A Model for Convexity-Based Cross-Hedges with Treasury Futures

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The conversion factoring system of the Chicago Board of Trade (CBOT) entails two rarely understood hedging implications. First, a convexity-based hedge may be essential because Treasury futures' convexity can be negative when multiple Treasuries closely compete to be the projected cheapest-to-deliver (CTD) Treasury. Second, a convexity-based hedge that may not require an option-theoretic solution is still possible because near the delivery month, a linear, arbitrage-free relationship should hold between a Treasury futures contract and its CTD Treasury.

The second hedging implication, arising from the linear, arbitrage-free relationship, was incorporated into Rendleman's [1999] model for a forward-looking duration hedge. However, the first hedging implication, arising from the extreme *optioness* in the futures market, has been neglected in the literature on cross-hedges and, as a result, incorrect/inefficient convexity-based hedges have been produced.[1] Although the option-theoretic literature recognizes the possibility of Treasury futures' negative convexity and the implications for pricing Treasury futures,[2] the neglect of the first hedging implication in the literature on cross-hedges is unsurprising, because imperfections in the CBOT conversion factoring system and their hedging implications are not well understood.[3]

Note that when changes in the term structure of interest rates are relatively large and less predictable, an accurate prediction of the projected CTD Treasury becomes more difficult. When multiple Treasuries closely compete to be the projected CTD Treasury, the increase in the optioness leads to the following: 1) Treasury futures tend to trade as if their underlying asset is a convex combination of the multiple Treasuries, 2) the Treasury futures' price tends to be non-linear in the price of par-delivery grade, and 3) the Treasury futures' convexity measure can be negative. Therefore, one hedging implication of the extreme optioness in the Treasury futures market is that convexity-based hedges may be essential for cross-hedging a bond portfolio.

This article extends Rendleman's [1999] framework for a duration hedge to develop a model for a forward-looking, convexity-based hedge that does not require option-theoretic duration and convexity matching between a bond portfolio and two Treasury futures contracts. The forward-looking, convexity-based hedge is of interest to both hedging practitioners and academic researchers for at least two reasons. First, such price-based, forward-looking hedge rules tend to be superior to regression-based, backward-looking hedges.[4] Second, the forward-looking hedge-ratio equations produced in this study provide information useful for less arbitrary pairing of long- and short-maturity Treasury futures contracts, which is essential for an effective convexity-based cross-hedge.

In this article, technical details about the numerical analysis of hedge cost performance are provided because such details are not available in the existing literature on cross-hedges. The numerical analysis considers scenarios involving different term structures of interest rates, hedge horizons, pairs of Treasury futures, and duration levels of both bond portfolio and CTD Treasuries. The numerical analysis indicates that a forward-looking hedge is, on average, 1.8% cheaper than a traditional hedge in most scenarios. It also indicates that the hedge cost saving is large when a convexity-based cross-hedge is most needed; that is, when high-duration bond portfolios are hedged, the term structure of interest rates is steep, and the CTD Treasury pays high coupons.

The remainder of this article is arranged as follows. In the next section, imperfections in the CBOT conversion factoring system and their hedging implications are explained. In the following two sections, a model for forwardlooking, convexity-based cross-hedges is developed, and the hedge cost performance of forward-looking hedges in various scenarios and robustness testing are discussed. The final section summarizes the study and discusses the practical implications of the main findings.

CBOT CONVERSION FACTORING SYSTEM AND HEDGING IMPLICATIONS

The Treasury futures market is unique in several ways. For example, short positions are allowed to deliver for physical settlement any Treasury that meets certain eligibility criteria. To facilitate short position choice among different coupon-maturity grades, CBOT computes the short's invoice quote as the futures settlement price multiplied by a conversion factor (CF).

The futures settlement price is the settlement quote multiplied by the futures' contract size, which is $100,000 for most Treasury futures. CF is the present value of a deliverable Treasury's future cash flows discounted at a 6% flat rate (to the first day of the delivery month).[5] Therefore, CF reflects the value of a deliverable Treasury relative to a 6% coupon notional Treasury. Short positions implicitly buy delivery-related options from long positions, and as a result the invoice quote tends to be smaller than the market quote of a deliverable Treasury. The positive difference (i.e., the deliverable Treasury's quote minus the futures' settlement price multiplied by the deliverable Treasury's CF) reflects the value of the delivery-related option or the delivery cost borne by short positions.

To compute CF, CBOT employs the following formula: $\text{CF} = a \times [0.5C + b + c] - d$, where $0.5C$ is the deliverable Treasury's first semiannual coupon in decimals, c captures the present value of all other coupons in decimals as of the first coupon date, and b captures the present value of $1 par Treasury as of the first coupon date. The bracket term $[0.5C + b + c]$ reflects the deliverable Treasury's present value as of the first coupon date. In addition, a is the discount factor for the period between the first coupon date and the first day of the delivery month, and d captures the accrued coupons as of the first day of the delivery month.

As a result, $\text{CF} = a \times [0.5C + b + c] - d$ reflects the Treasury's relative quote in decimals as of the first day of the delivery month. To simplify the CF computation, CBOT rounds down the remaining maturity of each deliverable Treasury to the nearest quarter for long-maturity (30- and 10-year) Treasury futures and to the nearest month for short-maturity (5- and 2-year) Treasury futures. The first coupon day for long-maturity futures is assumed to be the first day of the delivery month if the rounded-down maturity is n years and even quarters. If the rounded-down maturity is n years and odd quarters, the first coupon day is assumed to be the first day of the next March-cycle quarterly month. For short-maturity futures, the first coupon day is assumed to be the first day of the delivery month or the first day of one of the next 11 months.[6]

Note that the first coupon day is assumed to be the first day of March quarterly cycle delivery months, whereas actual Treasury coupons are paid in the middle of February quarterly cycle months. As a result, maturity rounding-down brings forward the future cash flows of Treasuries by 2.5 months in the case of long-maturity futures and 15 days in the case of short-maturity futures. It can be mathematically shown that maturity rounding-down causes a non-negligible upward bias in the CF measurement by approximately $\log(1.06^{5/24})$, or 120 basis points, in the case of long-maturity futures and approximately $\log(1.06^{2/52})$, or 24 basis points, in the case of short-maturity futures. This upward bias in CF does not, however, contribute to the determination of, or changes in, the CTD Treasury because its magnitude does not vary across deliverables within a deliverable basket.

