An Analysis of Portfolios of Insured Debts

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Growing sizes of investment projects fuelled by economic integration have created an increase in the demand for credit insurance (CI) products. Larger projects usually require long-term external financing and thereby generate the need for financial guarantees. For instance, Nortel Network Corporation gained a series of waivers from Export Development Canada to support its credit (Reuters, February 15, 2005) and in the United States in 2004, Ex-im Bank the official credit agency has provided loan guarantees and export CI to assist $17.8 billion US exports around the world.

CI is valuable to project sponsors, since it is an opportunity for them to improve their credit quality and to access to more funding at lower costs. From an insurance company viewpoint, portfolios of financial guarantees are complex because of their risk distribution structure and the systematic nature of credit risk, which underscores the importance of multiyear management.[1]

We distinguish between two types of insurance portfolios: "closed" and "opened." Closed portfolios consist of a fixed number of CI contracts to be carried until maturity. Opened portfolios on the other hand, allow the insurer to include new insurance contracts over time.

The objective of this article is to investigate the effect of multiyear risk-management decisions on CI portfolios. For this purpose, we contrast portfolio risk diversification with increased insurer's capital in a multiyear contractual environment. Using equivalent martingale measures (EMMs) in Harrison and Pliska [1981], we measure the value of CI through the market value of vulnerable financial guarantees, which accounts for all shortfalls, clients and insurers' specific risks, and capital as well as the correlations between the parties.

Merton [1977] established an isomorphism between a put option and a financial guarantee. Hence, a financial guarantee can be considered as a put option written by the insurer and given to the policy holder. Since this seminal work, several researchers have studied private financial guarantees provided by a vulnerable insurer (i.e., the insurer is subject to default risk). For instance, Johnson and Stulz [1987] and Klein and Inglis [1999] propose a model to price options subject to risk of financial distress on the part of the option writer. Lai and Gendron [1994] derive closed-form formulas for the value of private loan guarantees under stochastic interest rate regime. Gendron, Lai, and Soumaré [2002] use no-arbitrage pricing theory to evaluate portfolios of vulnerable private loan guarantees and investigate their risk diversification properties. All these studies assume the same maturity for all guarantees in the portfolio.

In this attempt at modeling multiple maturities in portfolios of CI, we use a no-arbitrage pricing model with realistic features such as coupon payments, stochastic interest rate, and stochastic volatilities. We rely on optimization by Monte Carlo simulations for numerical results.

We find that for a given riskiness level of insurer's capital, an optimal value of CI can be obtained by appropriate risk diversification and/or increased insurer's capital. Our simulation results show that for insurers with high risk exposure, portfolio risk diversification is more effective than increasing insurer's capital. For a creditworthy insurer, increasing the size of the insurer's capital can lead to significant improvement in the value of the CI portfolio. This suggests that alternative risk transfer techniques, which provide synthetic capital to the insurer, should be considered in an integrated risk management.

The article is organized as follows. We present the model in the next section and discuss the results from Monte Carlo simulations in Simulation Results section. Finally, we conclude in last section.

THE MODEL

As mentioned above, our model captures different types of risks inherent to portfolios of CIs through stochastic cash flows of the policyholders, net-assets of the insurer, stochastic interest rate and stochastic volatilities. In addition, default before the maturity of the debt is triggered by missing on a coupon payment.

The first subsection describes the different stochastic processes. The no-arbitrage pricing argument to value expected shortfalls (ESs) and CIs in a portfolio context is used in ES and CI Valuation section. The third subsection discusses the strategic risk management decisions for portfolios of CI.

Stochastic Processes

We make the standard assumptions of no-arbitrage pricing of Merton [1974] and Harrison and Kreps [1979], i.e., perfect, frictionless markets in which securities are traded in continuous time, with no tax, no transaction costs, no asymmetric information, and so on. For a standard exposition of this kind of framework, see a classical text such as Panjer [1998].

Assume a complete probability space $(\Omega, \mathbb{F}, Q)$ where $\mathbb{F} = \{\mathcal{F}_t;\ t \geq 0\}$ is the filtration, $\mathcal{F}_t$ is the set of information available at time t and Q the risk-neutral probability measure. The processes in this article are adapted to $\mathbb{F}$ and are measurable functions from $\Omega \times \mathbb{R}_+$ into $\mathbb{R}$. All equalities involving random variables are understood to hold P-almost surely. $E^Q[\,\cdot\,|\mathcal{F}_t]$ is the conditional expectation with respect to the available information set $\mathcal{F}_t$ at time t, and is denoted hereafter $E_t^Q[\,\cdot\,]$ for ease of notation. For any random variable X, we use $X(t)$ to represent $X(\omega,t)$: $\Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}$.

We consider a portfolio of N CI contracts. Each credit contract consists of insuring one debt. Let us denote by V_i the cash flows available for loan repayment by the policyholder i ($i = 1, \ldots, N$). The process describing the cash flows are

$$dV_i(t) = r(t)V_i(t)dt + \sigma_i(t)V_i(t)dZ_i(t), \quad i = 1, \ldots, N, \quad (1)$$

where r: $\Omega \times [0, +\infty) \rightarrow \mathbb{R}$ is the short-term interest rate, $\{\sigma_i;\ i = 1, \ldots, N\}$; $\Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}$ are the instantaneous volatilities of the cash flows returns described below (see Equation (4) for the volatilities and Equation (5) for the interest rate). $\{Z_i;\ i = 1, \ldots, N\}$: $\Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}$ are Wiener processes.

