Sources of Credit Risk: *Evidence from Credit Default Swaps*

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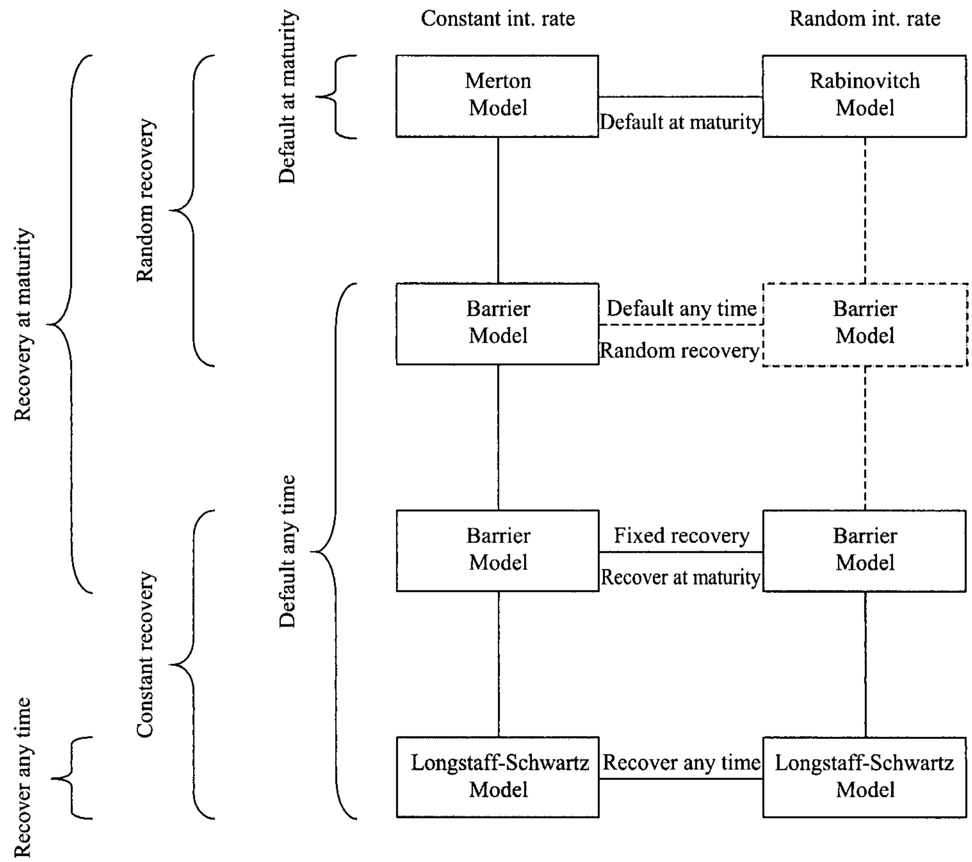
In this article, we test structural models of credit risk using transaction prices for credit default swaps. In particular, we examine four important sources that affect credit derivative prices: default likelihood, time of default, time and amount of recovery on default, and the evolution of interest rates. By comparing pairs of models that are nested in terms of their assumptions about these factors (one model is a special case, or restriction, of the other), we are able to understand what causes a model to succeed or fail. This improves the study by Wei and Guo [1997] who find that the Longstaff-Schwartz model [1995] and the Merton model [1974] do not outperform each other due to their non-nested assumptions.

Wei and Guo [1997] compare the Merton model (Merton [1974]) and the Longstaff-Schwartz model (Longstaff and Schwartz [1995]). The former model assumes a single default time and barrier, fixed interest rates, and random recovery, while the latter model assumes a continuous default barrier, random interest rates, and fixed recovery. Thus, as Wei and Guo note, the two models are not nested. Their main result is that the Longstaff-Schwartz model does not outperform the Merton model. This conclusion means that allowing interest rates to be random at the expense of random recovery does not improve the pricing performance of the Longstaff-Schwartz model and, consequently, random recovery in Merton's model plays an important role, at least as important as the role played by random interest rates. However, it is not possible to determine which assumption is the more important for pricing performance.

To carry out our tests, we must derive a continuous-time ("barrier") model with a fixed interest rate, in which a fixed recovery is paid once the barrier is triggered. This model, which we will refer to as the Fixed-Recovery-Barrier model, serves as a bridge between the fixed interest rate, random-recovery-barrier model, which we refer to as the Random-Recovery-Barrier model, and the Longstaff-Schwartz model. By comparing the Fixed-Recovery-Barrier and Longstaff-Schwartz models and also the Merton and Rabinovitch models, we can analyze the effect of random interest rates; by comparing the Fixed-Recovery-Barrier and Random-Recovery-Barrier models, we can assess the effect of random recovery. Finally, we assess the effect of continuous default by comparing the Merton and Random-Recovery-Barrier models. The classification of the nested models according to the three basic assumptions is summarized in Exhibit 1.

We find that random interest rates and random (asset value) recovery are important assumptions for achieving accurate pricing, while continuous default times, as opposed to a single default time, is not. We use credit default swap (CDS) prices rather than bond prices

EXHIBIT 1
Models Analyzed: Classified by Assumptions



Note: Shaded boxes represent models not included in the comparisons.

because they are a purer representation of credit risk, since bond prices embed convexity factors.[1] To achieve greater accuracy, we use actual transaction prices rather than the matrix prices that have been used in some other studies.

This article contributes to the literature in that we use actual transaction prices as opposed to matrix prices that suffer from misrepresentation of the true market,[2] and we use CDS spreads as opposed to bond prices that are less liquid and clean, in that only few bonds are transacted frequently in the market.

THE MODELS

For all the models in this article, we assume that the equity price follows the risk-neutral lognormal process

$$\frac{dS}{S} = rdt + \sigma dW_1 \tag{1}$$

where r is the short-term interest rate, σ is the volatility, and W_1 is a standard Brownian motion.