One important imperfection in the CBOT conversion factoring system, which contributes to the determination of a CTD Treasury, is the use of the 6% flat-discount rate. For example, when the forward rate curve lies above the 6% flat-term structure, the use of a flat 6% discount rate will produce a large upward-biased

CF for deliverable Treasuries with high duration (i.e., low coupon rate, long maturity, or high yield). When the forward rate curve lies below the 6% flat-term structure, the use of a 6% discount rate will produce a large downward-biased CF for deliverable Treasuries with high duration. This bias in CF contributes to the determination of the CTD Treasury because its magnitude varies across deliverable Treasuries within a deliverable basket.[7]

As the first day of the delivery month approaches, the optioness in the Treasury futures market will decline substantially because the uncertainty in predicting the projected CTD Treasury declines substantially. Cash-futures arbitrages around the first day of the delivery month will then force the adjusted basis for the projected CTD Treasury to shrink to zero; that is, the value of (the projected CTD Treasury quote minus the futures' settlement quote multiplied by the CTD Treasury's CF) should approach zero. In such an event, a Treasury future will trade almost as if it is a contract written on the projected CTD Treasury, and as a result a linear, arbitrage-free relationship between Treasury futures and the CTD Treasury should hold around the first day of the delivery month.[8]

In the next section, a forward-looking model for convexity-based hedges is developed using Rendleman's [1999] framework for a duration hedge.

A MODEL FOR CONVEXITY-BASED CROSS-HEDGES WITH TREASURY FUTURES

In order to develop the model for convexity-based hedges discussed in this article, it is assumed that 1) the term structure of interest rates will not be flat but will shift in parallel, 2) the forward price of a bond is an unbiased estimator of the expected spot price of the bond at the maturity of the forward contract, 3) neither the marking to market nor the margin requirement exists for Treasury futures contracts, and 4) hedgers' target hedge expirations and short positions' deliveries will occur on the first day of the delivery month.[9]

Interest Rate Sensitivity of Bond Prices

Under these assumptions, an investment strategy of buying a non-Treasury bond and selling the bond forward should generate a zero profit. Therefore, a simple carry-cost representation of an arbitrage-free relationship between a bond price and a forward bond price is possible. This arbitrage-free relationship dictates that the forward price should equal the cash bond price plus the net cost of carrying the cash bond position through the expiration of the forward contract, where the net carry cost is the cost of financing the bond purchase minus the revenue from the cash coupon received during the life span of the forward contract.

As a result, the full (i.e., dirty) price of a non-Treasury bond can be expressed as a sum of the certainty-equivalent values of the forward bond and a series of zero-coupon bonds:

$$P + A = (P' + A')e^{-rt} + \sum_{k=1}^{m} I_k e^{-r_k t_k} \tag{1}$$

where A is the accrued coupon as of now, P is the clean (i.e., quoted) price, $P + A$ is the full (i.e., dirty) price of a bond, $P' + A'$ is the dirty forward price of a bond, P' is the clean forward price of a bond, A' is the accrued coupon as of the maturity of a forward bond contract, e^{-rt} is the discount factor for the period t between now and the expiration of a forward bond contract, r is the spot interest rate continuously compounded for the period t, m denotes the number of cash coupon payments prior to the expiration of a forward bond contract, I_k denotes the k-th interest cash coupon payment occurring at the t_k-th time, and r_k is the continuously compounded spot interest rate for the period between now and the t_k-th time.

Equation (1) states a mathematical equivalence between the market price of a generic bond (the left-hand side) and the intrinsic value of a synthetic position (the right-hand side). The synthetic position is equivalent to selling the forward bond for the hedge horizon and lending, at a risk-free rate, the present value of all cash coupons to be received between now and the expiration of the forward bond. Therefore, the value of a synthetic position contains two parts: 1) the present price of a forward bond that matures at end of the hedge and 2) a series of short-term, zero-coupon bonds whose respective maturities match the respective coupon payment dates of a generic bond.

A Taylor-series expansion of a change in clean bond price with respect to a change in a short-term spot interest rate can be approximated as a simple expression containing only two terms. Using the definitions of forward-looking duration and convexity measure, Equation (2) is derived for a change in clean price:[10]

$$\begin{aligned} dP = {} & [-(P' + A')D'e^{-rt} - (P + A)D'']dr \\ & + [(P' + A')e^{-rt}(C' + tD') + (P + A)C''](dr)^2 \end{aligned} \tag{2}$$

where

$$D' \equiv -\frac{\mathrm{d}(P' + A')}{\mathrm{d}r}\frac{1}{P' + A'}$$

is a forward-looking Macaulay duration for a bond as of the maturity of a forward bond,

$$C' \equiv \frac{1}{2}\frac{\mathrm{d}^2(P' + A')}{\mathrm{d}r^2}\frac{1}{P' + A'}$$

is the corresponding convexity measure,

$$D'' \equiv \frac{t(P' + A')e^{-rt} + \sum_{k=1}^{m} t_k I_k e^{-r_k t_k}}{(P + A)}$$

is a forward-looking Macaulay duration for a synthetic bond that pays the forward price of a bond at the expiration of the forward bond and makes m coupon payments prior to the expiration of the forward bond, and

$$C'' \equiv \frac{(P' + A')e^{-rt}t^2 + \sum_{k=1}^{m} t_k^2 I_k e^{-r_k t_k}}{2(P + A)}$$

is the corresponding convexity measure of the synthetic bond position.