Let A be the total assets of the insurer and L the current existing liabilities. The total net-assets available to the insurance company to support CI claims is denoted by W:

$$W(t) = [A(t) - L(t)]^+. \quad (2)$$

This equation links conceptually the insurer's net-asset, capital, and insuring capacity. It is assumed to follow Ito diffusion process

$$dW(t) = r(t)W(t)dt + \sigma_W(t)W(t)dZ_W(t), \quad (3)$$

where σ_W: $\Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}$ (is the instantaneous volatility of the insurer's total net-assets described below in Equation (4) and Z_W: $\Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}$ a Brownian motion.

The volatilities of the cash flows ($\{V_i\}_{i=1,\ldots,N}$) and the insurer's total net-assets (W) are stochastic and their processes are given as follows:

$$d\sigma_k^2(t) = \nu_k\left(\beta_k - \sigma_k^2(t)\right)dt + \phi_k\sigma_k(t) \times \left[\rho_{k\sigma}dZ_k(t) + \sqrt{1-\rho_{k\sigma}^2}\,d\eta_k(t)\right], \quad k = 1, \ldots, N, W, \quad (4)$$

where $\{\eta_k;\ k = 1, \ldots, N, W\}$: $\Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}$ are Wiener processes. The coefficients $\{\nu_k, \beta_k$, and $\phi_k;\ k = 1, \ldots, N, W\}$ are respectively the speed of adjustment, the long-run mean and the variation of the diffusion volatility, and the constants $\{\rho_{k\sigma};\ k = 1, \ldots, N, W\}$ represent the correlations between the variables and their volatilities.[2]

The dynamic of the stochastic interest rate is

$$dr(t) = \kappa(\theta - r(t))dt + \sigma_r r(t)^\gamma dZ_r(t), \tag{5}$$

where κ is the speed of adjustment, θ is the long-run mean. σ_r is the volatility of the interest rate changes which is set to a constant, and $Z_r: \Omega \times \mathbb{R}_+ \to \mathbb{R}$ is a Wiener process.[3] The Cox, Ingersoll, and Ross (CIR) [1985] and Vasicek [1977] interest rate processes are particular cases of this process, with γ equal to 0.5 for the CIR [1985] process and 0 for the Vasicek [1977] process.

The Brownian motions satisfy the following conditions

$$E_t^Q[dZ_k(t)dZ_l(t)] = \rho_{kl}dt \quad \forall k,l \in \{1, \ldots, N, W, r\}, \; k \neq l, \tag{6a}$$

$$E_t^Q[dZ_k(t)d\eta_l(t)] = 0 \quad \forall k,l \in \{1, \ldots, N, W, r\} \times \{1, \ldots, N, W\}, \tag{6b}$$

$$E_t^Q[d\eta_k(t)d\eta_l(t)] = 0 \quad \forall k,l \in \{1, \ldots, N, W\}, \; k \neq l. \tag{6c}$$

ES and CI Valuation

The debt underwritten on the contract of policyholder i is represented by a coupon paying bond with coupon C_i at each interval Δ of time and face value F_i at maturity date T_i. The coupon payments are made semi-annually and subtracted from the policyholder's cash flows value denoted by V_i. The insurer seizes the policyholder's remaining cash flows at the first default date.

We use a no-arbitrage argument similar to Harrison and Kreps [1979] and Harrison and Pliska [1981], therefore the debts are priced using the EMM Q as follows.

In absence of default, the bond price at any time τ is

$$D_i(t) = E_t^Q\left[\sum_{k=0}^{[T-\tau/\Delta]} \exp\left(\int_\tau^{\tau+k\Delta} r(s)ds\right)C_i + \exp\left(-\int_\tau^T r(s)ds\right)F_i\right]. \tag{7}$$

From this formula, the default-free coupon bond price at time τ is the expected present value of all future payments until maturity. The first part of the expectation expression is the present value of all remaining coupons and the second part is the present value of the final payment.

The policyholder i defaults on its debt service if the cash flow V_i drops below the required payment. This occurs at the first exit $\tau_i: \Omega \times [0, +\infty) \to \mathbb{R}_+$

$$\tau_i = \inf(t : \text{s.t. } V_i(t) < C_i + F_i \quad \text{if } t = T_i). \tag{8}$$

Hence, the price of the risky debt at time zero without guarantee, $D_{i,NG}(0)$, is

$$D_{i,NG}(0) = E_0^Q\left[\sum_{k=1}^{[\tau_i/\Delta]-1} \exp\left(-\int_0^{k\Delta} r(s)ds\right)C + \exp\left(-\int_0^{\tau_i} r(s)ds\right)V_i(\tau_i)\right]. \tag{9}$$

Intuitively, from Equation (9), in the absence of a CI, upon default at time τ_i, all future payments are lost. The risky debt price is the expectation (under the risk-neutral probability Q) of the present value at time zero of all coupons received prior to default plus the value $V_i(\tau_i)$. Here, the default trigger date is random and can be any coupon date. For a zero coupon bond, the default trigger date has no random component, corresponding exactly to the maturity date of the debt.

