Default and Recovery Rates of Sovereign Bonds: *A Case Study of the Argentine Crisis*

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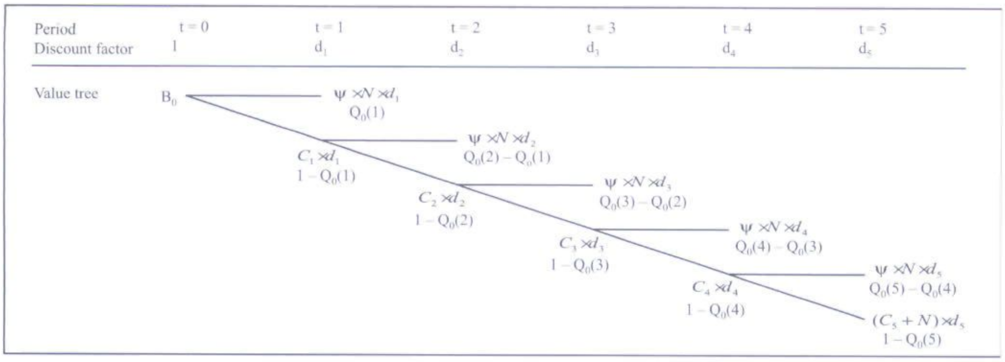
When economy minister Roberto Lavagna announced the result of the Argentine restructuring offer in March 2005, investors had tendered 76% of all eligible claims into the exchange. The restructuring forced creditors to accept the largest haircut seen so far on the sovereign bond market. Even though the offer was criticized by most players, including the IMF, it neatly matches what investors expected from the Argentine default in 2001. This analysis shows how the recovery expectation implied by Argentine global bond prices evolved during the crisis and finally hit the after-default price range in the mid-20s.

Once an issuer faces distress, the expected recovery value makes up a major portion of the total bond value. Traders are aware that after a deterioration of an issuer's credit quality, bonds are traded on a "price basis" rather than a "spread basis." This simply means that individual bond features such as coupon and maturity become less important for bond pricing. The intuition behind this is not of an economic nature but of a legal nature because in default a minority of bondholders can accelerate the respective issue, making it immediately repayable at par. The core legal claim against the sovereign consists therefore of the debt's nominal value. This fact is also reflected in coercive debt restructurings. Besides Argentina, Ecuador and Pakistan recently offered the same bond instruments to all holders of their original, defaulted securities, so that all investors received roughly the same compensation.

In terms of a yield curve analysis, this supplements the reasoning behind why yield curves are steeply inverted when default is close. This is especially the case if there is a bullet amortization due in the near future, making default likely at that point in time and causing the bond price not to converge to par. Conceptually, this behavior is not well described by today's most popular bond pricing framework, which defines recovery as a fraction of market value ("recovery of market value," RMV) and presumes bond prices to converge to par (see Duffie and Singleton [1999, p. 691]). Therefore, this article reinvigorates the "recovery of face value" (RFV) concept, which furthermore allows recovery and default probability parameters to be disentangled. This approach fits Argentine global bond data very well because Argentina's solvency was under dispute for quite a while. Its USD 128 billion public debt, which squeezed the fiscal budget during a prolonged recession, consisted of 73% of bonds, mostly foreign denominated. When down payment is not a viable option (especially under an unsustainable currency regime) and debt swaps only succeed in rolling over due debt at increased yields, default and a subsequent "hard" restructuring are a country's final choice. The estimates of the expected recovery ratio yielded in this article illustrate this evolution.

EXHIBIT 1
Event Tree



The estimates are useful for two purposes. First, expected recovery rates are necessary ingredients when pricing other instruments, such as derivatives, contingent on the recovery value. For instance, for credit default swaps, the presumed irrelevance of the recovery assumption (see Duffie [1999]) no longer holds when bonds trade far below par. Second, the relation between the parameter estimates and fundamental factors helps to establish a better picture of financial crises and appropriate policies to contain them. The results show, for instance, that the USD 40 billion rescue package put together by the IMF could not reduce the likelihood of insolvency but led to higher recovery rates. This means that the partial bailout made investors believe that they could pocket more if the government finally decided to restructure.

This article draws on the standard literature on reduced-form models that assume an exogenously specified bankruptcy process (see Heath et al. [1992]; Jarrow and Turnbull [1995]; Lando [1998]; Duffie and Singleton [1999], among others). Within this approach, *equilibrium models* follow the idea of term structure models for the risk-free rate and obey the no-arbitrage rule (see Dai and Singleton [2000]; Duffee [2002]). Although it is tractable under the RMV scheme, the closed-form solution for the RFV scheme applied in this article is much more complex. To provide a sufficient fit, affine models require at least a two-factor stochastic process for the default rate, making the model even more burdensome.

As opposed to equilibrium models, *empirical models* simply aim to provide a good fit to the observed term structure. Although they lack the no-arbitrage appeal, empirical models are handy for implementation and provide estimates of latent factors that can more easily be related to the course of other fundamental variables. Despite the functional form of Nelson and Siegel [1987] being the most popular among empirical models, this article uses the extreme value type I probability distribution to fit the default intensity term structure. This ensures the non-negativity of forward rates and provides a maximum degree of parsimony as only two parameters require estimation. The resulting daily estimates bear a straightforward economic interpretation and prove a good and robust fit to Argentine bond prices. This choice also allows for a price discount independent from maturity, reflecting uncertainty about the fulfillment of immediate payment obligations. This meets the empirical observation that maturing bonds do not necessarily converge to par during severe distress and default may be further delayed as a result of the grace period. The model remains computationally easy to handle since there is a closed formula for the risk-adjusted discount factor.