Equation (2) contains the first-order effect (the bracketed term in front of dr) and the second-order effect (the bracketed term in front of (dr)^2). These two terms are the same as the two terms in the corresponding equation for a generic bond. The first-order effect is a linear combination of the projected dollar durations of a forward bond and a synthetic bond, which is equivalent to the projected dollar duration of a generic bond. Similarly, the second-order effect is a linear combination of the projected dollar convexities of a forward bond and a synthetic bond as well as the projected dollar duration of the forward bond.

Unlike the traditional measure for dollar duration, the bracketed term in front of dr decomposes the projected dollar duration into two components: 1) the projected dollar duration with respect to changes in short-term forward rates beyond hedge expiration and 2) the projected dollar duration with respect to changes in short-term spot rates through hedge expiration. The bracketed term in front of (dr)^2 decomposes the projected dollar convexity in a similar manner. This explicit decomposition of a bond's sensitivities to both the spot and the forward rates, together with that for Treasury futures, will lead to cross-hedge equations that can generate information useful for less arbitrary pairing of long- and short-maturity Treasury futures for convexity-based hedges.

Interest Rate Sensitivity of a Treasury Futures Quote

A Taylor-series expansion of the change in a Treasury futures quote (dF') with respect to a spot interest rate change can be approximated in the following way:

$$\mathrm{d}F' = \frac{\mathrm{d}F'}{\mathrm{d}r}\mathrm{d}r + \frac{1}{2}\frac{\mathrm{d}^2F'}{\mathrm{d}r^2}(\mathrm{d}r)^2 \quad (3)$$

As explained in the previous section, the resolution of uncertainty for the projected CTD Treasury around the first day of the delivery month leads to a linear, arbitrage-free relationship between Treasury futures and their CTD Treasury. Therefore, the projected dollar duration (dollar convexity) of Treasury futures should equal the projected dollar duration (dollar convexity) of a CTD Treasury divided by its CF. Using this arbitrage-free, price-volatility relationship, Equation (3) takes the following explicit form:

$$\mathrm{d}F' = -\frac{(P'_{\mathrm{CTD}} + A'_{\mathrm{CTD}})}{f_{\mathrm{CTD}}}D'_{\mathrm{CTD}}\mathrm{d}r + \frac{(P'_{\mathrm{CTD}} + A'_{\mathrm{CTD}})}{f_{\mathrm{CTD}}}C'_{\mathrm{CTD}}(\mathrm{d}r)^2 \quad (4)$$

where F' is a Treasury futures quote, with a projected duration D'_F, a projected convexity C'_F, a projected dollar duration $F'D'_F$, and a projected dollar convexity $F'\ C'_F$; $P'_{\mathrm{CTD}} + A'_{\mathrm{CTD}}$ is the CTD Treasury's full price, with a projected dollar duration $D'_{\mathrm{CTD}}(P'_{\mathrm{CTD}} + A'_{\mathrm{CTD}})$ and a projected dollar convexity $C'_{\mathrm{CTD}}(P'_{\mathrm{CTD}} + A'_{\mathrm{CTD}})$; and f_{CTD} is the CTD Treasury's conversion factor.

Simultaneous Equations for Convexity-Based Hedge Ratios and Their Solutions

For convexity-based cross-hedge ratios, two hedging instruments are considered: long-term Treasury futures (indexed by the subscript 1) and short-term Treasury futures (indexed by the subscript 2). Their hedge ratios are denoted as h_1 and h_2, respectively. A positive value for a hedge ratio indicates the number of short positions in the corresponding Treasury futures relative to a unit position in a non-Treasury bond, and a negative value indicates the number of long positions in the corresponding Treasury futures.

Matching a bond's (or a bond portfolio's) forward-looking duration exposure [the first term in Equation (2)] to the two futures' forward-looking duration exposures [the first term in Equation (4)] yields Equation (5). Equating a bond's forward-looking convexity exposure [the second term in Equation (2)] to the two futures' forward-looking convexity exposures [the second term in Equation (4)] yields Equation (6):

$$(P'+A')D'e^{-rt}+(P+A)D''=h_1\frac{P'_{\mathrm{CTD}_1}+A'_{\mathrm{CTD}_1}}{f'_{\mathrm{CTD}_1}}D'_{\mathrm{CTD}_1}+h_2\frac{P'_{\mathrm{CTD}_2}+A'_{\mathrm{CTD}_2}}{f'_{\mathrm{CTD}_2}}D'_{\mathrm{CTD}_2} \tag{5}$$

$$(P'+A')e^{-rt}(C'+tD')+(P+A)C''=h_1\frac{P'_{\mathrm{CTD}_1}+A'_{\mathrm{CTD}_1}}{f'_{\mathrm{CTD}_1}}C'_{\mathrm{CTD}_1}+h_2\frac{P'_{\mathrm{CTD}_2}+A'_{\mathrm{CTD}_2}}{f'_{\mathrm{CTD}_2}}C'_{\mathrm{CTD}_2} \tag{6}$$

Using the forward-looking, arbitrage-free relationship between Treasury futures and CTD Treasuries, these equations can be simplified as follows:

$$(P'+A')D'e^{-rt}+(P+A)D''=h_1D'_{F_1}F'_1+h_2D'_{F_2}F'_2 \tag{7}$$

$$(P'+A')e^{-rt}(C'+tD')+(P+A)C''=h_1C'_{F_1}F'_1+h_2C'_{F_2}F'_2 \tag{8}$$

Simultaneous solutions for h_1 and h_2 are provided in Equations (9) and (10), respectively:

$$h_1=\frac{[(P'+A')D'e^{-rt}+(P+A)D'']C'_{F_2}-[(P'+A')e^{-rt}(C'+tD')+(P+A)C'']D'_{F_2}}{D'_{F_1}C'_{F_2}F'_1-C'_{F_1}D'_{F_2}F'_1} \tag{9}$$

$$h_2=\frac{-[(P'+A')D'e^{-rt}+(P+A)D'']C'_{F_1}+[(P'+A')e^{-rt}(C'+tD')+(P+A)C'']D'_{F_1}}{D'_{F_1}C'_{F_2}F'_2-C'_{F_1}D'_{F_2}F'_2} \tag{10}$$

The hedge ratio for long-maturity futures (h_1) is decomposed into two components: the first component refers to the portion of a future's hedge that is against long-term interest rate exposure, whereas the second component refers to the portion of a hedge that is against short-term interest rate exposure. The hedge ratio for short-term futures (h_2) is similarly decomposed into two components.[11]

NUMERICAL ANALYSIS OF HEDGE COST PERFORMANCE

The numerical analysis in this article compares the hedge cost performance of forward-looking hedges with that of traditional hedges under scenarios that involve different term structures of interest rates, hedge horizons, pairs of Treasury futures, and duration levels of both bond portfolio and CTD Treasuries.