For those models that assume random interest rates (Rabinovitch and Longstaff-Schwartz models), we assume the following one-factor model for the instantaneous rate:

$$dr = \alpha(\mu - r)dt + vdW_2 \tag{2}$$

where $dW_1 dW_2 = \rho dt$, μ is the risk-adjusted, long-term mean interest rate, α is the rate of mean reversion, and v is the short-rate volatility. Under this assumption, the default-free zero-coupon bond price, derived by Vasicek [1977], is

$$P(t,T) = e^{-r(t)F(t,T)-G(t,T)} \tag{3}$$

where

$$F(t,T) = \frac{1-e^{-\alpha(T-t)}}{\alpha}$$

$$G(t,T) = \left(\mu - \frac{\nu^2}{2\alpha^2}\right)(T-t-F(t,T)) + \frac{\nu^2 F(t,T)^2}{4\alpha}$$

The Merton Model

The Merton model is a one-period model, so the coupon debt of the company must be translated into a maturity-T (one-year) equivalent zero-coupon debt[3]

$$D(0,T) = A(0)[1-N(d_M^+)] + P(0,T)KN(d_M^-) \tag{4}$$

where $A(0)$ is the initial asset value, K is the face value of the debt, $P(0,T) = e^{-rT}$, and

$$d_M^{\pm} = \frac{\ln A(0) - \ln K - \ln P(0,T) \pm \frac{1}{2}V_M}{\sqrt{V_M}}$$

$$V_M = \sigma^2 T$$

The Rabinovitch Model

The Rabinovitch model replaces the constant interest rate from the Merton model with a random interest rate. The risky debt value from equation (4) becomes

$$D(0,T) = A(0)[1-N(d_R^+)] + P(0,T)KN(d_R^-) \tag{5}$$

where

$$d_R^{\pm} = \frac{\ln A(0) - \ln(P(0,T)K) \pm \frac{1}{2}V_R}{\sqrt{V_R}}$$

$$V_R = \sigma^2 T + \left(T - 2F(0,T) + \frac{1-e^{-2\alpha T}}{2\alpha}\right)\left(\frac{\nu}{\alpha}\right)^2 - 2\rho\sigma\frac{(T-F(0,T))\nu}{\alpha}$$

and $P(t, T)$ is given by equation (3).

The Barrier Models

We include two barrier models with fixed interest rates in our study, one with random (asset-value) recovery of the Merton type and the other with fixed recovery of the Longstaff-Schwartz type. When compared to the Merton model, the former model shows the significance of the continuous default assumption. The latter model, when compared to the Longstaff-Schwartz model, shows the significance of random interest rates. The comparison of these two barrier models shows the significance of random recovery.

The Random-Recovery-Barrier model is the popular down-and-out exotic equity option pricing model. This model is widely used in modeling the equity of the firm as a substitute for the Merton model because it allows for default to occur at any point in time, not just maturity. However, even though default can occur at any time, the recovery payoff must occur at the maturity date. (The down-and-out barrier exotic option model can be found in Hull [2006, p. 533].) Subtracting the option value that represents equity from the initial asset value of the firm, we arrive at the value for the risky debt as

$$\begin{aligned} D(0,T) = A(0)&\left[1-\left(N(d^+) - \left(\frac{H}{A(0)}\right)^{2q^+} N(h^+)\right)\right] \\ &+ P(0,T)K\left[N(d^-) - \left(\frac{H}{A(0)}\right)^{2q^-} N(h^-)\right] \end{aligned} \tag{6}$$

for the fixed barrier $H \geq K$ where

$$d^{\pm} = \frac{\ln A(0) - \ln H}{\sigma\sqrt{T}} + q^{\pm}\sigma\sqrt{T}$$

$$h^{\pm} = \frac{\ln H - \ln A(0)}{\sigma\sqrt{T}} + q^{\pm}\sigma\sqrt{T}$$

$$q^{\pm} = \frac{r}{\sigma^2} \pm \frac{1}{2}$$

and

$$\begin{aligned} D(0,T) = A(0)&\left[1-\left(N(x^+) - \left(\frac{H}{A(0)}\right)^{2q^+} N(y^+)\right)\right] \\ &+ e^{-rT}K\left[N(x^-) - \left(\frac{H}{A(0)}\right)^{2q^-} N(y^-)\right] \end{aligned} \tag{6A}$$

for $H < K$ where

$$x^{\pm} = \frac{\ln A(0) - \ln K + rT}{\sigma\sqrt{T}} \pm \frac{1}{2}\sigma\sqrt{T}$$

$$y^{\pm} = \frac{2\ln H - \ln A(0) - \ln K}{\sigma\sqrt{T}} + q^{\pm}\sigma\sqrt{T}$$

While the Fixed-Recovery-Barrier model does not appear in the literature, we can easily derive it. Since a payment is only due at maturity, we can write the debt value as the weighted average of the face value and recovery value as follows:

$$D(0,T) = P(0,T)K\{(\Pr[A(t) > H] + R\Pr[A(t) < H])\} \tag{7}$$

where R is a fixed recovery rate. From Black and Cox [1976] and Rubinstein and Reiner [1979], we know that the probability of crossing the fixed barrier is

$$N(d^-) - \left(\frac{H}{A(0)}\right)^{2q^+} N(h^-) \tag{8}$$

for $H \geq K$ and

$$N(x^-) - \left(\frac{H}{A(0)}\right)^{2q^-} N(\gamma^-) \tag{9}$$

for $H < K$. Substituting (8) and (9) into (7), we get

$$D(0,T) = P(0,T)\left\{K\left[N(d^-) - \left(\frac{H}{A(0)}\right)^{2q+} N(h^-)\right] + R\left[1 - N(d^-) - \left(\frac{H}{A(0)}\right)^{2q+} N(h^-)\right]\right\} \tag{10}$$

for $H \geq K$ and

$$D(0,T) = P(0,T)\left\{K\left[N(x^-) - \left(\frac{H}{A(0)}\right)^{2q^-} N(\gamma^-)\right] + R\left[1 - N(x^-) - \left(\frac{H}{A(0)}\right)^{2q^-} N(\gamma^-)\right]\right\} \tag{11}$$

for $H < K$.