The calculations in this article use the central assumption of investor risk neutrality, which is common to a number of models in finance. Implying that investors are usually risk averse, the risk-neutral default probability and loss rate would be greater in comparison to the real world measure. This caveat presents a trade-off made because there is no simple way to account for risk aversion. A calibration against observed default frequencies, for instance by rating class, or realized recovery rates appears inappropriate owing to the small number of observed sovereign defaults and the relatively young tradition of emerging country bond issuance.

THE MODEL

Following the standard bond pricing approach, this article derives the value of a bond from the discounted present value of the coupon payments and the nominal face value given no default occurs. If the bond defaults, the following recovery scheme applies: coupons are no longer paid, but the investor will receive a fractional recovery ψ of the remaining face value N upon default in accordance with the RFV concept. Assuming that investors are risk neutral, all payments are discounted at the risk-free rate. The current bond value is calculated from the sum of all discounted payments weighted by their probability of occurrence (with $Q_0(t)$ being the cumulative default probability). This is illustrated in a binomial event tree in Exhibit 1. The example assumes a bond that expires after five periods. The resulting discounted cash flows and their probabilities are marked at each node, adding up to the bond value B_0.

Analyzing a cross section of bonds issued by one debtor, the model assumes that there is a joint default probability. This means that the issuer defaults on all outstanding bonds at once, which follows from the standard cross-default clause included in boilerplate bond contracts. Modeling the joint default probability can be done in many different ways. Early empirical research applied constant default probabilities where default occurs each period with the same probability (see, e.g., Bhanot [1998]). Merrick [2001], who analyzes Argentine and Russian Eurobonds, uses a linear function of time for the default probability p. This approach has the disadvantage that when assuming a probability function of the form $p_t = \alpha + \beta t$, the probability p_t may exceed the interval [0,1].

In this article an instantaneous hazard rate is used, making it possible to embed the model in a time-continuous setting. The hazard rate thereby is the instantaneous probability of default at time t given survival until that time. Let Q_t be the cumulative probability of default during the time interval $[0,t]$. The hazard rate as viewed from time 0 is then

$$\mu_0(t) \equiv -\frac{d}{dt}\ln(1-Q_t) \tag{1}$$

The unconditional probability of default at a certain point in time t is

$$\mu(t)e^{-\int_0^t \mu(s)ds} \tag{2}$$

Since payments are not continuous but appear in certain but irregular time intervals, the default probability at payment time $t + \Delta$, $\Delta \geq 0$, considering the default probability until the last payment date t, takes the form

$$Q_{t+\Delta} - Q_t = e^{-\int_0^t \mu(s)ds}\left(1 - e^{-\int_t^{t+\Delta} \mu(s)ds}\right) \tag{3}$$

In Duffie and Singleton [1999], a constant hazard rate

$$\mu(t) = \lambda, \text{with } \lambda \geq 0 \tag{4}$$

is used as the base case. This is a function often applied in modeling default rates. Therefore, the constant hazard rate function is used in the next section to illustrate under which conditions the model can be estimated. For the empirical analysis in this article, however, this hazard function based is modified to become the extreme value type I distribution (*see Appendix*). Its hazard function is given by

$$\mu(t) = \frac{1}{\beta}\frac{e^{-(t-\alpha)/\beta}}{e^{e^{-(t-\alpha)/\beta}} - 1} \tag{5}$$

where α is the location parameter and $\beta > 0$ is the scale parameter. Analogously to Equation (2) the unconditional probability of default using the extreme value distribution becomes

$$\frac{1}{\beta}e^{-(t-\alpha)/\beta}e^{-e^{-(t-\alpha)/\beta}} \tag{6}$$

Since the mode of this density is α, this allows the modeling of a default probability term structure that shows a maximum at time $t = \alpha$ and has a standard deviation of $\frac{\pi}{\sqrt{6}}\beta$.

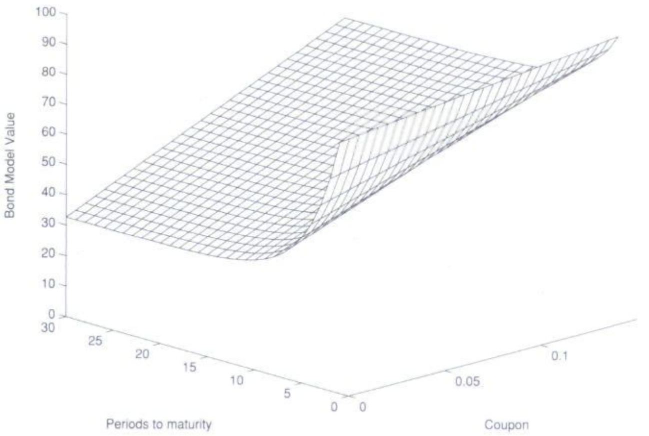
For $t \geq 0$, the unconditional default probability between times t and $t + \Delta$ from Equation (3) becomes

$$Q_{t+\Delta} - Q_t = \begin{cases} e^{-e^{-(\Delta-\alpha)/\beta}} & \text{for } t = 0 \\ e^{-e^{-(t+\Delta-\alpha)/\beta}} - e^{-e^{-(t-\alpha)/\beta}} & \text{for } t > 0 \end{cases} \tag{7}$$

This case differentiation is necessary because the extreme value distribution is defined for negative variates as well. Given values of α and β at which the distribution is positive for values of t below zero, this case differentiation results in an immediate non-zero unconditional default probability for any bond that has not matured yet. At $\alpha = 5$ and $\beta = 3$, for example, the immediate discount implied by Equation (7) would be 16%.