The shape of a term structure is captured by intercept (α) and slope (β) parameters of a general term-structure specification: $r(t) = \alpha + \beta\,[\ln(t + 5.0)-\ln(5.0)]$. For example, a term-structure specification with $\alpha = 6.5\%$ and $\beta = 0.5\%$ reflects a nearly flat term structure with high short-term spot rates, whereas a term-structure specification with $\alpha = 0.5\%$ and $\beta = 6.5\%$ reflects a very steep term structure with low short-term spot rates. A term structure with $\alpha = 3.5\%$ and $\beta = 3.5\%$ corresponds to the middle range in the shape of the term structure and reflects the typical term structure that existed until the end of 2000.

In this article, technical details about the numerical analysis are provided because such details are not available in the existing literature on cross hedges using Treasury futures. Hedge horizon and non-Treasury bond duration effects for various pairings of these hedge instruments are shown in Exhibit 1, and the term structure effect is provided in Exhibit 2. The CTD coupon effect is reported in Exhibit 3, and the bond coupon and maturity effects are shown in Exhibit 4 and Exhibit 5, respectively.

Exhibit 1 shows the hedge cost difference (in terms of absolute sum of hedge ratios) between traditional and forward-looking approaches for six different hedge horizons and three different bond durations. For a bond with a medium-level duration of 7.61 (Panel B), the percentage difference in mean hedge cost ranges from 1.57% to 1.81%, whereas the percentage difference in maximum hedge cost ranges from 2.53% to 3.99%. The hedge horizon does not have a statistically significant effect on the variability of the percentage difference in mean hedge cost. The hedge horizon effect is not statistically significant for bonds with a relatively high duration (9.90) and relatively low duration (3.28). However, bond duration does have a statistically significant effect on the percentage difference in mean hedge cost. For a bond with a relatively high duration, the mean percentage difference in mean hedge costs is approximately 1.9%, and it is approximately 1.3% for a bond with a relatively low duration. Although the percentage difference in mean hedge cost for a bond with a medium-level duration varies more across hedge horizons, the percentage difference in mean hedge cost increases with bond duration.[12]

Exhibit 2 shows the hedge cost difference (in terms of absolute sum of hedge ratios) between traditional and

EXHIBIT 1

Descriptive Statistics of the Difference between Traditional and Forward-Looking Cross-Hedge Ratios (by Various Hedge Horizons and Bond Durations)

Hedge Horizon	Mean	Median	Min	Max	%Diff_Mean	%Diff_Max	%Diff_Min
Panel A: High-Duration Bond (24.25-year maturity; 3% coupon; 9.90 Duration; 83.9 Convexity)							
6 months	0.9805	0.9799	0.9727	0.9925	1.9485	2.7346	0.7467
5 months	0.9808	0.9800	0.9733	0.9927	1.9244	2.6740	0.7324
4 months	0.9797	0.9792	0.9639	0.9908	2.0298	3.6106	0.9250
3 months	0.9813	0.9805	0.9742	0.9930	1.8745	2.5786	0.7021
2 months	0.9815	0.9809	0.9744	0.9931	1.8484	2.5645	0.6863
1 months	0.9805	0.9801	0.9643	0.9904	1.9501	3.5675	0.9614
Panel B: Medium-Duration Bond (14.25-year maturity; 8% coupon; 7.61 Duration; 42.3 Convexity)							
Hedge Horizon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
6 months	0.9843	0.9837	0.9689	0.9989	1.5656	3.1097	0.1100
5 months	0.9842	0.9834	0.9702	0.9986	1.5816	2.9807	0.1385
4 months	0.9825	0.9823	0.9614	0.9932	1.7540	3.8552	0.6818
3 months	0.9840	0.9827	0.9731	0.9980	1.5952	2.6919	0.1971
2 months	0.9840	0.9824	0.9747	0.9977	1.6039	2.5314	0.2270
1 months	0.9819	0.9814	0.9601	0.9916	1.8109	3.9906	0.8356
Panel C: Low-Duration Bond (4.25-year maturity; 13% coupon; 3.28 Duration; 6.40 Convexity)							
Hedge Horizon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
6 months	0.9880	0.9872	0.9767	0.9973	1.2021	2.3306	0.2693
5 months	0.9874	0.9864	0.9760	0.9964	1.2637	2.3952	0.3573
4 months	0.9839	0.9850	0.9546	0.9955	1.6062	4.5390	0.4471
3 months	0.9861	0.9848	0.9747	0.9950	1.3905	2.5328	0.5001
2 months	0.9854	0.9839	0.9739	0.9940	1.4604	2.6065	0.5955
1 months	0.9816	0.9826	0.9524	0.9931	1.8385	4.7571	0.6928

