Exchange-Traded Fixed-Income Derivatives in Asset Management and Asset-Liability Management

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Interest in the use of fixed-income derivatives in asset management has perhaps never been so high, both from a pure asset management perspective and from an asset-liability management perspective. On the one hand, from a pure asset management perspective, historically low levels of long-term yields, combined with the prospect of increases in short-term rates by central banks in Europe as well as in the United States, provide strong incentive for implementing strategies that aim to deliver downside protection by combining a long position in bond markets with a long position in put options. On the other hand, recent pension fund shortfalls have drawn attention to the risk management practices of institutional investors in general and defined benefit pension plans in particular, and have provided new incentives for institutional investors to use fixed-income derivatives in the management of their liability risk.

What has been labeled "a perfect storm" of adverse market conditions around the turn of the millennium has devastated many corporate defined benefit pension plans. Negative equity market returns have eroded plan assets at the same time as declining interest rates have increased the mark-to-market value of benefit obligations and contributions. In extreme cases, this has left corporate pension plans with funding gaps as large as, or larger than, the market capitalization of the plan sponsor. For example, in 2003, the companies included in the S&P 500 and the FTSE 100 index faced a cumulative deficit of $225 billion and £55 billion respectively (as per surveys by Credit Suisse First Boston [2003] and Standard Life Investments [2003] respectively), while the worldwide deficit reached an estimated 1,500 to 2,000 billion USD (Watson Wyatt [2003]). That institutional investors in general, and pension funds in particular, have been so markedly affected by recent market downturns, emphasizes the weakness of risk management practices. In particular, it has been argued that the kinds of asset allocation strategies implemented in practice, which were heavily skewed towards equities, in the absence of any protection with respect to their downside risk, were not consistent with a sound liability risk management process. In this context, new approaches, that are referred to as *liability driven investment* solutions, have rapidly gained interest with pension funds, insurance companies, and investment consultants alike, following recent changes in accounting standards and regulations that have led to an increased focus on liability risk management. Such strategies, which can add significant value in terms of liability risk management, lead to an increased focus on matching investors' portfolios via cash instruments (long-term real and nominal bonds) or via derivative instruments.

From an academic perspective, although an impressive amount of research is available on issues related to bond and fixed income derivative pricing and hedging, relatively few results can actually be found on how these instruments can be used to improve an investor's welfare in the context of portfolio strategies. This article precisely focuses on illustrating how various types of derivative securities can be used to shift the risk associated with investing in fixed income securities. In this article, we take an asset allocation perspective and assess in more detail the benefits of including bond futures and options on these futures in a fixed income portfolio. In addition to linear instruments such as futures, options have nonlinear payoffs that allow investors to construct funds with skewed or dissymmetric return profiles. In comparison with simple bond index exposure, these may offer an efficient way to limit the extreme values of a portfolio return distribution.

As such, our article is closely related to a number of papers that look at optimal portfolios when investors have access to an asset and options on this asset. On the basis of Merton's (Merton [1973]) replicating portfolio interpretation of the Black and Scholes [1973] option pricing model, one could argue that options are redundant assets, and as a result the optimal allocation to such assets should be either indeterminate (when an investor's expectation about parameter values match those of the markets), or infinite (when the investors' beliefs lead him to view the derivative asset as under-priced). It should however be recognized that this result critically depends on the concept of dynamic completeness (see Radner [1972] for a formal analysis), which can only be justified under a set of rather stringent assumptions that are never met in practice (continuous trading, the absence of transaction costs, leverage or short-sales constraints, constant volatility and drift of the underlying asset, symmetric information, etc.). When any of these assumptions are relaxed, it can actually be shown that the introduction of derivatives would improve the welfare of investors. A first strand of the literature, initiated by Brennan and Solanki [1981], has examined the question of optimal positioning in derivative assets from the static investor's standpoint. The question there is to determine the general shape of the nonlinear function of the underlying asset that would generate the highest level of expected utility for a given investor. Carr and Madan [2001] extend this early work by considering optimal portfolios of a riskless asset, a risky asset and derivatives on the latter in an expected utility framework. A second strand of the literature (starting with Leland [1980] and extended by Benninga and Blume [1985]) has studied the question from a different angle: taking as given a set of standard derivatives, these papers examine the features of (static) investors' preferences that would support a rational holding of these contracts.

This article, which distinguishes itself by exclusively focusing on bond derivatives, complements existing literature by focusing on the use of derivatives, not only in asset management, but also from an asset-liability management perspective. In an attempt to focus on plain vanilla products that can readily be used by investors, we take as objects of study a set of exchange-traded fixed-income derivatives traded on Eurex, the world's leading futures and options market for euro denominated derivative instruments. The use of futures and options will be organized according to the investment process and objectives and we will examine the value of introducing derivatives both in a pure asset management context and in an asset liability management context.

The rest of the article is organized as follows: First we introduce the theoretical model and show how we implement it. Second, we describe the implementation of the investment strategies and assess their benefits in a pure asset management context. Finally, we examine similar questions but from an asset liability management (ALM) perspective.

THE FRAMEWORK

To assess the different uses and benefits of derivatives in fixed income portfolios in the context of a full-blown numerical experiment, one needs to generate stochastic scenarios for asset prices. The scenario generation process will be similar for the whole paper, with additional assumptions on investors' preferences needed for some sections. In this section, we therefore describe our assumptions with respect to how fixed income markets are modelled.

Choice of a Model

The issue of modelling the future uncertainty is critical to any investment management problem. A stochastic forecasting model consists of multivariate time series models.

While generating stochastic scenarios for stock prices can be achieved in a straightforward manner, a specific challenge related to simulating bond prices is the need to ensure the absence of arbitrage between the prices of bonds with different maturities at a given point in time. In other words, one needs to impose some restrictions on the cross-section of bond prices. This can be achieved by using a consistent model of the term structure of interest rates. Term structure models imply that changes in yields of bonds of different maturities can be expressed as a function of changes in some underlying factors.

The first option would be to use an equilibrium model of the term structure. This type of model starts with diffusion processes for some state variables and specify endogenously a dynamic process for the short rate as well as a zero coupon curve. The obvious drawback of this class of model is that the simulated bond prices do not automatically fit those observed in the market. Arbitrage models, on the other hand, take as given the observed term structure of interest rates, which is then regarded as the underlying asset (see Ho and Lee [1986] for a discrete-time example, or Heath, Jarrow and Morton [1990, 1992] for a general formulation in continuous-time). In our context, since we do not deal with the problem of minimizing pricing errors in a practical hedging exercise, the drawback of equilibrium models does not seem to be a relevant one. It actually turns out that the calibration of popular arbitrage models would involve too much complexity given our specific requirements for the effects that the model should capture.

More specifically, the requirements we have are 1) stochastic volatility for bond prices, 2) nonredundancy of multiple bonds, and 3) the availability of closed form pricing formulas. Particular emphasis is placed on the fact that our model aims at capturing the stochastic volatility of bond prices, which is an important ingredient in the analysis of options available on bond and bond futures contracts. Further motivation for using a stochastic volatility model is related to the fact that the presence of time varying volatility of asset prices has been well documented in empirical studies (see e.g. Jones et al. [1998], Bollerslev, Cai, and Song [2000], and Reilly et al. [2000]). A further issue is that we need to avoid redundancy of multiple bonds, which rules out single factor term structure models, where bonds with different maturities are perfectly correlated. Finally, our task of testing different investment strategies will be greatly eased in terms of computational burden and complexity if we have closed form solutions.

