.2	0.8	0.5
Capital Goods	41	16.6%	2.7	-0.2	-0.1
Communications	54	18.3%	2.8	0.1	0.1
Consumer Cyclicals	126	8.7%	2.1	1.2	0.6
Consumer NonCyc	64	7.2%	2.2	-0.3	-0.1
Panel B: Euro IG					
Financials	515	12.8%	6.0	5.4	0.5
Banking	385	6.7%	2.7	15.1	0.5
Finance Companies	56	23.6%	5.4	0.1	0.0
Insurance	34	14.4%	1.6	-5.5	-0.2
Industrials	353	11.4%	1.4	-2.3	-0.1
Basic Industrials	29	14.2%	3.5	4.3	0.4
Capital Goods	38	20.0%	4.3	1.0	0.1
Communications	91	18.5%	2.1	0.5	0.0
Consumer Cyclicals	74	25.1%	1.5	44.9	0.7
Consumer NonCyc	62	10.2%	1.4	-3.1	-0.1
Utilities	78	9.8%	2.2	26.1	1.8
Electric	58	6.2%	2.5	24.0	2.0

Figures in bold are significant at the 5% level.

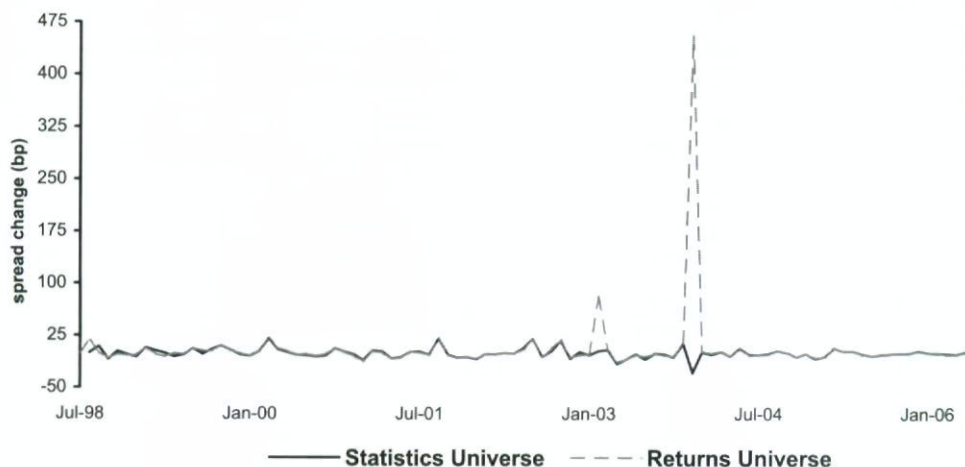
Electric sectors of the Euro Corporate Index (Panel B) have t -statistics of 2.1, 2.2, and 2.5, respectively. Using the 'Returns' universe the estimated parameters are no longer significant, with t -statistics of 1.8, 0.5, and 0.2, respectively.

Perhaps the most dramatic illustration is in the case of Consumer Non-Cyclical, where the estimate changes from 10.2% with a t -statistic of 1.4, to -98.5% (significant

at the 5% confidence level with a t -statistic of -2.5). The reason for the "flip" in the coefficient estimate is illustrated in Exhibit 10: spread widening in the sector in February 2003 and December 2003, caused by the Ahold and Parmalat events respectively. In the 'Statistics' universe, however, both Ahold and Parmalat are excluded from the index after the blow-ups. As a result, there is no significant change in spreads.

EXHIBIT 10

Spread Changes for the Consumer Non-Cyclical Sector of the Lehman Brothers Euro Credit Index



CONCLUSION

This article extends the study by Ben Dor et al. [2007] to credit default swaps, using a different estimation technique and higher data frequency. Both at the aggregate index and sector levels in European and U.S. markets, we find further evidence to support the hypothesis that systematic and idiosyncratic spread volatility is linearly proportional to the level of spreads, with only a few instances of significant systematic second-order effects. Furthermore, the estimated sensitivity of volatility to changes in spreads is generally consistent with those documented for corporate bonds when converted to a monthly frequency. We find these relationships to hold for both investment grade and crossover contracts. Data are insufficient to make any statements for the high yield asset class.

We also apply the maximum likelihood technique to aggregated spreads of cash bonds at the index and sector levels for investment grade and high yield securities. The results confirm previous findings in terms of both significance and magnitude of the relationship between spread volatility and initial spread level. It is worth noting that the coefficients for high-yield bonds are comparable in magnitude to those in investment grade, as was found to be the case in previous studies.