CI enhances the value of the risky debt. The price at time zero of the insured debt, $D_{i,G}(0)$, is

$$D_{i,G}(0) = D_{i,NG}(0) + E_0^Q \times \left[\exp\left(\int_0^{\tau_i} r(s)ds\right)\min(\alpha_i W(\tau_i), D_i(\tau_i) - V_i(\tau_i))\right], \tag{10}$$

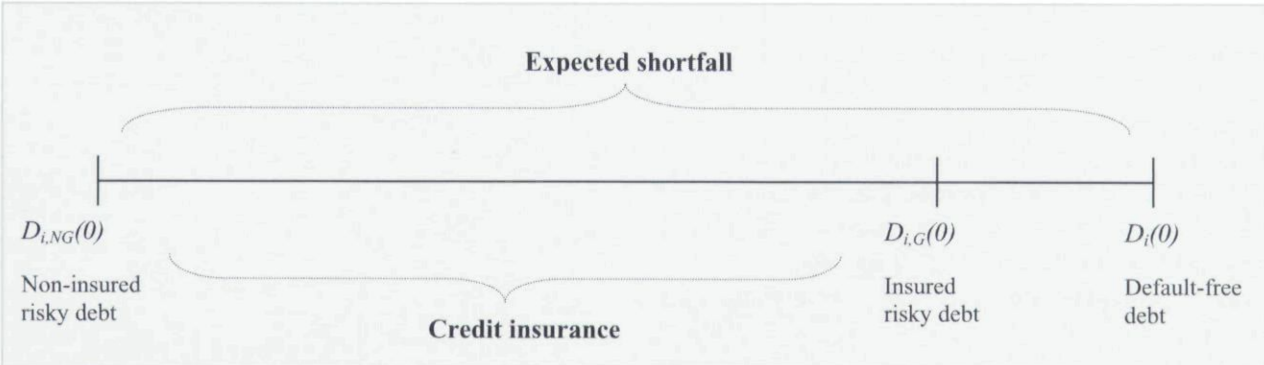
where α_i is the allocation percentage of the insurer's wealth to policyholder i.

In Equation (10), the price of the insured debt is equal to the price of the risky debt without CI plus an additional amount representing the value of the CI. Upon default at time τ_i, if the insurer's contribution ($\alpha_i W(\tau_i)$) combined with the value of the cash flows ($V_i(\tau_i)$) is sufficient to cover all remaining payments on the debt, then the lender receives full payment. Otherwise, the insurer can only provide partial payment.

Exhibit 1 illustrates the relation between the default-free debt ($D_i(0)$), the noninsured risky debt ($D_{i,NG}(0)$) and

E X H I B I T   1

Expected Shortfall and Credit Insurance



the insured risky debt ($D_{i,G}(0)$), given respectively by Equations (7), (9), and (10). The total ES on the loan is measured by the difference between the value of the default-free debt and the noninsured debt: $D_i(0)-D_{i,NG}(0)$. In the spirit of Merton [1977], the difference between the default-free debt and the insured risky debt comes from the fact that insurance companies can default. Some government debt can practically be considered as default-free. The value of CI is the difference between the insured risky debt and the noninsured risky debt:

$$D_i(0)-D_{i,NG}(0)=E_0^Q \times\left[\exp\left(-\int_0^{T_i} r(s)\mathrm{d}s\right)\min\left(\alpha_i W(\tau_i), D_i(\tau_i)-V_i(\tau_i)\right)\right].$$

In a portfolio context, our key variables are the ES and CI per unit of total risk-free debt value and are given by

$$\mathrm{ES}=\frac{\sum_{i=1}^{N}[D_i(0)-D_{i,NG}(0)]}{\sum_{i=1}^{N}D_i(0)}, \tag{11}$$

$$\mathrm{CI}=\frac{\sum_{i=1}^{N}[D_{i,G}(0)-D_{i,NG}(0)]}{\sum_{i=1}^{N}D_i(0)}. \tag{12}$$

ES captures not only the stochastic cash flows and insurer's net-assets risks, but also the interest-rate risk and the nexus of correlation risks among these sources of uncertainty. The difference between ES and CI represents what is not covered by the risky insurer. For a riskless insurer, e.g., government corporations, this difference is zero.

Multiyear Risk Management

In an insurance pricing context, profits are usually tied to premium levels. It is then reasonable to assume that insurance companies will seek to maximize the value of the portfolio of CI for a given level of ES under limited capacity (W).[4]

Since the objective of this article is to investigate the effect of intertemporal risk management on CI portfolios, we propose the following optimization program to find the combination of maturities which maximize CI:

$$\max_{\{T_i\}} \ \mathrm{CI} \quad \text{s.t. } \mathrm{ES}=\overline{\mathrm{ES}} \ \text{ and } \ W(0)=\overline{W}, \tag{13}$$

where $\{T_i: i = 1, \ldots, N\}$ are the arguments from the optimization and represent the maturities of the loans, $\overline{ES}$ is the maximum underwritable ES per dollar of debt and $\overline{W}$ the insurer's total net-assets limit.

For a given level of portfolio risk measured by ES, the insurer is to choose a pool of CIs with differing maturities to maximize CI. Since different combinations of maturities yield the same ES, the question is: which maturity structure yields the maximum premium levels to the insurer?

Two kinds of portfolios are considered, closed and opened. Closed portfolios consist of a fixed number of CI contracts to be carried until maturity. Opened portfolios on the other hand, allow the insurer to include new insurance contracts over time.

SIMULATION RESULTS

We conduct the optimization program (13) using Monte Carlo simulations to compute the values of the debts defined in Equations (7), (9), and (10).