Estimating the parameters of this hazard function offers insight into the expected default risk structure of Argentina. The parameter α roughly indicates at what point in time default was seen as most probable. This

EXHIBIT 2
Bond Model Value



unveils important information about a cross section of bonds with different maturities. Using a bootstrapping method, the zero curve implied by Argentine global bonds already indicates that the unconditional default probability has a hump-like shape with its maximum moving toward t_0 as the crisis evolves. Therefore, it is convenient to approximate the peak of the unconditional default probability by α using the extreme value default probability density. At this peak, the cumulative default function $Q(t)$ shows its steepest ascent.

As there are bonds in the estimation sample with both short and very long durations, the application of a constant hazard function could distort the estimation. Usually, such a model would underprice bonds with longer duration as it cannot reflect an expected economic recovery. The extreme value distribution function, in contrast, can be parameterized in such a way that the unconditional default probability decreases in the long-run. The scale parameter β refines the shape of the default probability density since its standard deviation is directly proportional to the scale parameter. In other words, the scale parameter determines the slope of the cumulative default function.

The "ingredients" can now be combined to calculate the value of a risky bond from its constituents, which are the coupon payments, the notional payment, and the recovery value. Therefore, imagine a bond with J coupon payments up to maturity at time $t = \tau$. The periodical coupon payments C_j are paid at times t_j with $j = 1, ..., J$. The face value N is paid at maturity τ. Discounting these payments back to time $t = 0$ using a continuous risk-free term structure r_t, the result would equal the value of a risk-free bond. The risk-neutral value of a risky claim can be obtained by weighting these payments with the probability of survival up to that time and adding the recovery value weighted by the default probability. For different hazard functions $\mu(t)$, the bond model value B_0 of a risky claim is calculated as follows:

$$
\begin{aligned}
B_0 = {} & \sum_{j=1}^{J} C_j \exp\left\{-\int_0^{t_j} [r_s + \mu(s)]ds\right\} + \\
& N \exp\left\{-\int_0^{\tau} [r_s + \mu(s)]ds\right\} + \\
& \psi N \sum_{j=1}^{J} \exp\left(-\int_0^{t_j} r_s ds\right) \\
& \left[\exp\left(-\int_0^{t_{j-1}} \mu(s)ds\right) - \exp\left(-\int_0^{t_j} \mu(s)ds\right)\right]
\end{aligned} \tag{8}
$$

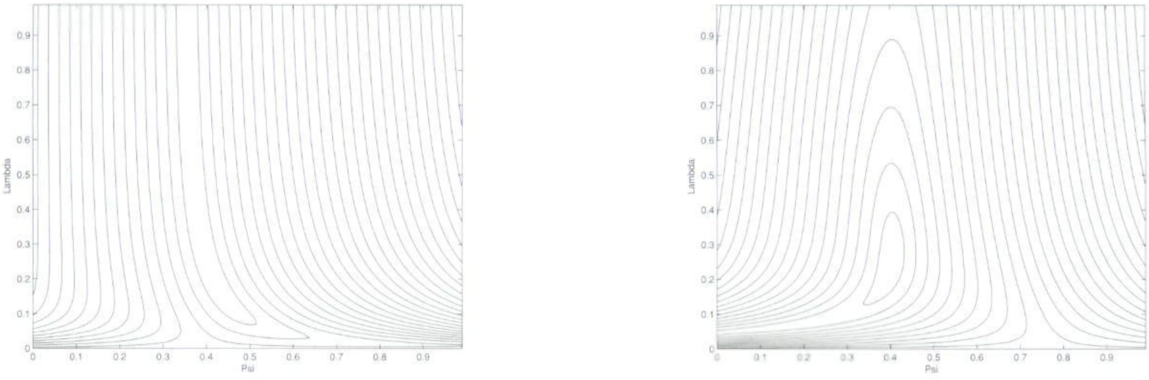
It has to be kept in mind that μ in this kind of framework constitutes an unobservable variable, resembling a number of factors. In a Duffie and Singleton [1999]–style framework, this parameter could be interpreted as a product of the risk-neutral default probability and the risk-neutral expected loss relative to the survival-contingent market value. Since disentangling these two factors is impossible, such an interpretation can never be defeated unless the default probability hits one, its upper bound. In the bond value model outlined earlier, this consideration would result in a mixed recovery layout where recovery upon default is constituted by both a fraction of face and a fraction of market value. For sovereign bonds, however, past experience has shown that once defaulted, *pari passu*, ranking bonds mostly receive the same recovery value, which complies with the Paris Club maxim of equal treatment. The *ex post* view on the Argentine restructuring affirms this observation. Nevertheless, to pay due regard to the two caveats of risk neutrality and market value recovery, the following discussion uses the term "default intensity" rather than "default probability" when referring to μ.

PARAMETER ESTIMATION

In general, there are two ways of empirically estimating the parameters from Equation (8) when risk-free discount factors are given:

1. Taking a time series of one or more bonds sharing the same cross-sectional default intensities and

EXHIBIT 3
RMSE Contour Plots for Two Cases



recovery rates, the intensity and recovery parameters can be estimated as constants during the observation period.

2. If there is a sufficient cross-sectional sample of bonds, an estimation can yield values for each parameter at any point in time.