The above considerations led us to choose the multifactor term structure model by Longstaff and Schwartz [1992], LS henceforth. Their model, 1) has stochastic volatility of bond prices which stems from the stochastic volatility of the process for the short rate; 2) incorporates more than one factor, which means that bonds of different maturities are not necessarily (dynamically) redundant assets; and 3) is tractable in the sense that it allows one to obtain an explicit closed-form solution for bonds of different maturities and for options on these bonds.

The LS model is a two factor extension of the Cox, Ingersoll, and Ross [1985] general equilibrium model. It can be seen that the dynamics of the short rate are similar to those in the CIR model, except for an additional factor of uncertainty given by the volatility of the short rate. The above equations allow writing the processes for the short rate r and the volatility of the short rate V as

$$\begin{aligned}
\mathrm{d}r_t &= \left(\alpha\gamma + \beta\eta - \frac{\beta\delta - \alpha\xi}{\beta - \alpha} r_t - \frac{\xi - \delta}{\beta - \alpha} V_t\right)\mathrm{d}t \\
&\quad + \alpha\sqrt{\frac{\beta r_t - V_t}{\alpha(\beta - \alpha)}}\mathrm{d}Z_t^{(1)} + \beta\sqrt{\frac{V_t - \alpha r_t}{\beta(\beta - \alpha)}}\mathrm{d}Z_t^{(2)} \\
\mathrm{d}V_t &= \left(\alpha^2\gamma + \beta^2\eta - \frac{\alpha\beta(\delta - \xi)}{\beta - \alpha} r_t - \frac{\beta\xi - \alpha\delta}{\beta - \alpha} V_t\right)\mathrm{d}t \\
&\quad + \alpha^2\sqrt{\frac{\beta r_t - V_t}{\alpha(\beta - \alpha)}}\mathrm{d}Z_t^{(1)} + \beta^2\sqrt{\frac{V_t - \alpha r_t}{\beta(\beta - \alpha)}}\mathrm{d}Z_t^{(2)}
\end{aligned}$$

where $Z^{(1)}$ and $Z^{(2)}$ are standard Brownian motions, and α, β, γ, δ, η, and ξ are constant parameters.

LS obtain the price of a zero coupon bond with time to maturity τ as functions of r and V.

$$B_t(\tau, r_t, v_t) = A^{2\gamma}(\tau)B^{2\eta}(\tau)\exp(\kappa\tau + C(\tau)r_t + D(\tau)v_t)$$

where

$$\begin{aligned}
&\upsilon = \lambda + \xi, \\
&\qquad A(\tau) = \frac{2\phi}{(\delta + \phi)(\exp(\phi\tau) - 1) + 2\phi} \\
&\phi = \sqrt{2\alpha + \delta^2}, \\
&\qquad B(\tau) = \frac{2\psi}{(\upsilon + \psi)(\exp(\psi\tau) - 1) + 2\psi} \\
&\psi = \sqrt{2\beta + \upsilon^2}, \\
&\qquad C(\tau) = \frac{\alpha\phi(\exp(\psi\tau) - 1)B(\tau) - \beta\psi(\exp(\phi\tau) - 1)A(\tau)}{\phi\psi(\beta - \alpha)} \\
&\kappa = \gamma(\delta + \phi) + \eta(\upsilon + \psi), \\
&\qquad D(\tau) = \frac{\psi(\exp(\phi\tau) - 1)A(\tau) - \phi(\exp(\psi\tau) - 1)B(\tau)}{\phi\psi(\beta - \alpha)}
\end{aligned}$$

and λ is the market price of risk.

LS also obtain the price of a call option on the zero coupon bond with payoff at expiration of $\max(0, F(r,V,T) - K)$ as

$$C(r,V,\tau,K,T) = F(r,V,\tau,+T)\Phi(\theta_1,\theta_2;4\gamma,4\eta,\varpi_1,\varpi_2) - KF(r,V,\tau)\Phi(\theta_3,\theta_4;4\gamma,4\eta,\varpi_3,\varpi_4),$$

where

$$\theta_1 = \frac{4\zeta\phi^2}{\alpha(\exp(\phi\tau)-1)^2 A(\tau+T)}$$
$$\theta_2 = \frac{4\zeta\psi^2}{\beta(\exp(\psi\tau)-1)^2 B(\tau+T)}$$
$$\theta_3 = \frac{4\zeta\phi^2}{\alpha(\exp(\phi\tau)-1)^2 A(\tau)A(T)}$$
$$\theta_4 = \frac{4\zeta\psi^2}{\beta(\exp(\psi\tau)-1)^2 B(\tau)B(T)}$$

and

$$\varpi_1 = \frac{4\phi\exp(\phi\tau)A(\tau+T)(\beta r - V)}{\alpha(\beta-\alpha)(\exp(\phi\tau)-1)A(T)}$$
$$\varpi_2 = \frac{4\psi\exp(\psi\tau)B(\tau+T)(V-\alpha r)}{\beta(\beta-\alpha)(\exp(\psi\tau)-1)B(T)}$$
$$\varpi_3 = \frac{4\phi\exp(\phi\tau)A(\tau)(\beta r - V)}{\alpha(\beta-\alpha)(\exp(\phi\tau)-1)}$$
$$\varpi_4 = \frac{4\psi\exp(\psi\tau)B(\tau)(V-\alpha r)}{\beta(\beta-\alpha)(\exp(\psi\tau)-1)}$$
$$\zeta = \kappa T + 2\gamma\ln A(T) + 2\eta\ln B(T) - \ln K$$

The function $\Phi(\theta_1, \theta_2; 4\gamma, 4\eta, \varpi_1, \varpi_2)$ is the joint distribution

$$\int_0^{\theta}\int_0^{\theta_2-\theta_2 u/\theta_1}\chi^2(u;4\gamma,\varpi_1)\chi^2(v;4\eta,\varpi_2)\,dv\,du$$

where $\chi^2(.; p,q)$ denotes the noncentral chi-square density with degrees of freedom parameter p and noncentrality parameter q. We rewrite this as a single integral by applying the Fubini theorem and solve the resulting integral numerically.

Scenario Generation

The model we use for interest rate dynamics involves quite a few parameters. One approach would consist of estimating these parameters by fitting the model to historical data. Alternatively, we have chosen to select parameter values consistent with the existing literature. Exhibit 1 contains information about the parameter values we use, and how they relate to existing literature.

We adopt the parameters in Jensen [2001]. These were estimated using the Efficient Method of Moments from a sample of T-Bill rates that spans a recent period. The parameters are estimated from data with weekly frequency, which corresponds to the frequency we choose for our discretization. In addition to the parameters in

EXHIBIT 1
Parameter Values

Parameter	Jensen [2001]	Longstaff and Schwartz [1992]	Longstaff and Schwartz [1993]	Navas [2004]	Munk [2002]	This Paper
α	0.0001	−0.0439	0.001149	2.6693 E-06	0.01	0.0001
β	0.00186	0.0814	0.1325	8.3633 E-05	0.08	0.00186
γ	70.3324		3.0493	12.3990	0.1	70.3324
δ	1.0872	0.3299	0.05658	9.0490 E-0.4	0.33	1.0872
η	8.7990		0.1582	0.0418	16	8.7990
ψ	0.8081		3.998	4.8949 E0.3	14	0.8081
λ	–		–	−0.2 to −0.4	0	−0.4
$\nu = \psi + \lambda$	–	14.4227	–	–	14	

The parameter values from Jensen (2001) are from Table 5. Longstaff and Schwartz (1992) only estimate four parameters, since this is sufficient if bond maturity is assumed to be given. Parameter estimates are from their Table 2. Parameter values from Longstaff and Schwartz (1993) are from Exhibit 3. Those from Navas (2004) are from Table 3 and Figure 2. Parameter values used by Munk (1999) are from page 173.