The applicability of the DTS paradigm to CDS has several important implications for portfolio managers and mixed portfolios with cash securities and credit derivatives. First, allocations to sectors across both products should be compared in terms of contribution to DTS. Second, risk models for both instruments can be based on relative spread volatilities allowing for dynamic risk updating and alleviating the need for using rating categories. Eliminating credit qualities, in turn, allows greater sector coverage as sector-quality based spread risk factors can be replaced by a purely sector-based approach. Furthermore, constructing "DTS-factors" based on relative spread changes allows for better risk forecasting as compared with measuring spread volatility in absolute terms (bp) as they exhibit more stability over time. Portfolio trades or security swaps can be hedged using DTS ratios of the two legs more effectively than using historical betas (which we demonstrate in our upcoming paper) and certainly better than the ratio of PV01s.

APPENDIX

Quasi-Maximum Likelihood Estimation

Maximum likelihood estimation begins with an assumption about the true distribution of the sample data. This probability density function is assumed to be known up to the value of certain parameters which need to be estimated from an available sample of observations. An optimal specification is sought for the distribution of those variables by searching for parameters that maximize an objective function—the likelihood. The MLE procedure chooses parameters in a way that maximizes the likelihood of drawing the sample under consideration from the calibrated distribution. This likelihood optimization is carried out by means of maximization, with respect to the parameters, of the so-called (log-) likelihood function of the observed data under the parameterized probability density function.

Specifically, let $f(y|x, \theta)$ denote the probability density function of a random variable y conditional on a random variable x and a set of parameters θ . In its application to our study, y represents the spread change, x the level of spread, and θ the set of parameters defining the dependence of spread change volatility on spread level. Given a sample of independent observations $y_1 \dots y_n$ we can write their joint probability density function, $L(\theta | x, y)$, in the following way:

$$L(\theta | x, y) \equiv f(y_1 \dots y_n | x_1 \dots x_n, \theta) \\ = \prod_{i=1}^n f(y_i | x_i, \theta) \quad (\text{A1})$$

This joint density function, when defined as a function of the unknown parameter vector θ , is called the likelihood function, where y and x indicate the collection of observations in the sample. The logarithm of the likelihood function is called the log-likelihood function:

$$\ln L(\theta | x, y) = \sum_{i=1}^n \ln f(y_i | x_i, \theta) \quad (\text{A2})$$

Parameter estimates θ can be obtained by maximizing the log-likelihood function with respect to the parameter set:

$$\hat{\theta} = \arg \max_{\theta} [\ln L(\theta | x, y)] \quad (\text{A3})$$

It can be shown that the estimates obtained by MLE procedure are asymptotically consistent and efficient. Namely, that as the number of sample observations increases, the MLE parameter estimates converge to their true values, and that the asymptotic variance of MLE estimates is the smallest possible in the class of consistent estimates.

One shortcoming of the MLE procedure is that it assumes a particular form for the probability density function of the

sample observations. It is usually the case that this distribution is not known a priori. Nevertheless, even assuming normal probability distribution for the data will, under certain conditions, lead to consistent results.^{17,18}

Estimation Methodology

In our analysis, we assume that the conditional volatility of spread change is a function of spread level, i.e.,

$$\sigma_t = \alpha + \beta s_{t-1} + \gamma \hat{P}_{s_{t-1}}^2 \quad (\text{A4})$$

in which $\hat{P}_{s_{t-1}}^2$ is the in-sample projection of s_t^2 to the linear space orthogonal to $L(1, s)$. In terms of an OLS regression, $\hat{P}_{s_{t-1}}^2$ is the residual term from regressing s_t^2 on s_t and a constant. The idea is that the estimate of the parameter γ would represent the potential non-linear relationship between conditional variance of spread changes and the level of spread. The reason for using the orthogonal projection of s_t^2 rather than simply s_t^2 is that the latter is highly correlated with the level of spread. Introducing such a high level of multi-collinearity into the model would severely reduce the significance level of the estimates of α and β . As a result, the linear coefficient becomes non-informative, as we cannot see whether the linear model per se explains the data well. To avoid this effect, we would like to split the contribution of the quadratic term into linear and non-linear components. Eventually, we would include only the non-linear component of the quadratic term into the specification, leaving out the linear component. To achieve this we use the in-sample projection of the quadratic term into the orthogonal space. The result is that estimates $\hat{\alpha}$ and $\hat{\beta}$ do not depend on the inclusion of the non-linear term. This construct is purely artificial and, in fact, does not affect the conclusion regarding the quadratic term in the specification of the model. Whether we include the quadratic term or its orthogonal projection does not affect parameter estimate $\hat{\gamma}$ and its t -statistic.

Using the Gaussian conditional probability density function, we can write the log-likelihood function in the form:

$$\ln L(\alpha, \beta, \gamma | \Delta s, s) = -\frac{T}{2} \ln(2\pi) \\ - \sum_{t=1}^T \ln \sigma_t - \frac{1}{2} \sum_{t=1}^T \frac{\Delta s_t^2}{\sigma_t^2} \quad (\text{A5})$$

The parameter estimates can be obtained by maximizing this likelihood function of the observed data with respect to parameters α , β , and γ .