Although many equilibrium models require a panel of bond data because of the large number of model parameters, the parameters of a parsimonious model can typically be estimated for every trading day. This reflects the daily adaptation of investors' expectations under a continuous flow of information. Daily parameter estimates can therefore be related to other fundamentals of similarly daily frequency, such as exchange rates or local stock indices. Although this is another extension not pursued in this article, the following section addresses the potential identification problem of default and recovery rates, which poses the main difficulty in the simultaneous estimation.

The most severe identification problem is embedded in the RMV framework in which the bond value formula contains only the product of the default hazard rate and fractional loss at default. Implied recovery rates can be estimated only when assuming a constant default intensity, as for instance in Duffie, Pedersen, and Singleton [2003, pp. 144ff]. But other functional forms of the bond value, applying different recovery frameworks, could suffer from similar problems. Even for the RFV framework, an empirical estimation may unveil very high positive correlation between the intensity and recovery parameters. This is, for instance, the case when bonds trade close to par. At par the recovery fraction of face becomes identical to the recovery of market value and poses the same identification problem. It is therefore important to know when to apply a model in the style of Equation (8). The following discussion highlights some rules of thumb with which data should comply to yield robust results.

Estimates of the implied parameters are obtained by comparing observed dirty bond prices and theoretical bond model values, which are calculated from Equation (8). Assuming that the dirty price of each bond follows the same distribution, the root mean squared error (RMSE) can be used as quasi-maximum likelihood estimator that can be minimized with a numerical optimization algorithm:

$$\min \sqrt{\frac{1}{n}\sum_{n=1}^{N}\left(\frac{V_n - B_n}{B_n}\right)^2} \tag{9}$$

Thereby, B_n is the bond model value of bond n in a cross section with N bonds and V_n is its observed dirty price. Since prices of bonds with higher duration usually are more sensitive to the input parameters, weighting the errors by the average life or the default-adjusted duration can be used to control for heteroscedasticity (see Bliss [1997]).

Exhibit 2 shows the behavior of the model price for bonds with different coupons and maturities at a constant hazard rate of $\lambda = 0.25$ and a recovery rate of 40%. The risk-free interest rate is flat at 5%. Looking at the curvature of the model value of a zero-coupon bond, the two elements of the bond value—the default intensity and the recovery rate—become more obvious. The chance of survival for longer maturities (in this case about 15 periods or longer) is extremely low for $\lambda = 0.25$; therefore the recovery value becomes the main constituent of the bond value. For shorter maturities, the model value consists to

Exhibit 4

Sample of U.S.-Dollar Denominated Global Bonds Issued by the Republic of Argentina

Name	Coupon	First coupon date	Maturity date	Par value ($mln)	Price range Min.	Mean	Max.
Arg01	9.250%	23-Feb-1996	23-Feb-2001	1000	98.16	99.85	100.56
Arg03	8.375%	20-Dec-1993	20-Dec-2003	1800	24.03	71.80	96.85
Arg06	11.000%	09-Oct-1996	09-Oct-2006	1300	24.64	71.64	99.25
Arg09	11.750%	07-Apr-1999	07-Apr-2009	1500	24.50	69.40	98.78
Arg10	11.375%	15-Mar-2000	15-Mar-2010	1000	20.50	67.38	96.15
Arg15	11.750%	15-Jun-2000	15-Jun-2015	2400	22.69	66.67	97.65
Arg17	11.375%	30-Jan-1997	30-Jan-2017	4600	22.50	66.79	95.50
Arg20	12.000%	03-Feb-2000	01-Feb-2020	1250	20.50	68.28	99.25
Arg27	9.750%	19-Sep-1997	19-Sep-2027	3400	23.50	60.52	85.93

Sources: Datastream, Bloomberg, Republic of Argentina.

a considerable extent of the bullet amortization weighted by the probability of survival $(1 - Q_0)$, which determines the characteristic curvature of the surface in Exhibit 2 for short maturities. Although different coupons also help to identify the model parameters explicitly, the difference in coupons has to be very substantial so that the resulting Macauley durations differ significantly.

Exhibit 3 illustrates an example in contour plots of the RMSE for simulated bond prices. The contour lines suggest lower levels of the RMSE in the middle of the plots. In the left-hand figure, the sample contains two zero-bonds with maturities of 20 and 30 years. In the right-hand figure, the two zero-bonds have a maturity of 1 and 11 years. Even if the difference in duration is the same in both plots, a minimization of the RMSE is hardly possible in the first case. The plot indicates that there is a narrow valley of combinations of λ and ψ for which the RMSE is very small. Since the value of these two bonds basically consists of the recovery value, the recovery fraction ψ may be inferred from them, but hardly λ. In contrast, the right-hand plot clearly hints at a global minimum at $\lambda = 0.25$ and $\psi = 0.40$, the true parameter values. The example shows how important it is to include in the sample shorter maturity bonds, which determine the characteristic curvature of the intensity function.

These kinds of experiments with the model have led to two more abstract rules of thumb with which the bond sample should comply:

1. For the determination of the recovery ratio, the bonds in the sample should contain considerable default risk, which causes long-term bonds with an average life of more than 20 years to trade below 85%. Otherwise the recovery value determines a portion of the bond value that is not large enough to identify the recovery ratio accurately.
2. To estimate the default parameters, it is important to catch the characteristics of the curvature of the cumulative intensity function $Q_0(t)$. This is achieved if the Macauley durations D_h of the $h = 1, \ldots, H$ bonds in the sample yield a wide range of different values of unconditional probabilities for $t = D_h$ in Equation (2).