Notes:
1. Selected statistics for the difference and percentage difference are computed for bonds with high, medium, and low durations for six different hedge horizons. The bond maturity is the remaining maturity at the end of the hedge horizon. The difference between traditional and forward-looking hedge ratios is computed as the absolute sum of traditional hedge ratios minus the absolute sum of forward-looking hedge ratios. The %Diff_Mean is computed as the mean difference divided by the absolute sum of traditional hedge ratios, and the %Diff_Min (%Diff_Max) is computed as the minimum (maximum) difference divided by traditional hedge ratios. These percentage differences reflect the respective mean and minimum (maximum) percentage differences in hedge costs of traditional and forward-looking hedges.
2. Hedge horizons at 1, 2, 3, 4, 5, and 6 months are considered.
3. Three pairs of α and β values are considered using a general term-structure specification. See text for details.
4. The remaining maturities of a CTD T-bond corresponding to 30-year T-bond futures are considered at the near-maximum (29.25), near medium (22.25 years), and near minimum (15.25 years). The remaining maturities of a CTD T-note corresponding to 10-year T-note futures are considered at the near maximum (9.25 years), near-medium (7.75 years), and near-minimum (6.25 years). The remaining maturities of a CTD T-note corresponding to 5-year T-note futures are considered at the near-maximum (4.75 years), near-medium (4.50 years), and near-minimum (4.25 years). The remaining maturity of a CTD T-note corresponding to 2-year T-note futures that seldom trade is considered only at the near-minimum (1.75 years) because the near-minimum is only 3 months away from the near-maximum (2.0 years).
5. CTD coupon rates at 3%, 8%, and 13% are considered.
6. There are 63 hedge-instrument pairs and three shapes of term structure for each horizon for each type of bond being hedged; therefore, the total number of observations used for the descriptive statistics in each row of Exhibit 1 is 189 (= 63 × 3).
7. All the mean statistics are statistically and significantly positive at the 1% level.

forward-looking approaches for seven different shapes of term structure and three different bond durations. For a bond with a medium-level duration of 7.90 (Panel B), the percentage difference in mean hedge cost ranges from 1.75% to 1.92%, whereas the percentage difference in maximum hedge cost ranges from 3.21% to 3.83%. Term-structure shape does not have a statistically significant effect on the variability of the percentage difference in mean hedge cost. The effect of the term-structure shape is not statistically significant for a bond with a relatively high duration (9.92) or a bond with a relatively low duration (3.48). Bond duration appears to have a weak but statistically significant effect on the variability of the percentage difference in mean hedge cost. For a bond with a relatively high duration, the mean percentage difference in mean hedge cost is approximately 1.9%, and it is approximately 1.7% for a bond with

EXHIBIT 2

Descriptive Statistics of the Difference between Traditional and Forward-Looking Cross-Hedge Ratios (by Various Shapes of Term Structure and Bond Durations)

Panel A: High-Duration Bond (24.25-year maturity; 3% coupon; 9.92 Duration; 83.9 Convexity)							
Term-Str. (α, β)	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
0.005, 0.065	0.9790	0.9784	0.9685	0.9875	2.1046	3.1524	1.2490
0.015, 0.055	0.9795	0.9789	0.9678	0.9882	2.0464	3.2213	1.1752
0.025, 0.045	0.9801	0.9794	0.9671	0.9891	1.9880	3.2904	1.0885
0.035, 0.035	0.9807	0.9800	0.9664	0.9902	1.9292	3.3594	0.9824
0.045, 0.025	0.9813	0.9804	0.9657	0.9911	1.8698	3.4285	0.8866
0.055, 0.015	0.9816	0.9807	0.9650	0.9920	1.8437	3.4979	0.8000
0.065, 0.005	0.9826	0.9817	0.9643	0.9928	1.7417	3.5675	0.7216
Panel B: Medium-Duration Bond (14.25-year maturity; 8% coupon; 7.90 Duration; 43.4 Convexity)							
Term-Str. (α, β)	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
0.005, 0.065	0.9808	0.9806	0.9677	0.9954	1.9215	3.2288	0.4599
0.015, 0.055	0.9812	0.9809	0.9679	0.9953	1.8783	3.2090	0.4713
0.025, 0.045	0.9816	0.9812	0.9666	0.9952	1.8375	3.3431	0.4773
0.035, 0.035	0.9820	0.9817	0.9650	0.9952	1.7999	3.5022	0.4779
0.045, 0.025	0.9825	0.9822	0.9633	0.9958	1.7469	3.6664	0.4190
0.055, 0.015	0.9823	0.9818	0.9617	0.9970	1.7697	3.8330	0.2974
0.065, 0.005	0.9808	0.9806	0.9677	0.9954	1.9215	3.2288	0.4599
Panel C: Low-Duration Bond (4.25-year; 13% coupon; 3.48 Duration; 7.39 Convexity)							
Term-Str. (α, β)	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
0.005, 0.065	0.9830	0.9830	0.9568	0.9973	1.7011	4.3180	0.2693
0.015, 0.055	0.9830	0.9830	0.9562	0.9972	1.7030	4.3830	0.2774
0.025, 0.045	0.9829	0.9830	0.9555	0.9971	1.7058	4.4486	0.2854
0.035, 0.035	0.9829	0.9831	0.9549	0.9971	1.7098	4.5145	0.2933
0.045, 0.025	0.9829	0.9831	0.9542	0.9970	1.7149	4.5808	0.3010
0.055, 0.015	0.9828	0.9830	0.9535	0.9969	1.7212	4.6473	0.3086
0.065, 0.005	0.9827	0.9830	0.9529	0.9968	1.7287	4.7141	0.3161

Notes:
1. See Exhibit 1, notes 1, and 4.
2. Seven combinations of α and β values are considered using various shapes of the term-structure specification. See text for details.
3. Hedge horizons at 1, 3, and 6 months are considered.
4. There are 63 hedge-instrument pairs and three hedge horizons for each type of bond being hedged under each term-structure shape; therefore, the total number of observations used for the descriptive statistics in each row of Exhibit 2 is 189 (= 63 × 3).
5. All the mean statistics are statistically and significantly positive at the 1% level.

a relatively low duration. The percentage difference in mean hedge cost increases with bond duration.[13]