We estimate the parameters in two steps, hoping to achieve greater robustness. First, we estimate parameters of the linear specification only. Then we fix the linear coefficients and

re-run the estimation with the non-linear term included. This procedure is justified since inclusion of the orthogonal projection, as discussed above, does not change the previously estimated linear coefficients $\hat{\alpha}$ and $\hat{\beta}$.

ENDNOTES

The authors would like to thank Adam Purzitsky for his advice and suggestions.

¹'DTS' is a registered service mark of Lehman Brothers.

²Similar to Ben Dor et al. [2007], we measure spread volatility as the volatility of absolute spread changes, as opposed to the volatility of log-changes in spreads used in continuous time models of derivative pricing.

³See for example Blanco, Brennan, and Marsh [2003].

⁴Bonds that experienced spread widening (tightening) migrate to higher (lower) spread range buckets.

⁵This result and its application is known as quasi (or pseudo)-maximum likelihood (QML). See Lee and Hansen [1994], who prove consistency of the quasi-maximum likelihood estimation for GARCH(1,1) specification of conditional volatility, which is methodologically similar to our specification.

⁶The value of $\hat{\sigma}_{t-1}^2$ is determined from an orthogonal in-sample projection of squared spreads on spread levels and the constant. See the appendix for a detailed explanation.

⁷The justification for using eight contracts is based on the assumption that idiosyncratic spread changes are uncorrelated with systematic spread changes but exhibit similar volatility. This assumption, backed up by our Global Risk Model, implies that we capture the systematic volatility to within approximately 5% of its true value on average.

⁸In general, these extreme spread movements are associated with highly idiosyncratic events, such as corporate mismanagement or sudden speculation about a pending LBO, and as such should not be expected to be part of the volatility anticipated by the market and reflected in a bond's spread.

⁹The existence of these two index universes addresses portfolio managers need to know the return of the index if bought at the beginning of the month as well as the composition of the up-and-coming index that will serve as the 'Returns' universe at the beginning of the next month.

¹⁰The one outlier is BAA plc, which had a low median spread of 28 bp and a relatively high volatility of 41 bp per year. The spreads on BAA plc widened almost 60 bp after a hostile takeover bid from Ferrovial and subsequently recovered when the company agreed to include a new clause into issued debt that allowed bondholders to sell bonds at their face value in case of rating downgrade resulting from merger or acquisition.

¹¹We use Markit Partners' sector classification for convenience.

¹²Volatilities based on weekly observations can be converted to a monthly frequency (assuming spread changes are

distributed independently over time) by approximately multiplying them by $\sqrt{4}$.

¹³See the appendix for details of the exact estimation procedure.

¹⁴Ben Dor et al. [2007] use Agency bond data and find that the linear relation between spread volatility and spread level holds for spreads above 20 bp. For spreads below 20 bp, spread volatility is roughly constant; the levels of systematic and idiosyncratic "structural" volatility are about 2.5–3.0 bp and 4.5 bp per month, respectively.

¹⁵Ben Dor et al. [2007] find that for HY bonds the proportionality factor is roughly 11.5%.

¹⁶The results for the entire sample reflect the effect of July–September 1998, which are excluded from the sub-periods.

¹⁷Unlike in our study, Ben Dor et al. [2007] examine the stability of the proportionality factor year-over-year without controlling for sector classification.

¹⁸The formal conditions under which consistency is assured are given, for example, in Huber [1967] or White [1982].

¹⁹The reason is that the estimators can be alternatively interpreted as extremum estimators with the resulting parameters converging in probability to some (pseudo-true) values. These parameters approximate the imposed model using the normal log-likelihood as a criteria function. These pseudo-true parameters often turn out to coincide with the true parameters of the model.

REFERENCES

- Ben Dor, A., L. Dynkin, P. Houweling, J. Hyman, E. Leeuwen, and O. Penninga. "DTS (Duration Times Spread): A New Measure of Spread Exposure in Credit Portfolios." *Journal of Portfolio Management* (Winter 2007), pp. 77–100.
- Blanco, R., S. Brennan, and I.W. Marsh. "An Empirical Analysis of the Dynamic Relationship between Investment Grade Bonds and Credit Default Swaps." *Journal of Finance*, 60 (2003), pp. 2255–2281.
- Huber, P.J. "The Behaviour of Maximum Likelihood Estimates Under Nonstandard Conditions." In *Proceedings of the Fifth Berkeley Symposium in Mathematical Statistics*, Vol. 1, Berkeley: University of California Press, 1967.
- Lee, S.W. and B.E. Hansen. "Asymptotic Theory for the GARCH(1,1) Quasi-Maximum Likelihood Estimator." *Econometric Theory*, 10 (1994), pp. 29–52.
- White, H. "Maximum Likelihood Estimation of Misspecified Models." *Econometrica*, 53 (1982), pp. 1–16.

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