DATA

The case study of the Argentine crisis requires two sets of data, Argentine global bond prices and the risk-free term structure implied from U.S. Treasury benchmark yields. The sample of Argentine global bonds includes the largest issues, which are denominated in U.S. dollars and were issued under New York law *(see Exhibit 4)*. Two bond issues are excluded from the data sample because they became extremely illiquid after the mega debt swap of USD 29.5 billion in June 2001. The three new issues offered in this swap are also not included since investors might have perceived them as already restructured bonds and therefore de facto senior. At the time of default, this sample nevertheless represents 81% of the outstanding principal of USD global bonds, excluding the mega swap issues. The time frame in question is June 8, 2000, to May, 6, 2002, including data for 498 trading days. Exhibit 5 shows the price chart for the 8.375% Arg03 (solid line) and the 9.750% Arg27 bond (dashed line).

ESTIMATION RESULTS

Exhibit 6 illustrates the resulting course of the three-year cumulative default intensity (labeled Q(3)) and the recovery rate (labeled ψ) of the Argentine global bonds.

Starting out at around 20%, the default intensity is creeping up throughout the sample and finally reaches the upper bound of 100% at the beginning of December 2001. Since the extreme value distribution is bound into [0,1] this implies that at this point the intensity measure cannot be diluted by any RMV factor. The recovery of face ratio stands at between 40 and 50% in the beginning and drops to a level below 40% in summer 2001. After default, a 20%-30% recovery of face value is expected by the market participants, which is astonishingly close to the value of the restructuring offer put forward three years later.

The exhibit shows some peaks with characteristically

EXHIBIT 5
Price Chart of Two Argentine Global Bonds

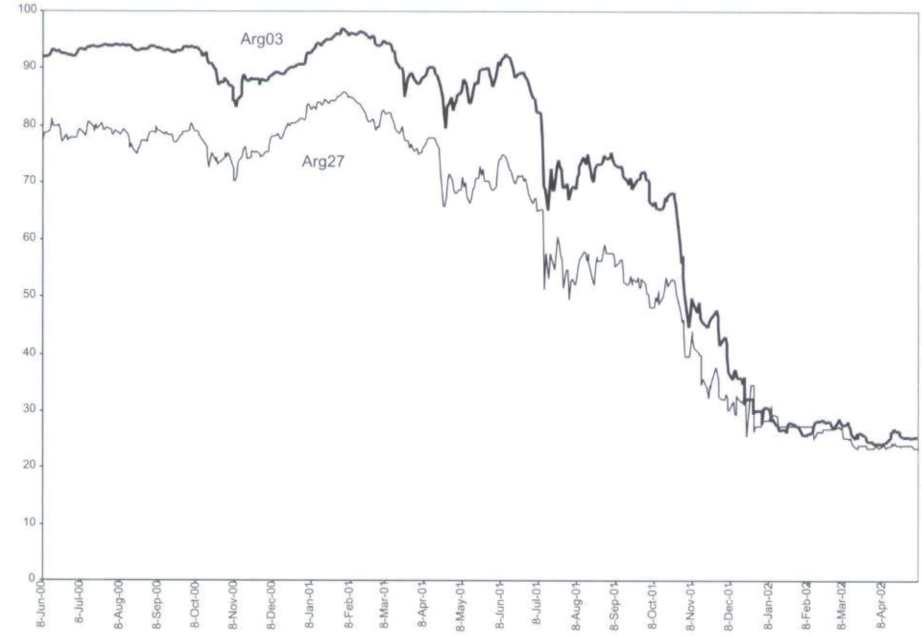
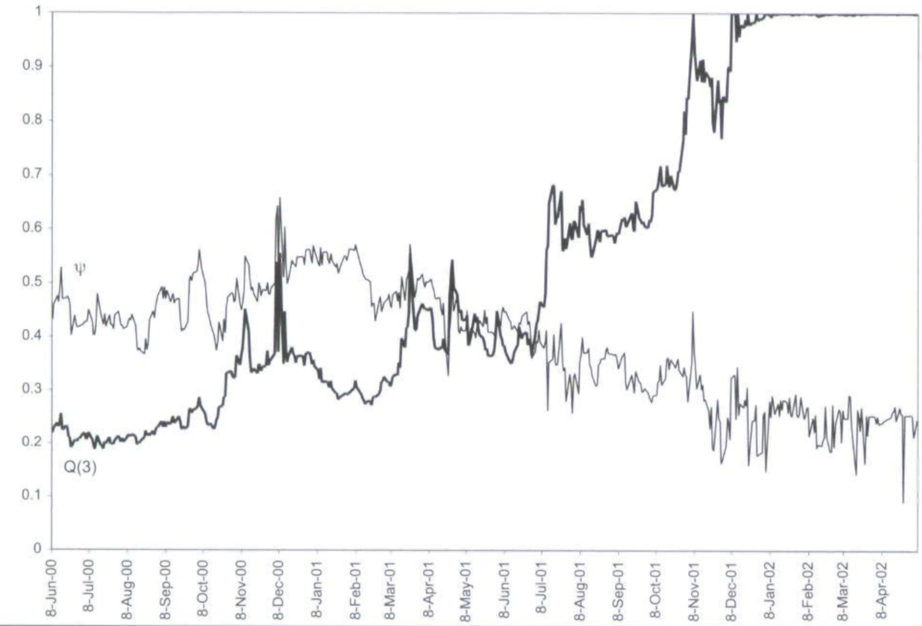


EXHIBIT 6
Three-Year Cumulative Default Intensity and Recovery Ratio



high correlation between the cumulative default intensity and the recovery ratio. Most of them can be explained by heavy market movements (as in the case of the peaks in November 2001) or—in some cases—by obviously mispriced quotes of single issues (as in the case of Arg06 around December 8, 2000).