The Longstaff-Schwartz Model

The Longstaff-Schwartz model generalizes the Fixed-Recovery-Barrier model by permitting interest rates to move randomly over time. In our notation, we can write the risky bond value as

$$D(0,T) = P(0,T)K[1 - (1-R)U(0,T)]$$

where $U(0, T)$ is the default probability between now and time T and

$$U(0,T) = \sum_{i=1}^{n} u_i$$

$$u_1 = N(a_1)$$

$$u_i = N(a_i) - \sum_{j=1}^{i-1} u_j N(b_{ij})$$

$$a_i = \frac{-\ln[A(0,T)/K] - M(\frac{iT}{n})}{\sqrt{S(\frac{iT}{n})}}$$

$$b_{ij} = \frac{M(\frac{jT}{n}) - M(\frac{iT}{n})}{\sqrt{S(\frac{iT}{n}) - S(\frac{jT}{n})}}$$

$$V(t) = \left(\frac{\rho\sigma\nu}{\alpha} + \frac{\nu^2}{\alpha^2} + \sigma^2\right)t - \left(\frac{\rho\sigma\nu}{\alpha^2} + \frac{2\nu^2}{\alpha^3}\right)(1 - e^{-\alpha t}) + \left(\frac{\nu^2}{2\alpha^3}\right)(1 - e^{-2\alpha t})$$

$$M(t) = \left(\frac{\alpha\mu - \rho\sigma\nu}{\alpha} - \frac{\nu^2}{\alpha^2} - \frac{\sigma^2}{2}\right)t + \left(\frac{\rho\sigma\nu}{\alpha^2} + \frac{\nu^2}{2\alpha^3}\right)e^{-\alpha T}(e^{\alpha t} - 1) + \left(\frac{r}{\alpha} - \frac{\alpha\mu}{\alpha^2} + \frac{\nu^2}{\alpha^3}\right)(1 - e^{-\alpha t}) - \left(\frac{\nu^2}{2\alpha^3}\right)e^{-\alpha T}(1 - e^{-\alpha t})$$

If we assume that $\nu = 0$ and $\alpha = \infty$, the interest rate process reduces to the constant interest rate case, $r = \mu$. In this special case, we have $V(t) = \sigma^2 t$ and $M(t) = (\mu - \frac{1}{2}\sigma^2)t$.

DATA

The CDS price data were obtained from Creditex for the period of February 15, 2000, through April 8, 2003. This is a transaction dataset where bid, offer, and trade prices are provided. The dataset contains 217,480 observations for 1,372 names. The most popular contract is the 5-year CDS (84.7%). From the dataset we can see that the real development of the CDS market is in 2002 where the number of observations (bid, offer, and trade) is nearly three times that of the prior year. More details about the dataset are given in appendix A.

Because the data are very diverse, we apply the following selection rules to obtain the firms used in our study. First, we select only U.S. names. Then, we remove sovereign CDS, and finally, we select only USD-denominated CDS. This leaves us with the observations in Exhibit 2.

Since most of the quotes (bids and offers) do not come in matched pairs, we only employ trade observations. Out of 7,108 trades, we further eliminate observations due to incorrect tickers (non-existent tickers, tickers with additional suffixes, and tickers containing numbers), companies with no financial data due to takeover or merger, and errors in the data, such as repetitions. There are 1,235 such unusable observations.

In order to apply the structural models, we need financial data of the companies. Given the unique balance sheet structure of financial firms, we decided to use only nonfinancial companies. Eliminating the 2,374 financial trades leaves 3,499 trades (7,108 − 1,235 − 2,374 = 3,499). We also eliminated three observations that had a maturity of zero. Our final dataset consisted of 3,496 trade observation points.

We collect rating, firm-size, and leverage information for the firms in our sample and also compute historical equity variance. Since the CDS are written on names rather than specific bond issues, the credit ratings should be at the company level. We obtained rating information from the Fixed Investment Securities Database (FISD) from Mergent, Inc. This database has complete at-issue information on 162,593 bonds issued by 10,177 entities. Issue dates range from 1894 (a 100-year bond) to June 2004. About 90% of the bonds have issue dates in 1986 or later (for another 5%, the issue date is not available). Current (latest up to June 2004) and historical ratings from four agencies are included in FISD: Moody's, Standard & Poor's, Fitch, and Duff & Phelps.[4] We use only Standard and Poor's ratings for consistency. FISD gives the ratings on a per-issue basis, not on a company basis. We checked a sample of companies and found that in most cases, all bond issues had the same current rating. In a few cases, there was one bond with a rating one notch different from the rest. Therefore, we use the rating of a company's first issue in the list as a company rating.[5]

Exhibit 3 provides a summary of the characteristics of the CDS spreads used in this study. The majority of the trades are for CDS on senior unsecured debt of companies with credit ratings in the A and BBB ranges. Almost all the CDS have maturities of three to six years, with the majority being five years.

We obtain monthly equity prices from Center for Research in Security Prices (CRSP) and quarterly financial data from Compustat for the study period. We compute the equity volatility using the past 12 discrete monthly returns.

For the benchmark Treasury rates and to estimate the term-structure model, we use the daily Constant Maturity Treasury (CMT) rates published by the U.S. Department of the Treasury. These rates are available from the website of the Federal Reserve Bank of St. Louis.[6] The period of the CMT matches with that of the tenor of the CDS.

EXHIBIT 2
Bid, Offer, and Trades in Database by Sector

Sector	Bid	Offer	Trade
Nonfinancial	21,079	20,572	4,734
Financial	15,461	15,382	2,374
Total	36,540	35,954	7,108