Exhibit 3 shows the hedge cost difference (in terms of absolute sum of hedge ratios) between traditional and forward-looking approaches for six different CTD coupon rates and three different bond durations. For a bond with a medium-level duration of 7.90 (Panel B), the percentage difference in mean hedge cost ranges from 1.26% to 2.22%, whereas the percentage difference in maximum hedge cost ranges from 1.96% to 4.00%. The CTD coupon has a statistically significant positive effect on the variability of the percentage difference in mean hedge cost. This positive effect of the CTD coupon carries over to the case of a bond with a relatively high duration (9.93) and a bond with a relatively low duration (3.47).[14]

Exhibit 4 shows the hedge cost difference (in terms of absolute sum of hedge ratios) between traditional and forward-looking approaches for six different coupon rates and three different bond maturities. Given the strong CTD coupon effect reported in Exhibit 3, identical coupon rates for CTD Treasuries are no longer assumed for the analysis reported in Exhibit 4. For a bond with a medium remaining maturity of 14.25 years (Panel B), the percentage difference in mean hedge cost ranges from 1.81% to 1.86%, whereas the percentage difference in maximum hedge cost ranges from 3.75% to 4.11%. Bond coupon rates do not have a statistically significant effect on the variability of the percentage difference in mean hedge cost. The bond coupon effect does not exist for a bond with a relatively short remaining maturity (4.25

EXHIBIT 3

Descriptive Statistics of the Difference between Traditional and Forward-Looking Cross-Hedge Ratios (by Various Coupon Rates of CTD and Bond Durations)

Panel A: High-Duration Bond (24.25-year maturity; 3% coupon; 9.93 Duration; 84.0 Convexity)							
CTD Coupon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
3%	0.9853	0.9849	0.9792	0.9928	1.4737	2.0776	0.7217
5%	0.9818	0.9821	0.9746	0.9913	1.8168	2.5414	0.8685
7%	0.9803	0.9806	0.9748	0.9905	1.9717	2.5232	0.9505
9%	0.9793	0.9792	0.9708	0.9900	2.0703	2.9238	1.0009
11%	0.9781	0.9779	0.9673	0.9897	2.1940	3.2676	1.0337
13%	0.9772	0.9770	0.9643	0.9894	2.2788	3.5675	1.0560
Panel B: Medium-Duration Bond (14.25 years; 8% coupon; 7.90 Duration; 43.4 Convexity)							
CTD Coupon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
3%	0.9874	0.9878	0.9804	0.9980	1.2625	1.9639	0.1971
5%	0.9836	0.9842	0.9750	0.9971	1.6404	2.4960	0.2906
7%	0.9810	0.9818	0.9712	0.9965	1.8969	2.8836	0.3529
9%	0.9792	0.9801	0.9668	0.9960	2.0798	3.3227	0.3969
11%	0.9779	0.9789	0.9631	0.9957	2.2141	3.6882	0.4293
13%	0.9778	0.9779	0.9600	0.9955	2.2230	3.9992	0.4540
Panel C: Low-Duration Bond (4.25-year maturity; 13% coupon; 3.47 Duration; 7.40 Convexity)							
CTD Coupon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
3%	0.9905	0.9914	0.9802	0.9973	0.9475	1.9797	0.2693
5%	0.9867	0.9880	0.9756	0.9951	1.3302	2.4381	0.4902
7%	0.9837	0.9845	0.9685	0.9962	1.6306	3.1510	0.3809
9%	0.9812	0.9820	0.9625	0.9960	1.8844	3.7530	0.3955
11%	0.9790	0.9802	0.9573	0.9959	2.0999	4.2682	0.4081
13%	0.9771	0.9780	0.9529	0.9958	2.2862	4.7141	0.4191

Notes:
1. See Exhibit 1, notes 1 and 4.
2. CTD coupon rates at 3%, 5%, 7%, 9%, 11%, and 13% are considered.
3. Hedge horizons at 1, 3, and 6 months are considered.
4. Three pairs of α and β values are considered using the term-structure specification described in the text.
5. There are 27 hedge-instrument pairs, three hedge horizons, and three shapes of term structure for each type of bond being hedged for each CTD coupon rate; therefore, the total number of observations used for the statistics in each row of Exhibit 3 is 189 (= $27 \times 3 \times 3$).
6. All the mean statistics are statistically and significantly positive at the 1% level.

years) or a bond with a long remaining maturity (24.25 years). For a bond with a relatively long maturity, the percentage difference in mean hedge cost ranges from 1.75% to 1.93%, and for a bond with a relatively short maturity of 4.25 years, the percentage difference does not vary across bond coupon rates.[15]

Exhibit 5 describes the hedge cost difference (in terms of absolute sum of hedge ratios) between traditional and forward-looking approaches for six different maturities and three different bond coupon rates. Although the analysis is similar to that reported in Exhibit 4, an additional robustness test on a lack of bond coupon and maturity effects is provided in Exhibit 5. This exhibit also shows the relative strength, if any, between bond coupon and bond maturity effects. For a bond with a medium-level coupon rate of 8% (Panel B), the percentage difference in mean hedge cost ranges from 1.79% to 1.85%, whereas the percentage difference in maximum hedge cost ranges from 3.42% to 4.80%. Bond maturity does not have a statistically significant effect on the variability of the percentage difference in mean hedge cost. A positive bond maturity effect does exist, however, for a bond with a relatively high coupon rate (13%) and a bond with a relatively low coupon rate (3%). For a relatively high coupon bond, the percentage difference in mean hedge cost ranges from 1.47% to 1.75%, and for a relatively low coupon bond, the percentage difference ranges from 1.66% to 1.87%.[16]

The numerical analysis described above indicates that a forward-looking hedge is, on average, 1.8% cheaper than a traditional hedge. It also indicates that the hedge cost saving is substantial when the convexity-based hedge is

EXHIBIT 4

Descriptive Statistics of the Difference between Traditional and Forward-Looking Cross-Hedge Ratios (by Various Coupon Rates and Remaining Maturities of Bond Being Hedged)