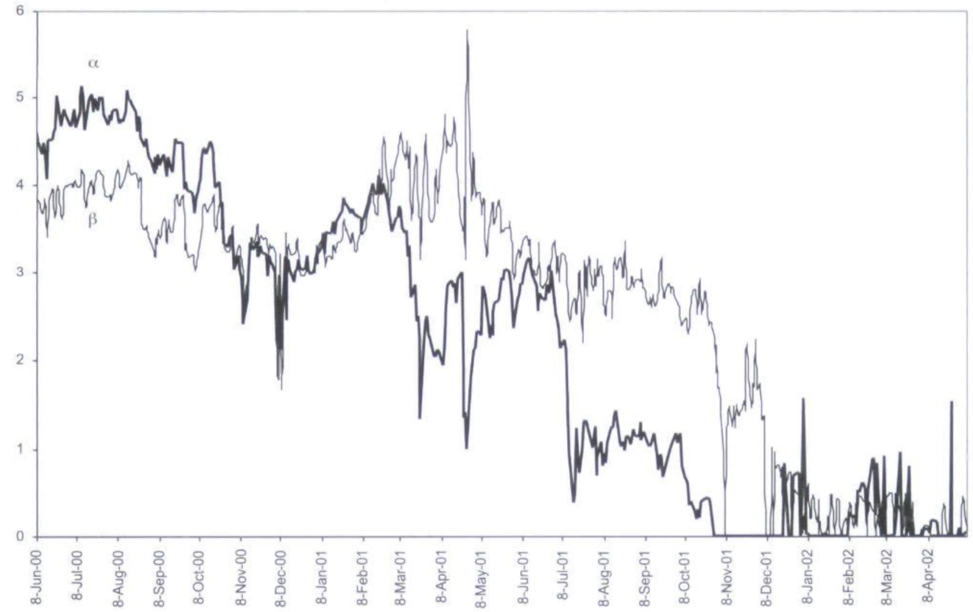
Exhibit 7 shows the resulting estimates of the two parameters describing the extreme value hazard function. The parameter α is the location parameter and serves to visualize the mode of the default intensity. If the default intensity is interpreted as a real probability measure, this line shows after which period of time default was seen as most probable. The scale parameter β determines the standard deviation of the cumulative extreme value density. It is very obvious that during times of turbulent market movements β goes up. This is the case, for instance, in spring 2001. After market disappointment about lower tax revenues than expected, a political crisis evolved. In the end, a shuffle of the cabinet brought Domingo Cavallo back into power. This period of uncertainty is reflected by very high values of β.

From the path of the default probability, one can identify periods of rather stable default probabilities followed by level jumps. Without proving the statistical significance of these jumps, the Argentine crisis is divided into five episodes. Exhibit 8 states means, variances, and correlations for the five episodes as well as for the whole sample. Mean values for α decline during the course of the episodes. Except for the last episode, the scale parameter β remains high. The recovery ratio ψ drops to a realistic level of 25% in the fifth episode. In all single periods the daily estimates of α and ψ are negatively correlated, contrasting the high positive correlation in the total sample, which supports the distinctiveness of the five different episodes.

During the first episode, in summer 2000, spreads came down in favor of Argentina and the government suc-

EXHIBIT 7

Location and Scale Parameter of the Extreme Value Hazard Function



cessfully managed to place a number of new bond issues. In the model, these circumstances are reflected by a mean default intensity of 23% for a three-year horizon and a mean recovery rate of 44%. The high values of α and β together with their high correlation cause this model to behave more like a constant hazard rate model where $\mu(t) = \lambda$. This is confirmed when looking at the implied zero curve for the first period, which can be obtained by bootstrapping. This curve is almost flat around a mean yield of 12.4%, showing that bonds were priced on a spread basis at that time.

In October (the beginning of the second episode), rumors spread about the necessity of a moratorium. The cumulative three-year default intensity jumps up to 34% almost at the same time as Standard & Poor's assigns a negative outlook to Argentina's double-B rating. Until February 2001 prices recovered because international lenders approved a USD 39.7 billion aid package by the end of December 2000. It is striking that this bailout package is associated with an increase of the recovery ratio to the highest level in the sample. This could have indicated very early that investors doubted the sustainability of the Argentine situation—a high level of foreign denominated indebtedness combined with a currency board (see Manasse and Roubini [2005, p. 20]). Since a partial bailout might work only in a pure liquidity crisis, the injection of this rescue loan seems solely to have nurtured an improved recovery perspective while having failed to make investors believe in a turnaround of the crisis.

At the beginning of the third episode, contagion effects from Turkey kicked in while Argentina successfully closed a mega debt swap to defer due payments of a total amount of USD 16 billion through 2005. Although this clearly improved Argentina's liquidity position, the swap added to its average cost of borrowing and failed to improve the country's solvency. Nevertheless, this appears to stabilize both the intensity and the recovery parameters at their current level for a short while.

But after July 10, 2001 (the beginning of the fourth episode), prices—especially those of long-term issues—dropped significantly, partly associated with downward rating actions by S&P, Moody's, and Fitch. The spread over U.S. Treasuries widened to more than 13 percentage points as a result of warnings of further rating cuts. The mean recovery ratio of 34% makes up the main portion of the average bond price, showing that the bonds are increasingly traded on a price basis.

In October (the final, fifth episode), the parameter estimations suggest another deterioration of the crisis. The cumulative intensity of default reaches 100% first on November 6, 2001, the day when Standard & Poor's put Argentina's credit rating to "selective default" in response to a planned debt exchange that would violate intercreditor equity. But on the following day, the announcement of a spending deal with the Argentine provinces raised hopes for immediate IMF help and the avoidance of default, causing the default probability to drop again. The recovery ratio drops well below 30% and remains in a range between 20 and 30% for the remainder of the sample.

