Does Mortgage Hedging Raise Long-Term Interest Rate Volatility?

YAN CHANG, DOUGLAS MCMANUS, AND BUCHI RAMAGOPAL

YAN CHANG
is an economist at Freddie Mac in McLean, VA.
yan_chang@freddiemac.com

DOUGLAS MCMANUS
is a principal economist at Freddie Mac.
douglas_mcmanus@freddiemac.com

BUCHI RAMAGOPAL
is director of financial research at Freddie Mac.
buchi_ramagopal@freddiemac.com

Volatility is an ever-present element of financial markets. Recent history features several episodes of heightened volatility. Events such as the Russian debt crisis, the collapse of Long-Term Capital Management, and the disruption of the Asian markets in the late 1990s were all responsible for periods of heightened volatility. While from a macroeconomic perspective we can see such volatility as the source of economic instability, from a complete markets perspective, it represents the efficient functioning of markets in the face of uncertainty.

Markets, institutions, and practices are typically examined as possible sources of extremes in volatility. We examine the possible role of mortgage hedging in creating volatility in interest rate markets.

Since 1990, 30-year fixed mortgage rates have declined from a high of 10.5% to a low of 5.2% in June 2003. This decline in rates prompted households to refinance mortgages in record numbers. The refinancing activity peaked during 1992-1993, 1998, and, most recently, in 2003. By the end of 2003, up to 40% to 45% of the $7.0 trillion single-family debt outstanding had been refinanced in 2003 alone. In 2003, the volume of refinancing activity saw close to $3 trillion of mortgage debt outstanding restruck at lower rates. Yet despite the huge volume, the U.S. mortgage market managed to respond flexibly, providing homeowners the benefit of lower rates.

The decline in rates and the restriking of mortgage debt has required an increasing number of mortgage market participants such as originators, mortgage servicers, and portfolio managers to manage a variety of risks. Fixed-rate mortgages have positive duration and negative convexity; that is, as market interest rates rise, the value of a security declines linearly with duration and quadratically with convexity. Market participants use a mixture of static and dynamic hedging strategies to mitigate this interest rate risk.

Static hedging of mortgage interest rate risk can be accomplished by issuing callable debt or through synthetic callable debt created by the purchase of swaptions and the issuance of bullet debt. Dynamic hedging calls for constant management of the relative durations of the assets and liabilities, typically through swap transactions. Counterparties to these derivative transactions have diversified books of business and can thus manage these risks. They also serve as intermediaries by identifying other parties that might naturally offset these mortgage market risks.

The nature and extent of mortgage hedging activities has raised the issue of its impact on market interest rates. Kambhu and Mosser [2001, p. 51] observe that "disruptions to liquidity can affect the short-term dynamics of interest rates and the shape of the curve independently of fundamentals." Their analysis shows that the dynamics of the yield curve have changed in ways that seem related to the growth of the interest rate options market.

Researchers have taken a variety of approaches to evaluating the impact of large mortgage market participants on rate volatility. Gonzalez-Rivera [2001] and Naranjo and Toevs [2002] suggest that large mortgage market participants reduce volatility in mortgage rate spreads as a result of their participation in the market. Perli and Sack [2003] and Goodman and Ho [2004], however, suggest that mortgage hedging increases long-term rate volatility, although they have different estimates of the extent of amplification of rates resulting from hedging. Perli and Sack suggest that the tools of financial engineering, as applied to mortgage risk management, might be responsible for heightened volatility. This article investigates that claim.

Whether rational speculators or risk management frameworks can destabilize prices or exacerbate volatility is a constant question for research. One position, starting with Friedman [1953], is that rational speculators act to stabilize asset prices. In Friedman's view:

> Speculators who destabilized prices by, on average, buying when prices were high and selling when they were low would find this unsustainable and be eliminated from the market. The more rational speculators who moved prices back toward fundamentals would eliminate [the speculators] who are less than rational and [move] prices away from fundamentals [1953, p. 175].

Several recent researchers have developed theoretical examples in which trading activities can destabilize prices. In this case, rational speculators might not easily stabilize asset prices even when they have access to identical information and have identical priors as shown by Hart and Kreps [1986]. Similarly, De Long, Shleifer, Summers, and Waldmann [1990] also show that investors with extrapolative expectations, those who chase trends, or who have stop-loss orders in place (all logically positive feedback investment strategies), might contribute to destabilizing market movements.