For the closed portfolios, we consider two CI contracts similar in size but with different cash flows risks. For the opened portfolios, we also have two CI contracts similar in size and different in cash flows risks, but we consider them sequentially. In this case, the second contract comes into effect only when the first one expires. With the no-arbitrage argument, we can price all contracts today based on the available information set. The maturities of the debts that maximize the value of the CI portfolios are given in Equation (13).

Baseline Parameters Values

The initial cash flows values ($V_i(0)$) are 250 and the insurer's initial net-asset value ($W(0)$) is 100 or 200. All debt face values are fixed to 100. The coupon rate of 6% is the annual rate payable each 6 months up to the maturity of the debts. The insurer's wealth is assigned in equal percentage to each policyholder.

Exhibit 2 gives the parameters values for the stochastic volatility models. Initial values for the volatilities ($\sigma_i(0)$) are 0.20 for low-risk and 0.70 for high-risk. We choose a ν of 0.50, β equal to $\sigma^2(0)$ (the square of the initial volatility), and ϕ equal to 0.15. The correlations between asset returns and volatility changes, $\rho_{i\sigma}$, are set to zero.

Exhibit 3 presents the parameters values of the interest-rate process.[5] The initial interest rate, $r(0)$, is 3%, the long-run mean, θ, is 3%, the adjustment speed, κ, is 0.20, and the interest rate volatility, σ_r, is 0.10. The value of γ is 0.5, which corresponds to the CIR (1985) process.

For an insurer specializing in a specific sector, returns of cash flows may be highly correlated. The correlations between the two cash flows and between the insurer and

EXHIBIT 2
Parameters Values for Stochastic Volatility Processes

	$\sigma(0)$	ν	β	ϕ	ρ
Values	[0.20, 0.70]	0.50	$\sigma^2(0)$	0.15	0

This exhibit presents the parameters values for mean-reverting stochastic volatility processes. β is the long-run mean, ν the speed of adjustment, ϕ the variation coefficient of the diffusion volatility, ρ the correlation between the cash flows (V_i) or insurer's total net-assets (W) returns and volatility processes, and $\sigma(0)$ is the initial volatility. Z and η are independent Brownian motions. The volatilities are annualized:

$$d\sigma^2(t) - \nu(\beta - \sigma^2(t))dt - \phi\sigma(t)\left[\rho dZ(t) + \sqrt{1-\rho^2}\, d\eta(t)\right].$$

EXHIBIT 3
Parameters Values for the Interest Rate Process

	$r(0)$	κ	θ	σ_r	γ
Values	0.03	0.20	0.03	0.10	0.50

This exhibit shows the parameters values for the interest rate process, r, $r(0)$ represents the initial interest rate value, κ the speed of adjustment, θ the long-run mean and σ_r the interest rate volatility. We use the CIR (1985) process, i.e., $\gamma = 0.50$. Z_r is a standard Wiener process. All values are annualized:

$$dr(t) = \kappa(\theta - r(t))dt + \sigma_r r(t)^{\gamma} dZ_r(t).$$

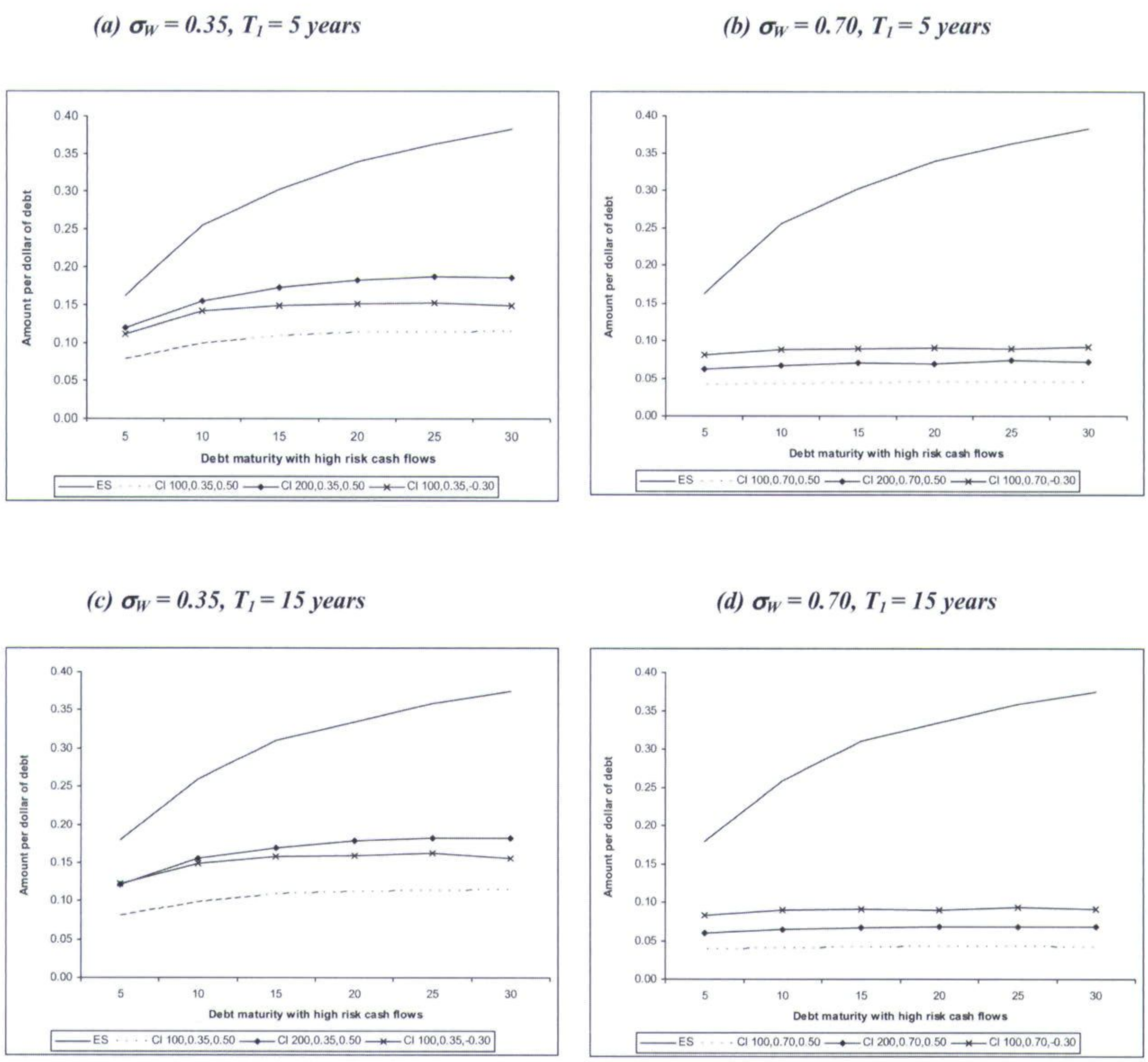
the cash flows, $\{\rho_{ij}, i, j = 1, \ldots, N, W\}$, are chosen from the set $\{-0.30, 0.50\}$.