ANALYSIS OF SINGLE ISSUES

This section compares the market prices of single issues with their fitted values resulting from daily estimates in the previous section. This offers valuable insight into the performance of the model during the Argentine crisis.

Throughout the sample, the RMSE of daily estimates is well below 2% most of the time. Increased pricing

EXHIBIT 8

Means, Variances, and Correlations of the Parameter Estimates

Total sample					
	Mean	Variance	Correlation Matrix		
α	2.19	2.91	1.00	0.74	0.80
β	2.64	2.01		1.00	0.72
ψ	0.39	0.012			1.00
Episode 1 (08-Jun-2000–30-Oct-2000)					
	Mean	Variance	Correlation Matrix		
α	4.48	0.17	1.00	0.87	-0.64
β	3.75	0.10		1.00	-0.80
ψ	0.44	0.002			1.00
Episode 2 (31-Oct-2000–23-Mar-2001)					
	Mean	Variance	Correlation Matrix		
α	3.28	0.23	1.00	0.59	-0.40
β	3.44	0.27		1.00	-0.74
ψ	0.52	0.002			1.00
Episode 3 (26-Mar-2001–10-Jul-2001)					
	Mean	Variance	Correlation Matrix		
α	2.53	0.19	1.00	-0.36	-0.38
β	3.65	0.32		1.00	0.41
ψ	0.44	0.002			1.00
Episode 4 (11-Jul-2001–26-Oct-2001)					
	Mean	Variance	Correlation Matrix		
α	0.93	0.11	1.00	0.31	-0.07
β	2.77	0.06		1.00	-0.16
ψ	0.34	0.001			1.00
Episode 5 (29-Oct-2001–06-May-2002)					
	Mean	Variance	Correlation Matrix		
α	0.14	0.09	1.00	-0.19	-0.65
β	0.53	0.38		1.00	0.05
ψ	0.25	0.002			1.00

EXHIBIT 9

Pricing Errors During Five Episodes

Episode 1 (08-Jun-2000–30-Oct-2000)										
	Arg01	Arg03	Arg06	Arg09	Arg10	Arg15	Arg17	Arg20	Arg27	RMSE
Mean	1.29	-0.46	-0.31	-0.96	-0.54	0.57	0.84	2.06	-1.48	0.014
S.D.	0.54	0.72	0.67	0.74	0.86	0.89	0.79	0.79	0.68	0.004
Episode 2 (31-Oct-2000–23-Mar-2001)										
	Arg01	Arg03	Arg06	Arg09	Arg10	Arg15	Arg17	Arg20	Arg27	RMSE
Mean	1.60	-0.70	-0.40	-0.06	-0.49	0.52	0.60	1.35	-1.18	0.014
S.D.	0.75	1.27	0.77	1.11	1.14	0.99	1.05	0.99	0.57	0.004
Episode 3 (26-Mar-2001–10-Jul-2001)										
		Arg03	Arg06	Arg09	Arg10	Arg15	Arg17	Arg20	Arg27	RMSE
Mean		-0.27	1.17	-1.08	-0.95	-0.04	1.17	0.84	-0.55	0.015
S.D.		0.79	1.77	0.74	0.75	1.09	0.91	1.06	0.58	0.006
Episode 4 (11-Jul-2001–26-Oct-2001)										
		Arg03	Arg06	Arg09	Arg10	Arg15	Arg17	Arg20	Arg27	RMSE
Mean		-0.30	2.29	-0.69	-1.25	-0.83	0.87	-0.28	0.81	0.024
S.D.		0.46	1.62	1.02	0.98	0.85	1.26	2.06	1.03	0.010
Episode 5 (29-Oct-2001–06-May-2002)										
		Arg03	Arg06	Arg09	Arg10	Arg15	Arg17	Arg20	Arg27	RMSE
Mean		0.12	1.93	0.74	-0.26	-1.32	0.70	-0.81	0.39	0.056
S.D.		1.36	2.14	1.98	1.26	1.67	1.60	2.04	1.78	0.023

errors occur in July 2001 and during the volatile period after November 2001. Excessive errors are sometimes caused by single bonds that show extraordinary high price deviations, driving up the mean RMSE. Taking the simple mean of the pricing error, calculated as dirty market price minus fitted model price, and its standard deviation, pricing failures from Exhibit 9 can be compared between the different periods. Up to its maturity, the Arg01 bond is traded higher than the theoretical model value suggests. Although this deviation is certainly not negligible, the model's performance with regard to a maturing bond is comparatively good. Bonds close to maturity are often not well modeled by term structure models and are also excluded from bond indices such as the EMBI Global.

Other price deviations appear to be related to bond liquidity (see Subramanian [2001]): thus, the Arg17—by outstanding amount the largest issue—is trade rich, whereas the smaller Arg10 issue (despite having the same coupon) trades cheap on average for all periods. The Arg20 shows positive pricing errors through the first three periods and negative pricing errors after the mega debt swap in June 2001, as most of the Arg20 bondholders tendered into the swap and trading prices became sticky. This outcome is opposite to that of the Arg27 issue, which turned into the long-maturity benchmark bond after the debt swap since it became the largest issue by outstanding face value (apart from the new bonds not considered here). Aside from this statistically insignificant relation, Exhibit 9 does not suggest that the pricing errors show a particular pattern or significant biases with respect to price levels, maturities, or coupons.