EMPIRICAL RESULTS

Model Parameters

Both the Rabinovitch and Longstaff-Schwartz models assume random interest rates of the Vasicek type. Hence, we first estimate the one-factor Vasicek model, described by equations (2) and (3), for the term structure. Details of the estimation procedure are given in appendix B. The parameter estimates we obtain are

$$\alpha = 0.7116$$
$$\mu = 0.0352$$
$$\nu = 0.0437$$

EXHIBIT 3

Arbitrage Credit Default Swap Spreads in Ratings Groups

Rating	AAA	AA	A	BBB	BB	B	All
A: All CDS							
	24.94	51.82	101.34	204.66	451.20	816.63	157.23
	(18)	(197)	(1425)	(1808)	(40)	(8)	(3496)
B: Maturity in years							
<5	–	32.88	134.97	236.46	549.50	390.00	202.35
	–	(8)	(107)	(118)	(12)	(2)	(247)
=5	24.94	52.63	98.67	202.43	409.07	958.83	153.86
	(18)	(189)	(1315)	(1689)	(28)	(6)	(3245)
>5	–	–	71.67	220.00	–	–	108.75
	–	–	(3)	(1)	–	–	(4)
C: Year of transaction							
2000	16.00	–	64.13	–	109.06	–	80.58
	(4)	–	(2)	–	(16)	–	(32)
2001	28.00	56.57	114.28	178.27	336.53	240.00	142.40
	(4)	(79)	(376)	(402)	(17)	(1)	(879)
2002	22.44	52.94	98.25	220.54	523.82	982.17	166.21
	(9)	(90)	(804)	(1015)	(17)	(6)	(1941)
2003	27.00	38.00	93.02	194.07	570.33	400.00	154.22
	(5)	(24)	(233)	(375)	(6)	(1)	(644)
D: Leverage (debt ÷ (debt + size), debt = 0.5 × (current liability + total liability))							
[0, 0.3]	19.45	40.63	84.27	145.39	337.50	555.00	90.46
	(11)	(110)	(416)	(135)	(2)	(2)	(676)
(0.3, 0.6]	26.00	65.98	108.04	191.99	441.93	–	153.83
	(5)	(87)	(740)	(992)	(15)	–	(1839)
(0.6, 1]	52.50	–	109.30	234.88	467.13	903.83	209.61
	(2)	–	(269)	(681)	(23)	(6)	(981)
E: Firm size (stock price × outstanding shares, unit = $10,000,000)							
[0, 10]	26.50	55.52	102.80	206.41	460.85	816.63	161.10
	(4)	(75)	(1347)	(1779)	(9)	(8)	(3352)
(10, 20]	16.67	23.05	77.26	97.21	75.00	–	74.14
	(6)	(11)	(76)	(29)	(1)	–	(123)
(20, –]	30.38	21.86	29.50	–	–	–	25.83
	(8)	(1)	(2)	–	–	–	(21)
F: Equity variance (weekly)							
[0, 0.05]	23.77	40.25	78.67	148.01	140.00	191.50	103.44
	(13)	(100)	(622)	(505)	(2)	(2)	(1244)
(0.05, 0.1]	28.00	63.76	110.09	215.62	466.92	240.00	172.54
	(5)	(7)	(769)	(1211)	(24)	(1)	(2107)
(0.1, –]	–	–	318.10	371.46	468.71	1182.00	396.29
	–	–	(34)	(92)	(14)	(5)	(145)
G: Sector							
Corporate	24.94	48.30	100.50	201.73	451.20	816.63	154.88
	(18)	(179)	(1391)	(1706)	(40)	(8)	(3342)
Telecomm	–	86.83	135.82	253.65	–	–	208.14
	–	(18)	(34)	(102)	–	–	(154)
H: Debt priority							
Senior	24.94	51.82	101.34	203.94	451.20	816.63	156.73
	(18)	(197)	(1425)	(1799)	(40)	(8)	(3487)
Subordinated	–	–	–	243.00	–	–	243.00
	–	–	–	(8)	–	–	(8)

Note: Each ratings group includes the corresponding + or – ratings. The numbers in parentheses represent the number of observations in each group.

Note that the parameter μ contains the market price of risk.[7] Since the CMT rates are par bond rates, they resemble the (semi-annual discrete) zero-coupon rates. For the sake of convenience and consistency, we treat them as continuously compounded yields to compute the zero-coupon bond prices. The discrepancy between the two should be negligible for our purposes.

We make a number of assumptions in comparing the five models analyzed in this study:

- For the fixed recovery models (Fixed-Recovery-Barrier and Longstaff-Schwartz), the recovery rate is 0.4.[8]
- For the random interest models, the correlation between the credit spread and risk-free rate is zero.
- For the barrier models, the barrier is assumed to be half of the sum of the short-term debt and the total debt.[9]
- For all the models, the maturity structure of the liabilities matches exactly what the model assumes. For example, in the Merton model, we assume that the company has a single debt (with a zero-coupon rate) that matures at the same time as the CDS contract. Liabilities of various maturities are converted into a single equivalent debt.[10] This assumption corresponds to the assumption that the risky debt of the company and the CDS contract sum to a risk-free debt. Hence, for a given CDS spread, the total protection value is compared to the model protection value given by various models.
- We obtain historical equity volatility from equity prices and solve for the asset price and asset volatility using the following simultaneous equations:

$$E = A - D_i(A,\sigma)$$
$$\sigma_E = \frac{A}{E}\frac{\partial E}{\partial A}\sigma$$

where i is the choice of the model (Merton, Barrier, Longstaff-Schwartz, and Rabinovitch) and the equity volatility is based upon past 12 discrete returns.
- For all the models, we compute the zero-coupon spread as

$$s(t,T) = -\frac{1}{T-t}\ln\frac{D(t,T)}{P(t,T)}$$

which approximates the CDS spread in the dataset.

Model Comparisons

Root mean square error. Exhibit 4 reports summary statistics of the entire sample of the actual CDS spreads and model-predicted CDS spreads. Consistent with Eom, Helwege, and Huang [2004], we find that some models overestimate the spreads (Fixed-Recovery-Barrier and Longstaff-Schwartz models) and some underestimate the spreads (Merton, Rabinovitch, and Random-Recovery-Barrier models).

Exhibit 5 reports the overall pricing errors of the selected models. Panel A of the exhibit shows the summary statistics for the pricing error (PE) and Panel B shows the summary statistics for the absolute pricing errors (APE), where we compute the pricing error (PE) and absolute pricing error (APE) as

$$\text{PE} = \text{predicted spread} - \text{observed spread}$$
$$\text{APE} = |\text{predicted spread} - \text{observed spread}|$$

We use MPE and MAPE to denote the means of these two error measures.