Jensen, we have to specify the risk aversion parameter λ. We choose $\lambda = -0.4$, which is in the range of parameters estimated by Navas [2004]. In addition, our choice of λ allows us to obtain returns for the bonds that correspond to recent data for German Government bonds. For example, the iboxx Euro Sovereign Germany index for long maturity bonds shows an annualized return of 5.12% since its inception in 1999, which is in line with the mean return of 4.96% which we obtain in our scenarios.

Using these parameter values, which are depicted on the right-most column of Exhibit 1, we simulate 1,000 paths for the short rate r and its volatility V according to the equations for dr and dV in the LS model. The next step is the time-discretization of these processes for simulating paths so that we can generate scenarios to represent the future uncertainty. We generate paths with 52 observations per year over a horizon of one year, and chose starting values for interest rate and interest rate volatility to be equal to their long term mean given our chosen parameter values. We obtain the long term mean by determining the equilibrium point of the differential equations for r and V above. With this approach, we obtain an initial value of 2.1% for the interest rate and of 1.654% for interest rate volatility.

Once the paths for interest rates and interest rate volatility are simulated, returns for different asset classes are calculated on each path in order to generate the scenarios. We describe the assets in the following subsection.

Asset Universe

For reasons outlined in the introduction, we use as benchmark instruments Eurex futures contracts on notional debt instruments issued by the Federal Republic of Germany with different remaining terms, as well as options on these futures contracts.

To model the price evolution of these instruments, we generate scenarios for the price of a zero coupon notional bond with a long-term maturity. The position in this fictitious bond is rolled over every three months. As a result, the maturity of the strategy does not decline constantly, but is instead reset every three months. We also consider options on this notional bond.[1]

Futures. In this section, we consider the futures contract on a long-term instrument with a six percent annual coupon (Bund Futures). In the section *Managing Liabilities with Interest Rate Derivatives*, we will introduce an additional contract on very long term bonds with a four percent annual coupon (Buxl Futures).

We want to ensure that the instruments we model have a sensitivity to interest rate changes that correspond to the underlyings of the Eurex futures. Since the duration of a pure discount bond is equal to its maturity, we use the calculated duration of the underlyings to specify the maturity of the bonds in our simulation model. For the sake of simplicity, we calculate duration as

$$D = \frac{\sum_{t=1}^{T} \frac{tC(t)}{(1+r)^t}}{P_\tau}$$

where P_τ is the bond price at expiration of the future, $C(t)$ is the cash flow in period t and r is the discount rate. For the purpose of calculating duration, we set P_τ to the nominal value and r to the coupon rate.

Exhibit 2 gives an overview of the contract specifications and the calculated duration.[2]

Options on futures. We consider the Eurex options on the Bund futures contract. These options are available with expiry dates up to 6 months in the future. Available expiry dates are the three nearest calendar months, as well as the following quarterly month of the March, June, September, and December cycle thereafter. A number of strike prices are available with intervals of 0.5 points or 50 ticks. For each contract month, there are at least nine call and nine put series, each with an at-the-money strike price as well as four in-the-money and four out of-the-money strike prices. There are no options available on the Buxl futures contract currently.

EXHIBIT 2
Contract Specifications

Contract	Remaining Term Defined in Contract	Remaining Term Assumed by us	Calculated Duration Used in this Paper
Euro-Bund Futures (FGBL)	8.5–10.5	10	7.8017
Euro-Buxl Futures (FGBX)	24–35	30	17.984

DERIVATIVES STRATEGIES IN FIXED INCOME PORTFOLIO MANAGEMENT

The management of fixed-income portfolios comprises a variety of tasks and involves both passive (indexing) and active (bond timing and bond picking) strategies. All these tasks may effectively be facilitated by using derivatives. In particular, futures on fixed income instruments help investors or managers to neutralize biases in terms of factor exposure (changes in level or slope of the yield curve) that may result from bond picking bets. Conversely, an investor may decide to protect himself from market risk in order to conserve only the return linked to the active bets taken by a manager. Another natural use of bond futures is for timing strategies between different maturity segments of the bond market. In addition, it is possible to create leveraged positions through futures, following arbitrage strategies between the futures contract and the underlying or hedging interest rate risk.

In what follows, we focus on how the use of options can further improve risk management, thanks to their ability to generate nonlinear, convex payoffs that offer downside risk protection. Most fixed-income strategies rely on static exposure to the returns of the different instruments. In fact, it can be argued that the implementation of dynamic strategies face a number of barriers stemming from the particular characteristics of bond markets (infrequent pricing and low liquidity compared with stock and currency markets). Under these circumstances, using derivatives constitutes an alternative to the direct implementation of dynamic trading strategies. In fact, from the derivation of no arbitrage pricing formulas, it is well known that a payoff that is a nonlinear function of the price of the underlying asset may be achieved by appropriate dynamic trading strategies in complete markets. This logic can be inversed in order to examine the equivalence of nonlinear payoffs at a point in time with a dynamic trading strategy over time (see in particular Haugh and Lo [2001]). From the investor's perspective, it is interesting to assess which benefits may be expected from investing in such instruments.

In a first subsection, we analyze the benefits of option-based strategies on a standalone basis. In the next subsection, we consider the same question from a portfolio standpoint.

Option Strategies

Description of Strategies. A favorite strategy with investors and asset managers alike is a protective put buying (PPB) strategy. This strategy consists of a long position in the underlying asset and a long position in a put option, which is rolled over as the option expires. It should be underlined that PPB is different from portfolio insurance, since the put is rolled over in each subperiod. Therefore, the payoff at the end of a total period with multiple subperiods does not simply correspond to a guaranteed minimum payoff, as in the case of portfolio insurance. At the end of every subperiod, however, the long position in the put option offers a protection against downside risk, which leads to avoiding the left tail of the returns distribution.

The PPB strategy has been widely studied in the context of equity portfolio management (see Merton et al. [1982] and Figlewski et al. [1993]). However, such an option strategy has not been analyzed in any detail for the case of fixed-income derivatives. This may be surprising since downside risk is also very significant in fixed-income markets. In the theoretical setup that we have chosen, changes in the short rate and changes in interest rate volatility lead to fluctuations of bond prices. This reflects the fact that, in practice, these two factors are important determinants of the value of bond portfolios. We introduce the PPB strategy in our simulation analysis in order to assess the benefits that investors can expect from such a strategy by comparing the strategy to a position in the Bund futures contract.

τ denotes the time until expiration of the option and T the remaining term of the bond at the initial date. We model the position in the Bund futures as a position in the bond that is held from t_0 to $t_0 + \tau$. Hence $T - \tau$ is the remaining term of the discount bond (i.e., the duration of the coupon bond) at option expiration. At t_0 the maturity of the bond is equal to T. At $t_0 + \tau$ the bond with remaining maturity $T - \tau$ is sold and a new position is taken up in a bond with maturity equal to T, which is held up to $t_0 + 2\tau$ and so forth. This corresponds to a futures position that is held for three months and then rolled over. In our simulations, we take $\tau = 3$ months and $T - \tau = 7.8017$.

We construct the PPB strategy in the following way, which is similar to Merton et al. [1982], p.35. The portfolio held is made up of a number N of the underlying bond plus the same number of put options. We then scale the initial investment to be equal to an amount I, say €100. Put options with time to expiration equal to τ are bought at t_0 so that they expire at $t_{0+\tau}$. Hence, an option pricing formula is only needed to establish the premium at the initial investment, not for the payoff at expiry. We choose the strike price K of the put option as a function of the price of the bond $F(r, V, T)$ at the date when the option is bought.