We next present simulation results pertaining to our research article's question.

Closed Portfolio of CI

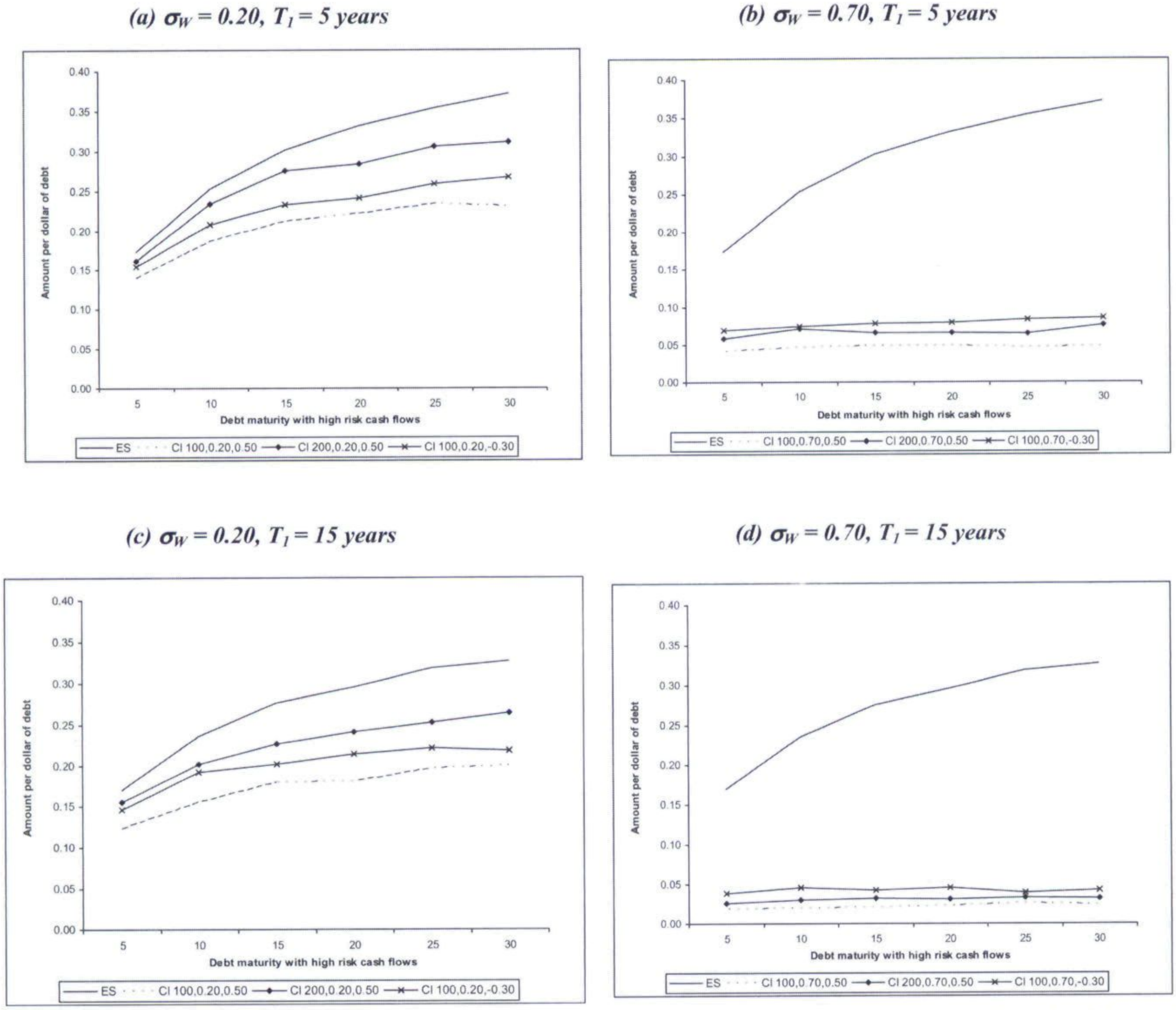
Exhibit 4 contains four graphs. The maturity of the debt with low risk cash flows is set to 5 years for the top two graphs ((a) and (b)) and 15 years for the bottom graphs ((c) and (d)). The graphs on the left side ((a) and (c)) are for an insurer with low risk exposure ($\sigma_W = 0.35$) and