We observe several important results. From Panel A, we see that none of the models correctly predicts the CDS

Exhibit 4

Summary Statistics for Actual CDS Spreads from Transaction Data and Predicted CDS Spreads by Model

Data source or Model	N	Mean	Std	Median	Minimum	Maximum
CDS transaction data	3496	157.23	184.78	100.00	10.00	3600.00
Merton	3496	78.04	163.57	25.81	0.00	2632.00
Rabinovitch	3496	82.45	165.79	28.92	0.00	2674.00
Barrier (random recovery)	3496	23.88	24.52	16.85	0.00	141.36
Barrier (fixed recovery)	3496	206.77	367.93	86.57	0.00	10217.00
Longstaff and Schwartz	3496	172.93	330.96	65.31	0.00	7609.00

EXHIBIT 5

Summary Statistics for Performance of the Five Structural Models

(A): Pricing Error (PE)

Model	N	Mean (MPE)	Std	Median	Minimum	Maximum	t-value
Merton	3496	–79.19	198.17	–62.61	–3592.69	2427.41	–23.62
Rabinovitch	3496	–74.78	198.35	–60.42	–3590.79	2468.97	–22.29
Barrier (random recovery)	3496	–133.35	177.73	–83.12	–3593.85	68.49	–44.35
Barrier (fixed recovery)	3496	49.54	341.92	–19.58	–3566.91	9266.51	8.57
Longstaff and Schwartz	3496	15.70	321.11	–29.86	–3579.13	7424.46	2.89

(B): Absolute Pricing Error (APE)

Model	N	Mean (MAPE)	Std	Median	Minimum	Maximum	t-value
Merton	3496	119.46	176.83	71.51	0.19	3592.69	39.94
Rabinovitch	3496	117.38	176.51	70.00	0.05	3590.79	39.31
Barrier (random recovery)	3496	133.55	177.58	83.12	0.27	3593.85	44.46
Barrier (fixed recovery)	3496	148.95	311.72	74.84	0.01	9266.51	28.25
Longstaff and Schwartz	3496	130.45	293.83	72.42	0.09	7424.46	26.25

Note: Performance measured by pricing error (PE = predicted spread – observed spread) and absolute pricing error (APE = |predicted spread – observed spread|). The t-values are for the test of the mean being equal to zero.

spreads. Furthermore, the random recovery models underestimate the market price (i.e., MPEs are negative) and the fixed recovery models overestimate the market price (i.e., MPEs are positive). From Panel B, we see that the simple models (Merton and Rabinovitch) outperform more complex models (barriers and Longstaff-Schwartz), although the differences are not large. Except for the Random-Recovery-Barrier model, these comparisons seem to suggest that random recovery is the most important assumption in valuing CDS contracts.

This result is of particular importance because many reduced-form models assume a fixed recovery for the CDS contracts. However, actual CDS contracts have delivery options similar to those of Treasury bond and note futures contracts. The buyer of a name-based CDS contract, like the ones we examine in this study, can deliver any bond that satisfies the contractual criteria under the same credit entity, not necessarily the reference bond. Consequently, actual CDS spreads must reflect random recovery.

Comparisons of the Merton model with the Rabinovitch model and the Fixed-Recovery-Barrier model with the Longstaff-Schwartz model suggest that random interest rates are important as well. In both comparisons, the difference between the models is the assumption of random interest rates. The models that employ random interest rates (Rabinovitch and Longstaff-Schwartz models) outperform those that assume a constant interest rate (Merton and Fixed-Recovery-Barrier models).

The purpose of comparing the Merton and Random-Recovery-Barrier models is to test for the importance of the latter's assumption of continuous default: default may occur at any time during the life of the bond. The Merton model converts the stream of liabilities into a one-time balloon payment at $T = 1$. The fact that the Merton model has a lower mean APE than the barrier model suggests that continuous default does not play an essential role in explaining CDS prices. Moreover, the fact that single-time default models (Merton and Rabinovitch models) in general outperform the continuous default models (barriers and Longstaff-Schwartz models) strengthens this observation. There was no qualitative change in the results when we investigated sub-samples based on rating, leverage, size, or volatility.[11]

Exhibit 6 reports the Z-score test of each pair of models. The Z score between any two models, i and j, is[12]

$$Z_{ij} = \frac{\text{mean}(APE_{i,t} - APE_{j,t})}{\text{std}(APE_{i,t} - APE_{j,t})}$$

where the mean and standard deviation are computed across all 3,496 observations. For each pair of models, we computed the difference between the APEs of their

EXHIBIT 6
Comparison of Models' Absolute Pricing Errors Using the Paired Z-Test

Model	Rabinovitch	Barrier (random recovery)	Barrier (fixed recovery)	Longstaff and Schwartz
Merton	2.09	−14.08	−29.48	−10.99
	(19.55)	(−9.04)	(−6.36)	(−2.48)
Rabinovitch		−16.17	−31.57	−13.08
		(−10.13)	(−6.81)	(−2.95)
Barrier (random recovery)			−15.4	3.09
			(−3.29)	(0.69)
Barrier (fixed recovery)				18.49
				(6.05)

Note : The Z score between two models, i and j, is computed as Z_{ij} = mean ($APE_{i,t} - APE_{j,t}$)/std($APE_{i,t} - APE_{j,t}$). We compute APE difference between two models for every observation, and then compute the mean and standard deviation.

predictions for every observation and then computed the mean and standard deviation of the differences. The ranking of the models, as indicated in Exhibit 6, is Rabinovitch, Merton, Longstaff-Schwartz, Random-Recovery-Barrier, and Fixed-Recovery-Barrier. This confirms the observations we made earlier based on the results reported in Exhibit 5. Furthermore, the fact that the Longstaff-Schwartz and Random-Recovery-Barrier models achieve similar results, yet they both outperform the Fixed-Recovery-Barrier model, demonstrates that both random interest rates and random recovery are important but one does not dominate the other. This result confirms the findings of Wei and Guo [1997]. However, via our nested models, we can now see more clearly the effect of each assumption.

Likelihood ratio test. The likelihood ratio test (LRT) is a statistical test that can determine the relative goodness-of-fit of two nested models. A relatively more complex model is compared to a simpler model which is a special case (restriction) of it, to see if it fits a particular dataset significantly better. If so, the more complex model can be used in subsequent analyses. The restrictions that define the nested model may take the form of any relation among the model parameters. In the simplest case, these restrictions take the form of setting a parameter of the nesting model to a fixed value. Adding additional parameters will always result in a higher likelihood score.