One outstanding question is the choice of the strike price of these options. Given the fact that exchange traded options are issued with strike prices rather close to the current price of the underlying asset, an investor will choose from this proposed range of strike prices and end up with options that are not too far in- or out-of the-money. The typical range of moneyness considered in the literature on options strategies is from 10% out of the money to 10% in the money. Hence, Merton, Scholes and Gladstein [1982] consider moneyness of −10, 0, and 10%. Likewise, Figlewski, Chidambaran, and Kaplan [1993] consider moneyness of −10, −5, 0, 5, and 10%. As outlined in Merton, Scholes and Gladstein [1982], there is no single best alternative for the strike price. Instead, the choice depends on the preferences of the investor, such as his risk tolerance. In what follows, we decide to set the strike price to 10% out of the money in our base case so that $K/F(r, V, T) = 0.9$. This corresponds to an investor who is willing to take on some downside risk in any subperiod and is concerned over decreasing profitability of the strategy when the strike price is increased (see e.g. Mcmillan [2001], Chapter 17). Fabozzi [1996], Chapter 16 highlights that in the context of bond portfolio management, protective puts are usually implemented with out-of-the money puts on bonds or futures.

We generate scenarios for the position in the Bund futures and for the PPB strategy over one year with rebalancing taking place at the beginning of months 1, 4, 7, and 10.

Results. We first conduct the base case experiment using given parameter values, and then perform a variety of robustness checks.

To assess the results for the PPB strategy with Bund futures and options on Bund futures when compared with the position in the Bund futures only, we look at the portfolio returns after one year. Exhibit 3 shows typical performance statistics. It should be underlined that all these statistics are based on the distribution of the final portfolio value across the 1,000 scenarios that we generate. This is different from calculating such statistics from a time-series of asset returns, as is done in empirical studies.

Examining the performance statistics leads to the conclusion that the PPB strategy is largely favorable. In particular, the mean return also increases. This stems from the fact that the put option is exercised in scenarios with strongly negative returns. Consequently, the left tail of the returns distribution is cut off, which increases the mean return. This effect is equally apparent from the higher skewness of the PPB strategy. Exhibit 4 shows the return distributions for both the futures strategy ("Bond") and the PPB strategy.

EXHIBIT 3

Risk and Return of a Bund Futures Strategy and Protective Put Buying Strategy for a 1-year Investment Horizon

Performance Statistics	Bond	PPB
Mean	4.96%	5.98%
Sharpe-Ratio (2%)	0.52	0.70
VaR (95%)	4.59%	3.20%
Skewness	−0.28	0.14
Information Ratio	0.00	0.41

The left part of the table indicates the percentiles of the return distribution over 1,000 scenarios that we generated. The right part of the table indicates some standard performance measures based on this distribution. The information ratio is calculated with respect to the Bund futures strategy. "Bond" denotes the Bund futures strategy. "PPB" denotes the protective put buying strategy.

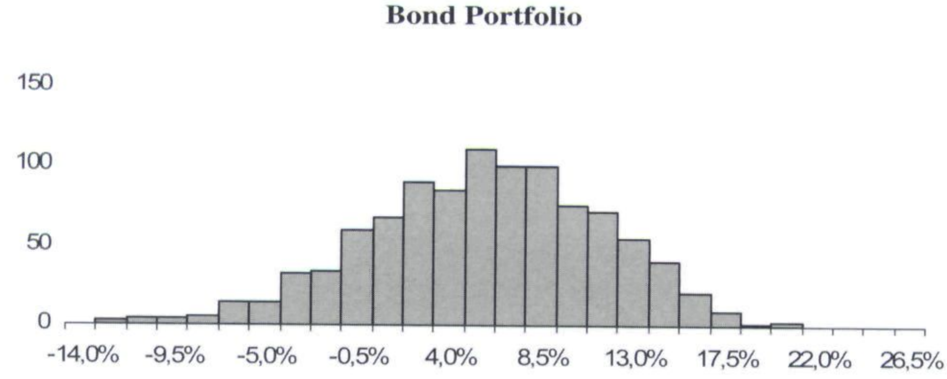
Inspection of the probability distribution functions of annual returns confirms the results in Exhibit 3. Focussing on negative returns below 7%, it can be seen that the PPB strategy has less frequent losses of this magnitude.

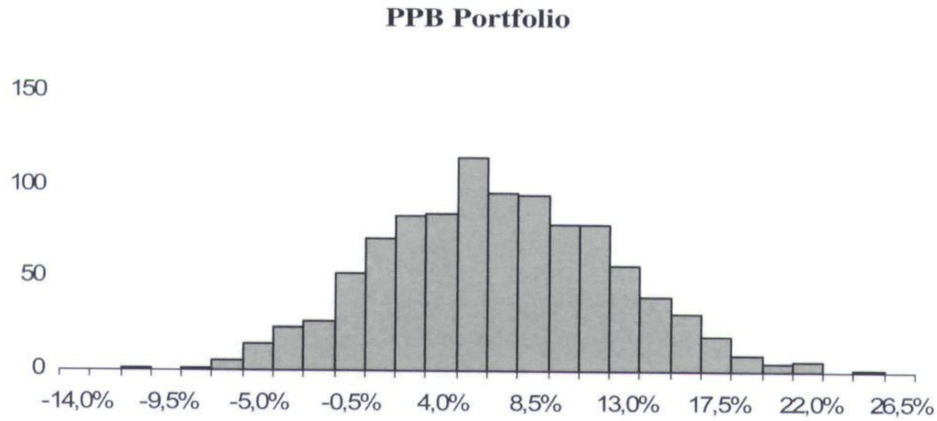
It is not true, however, that "PPB" dominates "Bond Only" for each single scenario. For a given scenario, the return of "PPB" could be lower than that of "Bond Only". Intuitively speaking, one may actually expect that the cost of purchasing the downside protection will have an impact on the performance. A straightforward calculation actually shows that the probability of underperforming the bond futures strategy is as high as 73.20% when the investor purchases the puts. This results from paying the option price at the beginning of every three month period. However, the negative effect of paying the option price does not outweigh the benefits of avoiding the most negative drawdowns, as can be seen from the complete returns distribution. In other words, the PPB does under-perform slightly when bond markets are doing well, and outperforms significantly when bond markets are doing poorly.

Since we are concerned that our results may be driven by our choice of the strike price, we vary K and compare the PPB strategy to the simple futures strategy for various values of K. From a different standpoint, one can argue that rather than choosing a given value for K, an investor would be interested in choosing the optimal value. This issue is also addressed by our robustness analysis.

EXHIBIT 4

Return Distributions for both the Futures Strategy ("Bond") and the PPB Strategy





Changing the values of K does not lead to a significantly different evaluation of the PPB Strategy. Across a wide range of strike prices, the strategy actually stays favorable when compared with the simple futures strategy. The graph in Exhibit 5 shows the evolution of the conditional relative deficit, i.e., average under-performance of PPB across scenarios that are such that PPB out-performs the bond only portfolio, as a function of the degree of moneyness chosen for the put options. (As recalled earlier, our base case corresponds to a level of moneyness equal to -10%).