EXHIBIT 4
ES and CI for Closed Portfolios



These graphs plot the ES and CI for different parameters values. This graph is constructed for closed portfolios, which combines low-risk and high-risk cash flows. The correlations between the cash flows, the insurer's net asset and the interest rate, $\{\rho_{ir}, i = 1, \ldots, N, W\}$, are set equal to -0.25. The correlations between the two cash flows, and between the cash flows and the insurer, $\{\rho_{ij}, i, j = 1, \ldots, N, W\}$, are selected from $\{-0.30, 0.50\}$. The maturity of the debt with low risk cash flows (T_2) is 5 years for the top graphs and 15 years for the bottom graphs. In the legend, "CI 100,0.35,0.50" stands for CI with $W = 100$, $\sigma_W = 0.35$, and $\rho_{ij} = 0.50$.

EXHIBIT 5
ES and CI for Opened Portfolios



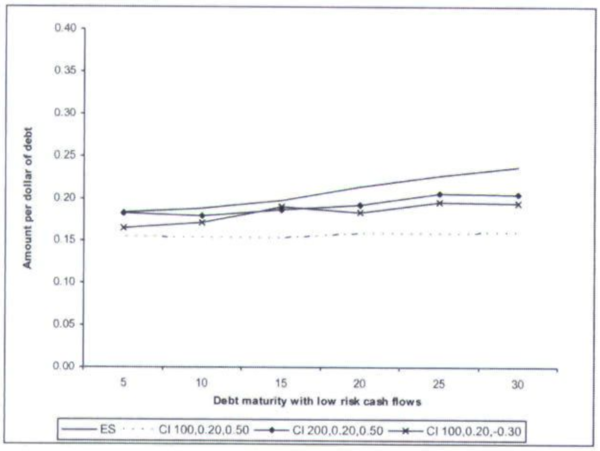
These graphs plot the ES and CI for different parameters values. The graphs are constructed for opened portfolios, which start with low-risk cash flows and end with high-risk cash flows. The correlations between the cash flows, the insurer's net asset and the interest rate, $\{\rho_{ir}, i = 1, \ldots, N, W\}$, are set equal to -0.25. The correlations between the two cash flows, and between the cash flows and the insurer, $\{\rho_{ij}, iI, j = 1, \ldots, N, W\}$, are selected from $\{-0.30, 0.50\}$. The maturity of the debt with low risk cash flows (T_1) is 5 years for the top graphs and 15 years for the bottom graphs. In the legend, "CI 100,0.20,0.50" stands for CI with $W = 100$, $\sigma_W = 0.20$, and $\rho_{ij} = 0.50$.

the other two graphs ((b) and (d)) are for an insurer with high risk exposure ($\sigma_W = 0.70$).

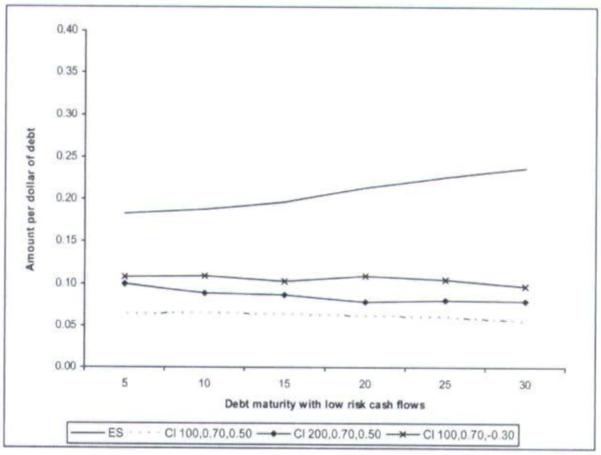
In each graph, the upper line represents the ES, i.e., the maximum CI value for an insurer with low risk exposure. The other three lines are characterized by the vector (W, σ_W, ρ) where W is the initial net-asset value of the insurer, σ_W is the initial volatility of the insurer, and ρ represents the value of the correlations between the cash flows, and between the cash flows and the insurer. The dotted line is the CI value for the parameters vector (100, 0.35, 0.50). The line with the '×' mark gives CI for