The LRT begins with a comparison of the maximum log likelihood scores of the two models

$$LR = 2 \times (\ln\Lambda_1 - \ln\Lambda_2)$$

where Λ is the value of the likelihood function. Since the model restrictions are linear, assuming that the errors are normally distributed, the Λ values are proportional to the sum of the squared errors for each model. This LR statistic approximately follows a chi-square distribution, where the number of degrees of freedom is equal to the number of additional parameters in the more complex model. Using this information, we can use standard statistical tables to determine the critical value of the test statistic for the statistical significance of LR. When LR is significant, we reject the null hypothesis that the more restricted model and the less restricted model are the same.

Exhibit 7 presents results of the LRTs of the nested models. The Rabinovitch model nests the Merton model by setting the parameters in the interest rate process to generate a constant interest rate ($\alpha = \infty$ and $\nu = 0$). When α and ν are permitted to vary, then surely the Rabinovitch model will perform better. There are, therefore, two degrees of freedom. The chi-square statistic is 0.002, which is not statistically significant. This finding strongly suggests that the contribution of random interest rates is insignificant with respect to the pricing of CDS. However, we can test the same impact by comparing the Longstaff-Schwartz model with the Fixed-Recovery-Barrier model. The two models are nested in the same way as the Merton and the Rabinovitch models.[13] Although the number of converged cases decreases substantially because of a larger number of parameters, the chi-square statistic is 5.661 which is significant at the 6% level. The difference in the results indicates that the random interest rate assumption causes an important impact only when we permit default

EXHIBIT 7
Likelihood Ratio Test

	Likelihood function (negative log)	Likelihood Ratio χ^2 (p)
Likelihood Ratio Test - Test of Random Interest Rates		
Longstaff-Schwartz	−368.825	5.66 (0.059)
Fixed Recovery Barrier	−363.169	
(N = 59; d.f. = 2)		
Rabinovitch	−1,895.737	0.00 (1.000)
Merton	−1,895.735	
(N = 273; d.f. = 2)		
Likelihood Ratio Test - Test of Continuous Default Barrier		
Random Recovery Barrier	−1,890.763	4.97 (0.026)
Merton	−1,895.735	
(N = 273; d.f. = 2)		
Likelihood Ratio Test - Test of Random Recovery		
Random Recovery Barrier	−1,160.977	24.15 (0.00)
Fixed Recovery Barrier	−1,185.127	
(N = 167; d.f. = 1)		

Note: The likelihood ratio test is $\chi^2 = 2(\ln \Lambda_1 - \ln \Lambda_2) = 2[(-\ln \Lambda_2) - (-\ln \Lambda_1)]$ *where model 1 is the more restricted model. This is a chi-square distribution with the degrees of freedom equal to the additional parameters in model 2.*

to be continuous. This is particularly interesting because for single-period default models, such as the Merton and Rabinovitch models, the impact of random interest rates on prices is through discounting. In contrast, for continuous-default models, random interest rates can also affect the occurrence of default itself.

The test of the Random-Recovery-Barrier and the Merton models shows the impact of the default barrier. The Random-Recovery-Barrier model degenerates to the Merton model if the barrier is set to 0 ($H = 0$). The chi-square statistic is 4.972, which is significant at the 3% level at one degree of freedom, showing support for continuous default. This result echoes the previous result on random interest rates and shows that continuous default increases the impact of random interest rates.

Lastly, we examine the random recovery assumption by testing the Fixed-Recovery-Barrier and Random-Recovery-Barrier models. The two models differ by the recovery assumption, everything else equal.[14] The chi-square statistic is 24.152, which not only is significant at the 1% level, but also is the most significant statistic of all the tests. This result strongly supports the importance of the random recovery assumption.

In sum, by comparing nested models using the LRT, we find that the random recovery assumption is the most important one. It generates the highest likelihood ratio. The next most important assumption is random interest rates. The least important assumption is a continuous default boundary.

Source of mispricing. Next, we use absolute pricing errors to investigate the sources of mispricing. We separate the APEs for each model into two groups of equal size—a high APE group (those higher than the median) and a low APE group (those lower than the median). Then, we compute the means and standard deviations of the numeric ratings, leverages, sizes, and volatilities for both groups for each model. Finally, we compute the differences of the sample means from the two groups (low minus high).

To test for the means being different, we calculate the standard deviations of the differences of the sample means by assuming that the high and low APE groups are independent.

The results are shown in Exhibit 8. For all five models, smaller absolute pricing errors are associated with 1) lower (better) numeric ratings, 2) lower financial leverage, 3) larger firm size, and 4) smaller equity volatility. These results confirm the common wisdom that high-quality firms are more accurately assessed by the market. Another way to see these results is by regressing the APE differences on the company characteristics and controlling for market-wide effects using dummy variables for the years covered by the dataset. The regression results in Exhibit 9 confirm the previous results except for the size variable.

So far we have used the pricing errors of a set of nested models to examine the relative performance of these models. It is widely acknowledged that structural models cannot price credit derivatives accurately. This is primarily attributable to market segmentation and inefficiency. Indeed, while we are able to compare different models, all of the models are rejected by the market (high t statistics). While structural models do not accurately price credit derivatives, many studies (e.g., Moody's KMV) confirm that they are effective in predicting defaults. In other words, although structural models miss the absolute level of risk, they provide accurate credit rankings. For this reason, we compute standard and rank correlations between the actual spreads and predicted spreads generated by the models. The correlation results shown in Exhibit 10 indicate that the models are similar to one another. The Pearson correlation takes the level into account, so it produces the lowest correlation numbers. The Spearman rank correlation is based only on the order; all of the models show very high correlations. The Longstaff-Schwartz and Fixed-Recovery-Barrier models are slightly better than the others in this test, but the differences are not significant.