CONCLUSION

This article suggests adjustments of reduced-form models in the context of sovereign bonds. Since bondholders mostly receive equal compensation after a sover-

EXHIBIT 10

Extreme Value Type I Distribution for Different Parameters

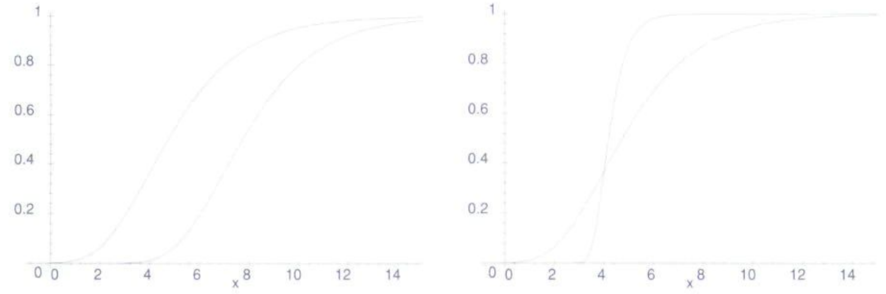
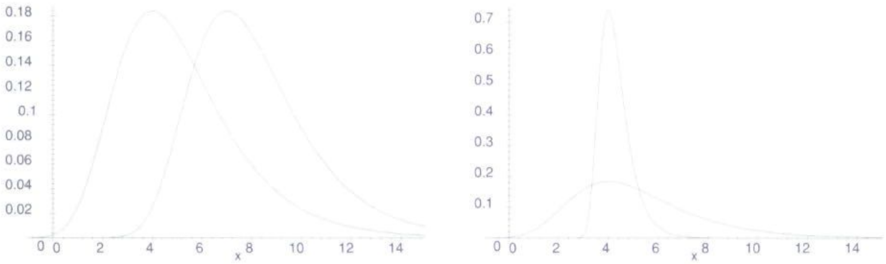


EXHIBIT 11

Extreme Value Type I Density for Different Parameters



eign default (which is in line with intercreditor equality), bond prices should be modeled allowing for fractional recovery of face value (RFV) rather than using a pure recovery of market value (RMV) approach. This allows the default intensity to be disentangled from the recovery rate of face so that in an empirical application estimates can be gained for both parameters. These parameter estimates are useful for economic analysis or asset pricing of other credit-contingent claims such as credit default swaps.

The RFV framework is embedded in an empirical model of the default intensity term structure, using the functional form of the extreme value probability distribution. Such a parsimonious setting requires the estimation of only three parameters. Furthermore, the intensity function used in this study ensures positive forward rates and shows a survival function converging to zero at infinity. In combination with the RFV scheme, it mimics well the inverse term structure curve typically observed during distress and effectively allows for a price discount on maturing issues that is consistent with empirical reality.

When estimating such a model to fit a cross section of bond prices, some rules of thumb help to avoid an identification problem. The sample of equally ranking bonds should contain at least one long-term bond trading at below 85%. To correctly parameterize the intensity function it is advisable to include bonds in the estimation that describe the characteristic curvature of the default intensity function. This is achieved by ensuring that their durations have significantly different unconditional default probabilities. The parsimony of the model allows daily estimates to be derived by numerically minimizing RMSEs, which is computationally simple in comparison with the likelihood estimators of affine spread models.

The results can then be related to economic or political events or used in econometric frameworks with other fundamental variables of similar frequencies. A case study of Argentine global bonds during Argentina's latest financial crisis, 2000-2002, yields an illustrative picture of the crisis consistent with major events during that period. The default intensity is found to evolve in jumps, which suggests the segregation of the Argentine crisis into five episodes. Most jumps are associated with rating actions. After the IMF and other international lenders approved a USD 40 billion aid package, the recovery ratio rises to well above 50% at the beginning of 2001, while leaving the default intensity almost unaffected. This indicates that solvency concerns were not resolved by the package although investors welcomed the IMF support as softeners in the foreseeable bond restructuring. Pricing errors of single bonds do not show particular patterns with respect to bond characteristics and therefore support the usefulness of the model.

APPENDIX

The Extreme Value Distribution

The extreme value type I distribution

$$G(x) = exp\left\{-\exp\left[-\frac{x-\alpha}{\beta}\right]\right\},\ x \in \qquad \text{(A-1)}$$

is a special type of the general extreme value distribution (see Gumbel [1958]). The use of the extreme value type I distrib-

Gumbel [1958]). The use of the extreme value type I distribution in this article is motivated by the characteristic shape of its density, which is determined by two parameters, α and β.

Exhibit 10 illustrates the distribution for some arbitrary parameters. In the left graph, $\alpha_1 = 4$ (left line) and $\alpha_2 = 7$ (right line) while $\beta = 2$ is constant. The graph on the right-hand side illustrates two distribution shapes for $\alpha = 4$ with $\beta_1 = 2$ (flat line) and $\beta_2 = 0.5$ (steep line). From the distribution function G follows the probability density function that is illustrated in Exhibit 11 (with parameter values according to Exhibit 10):

$$g(x) = \frac{1}{\beta} e^{-(x-\alpha)/\beta} e^{-e^{-(x-\alpha)/\beta}} \tag{A-2}$$

The location parameter α describes the mode of the density, whereas the shape parameter β is proportional to the density's standard deviation. Thereby, each parameter determines a different characteristic of the distribution. For instance, a change of α leads only to a change in the location of the distribution function without affecting the shape of the function. This is a very convenient effect, especially for illustrative purposes.

ENDNOTE

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