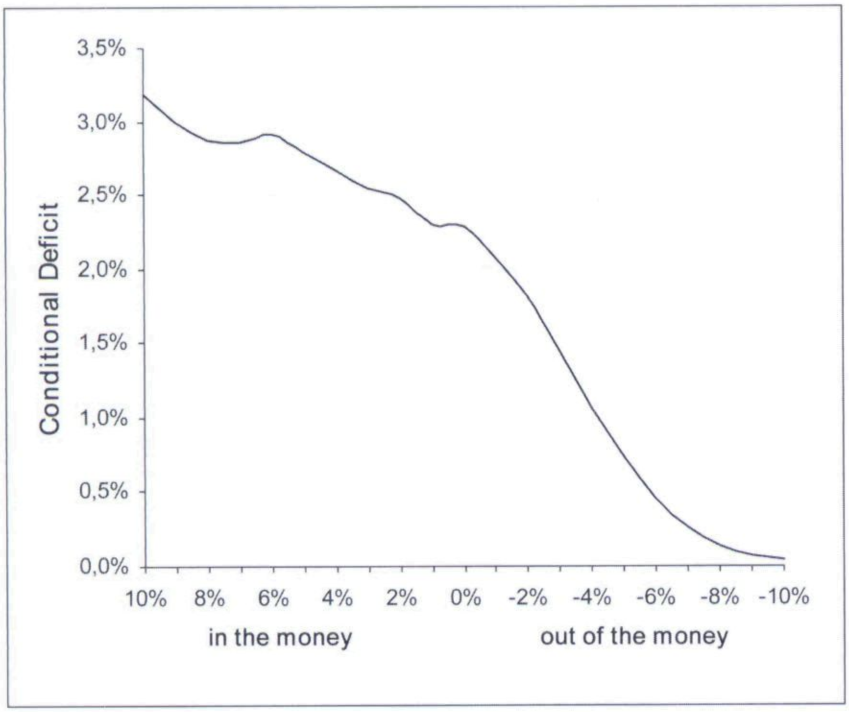
The cost of the protection is apparent from this graph. It can be seen that with rising levels of protection (i.e. as the strike for the put option is chosen more and more into the money), the underperformance increases. Again, it should be noted that this is the magnitude of underperformance conditional on the fact that PPB underperforms, i.e., given that the put option is not exercised and thus the returns of PPB are inferior to the case where no option is bought. Our choice of a level of protection is consistent with a -10% level of moneyness, which is consistent with related papers, is supported by this analysis since it leads to a conditional deficit converging to zero.

Optimal Allocation

The previous simulations considered stand alone investments into either the options strategy or the bond futures strategy. It appears that the PPB strategy clearly dominates the bond futures strategy. More precisely, the PPB strategy has lower downside risk, while achieving returns that are considerably above those for the bond futures strategy. Investors looking for capital preservation would naturally favor such an investment. The assessment of the stand-alone benefits would ultimately suggest that an investor should replace its bond portfolio by a suitably designed option strategy.

EXHIBIT 5

Conditional Under-Performance of the Option Strategy for Different Strike Prices



However, instead of looking at choices between single assets, a more relevant question is to ask what benefits arise from investing into the option strategy in a portfolio context, as an addition to the bond futures strategy. This subsection turns to this issue.

Description of the Optimization Problem. Benefits of the asymmetric characteristics of options are expected to arise in a framework that explicitly takes into account the asymmetry of the returns distribution, such as in the context of quintiles of the return distribution (such as Value-at-Risk). Since long positions in put options can truncate the returns distribution at a given level, they are expected to constitute a significant portion of the optimal portfolios of investors with downside risk preferences.

Our portfolio choice problem is between the bond futures strategy and the PPB strategy described earlier. We test two optimization objectives: Our first risk measure is the Value at Risk (at 95% confidence) and the second one is the variance of portfolio returns. The time horizon is one year, as in the earlier section.

The Results. The aim of our exercise is to show the risk return characteristics of minimum risk portfolios with respect to the two risk measures. We also want to analyze the resulting optimal allocation decision, i.e., the weights given to protective put buying and the simple bond futures strategy. It should be kept in mind that the PPB strategy itself contains a position in the bond futures and just adds the long position in the put option.

Exhibit 6 below shows the risk and return characteristics of the minimum risk portfolios. In addition, the last two lines of the table show the improvement over the case where the portfolio consists of 100% invested in the bond futures strategy. It can be seen that the reduction of the volatility (standard deviation of returns across simulated portfolio wealth at a horizon of one year on our 1,000 paths) is present, but not very significant in economic terms. However, the reduction of Value at Risk is very pronounced, and this even in the case of the minimum variance portfolio, which reduces the Value at Risk by 15%. The Minimum Value at Risk Portfolio even reduces the Value at Risk by 32%.

These results clearly show that assessing the value of the option strategy in terms of standard deviation is misleading, since the standard deviation does not fully describe the risk in the case of asymmetric return distributions. The main benefit of including the option strategy comes from the fact that it introduces positive asymmetry in the portfolio. This only becomes fully apparent when

EXHIBIT 6

Risk and Return Characteristics of Minimum Risk Portfolios

	Min VaR	Min Variance
Mean	5.92%	5.44%
Std Dev	5.64%	5.52%
VaR (95%)	3.20%	3.90%
Sparpe-Ratio (2%)	0.69	0.62
Skewness	0.11	−0.10
Reduction of Volatility	0.04%	2.21%
Reduction of VaR	30.36%	15.13%

looking at the Value at Risk, which captures the asymmetry of the returns distribution.

In addition to assessing the resulting risk return characteristics, we are interested in what percentage an investor would allocate to the option strategy given that one is averse to normal risk (standard deviation) or extreme risks. The graphs in Exhibit 7 below show that the PPB strategy is included with a significant weight in both the minimum variance and the minimum Value-at-Risk portfolio. However, the allocation in the latter case is far more important. This is not surprising given that the optimization objective in the Value-at-Risk case leads to a preference for the asymmetry implicit in the PPB strategy. On the other hand, it appears that even under the volatility objective, the weight attributed to the PPB strategy is significant.

In summary, this section concludes that option strategies are a useful supplement to a pure linear exposure to fixed income instruments. It should be noted that rather than designing an optimal options strategy, we define the strategy ex-ante, which suggests that even higher benefits may be obtained by an investor if one achieves an optimal design. In other words, the fact that the optimal allocation to the PPB strategy is significant even though such payoffs may not necessarily be optimal, strongly suggests that the use of derivatives can significantly improve investors' welfare.

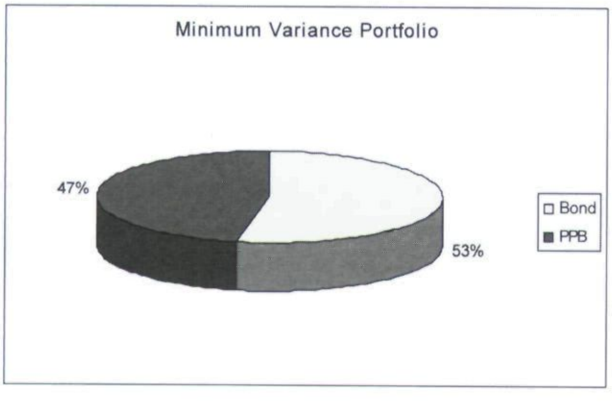
In light of the academic research (see in particular Leland [1980] as well as subsequent theoretical studies on the subject), it actually appears that a linear payoff being optimal is the exception rather than the rule (which of course is obvious from the fact that the set of nonlinear payoffs trivially encompasses the linear ones). Although this is true in a pure asset management context (such as an investment process of a mutual fund), it is desirable to test whether nonlinear payoffs will also be attractive for liability-driven forms of portfolio management. We now turn to an assessment of the strategy used in this section in an asset liability management context.