EXHIBIT 6
ES and CI for Opened Portfolios

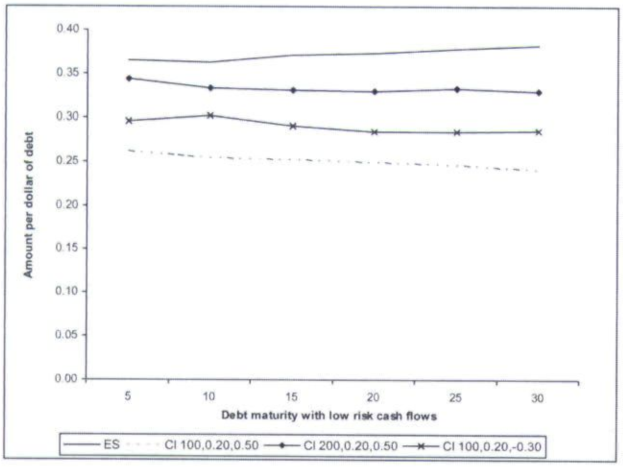
(a) $\sigma_W = 0.20$, $T_2 = 5$ years



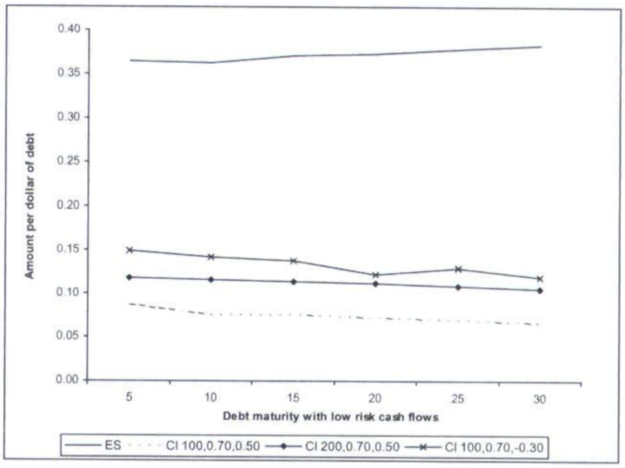
(b) $\sigma_W = 0.70$, $T_2 = 5$ years



(c) $\sigma_W = 0.20$, $T_2 = 15$ years



(d) $\sigma_W = 0.70$, $T_2 = 15$ years



These graphs plot the ES and CI for different parameters values. The graphs are constructed for opened portfolios, which start with high-risk cash flows and end with low-risk cash flows. The correlations between the cash flows, the insurer's net asset and the interest rate, $\{\rho_{ir}, i = 1, \ldots, N, W\}$, are set equal to 0. The correlations between the two cash flows, and between the cash flows and the insurer, $\{\rho_{ij}, iI, j = 1, \ldots, N, W\}$, are selected from $\{-0.30, 0.50\}$. The maturity of the debt with high risk cash flows (T_2) is 5 years for the top graphs and 15 years for the bottom graphs. In the legend, "CI 100,0.20,0.50" stands for CI with $W = 100$, $\sigma_W = 0.50$, and $\rho_{ij} = 0.50$.

the negatively correlated cash flows, i.e., $\rho = -0.30$. We observe that risk diversification through negative correlations increases the value of CI. The line with the '◆' mark gives the CI value for a larger insurer, $W = 200$. As expected, a large insuring capacity increases the value of CI. Note that the increase in the CI value is not proportional to the percentage variation of the insurer's net asset.

Moreover, with our set of chosen parameters, for an insurer with low risk exposure, the value of CI is much higher with increased insuring capacity than with risk diversification. In contrast, when the insurer is riskier, there is more to be gained through risk diversification than by doubling the insuring capacity.

Opened Portfolio of CI

As above, Exhibits 5 and 6 contain four graphs. Exhibit 5 gives the results for opened portfolios where the insurer starts with a debt with low-risk cash flows and has to offer CI to a policy holder with high-risk cash flows. In Exhibit 6, the insurer has a debt with high-risk cash flows and can accept any client with low-risk cash flows debt. In Exhibits 5 and 6 respectively, the maturity of the debt with low-risk (high-risk) cash flows is set to 5 years for the top two graphs ((a) and (b)) and 15 years for the bottom graphs ((c) and (d)). The graphs on the left side ((a) and (c)) in both figures, are for *n* insurer with low risk exposure ($\sigma_W = 0.20$) and the other two graphs ((b) and (d)) are for an insurer with high risk exposure ($\sigma_W = 0.70$).

Similar to the case of closed portfolios, decreasing the correlations between the cash flows, and between the cash flows and the insurer, improves substantially CI, especially for risky insurers. For example when the insurer's net asset volatility is 0.70, decreasing the correlations from 0.50 to -0.30, we obtain a CI superior to the one we would obtain by doubling the net asset level of the insurer from 100 to 200.

Note finally that the overall riskiness level of the portfolio is mainly driven by the maturity structure of the debt with high-risk cash flows in both types of portfolios (closed and opened).

CONCLUSIONS

As the use of CI becomes more widespread and pervasive, for instance in fostering exports, economic growth, and mitigating financial crises, the management of portfolios of CI has become important. In this article, we analyze the multiyear risk management decisions of credit-enhanced debt contracts on the value of an insurer's CI portfolio.

To achieve our goal, we propose a contingent-claims model that includes many realistic features such as coupon payments, stochastic interest-rate, and stochastic cash flows volatility. Using optimization by Monte Carlo simulations, we compute the value of total ES and CI for portfolios of CI.

We distinguish between two types of insurance portfolios: "closed" and "opened." Closed portfolios consist of a fixed number of CI contracts with different cash flows risks to be carried until maturity. Opened portfolios on the other hand, allow the insurer to add new insurance contracts sequentially. In this case, the next contracts come into effect only when the previous ones expire. By the no-arbitrage argument, all contracts are priced based on today available information set. For both closed and opened portfolios, by selecting the maturities of the debts that maximize the value of CI, the overall risk of the portfolio is mainly driven by the maturity structure of the debt with high-risk cash flows.