SUMMARY

Previous studies of structural models of credit risk have lacked the consistent modeling framework and high quality data needed to perform model comparisons. We have defined a set of nested models and compared them using a CDS dataset. The nested models allowed us to examine which model assumptions (continuous default, random recovery, and random interest rates) play an essential role in explaining CDS spreads. The transaction data that we use are far superior to the matrix pricing data used by Eom, Helwege, and Huang [2004] or the narrowly focused data used by Wei and Guo [1997].

Our empirical results indicate that random interest rates and random recovery are important assumptions, while continuous default is not. This is consistent with the finding

EXHIBIT 8

Differences in Rating, Leverage, Size, and Volatility between Companies with High and Low Absolute Pricing Errors for Each Model

	Merton	Rabinovitch	Barrier	Barrier (fixed recovery)	Longstaff and Schwartz
Rating	−1.36	−1.34	−1.59	−1.03	−1.1
	(−23.25)	(−22.84)	(−28.09)	(−17.17)	(−18.42)
Leverage	−0.07	−0.07	−0.11	−0.12	−0.1
	(−12.29)	(−11.40)	(−20.04)	(−21.27)	(−17.43)
Size	6991.43	6490.23	10072.01	7346.67	7559.55
	(6.02)	(5.58)	(8.72)	(6.33)	(6.51)
Volatility	−0.01	−0.01	−0.01	−0.01	−0.01
	(−11.25)	(−11.66)	(−16.00)	(−19.61)	(−16.89)

Note: We separate the absolute pricing errors (APE) for each model into two equal size groups—high APE and low APE. Then, we compute the means and standard deviations of the numeric ratings, leverages, sizes, and volatilities. The numeric version of the credit rating ranges from 2 for AAA to 25 for D. The numbers reported are the difference between the means in the two groups. The numbers in parentheses are t-values based upon the standard deviation that assumes the two groups are independent.

EXHIBIT 9

Regression of Absolute Pricing Errors on Company Characteristics for Each Model

Model	Merton	Rabinovitch	Barrier	Barrier (fixed recovery)	Longstaff and Schwartz
Intercept	–239.65	–234.28	–280.10	18.52	101.44
	(–6.23)	(–6.09)	(–7.44)	(0.28)	(1.56)
Year 2001	49.17	50.26	57.28	105.25	101.34
	(1.67)	(1.71)	(1.99)	(2.08)	(2.04)
Year 2002	112.84	113.09	131.74	150.68	141.27
	(3.87)	(3.88)	(4.61)	(2.99)	(2.86)
Year 2003	92.97	93.45	98.97	143.80	155.98
	(3.13)	(3.15)	(3.41)	(2.81)	(3.11)
Maturity	–21.29	–21.83	–21.47	–96.73	–93.95
	(–4.75)	(–4.87)	(–4.9)	(–12.51)	(–12.4)
Rating	18.33	17.83	21.29	1.62	4.96
	(10.03)	(9.76)	(11.91)	(0.52)	(1.61)
Leverage	170.51	156.45	216.14	620.62	451.98
	(9.61)	(8.82)	(12.46)	(20.29)	(15.08)
Firm size	0.00	0.00	0.00	0.00	0.00
(million)	(2.89)	(2.72)	(3.05)	(0.73)	(1.28)
Equity volatility	2173.99	2264.93	2101.96	3485.99	2140.94
(weekly)	(13.44)	(14.01)	(13.29)	(12.5)	(7.83)
R-Square	0.1670	0.1638	0.2093	0.2026	0.1379
Adj R-Sq	0.1651	0.1619	0.2075	0.2007	0.1359

Note: The dependent variable is the APE for a particular CDS transaction. The dependent variables are: dummy variables for the year of the transaction (2001, 2002, and 2003), maturity in years, a numeric rating, leverage ratio (debt/assets), firm size (market capitalization in $million), and weekly equity volatility.

EXHIBIT 10

Measures of Correlation between Actual and Predicted CDS Spreads for Each Model

Model	Pearson Correlation	Kendall's Rank Correlation	Spearman's Rank Correlation
Merton	0.3578	0.4073	0.5752
	(<0.0001)	(<0.0001)	(<0.0001)
Rabinovitch	0.3638	0.4143	0.5842
	(<0.0001)	(<0.0001)	(<0.0001)
Barrier (random recovery)	0.3483	0.3982	0.5656
	(<0.0001)	(<0.0001)	(<0.0001)
Barrier (fixed recovery)	0.3869	0.4510	0.6291
	(<0.0001)	(<0.0001)	(<0.0001)
Longstaff and Schwartz	0.3317	0.4495	0.6258
	(<0.0001)	(<0.0001)	(<0.0001)

Note: The Pearson correlation is the standard correlation defined in terms of covariance, while the Kendall and Spearman correlations are nonparametric and depend only on the ordering of the data.

by Wei and Guo [1997]. We also find evidence consistent with Eom, Helwege, and Huang [2004] that structural models may overestimate or underestimate actual price levels. The correlation rankings generated by the structural models are similar to one another, suggesting that all structural models have similar default prediction power.

Appendix A

Data Description

Date Range

Beginning Date	Ending Date
2/15/2000	4/8/2003

Firm numbers by region:

REGION	Frequency	Percent
AME	730	53.2
ASI	71	5.2
AUS	43	3.1
EUR	457	33.3
JPN	71	5.2
Total	1372	100.0

Firm numbers by currency:

Currency	Frequency	Percent
AUD	1	0.1
EUR	262	19.1
GBP	11	0.8
JPY	49	3.6
USD	1049	76.5
Total	1372	100.0

Firm numbers by sector:

SECTOR	Frequency	Percent
CORP	935	68.1
FIN	328	23.9
SOV	61	4.4
TEL	48	3.5
Total	1372	100.0

Firm number by debt priority:

DEBT	Frequency	Percent
CB	2	0.1
SNR	1360	87.2
SUB	198	12.7
Total	1560	100.0

**188 firms issue both SNR and SUB swap*

Trade Frequency (number of bid, offer, and trade)

MONTH	2000	2001	2002	2003	2000 -2003
1	-	2959	6103	14184	23246
2	496	3275	6417	14067	24255
3	904	2656	5248	14992	23800
4	1373	2859	7466	3731	15429
5	2286	3097	8055	-	13438
6	1858	3141	9632	-	14631
7	2514	2863	15062	-	20439
8	4101	3712	9693	-	17506
9	4240	2470	9913	-	16623
10	3810	4840	11662	-	20312
11	2376	4162	8753	-	15291
12	1581	2370	8557	-	12508
Total	25539	38404	106561	46974	217478