Arguments similar to destabilizing speculation have been used to make the case that risk management in general or portfolio insurance in particular contributes to greater volatility in rates or asset prices. Portfolio insurance, for example, according to the 1988 Brady Commission report, was considered the exacerbating factor in the crash of 1987.

In a similar vein, the value at risk (VaR) methodology has also been blamed for amplifying volatility (see Persaud [2000] and Dunbar [2000]). The argument is that shocks might increase volatility initially, leading to a breach of VaR positions, and require investors to add capital or liquidate some part of their portfolios. Jorion [2002, p. 2] asks whether large institutions that exceed predetermined or required VaR levels might then simultaneously "sell the same asset at the same time, creating higher volatility and correlations, which exacerbate the initial effect, forcing additional sales." He sees such criticisms relating to the VaR framework as misplaced.

Perli and Sack [2003] speculate that when interest rates rise, mortgage hedgers increase the duration of their liabilities by executing a pay-fixed swap; that is, they synthetically exchange short-term debt for longer-maturity obligations. Conversely, in response to declining rates, mortgage hedgers reduce the duration of their liabilities through receive-fixed swaps. The extent of the amplification effects will depend on the degree to which mortgage hedgers adopt dynamic rather than static hedging strategies. It is assumed that some constant proportion of mortgage interest rate risk is actively managed through dynamic and not static hedging strategies.

A recent line of argument has been to push for the increased use of adjustable-rate mortgages, which would reduce the need to hedge dynamically. While the adoption of adjustable-rate mortgages would eliminate the need for large mortgage holders to hedge dynamically, there remains a different impact on individual homeowners who may not be in a position to manage such interest rate risk, as evidenced by higher ARM default rates for otherwise identical mortgages.

It is ironic that ARMs are promoted as an alternative to fixed-rate mortgages in the U.S. market when countries such as the United Kingdom see the prevalence of ARMs as one reason behind their boom and bust housing cycles, which increase the swings in business cycles. Miles [2004] highlights the need for the U.K. housing market to introduce longer-term fixed-rate mortgages with prepayment options, much like those in the U.S. mortgage market.

The question remains as to whether dynamic hedging by mortgage market participants creates volatility, and, if so, the extent to which this is the case. Ultimately, the impact of mortgage hedging on interest rate volatility must be determined empirically.

I. MODEL

We assume changes in long-term rates are driven by a data-generating process. The shocks generated by this process emanate from monetary policy innovations, news, and changes in risk preferences. The long-term rate is the ten-year swap rate, and changes in this rate between time t and t + n are generated by a fundamental shock $\varepsilon(t, t + n)$.

The changes in the ten-year swap rate are characterized as:

$$\Delta r(t, t + n) = \sqrt{\gamma_t}\varepsilon(t, t + n) \tag{1}$$

In this setting, the fundamental shock $\varepsilon(t, t + n)$ is exacerbated by a factor $\sqrt{\gamma_t}$ whose value is known at time t and whose magnitude is determined by the degree of mortgage hedging activity. This amplification factor is supposed to characterize the state of the mortgage market at the beginning of the period. It takes on a value that could be close to one if there is little prepayment risk, and values that are higher if there is greater prepayment risk. Thus, shocks during periods of high prepayment risk are amplified, and shocks during periods of low prepayment risk are dampened.

Using the specification in (1), the variance of the interest rate is characterized as:

$$\sigma^2_{\Delta r,t} = \gamma_t \sigma^2_{\varepsilon,t} \tag{2}$$

Equation (2) describes the second moment of the ten-year swap rate as a function of the second moment of the fundamental shock and the amplification factor.

The conditional variance of the fundamental shocks is described as following an AR(1) process:

$$\sigma^2_{\varepsilon,t} = \alpha_0 + \alpha_1\sigma^2_{\varepsilon,t-1} + u_t \tag{3}$$

Using Equation (2) to rewrite last period's fundamental variance as a ratio of last period's variance of the swap rate to the amplification factor, and using (3), Equation (2) is reexpressed as:

$$\sigma^2_{\Delta r,t} = \alpha_0\gamma_t + \alpha_1\frac{\gamma_t}{\gamma_{t-1}}\sigma^2_{\Delta r,t-1} + \gamma_t u_t \tag{4}$$

During periods of higher prepayment risk, a higher γ raises the average level of volatility, leading to a larger value of the first term on the right-hand side of (4). The second term highlights the autocorrelation of volatility, and the third term describes the *volatility of volatility* effect, or that changes in expected volatility are greater when prepayment risk is high.