We find that for a given riskiness level of insurer's capital, an optimal value of CI can be obtained by appropriate risk diversification and/or increased insurer's capital. Our simulation results show that for insurers with high risk exposure, portfolio risk diversification is more effective than increasing insuring capacity. For a creditworthy insurer, increasing the size of the insurer's capital can lead to significant improvement in the value of the CI portfolio. This suggests that alternative risk transfer techniques, which provide synthetic (or contingent) capital to the insurer, should be considered in an integrated risk management.

ENDNOTES

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[1]Ramaswamy [2002]: "Managing credit risk in a bond portfolio is usually more difficult than managing market risk. The reason is that credit risk is a low-probability default event that leads to highly skewed return distributions . . . To a certain extent, managing credit risk is similar to managing event risk when pricing an insurance premium." See also 'Credit Insurance: Rating Methodology', Moody's Investors Service, Global Credit Research, July 2001.

One of the channels of risk management is the use of alternative risk transfer strategies (see for instance Banks [2004], Culp [2002] and Lane [2002]). Alternative risk transfers such as recapitalization, contingent capital, reinsurance or captives are captured in our model by an increase in the insurer's total net assets W and volatility σ_W. For a description of techniques and strategies for managing corporate risk see Doherty [2000].

[2]The volatility risk is rewarded in the sense that it earns a risk premium proportional to the factor itself, $\lambda_k \sigma^2_{kt}$, and it is incorporated into the stochastic process through the coefficient ν_k; or we can assume the risk premium of the volatility risk (λ_k) to be zero. See Bakshi, Cao, and Chen [1997] for a characterization of these processes.

[3]The interest rate risk is rewarded, i.e., the risk factor earns a risk premium proportional to r_t ($\lambda_r r_t$). This is implicitly reflected in Equation (5) and adjusted through κ, or we can assume the risk premium of the interest rate risk (λ_r) to be zero.

[4]Alternatively, in a financial equilibrium context, this behaviour can also be justified through the existence of tax shields related to debt and the time deductibility of premiums, since credit insurance can be viewed as synthetic equity financing (see Culp [2002]).

[5]See Chan et al. [1992] and Nowman [1997] for the estimation of the interest-rate process parameters.

REFERENCES

Bakshi, G., C. Cao, and Z. Chen. "Empirical Performance of Alternative Option Pricing Models". *The Journal of Finance*, 52 (1997), pp. 2003–2049.

Banks, E. *Alternative Risk Transfer: Integrated Risk Management through Insurance, Reinsurance, and the Capital Markets*. Wiley, 2004.

Chan, K.C., G.A. Karolyi, F.A. Longstaff, and A.B. Sanders. "An Empirical Comparison of Alternative Models of the Short-Term Interest Rate." *The Journal of Finance*, 47 (1992), pp. 1209–1227.

Cox, J.C., J.E. Ingersoll Jr., S.A. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*, 53 (1985), pp. 385–407.

Culp, C.L. *The ART of Risk Management: Alternative Risk Transfer, Capital Structure, and the Convergence of Insurance and Capital Markets*. Wiley, 2002.

Doherty, N.A. *Integrated Risk Management: Techniques and Strategies for Reducing Risk*. McGraw-Hill, 2000.

Gendron, M., V.S. Lai, I. Soumaré. "An Analysis of Private Loan Guarantee Portfolios." *Research in International Business and Finance*, 16 (2002), pp. 395–415.

Harrison, J.M., and D.M. Kreps. "Martingales and Arbitrage in Multiperiod Securities Markets." *Journal of Economic Theory*, 20 (1979), pp. 381–408.

Harrison, J.M., and S.R. Pliska. "Martingales and Stochastic Integrals in the Theory of Continuous Trading." *Stochastic Processes and their Applications*, 11 (1981), pp. 215–260.

Johnson, H., and R. Stulz. "The Pricing of Options with Default Risk." *The Journal of Finance*, 42 (1987), pp. 267–280.

Klein, P., and M. Inglis. "Valuation of European Options Subject to Financial Distress and Interest Rate Risk." *Journal of Derivatives*, 6 (1999), pp. 44–56.

Lai, V.S., and M. Gendron. "On Financial Guarantee Insurance under Stochastic Interest Rates." *The Geneva Papers on Risk and Insurance Theory*, 19 (1994), pp. 119–137.

Lane, M. (ed.). *Alternative Risk Strategies*. London: Risk Books, 2002.

Merton, R.C. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *The Journal of Finance*, 29 (1974), pp. 449–470.

——. "An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees: An Application of Modern Option Pricing Theory." *Journal of Banking and Finance*, 1 (1977), pp. 3–11.

Nowman, K.B. "Gaussian Estimation of Single-Factor Continuous Time Models of the Term Structure of Interest Rates." *The Journal of Finance*, 52 (1997), pp. 1695–1706.

Panjer, H. (ed.). *Financial Economics with Applications to Investments, Insurance and Pensions*. Schaumburg, IL: The Actuarial Foundation, 1998.

Ramaswamy, S. "Managing Credit Risk in a Corporate Bond Portfolio." *The Journal of Portfolio Management*, Spring (2002), pp. 67–72.

Vasicek, O. "An Equilibrium Characterization of the Term Structure." *Journal of Financial Economics*, 5 (1977), pp. 177–188.