Panel A: Long-Maturity Bond (24.25 years)							
Bond Coupon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
3%	0.9807	0.9801	0.9643	0.9928	1.9251	3.5675	0.7217
5%	0.9811	0.9807	0.9651	0.9943	1.8936	3.4851	0.5725
7%	0.9818	0.9810	0.9658	0.9953	1.8238	3.4188	0.4715
9%	0.9821	0.9814	0.9664	0.9959	1.7886	3.3642	0.4099
11%	0.9823	0.9815	0.9668	0.9963	1.7705	3.3184	0.3684
13%	0.9825	0.9818	0.9672	0.9966	1.7537	3.2796	0.3385
Panel B: Medium-Maturity Bond (14.25 years)							
Bond Coupon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
3%	0.9818	0.9810	0.9625	0.9968	1.8244	3.7523	0.3194
5%	0.9819	0.9816	0.9612	0.9974	1.8069	3.8802	0.2572
7%	0.9818	0.9814	0.9603	0.9979	1.8190	3.9660	0.2140
9%	0.9814	0.9813	0.9597	0.9982	1.8567	4.0276	0.1823
11%	0.9814	0.9814	0.9593	0.9984	1.8552	4.0740	0.1581
13%	0.9815	0.9814	0.9589	0.9986	1.8538	4.1101	0.1389
Panel C: Short-Maturity Bond (4.25 years)							
Bond Coupon	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
3%	0.9833	0.9825	0.9524	0.9987	1.6728	4.7586	0.1324
5%	0.9833	0.9830	0.9525	0.9981	1.6706	4.7477	0.1917
7%	0.9833	0.9833	0.9526	0.9976	1.6686	4.7380	0.2353
9%	0.9833	0.9834	0.9527	0.9973	1.6666	4.7292	0.2747
11%	0.9833	0.9834	0.9528	0.9967	1.6671	4.7213	0.3285
13%	0.9833	0.9835	0.9529	0.9973	1.6656	4.7141	0.2693

Notes:
1. See Exhibit 1, notes 1 and 4.
2. Bond coupon rates at 3%, 5%, 7%, 9%, 11%, and 13% are considered.
3. Hedge horizons at 1, 3, and 6 months are considered.
4. Three pairs of α and β values are considered using the term-structure specification described in the text.
5. CTD coupon rates at 3%, 8%, and 13% are considered.
6. There are 63 hedge-instrument pairs, three hedge horizons, and three shapes of term structure for each coupon rate and remaining maturity of a bond; therefore, the total number of observations used for the descriptive statistics in each row of Exhibit 4 is 567 (= 63 × 3 × 3).
7. All the mean statistics are statistically and significantly positive at the 1% level.

most needed, that is, when high-duration bonds are hedged, the term structure of interest rates is steep, and the CTD Treasury pays high coupons. These key results are robust to, among other things, various hedge horizons, different shapes of term structure, and various pairing of different types of Treasury futures.

SUMMARY AND PRACTICAL IMPLICATIONS

Convexity-based cross-hedges with Treasury futures may not require option-theoretic duration-convexity matching because near the delivery month, a linear, arbitrage-free relationship should hold between Treasury futures and their CTD Treasuries. Using these hedging implications of the CBOT conversion factoring system and Rendleman's [1999] framework for duration hedges, a model was developed for forward-looking, convexity-based cross-hedges.

The numerical analysis indicates the following: 1) a forward-looking hedge is cheaper in scenarios that involve different term structures of interest rates, hedge horizons, pairs of Treasury futures, and duration levels of both bond portfolio and CTD Treasuries, and 2) the cost saving is large when the convexity-based hedge is most needed; that is, when high-duration bonds are hedged, the term structure of interest rates is steep, and the CTD Treasuries pay high coupons.

Such forward-looking, convexity-based cross-hedges can be particularly useful for less arbitrary pairing of different Treasury futures contracts because the hedge-ratio equations show explicit channels through which short-term spot rates and forward interest rates beyond the hedge

EXHIBIT 5

Descriptive Statistics of the Difference between Traditional and Forward-Looking Cross-Hedge Ratios (by Various Remaining Maturities and Coupon Rates of Bond Being Hedged)

Panel A: High-Coupon (13%) Bond							
Bond Maturity	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
24.25	0.9825	0.9818	0.9672	0.9966	1.7537	3.2796	0.3385
20.25	0.9828	0.9816	0.9645	0.9977	1.7240	3.5501	0.2298
16.25	0.9836	0.9826	0.9608	0.9989	1.6385	3.9235	0.1138
12.25	0.9845	0.9833	0.9575	0.9998	1.5458	4.2528	0.0192
8.25	0.9853	0.9842	0.9547	0.9999	1.4675	4.5342	0.0147
4.25	0.9854	0.9855	0.9524	0.9970	1.4648	4.7571	0.3050
Panel B: Medium-Coupon (8%) Bond							
Bond Maturity	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
24.25	0.9817	0.9811	0.9658	0.9980	1.8314	3.4217	0.1977
20.25	0.9815	0.9811	0.9655	0.9986	1.8463	3.4537	0.1388
16.25	0.9818	0.9813	0.9621	0.9974	1.8158	3.7883	0.2630
12.25	0.9818	0.9816	0.9581	0.9987	1.8223	4.1926	0.1311
8.25	0.9821	0.9820	0.9547	0.9999	1.7887	4.5280	0.0144
4.25	0.9815	0.9821	0.9520	0.9944	1.8528	4.7976	0.5626
Panel C: Low-Coupon (3%) Bond							
Bond Maturity	**Mean**	**Median**	**Min**	**Max**	**%Diff_Mean**	**%Diff_Max**	**%Diff_Min**
24.25	0.9813	0.9808	0.9642	0.9946	1.8694	3.5805	0.5441
20.25	0.9815	0.9809	0.9654	0.9944	1.8503	3.4605	0.5620
16.25	0.9817	0.9810	0.9666	0.9963	1.8270	3.3352	0.3663
12.25	0.9825	0.9815	0.9602	0.9981	1.7496	3.9849	0.1906
8.25	0.9834	0.9821	0.9556	0.9996	1.6636	4.4394	0.0355
4.25	0.9833	0.9825	0.9524	0.9987	1.6728	4.7586	0.1324