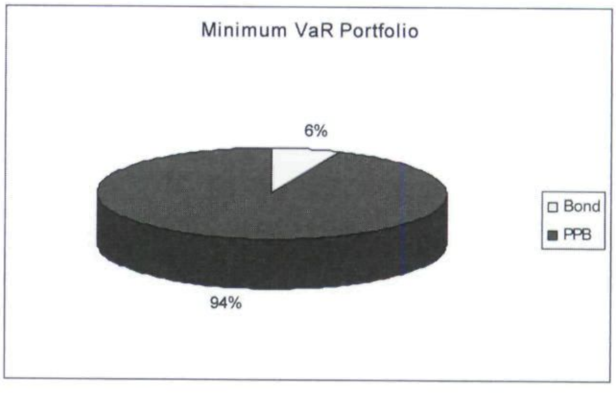
MANAGING LIABILITIES WITH INTEREST RATE DERIVATIVES

As recalled in the introduction, the recent pension fund crisis and the emergence of new regulatory and accounting standards have induced a heightened level of scrutiny on risk management practices of institutional investors. A paradigm change is currently taking place in asset management that puts the relative performance in relation to the institution's liabilities in the forefront of the decision-making process. In the face of the dramatic impact of equity bear markets at the beginning of the new millennium, combined with a decreasing interest rates environment that has led to a sharp increase in liabilities, the recognition by asset managers of the specific nature of the investment decision of such investors has led to the emergence of a variety of *liability driven investment* solutions.

A typical pension plan is exposed to significant interest rate risk emanating from a duration mismatch between assets and liabilities. This exposure is permanent and represents a large, unacknowledged, strategic bet on interest rates and the mismatch in duration exposes the plan to uncompensated risk. Assuming that most pension

EXHIBIT 7
Composition of Minimum Risk Portfolios





plans have an average duration between 10 and 15 years, it is obvious that there is an opportunity cost and dramatic risk/reward difference of using government bonds with a duration that will be significantly lower than that of the liabilities. A given decrease in the level of interest rates will have a more marked impact on the value of the liabilities than on the value of the assets, with a marked decrease of surplus size as a consequence. A positive duration gap indicates that assets are more interest rate sensitive than liabilities, on average. Thus, when interest rates rise (fall), assets will fall proportionately more (less) in value than liabilities and the market value of equity will fall (rise) accordingly. On the other hand, a negative duration gap indicates that weighted liabilities are more interest rate sensitive than assets. Thus, when interest rates rise (fall), assets will fall proportionately less (more) in value than liabilities and the market value of equity will rise (fall).

The most straightforward approach to asset-liability management, explicitly focusing on risk management, is a simple matching strategy, where the investor tries to hold assets that replicate his liabilities. If exact replication of the cash flows is achieved, the assets will perfectly match the liabilities and there is no risk of shortfall, as well as no chance for surplus. Let us assume for example that a pension fund has a commitment to pay out a monthly pension to a retired person. Leaving aside the complexity relating to the uncertain life expectancy of the retiree, the structure of the liabilities is defined simply as a series of cash outflows to be paid, the real value of which is known today. It is possible in theory to construct a portfolio of assets whose future cash flows will be identical to this structure of commitments.

Instead of perfectly matching the cash flow, institutional investors typically may instead follow a duration matching approach, where the interest rate sensitivity of the assets matches that of the liabilities. Although OTC contracts such as inflation and interest-rates swaps can prove useful in the implementation of liability-matching portfolios, we argue in this section that exchange-traded alternatives, such as futures, can be regarded as natural cost-efficient alternatives. In a nutshell, interest rates swaps are naturally fitted for implementing *cash-flow matching* strategies, which involves ensuring a perfect *static* match between the cash flows from the portfolio of assets and the commitments in the liabilities, while exchange-traded futures should be the instruments of choice in the implementation of *immunization* strategies, which allows the residual interest rate risk created by the imperfect match between the assets and liabilities to be managed in a *dynamic* way.

In the context of our simulation model, we evaluate the benefits of different derivatives strategies by observing the impact of price fluctuations of fixed income instruments. To analyze derivatives strategies in an asset liability management framework, we have to simulate paths for liabilities in addition to the paths for asset prices from the section *Derivatives Strategies in Fixed Income Portfolio Management*.

The Investor's Liabilities

On the liability side, in an attempt to focus on a stylised institutional investor, we model the liabilities as a short position in a global bond index, which can be represented by a zero-coupon bond with constant time-to-maturity. The Longstaff–Schwartz framework allows us to easily price liabilities modelled in this manner. Our way of representing liabilities reflects that most institutional investors' liabilities are impacted by changes in interest rates and changes in interest rate volatility. As a result, liabilities would be perfectly correlated with return on a bond index. In practice, there are certainly other factors, such as actuarial uncertainty, that determine the returns on liabilities. This is the reason why we introduce a disturbance term in our liability process l_t, as a convenient reduced-form way to achieve a target correlation between liabilities and the discount bond. In our model, liabilities then correspond to the returns of the discount bond with price L plus a Gaussian white noise:

$$\frac{\mathrm{d}l_t}{l_t} = \rho \frac{\mathrm{d}L_t}{L_t} + (1-\rho)a_t \quad \text{with} \quad a_t \sim N\left(\mu_l, \sigma_a^2\right)$$

where a is orthogonal to $\mathrm{d}L_t/L_t$, the return process of the bond, and where we take $\rho = 0.8$, $\sigma_a = 0.05$, and we set μ_l to be equal to the mean of L_t.[3]

The duration of the discount bond is chosen to be equal to 10 years in a first case and equal to 15 years in a second case. This choice corresponds to the typical pension plan liability duration of 10–15 years (see Farley [2003]). According to van Dootingh [2005], the average duration of liabilities is approximately 15 years.

In the following subsection, we consider the effect of investing into different derivatives strategies on the performance relative to liabilities. To isolate the effects of short term changes in the interest rate and interest rate volatility, we assess the gap between assets and liabilities after a three months horizon.

Bund Futures and Options

We will first look at our base case, where the investor just holds a position in the Bund future. We assess the outcome of investing 100% of the assets into this strategy in terms of shortfall measures. Since the duration of the Bund future is lower than that of the liabilities we model, it can be expected that the shortfall risk will be significant. A major problem in the management of liability risk is actually that most institutional investors face liabilities that have very long maturities (as in the example of pension funds). Although in principal the liability position of such an investor can be associated with a short position in a bond, the bond market may not offer any instruments of such long maturity. Therefore, it is difficult to find an appropriate instrument for hedging purposes (see next subsection for the benefits of using futures available on longer-term bonds in that perspective).

An alternative to managing expected shortfall on the asset side may lie in using options on interest rate futures, as in the PPB strategy introduced earlier in this article. Because of the convex nature of the payoffs to the strategy, one may expect a reduction in expected shortfall from employing the PPB strategy.

Exhibit 8 shows the percentiles and the mean of the difference between assets and liabilities at the three months horizon.

The shortfall risk is significant, as can be seen from the fact that with 25% probability, the assets will be lower than the liabilities both for the simple futures strategy and for the PPB strategy. This has been anticipated, as there is an important duration gap. It can also be seen that, somewhat surprisingly, the PPB strategy does not reduce the shortfall risk, but actually increases significantly the performance of the portfolio. This latter statement can be verified when looking at the mean surplus and the maximum surplus. When thinking about the impact of an isolated shock to interest rates, these results become more intuitive.