Term to maturity distribution

Schedule	Frequency	Percent	Schedule	Frequency	Percent
0.00	138	0.1	7.50	26	0.0
0.50	709	0.3	8.00	526	0.2
1.00	4878	2.2	8.50	63	0.0
1.50	546	0.3	9.00	152	0.1
2.00	3805	1.7	9.50	25	0.0
2.50	851	0.4	10.00	4805	2.2
3.00	9242	4.2	11.00	5	0.0
3.50	598	0.3	11.50	2	0.0
4.00	3674	1.7	12.00	88	0.0
4.50	1127	0.5	15.00	155	0.1
5.00	184142	84.7	15.50	12	0.0
5.50	114	0.1	18.00	9	0.0
6.00	375	0.2	20.00	4	0.0
6.50	40	0.0	25.00	1	0.0
7.00	1363	0.6	30.00	5	0.0
			Total	217480	100.0

Amount distribution

Amount	Frequency	Percent
< 5	2641	1.2
< 50	12	0.0
> 15	3693	1.7
5-15	210036	96.6
500–	1098	0.5
Total	217480	100.0

Time distribution of trades

Hour	Frequency	Percent	Hour	Frequency	Percent
0	16660	7.7	12	5773	2.7
1	518	0.2	13	4442	2
2	5955	2.7	14	4467	2.1
3	33909	15.6	15	3868	1.8
4	27251	12.5	16	2411	1.1
5	16725	7.7	17	3843	1.8
6	10757	4.9	18	120	0.1
7	10618	4.9	19	702	0.3
8	21802	10	20	2309	1.1
9	18414	8.5	21	1692	0.8
10	14174	6.5	22	705	0.3
11	10039	4.6	23	326	0.1

APPENDIX B

Vasicek Model Estimation

Both the Rabinovitch and Longstaff-Schwartz models assume the Vasicek [1987] model for the risk-free term structure. The Vasicek model is a one-factor, mean-reverting Gaussian model with the following short-rate dynamics:

$$dr = \alpha(\mu - r)dt + \nu dW_2$$

under the risk-neutral measure. The solution to the above stochastic differential equation is an AR(1) process in discrete time

$$r_t = (1 - e^{-\alpha h})\,\mu + e^{-\alpha h} r_{t-h} + e_t \tag{1B}$$

where h is the time interval and e_t has mean 0 and variance $\frac{1-e^{-2\alpha h}}{2\alpha}\nu^2$.

The closed-form solution from equation (3) in the text shows that

$$P_{t,T} = e^{-r_t F_\tau - G_\tau}$$

where $t + \tau = T$ represents the maturity of the bond. Note that the 5-year CMT rates have a constant rolling maturity of 5 years. Hence $\tau = 5$ and remains fixed. Inverting the pricing formula we get the instantaneous rate

$$r_t = -\frac{\ln P_{t,T} + G_\tau}{F_\tau} \tag{2B}$$

Substituting this into the regression equation (1B) gives

$$\begin{aligned} -\ln P_{t,T} &= (1 - e^{-\alpha h})(\mu F_\tau + G_\tau) \\ &\quad + e^{-\alpha h}(-\ln P_{t-h,T-h}) + F_\tau e_t \end{aligned} \tag{3B}$$

and solving the resulting simultaneous equations, we obtain

$$\text{Slope} = e^{-\alpha h}$$
$$\text{Intercept} = (1 - e^{-\alpha h})(\mu F_\tau + G_\tau)$$
$$\text{MSE} = \nu^2 F_\tau^2 \frac{1 - e^{-2\alpha h}}{2\alpha}$$

where $h = \frac{1}{252}$ for daily data. In fact, there is no need to solve simultaneously for all three parameter values. The slope gives the solution for α, then the MSE gives the solution for ν, and finally the intercept gives the solution for μ.

ENDNOTES

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[1]Convexity is a concern because bonds trading at the same yield may have different durations and total returns.

[2]Sarig and Warga [1989] found that matrix prices may not be accurate estimates of market prices. Many more recent studies have eliminated matrix prices from their datasets for this reason (e.g., Elton, Gruber, Agrawal, and Mann [2001] and Chen, Lesmond, and Wei [2007]).

[3]This type of debt transformation is used by Moody's KMV, the first commercial provider of the Merton model. See Crosbie and Bohn [2003].

[4]Duff & Phelps Credit Rating Company was acquired by Fitch (Fitch Ratings) in 2000.

[5]One could try to take an average rating for all the issues of a company, but we found that this does not provide much additional information.

[6]http://research.stlouisfed.org/fred2/categories/22

[7]In the Vasicek model and under log utility, the market price of risk is constant and independent of the risk level. Usually the risk premium is written as νq, so $\mu = \mu^* + \nu q$, where μ^* is the mean level of the instantaneous rate under the original measure and q is the market price of risk.

[8]According to Moody's 2002 survey, the average recovery rate for senior unsecured debts is 41%. Many banks report higher recovery rates for similar debts. Hence, we use 40% in our comparisons. For robustness checking, we also run the comparisons with 60% recovery and the results are similar and available upon request.

[9]This is the assumption that was made by Moody's KMV.

[10]For example, one commercial provider of structural models, Moody's KMV, assumes that such debt should have a face value approximately equal to the company's total current liabilities plus half of its long-term liabilities.

[11]The subsample results are available upon request.

[12]We do not present the results based upon PE because equal overestimates and underestimates will be aggregated rather than cancelled.

[13]The Longstaff-Schwartz model has an additional flexibility that permits the recovery to be paid upon default (whereas the Fixed-Recovery-Barrier model permits only recovery at maturity).

[14]There is a technical difficulty when comparing these two models because the random recovery in the Random-Recovery-Barrier model is implicit. Nevertheless we still perform the LRT because in theory the Random-Recovery-Barrier model is more flexible than the Fixed-Recovery-Barrier model.

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