Notes:
1. See Exhibit 1, notes 1 and 4.
2. Six different remaining maturities of a bond being hedged (4.25, 8.25, 12.25, 16.25, 20.25, and 24.25 years) are considered.
3. Hedge horizons at 1, 3, and 6 months are considered.
4. Three pairs of α and β values are considered using the term-structure specification described in the text
5. CTD coupon rates at 3%, 8%, and 13% are considered.
6. There are 63 hedge-instrument pairs, three hedge horizons, and three shapes of term structure for each coupon rate and remaining maturity of a bond; therefore, the total number of observations used for the descriptive statistics in each row of Exhibit 5 is 567 ($= 63 \times 3 \times 3$).
7. All the mean statistics are statistically and significantly positive at the 1% level.

horizon affect the hedge ratios. Such forward-looking cross-hedges can also apply to other developed fixed-income markets because these markets adopt a similar conversion factoring system. To the extent that the interest rate cycles in these markets closely follow the interest rate cycles in the United States, which are quite often due to an international interest rate parity condition, the results of this study may also apply to these fixed-income markets.

ENDNOTES

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[1]Such convexity-based hedges are surveyed in Goodman and Vijayaraghavan [1988] and Lien and Tse [2002]. For a comparison between forward-looking and incorrect/inefficient duration-based hedge ratios, see Daigler and Copper [1998] and exhibit 1 in Rendleman [1999]. For a recent study on the interrelated nature of delivery-related options and implications on trading strategies, see Daigler [2005].

[2]See Boyle [1989] and Hemler [1990] for early studies on pricing delivery-related options.

[3]For early discussions about the nature of the CBOT conversion factoring system and its implications for determining

the CTD Treasuries, see Kane and Marcus [1984] and Benninga and Wiener [1999]. For a recent study on the interrelated nature of delivery-related options and implications on trading strategies, see Daigler (2005).

[4]For empirical evidence and related arguments, see Sercu and Wu [2000].

[5]The flat rate was 8% until the issue of 2000 March Treasury futures contracts.

[6]Let n be the number of whole years from the first day of the delivery month to a Treasury's maturity and z the number of months between n years and the Treasury's maturity. Due to maturity rounding down, z takes on the values 0, 3, 6, or 9 for long-maturity futures, whereas for short-maturity futures, z can be any integer from 0 to 11. It can be shown that $b = (1 + 6\%/2)^{-2n}$ if z is less than 7; otherwise, $b = (1 + 6\%/2)^{-(2n+1)}$ and $c = (C/0.06) \times (1 - b)$. Also, let $v = z$ if z is less than 7; otherwise, $v = 3$ for long-maturity futures, but $v = z - 6$ for short-maturity futures. As a result, $a = (1 + 6\%/2)^{-(v/6)}$ and $d = 0.5C(1 - v/6)$. An alternative explanation is available in Daigler [2005].

[7]In general, the projected CTD Treasury tends to have the lowest (highest) duration when the yield is lower (higher) than 6%. Given the duration, however, the Treasury with the higher yield or with the highest repo rate tends to be the projected CTD Treasury. For details, see Kane and Marcus [1984], Livingston [1984], and Burghardt et al. [1994]. However, when the term structure is nearly flat, the CTD Treasury tends to be a high-coupon Treasury when the market interest rate for long bonds is greater than 6% and the coupon rate of an eligible Treasury exceeds the market interest rate. For the implications, see Benninga and Wiener [1999].

[8]The adjusted bases for non-CTD Treasuries are positive; that is, (a non-CTD Treasury quote minus the futures' settlement quote multiplied by the non-CTD Treasury's CF) > 0. For alternative explanations on these bases around the first day of the delivery month, see Kishimoto [1998], Rendleman [1999] and Daigler [2005].

[9]As with the first three assumptions, the fourth assumption is a reasonable one given the fact that both long and short positions in Treasury futures prefer to avoid extreme price volatilities during the delivery month and their unfavorable implications on hedging or delivery decisions. These four assumptions are, therefore, essential for developing a model for forward-looking, convexity-based cross-hedges.

[10]In this article, the higher-order bond price-volatility measures and the bond theta effect are not considered. For higher-order measures of bond price volatility, see Nawalkha and Lacey [1990]. For bond theta effect, see Kang and Chen [2002].

[11]Unlike traditional hedge ratios, forward-looking hedge ratios can provide additional information useful for less arbitrary pairing of short-maturity and long-maturity Treasury futures. Note that the second hedge ratio (h_2) formula can be further simplified when one-year or shorter-term futures are employed because the price and price-volatility measures for such futures take simpler expressions.

[12]To check the robustness of insignificant hedge-horizon effect, statistics were obtained that did not control the duration level but considered hedge horizons from 1 to 12 months. These statistics (not reported here) indicate that the hedge-horizon effect does not exist, even when the duration level is not controlled. It also indicates that the lack of a hedge-horizon effect becomes even more distinct when the horizon becomes longer.

[13]To check the robustness of insignificant effect from a term-structure shape, statistics were obtained that did not control the duration level. These statistics (not reported here) indicate that the effect of term-structure shape does not exist, even when the duration level is not controlled. However, a further analysis indicates a statistically significant positive relationship between the term-structure slope parameter (β) and the mean difference in mean hedge cost.

[14]To check the robustness of the CTD coupon effect, statistics were obtained that did not control the duration level but considered 11 levels of CTD coupon rates. These statistics (not reported here) indicate that the CTD coupon effect does exist, even when the duration level is not controlled.

[15]The positive duration effect implies a positive maturity effect and a negative coupon-rate effect. To check the robustness of insignificant bond coupon effect, statistics were obtained that did not control for bond maturity. These statistics (not reported here) indicate that the bond coupon effect does not exist, even when bond maturity is not controlled.

[16]The robustness test of insignificant bond maturity effect (not reported here) also indicates that the bond maturity effect does not exist, even when the bond coupon rate is not controlled.

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