Consider a steep rise in interest rates. This will lead to a decline in the value of the Bund futures and to an even steeper decline in the value of the liabilities, due to their longer duration. Therefore, we will observe a surplus after the interest rate shock for the Bund futures strategy. For the PPB strategy, the surplus will be even more positive. This comes from the fact that the decline in value of the PPB portfolio will be limited because of the protection from the put options. Next consider a steep decline in interest rates. The value of the Bund futures strategy will rise. Liabilities will rise even more in value, so that we will observe a shortfall for the Bund futures strategy after the interest rate rise. If the investor holds the PPB strategy, the value of his/her assets will also rise, but less than for the simple futures strategy. The difference is given by the option premium that the investor has to pay.

In summary, the PPB strategy actually serves as a return enhancer, while being exposed to roughly the same risk of drawdown relative to liabilities upon a negative interest rate shock (decline in the interest rate). Although it may be interesting to look at alternative option strategies in the context of asset liability management, the most straightforward way to manage the shortfall risk is to pursue simple duration matching strategies.

EXHIBIT 8

Surplus/Deficit between Assets and Liabilities after a 3-month Period for the Bund Futures Strategy and Protective Put Buying Strategy

	Bund (%)	PPB (%)
Minimum	−0.77	−0.78
5%	−0.56	−0.56
25%	−0.16	−0.17
Median	0.09	0.08
Mean	0.34	0.70
75%	0.33	0.32
95%	0.74	0.76
Max	1.14	6.97

The percentiles of the surplus/deficit distribution over 1,000 scenarios that we generated are shown. The results are for liabilities with 15 year duration.

Introducing the Buxl Contract

As highlighted earlier, having long term maturity instruments for hedging purposes at their disposal is a critical need for institutional investors since the convexity of the price-yield relation for a bond increases with longer maturities. The significant shortfall risk for the two strategies based on the Bund futures actually stems from the lower duration compared with the liabilities. This problem is all the more significant in times of low interest rate coupons, which also have a positive impact on convexity. Given these problems, we propose to assess the usefulness of a 30 year bond future for hedging purposes. This contract corresponds to a duration of roughly 18 years, which is actually higher than the duration for the liabilities in our model (10 years and 15 years), while the duration of the Bund contract is lower in both cases.

Exhibit 9 shows the distribution of the difference between assets and liabilities at the three months horizon. Again, this allows us to assess the impact of interest rate shocks on the investor's situation, given that he/she faces liability constraints. The left hand histograms show the surplus/deficit distribution in the case where the liabilities correspond exactly to a short position in a bond index. The histograms on the right hand side show the surplus/deficit distribution in the case where the liabilities only have a correlation of 0.8 with the bond index, i.e., where we introduced a white noise perturbation. The upper graphs show the case where the investor holds the simple Bund futures strategy, the middle graphs correspond to the Buxl futures strategy, and the lower graphs show the case where the investor tries to match the duration of liabilities by mixing both strategies. It can be seen that the Buxl futures strategy leads to a lower variability of the distribution, while the duration matching strategy achieves the lowest distribution. In addition, it appears that even if the liabilities deviate significantly from a zero coupon bond, the duration matching technique is useful, i.e., the main risk stems from the interest rate changes.

We obtain the weights of the Bund and Buxl strategy by minimizing the shortfall variance. We favor this approach to simple duration calculations, which rely on certain assumptions, notably that the yield curve is only affected by small parallel shifts, which may not hold in our setup.

EXHIBIT 9

Surplus/Deficit between Assets and Liabilities after a 3-month period (10 year duration for liabilities)

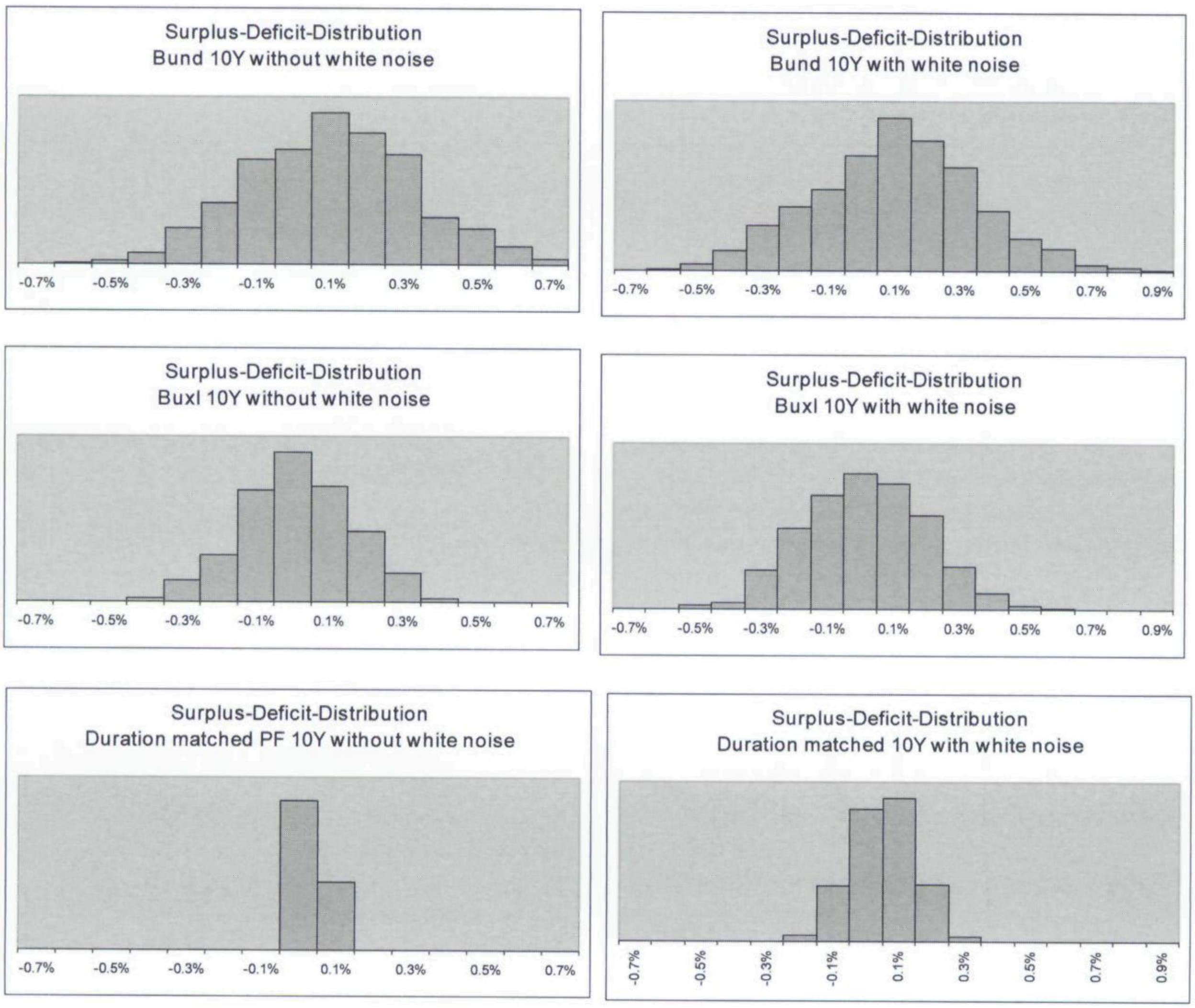


Exhibit 10 shows the results for the case, where liabilities have a duration of 15 years. The Buxl future allows for even more significant reduction in this case, which is not surprising given that its duration is close to those for the liabilities.

Exhibit 11 shows the weights that we obtain for the different cases. The positive contribution of the Buxl future is obvious. The longer duration of this futures contract leads to significant reduction of the surplus/deficit variance, allowing investors with liability constraints to achieve enhanced matching when compared with the case where the Buxl future is not available. This is reflected in the significant allocations the Buxl futures strategy obtains in the duration matching portfolios, which range between 60 and 98.8%.

EXHIBIT 10

Surplus/Deficit between Assets and Liabilities after a 3-month Period (15 year duration for liabilities)

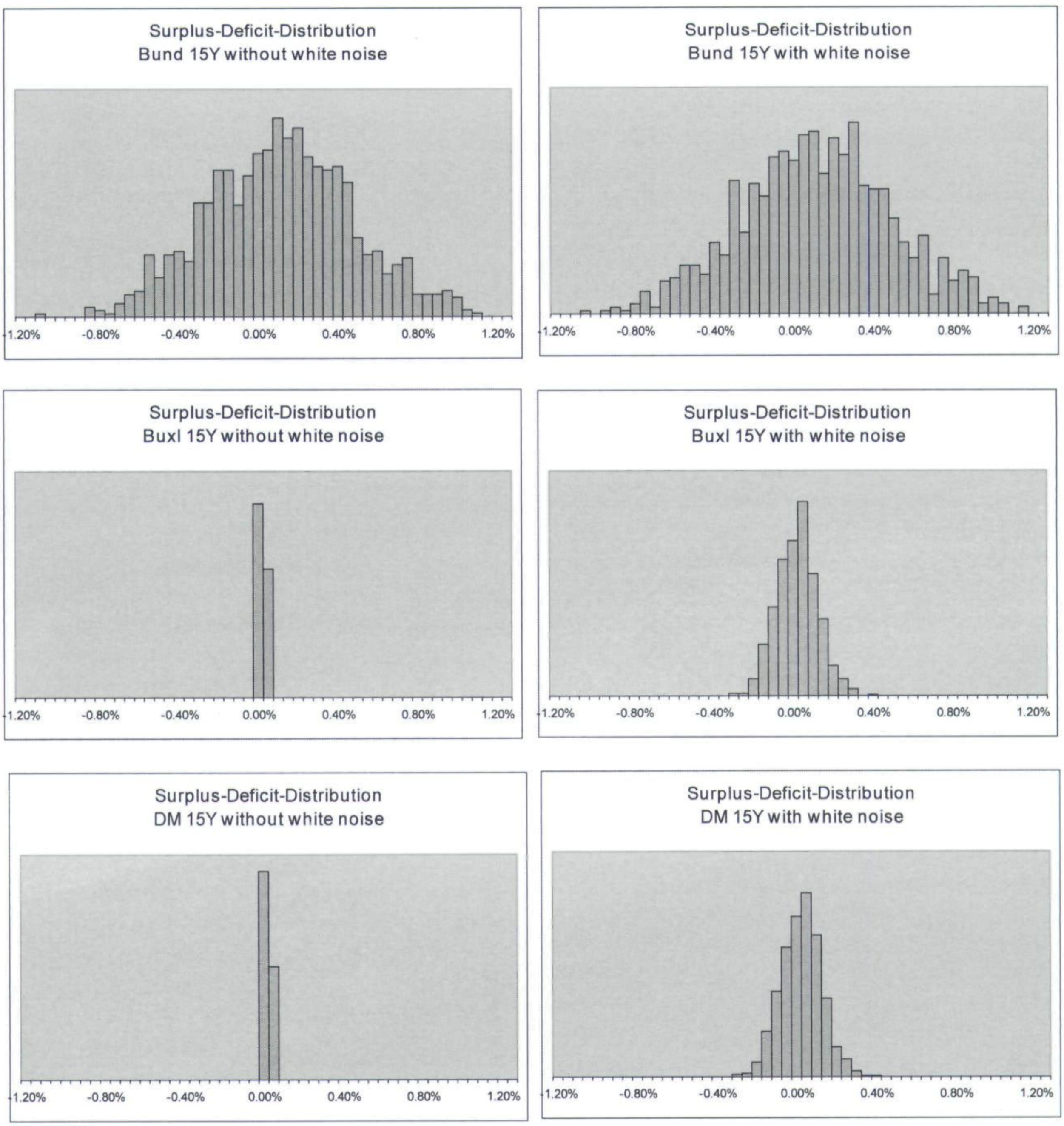


EXHIBIT 11
Composition of Duration Matching Portfolios

	Duration of Liabilities (years)	Bund (%)	Buxl (%)
Without White Noise	10	39.7	60.3
	15	3.8	96.2
With White Noise	10	39.2	60.8
	15	1.2	98.8

A comment on the limits of our ALM risk management analysis is in order. In practice, simultaneous management of interest rate risk and other risk factors is known to be extremely difficult. This is captured in our model by the error term, which reflects non-interest rate related risk. It can be checked, as expected, that results with the white noise perturbation are less favorable than in the case without the white noise perturbation.

Another, probably more important, disadvantage of the risk minimization strategy considered here (as well as its more standard and specific applications such as cash-flow matching strategies, or immunisation strategies) is that it represents a positioning that is extreme and not necessarily optimal for the investor in the risk/return space. In fact, we can say that the surplus risk minimization approach in asset-liability management is the equivalent of investing in a low risk asset in an asset management context. It allows for tight management of the risks. However, the lack of return, related to the absence of risk premia, makes this approach very costly, which leads to an unattractive level of contribution to the assets. The PPB strategy presented earlier may be a partial answer to this concern.

CONCLUSION

In this article, we assess the value of using bond futures and options on these futures. We examine the value of introducing derivatives both in a pure asset management context and in an asset liability management context. We conduct a simulation study based on the Longstaff and Schwartz [1992] interest rate model by generating stochastic scenarios for instruments that correspond to Eurex contracts on long term German government bonds. In particular, we look at the Bund futures and Buxl futures contracts, as well as options on the Bund futures contract. Our results show that the nonlinear character of the returns on put option buying strategies offers appealing risk reduction properties in the pure asset management context. Consequently, such strategies obtain a high weighting in an optimal portfolio, especially when investors are concerned with minimizing the extreme risks. In an asset liability management context, we show that such strategies offer surplus benefits. To achieve significant shortfall reduction, the investor typically needs to invest in very long-term contracts that reflect the long duration of most liability constraints. Our results underline the importance of such contracts for asset liability management.

Future research may take a similar perspective in order to assess the value of different types of options strategies. In this article we have limited the investor's asset menu to derivatives strategies that are defined ex ante. An extremely relevant research topic, both from a practical and from a theoretical point of view, is the question of optimal product design, given the preferences of asset managers or institutional investors.

ENDNOTES

[1]In practice, using futures contracts and options on futures is usually considered as a cheaper alternative to the use of bond and bond options. The reason why we have chosen to model bond and bond option prices, as opposed to futures and futures option prices, is because of the increased tractability required for the numerical exercises performed below. In particular, in the Longstaff and Schwartz framework outlined above, we are able to use closed-form solutions for zero coupon bonds and options on these bonds. It should be stressed that this is done for modelling reasons only and has no conceptual consequences. Because of only minor differences in the contractual payoff of these contracts, our conclusions can be considered as directly carrying over to the use of bond futures and options on these futures.

[2]Of course, the results of the numerical exercise could easily be adjusted to reflect the exact characteristics of the actual contracts on a given trading date.

[3]It should be noted that the return on liabilities is usually considered to be higher than that of a bond with similar duration. Since the difference in performance is typically compensated by additional contributions, which we do not model here, we have chosen to abstract from this added complexity in this article.

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