The framework for determining the size of the amplification factor $\sqrt{\gamma_t}$ is described as:

$$\gamma_t = 1 + \beta X_t \tag{5}$$

where X_t is a measure of hedging intensity. When there is no hedging pressure, the value of γ_t is one; when there is an increased need for greater mortgage market hedging, the value is higher.

Using the specification for the hedging amplification factor in (5), (4) can be rewritten as

$$\sigma^2_{\Delta r,t} = \alpha_0 + \alpha_0\beta X_t + \alpha_1\frac{1 + \beta X_t}{1 + \beta X_{t-1}}\sigma^2_{\Delta r,t-1} + (1 + \beta X_t)u_t \tag{6}$$

The final specification uses a lagged value of the hedging amplification factor to avoid endogeneity that arises from the possibility that volatility both causes and is caused by hedging. This produces:

$$\sigma^2_{\Delta r,t} = \alpha_0 + \alpha_0\beta X_{t-1} + \alpha_1\sigma^2_{\Delta r,t-1} + (1 + \beta X_{t-1})u_t \tag{7}$$

The product of the parameters $\alpha_0\beta$ provides a measure of the degree of sensitivity of volatility to changes in hedging intensity. Note that the error term is multiplied by a factor relating to hedging intensity that induces heteroscedasticity into the estimated equation, which is interpreted as a volatility of volatility effect.

Equation (7) is estimated using the maximum-likelihood framework, where the likelihood function is given by:

$$L(\alpha_0, \alpha_1, \beta, \upsilon) = -\frac{1}{2}\ln\left(\upsilon(1 + \beta X_{t-1})^2\right) - \frac{1}{2\upsilon}\left(\sigma^2_{\Delta r,t} - \alpha_0(1 + \beta X_{t-1}) - \alpha_1\sigma^2_{\Delta r,t-1}\right)^2 \tag{8}$$

II. DATA

Volatility in financial markets is typically measured in one of two ways. The first examines the ex post historical volatility of rates. The second uses financial derivative pricing models to ascertain the ex ante volatilities consistent with derivative asset prices. Like Perli and Sack [2003], we focus on the volatilities implied by the prices of swaption derivative contracts.[1] We obtain these implied volatilities by applying Black's [1976] model of interest rate option pricing to translate market data on prices into volatility.

The Perli and Sack methodology seeks to explain short-term movements in volatilities unrelated to longer-term shifts in the underlying interest rate process. To implement this measure, they derive the residual component of short-term volatility orthogonal to long-term volatility. This step involves regressing a squared measure of short-term implied volatility on squared values of longer-term implied volatility, and using the residual as the dependent variable in the analysis.

The residual η_t in Equation (9) measures the part of short-term volatility not explained by long-term volatility:

$$\sigma^2_{\Delta r,t}(3m \times 10y) = \psi_0 + \psi_1\sigma^2_{\Delta r,t}(2y \times 10y) + \eta_t \quad (9)$$

where $\sigma^2_{\Delta r,t}(3m \times 10y)$ is the square of the implied volatility of a three-month swaption on a ten-year swap, and $\sigma^2_{\Delta r,t}(2y \times 10y)$ is the square of the implied volatility on a two-year option on a ten-year swap. The results from (9) are used to build the orthogonalized measure of variance, which is constructed as follows:

$$\sigma^2_{\Delta r,t} = \eta_t + \bar{\sigma}^2_{\Delta r,t}(3m \times 10y) \quad (10)$$

where η_t is the time series of residuals, and $\bar{\sigma}^2_{\Delta r,t}(3m \times 10y)$ is the sample average of the variance of the three-month × ten-year swaptions.

We use a time series of the implied volatility for four swaptions: three-month × five-year, three-month × ten-year, six-month × five-year, and six-month × ten-year. The implied volatilities are measured in basis points (bp), i.e., 100th of 1%. The sources for the option volatilities are Salomon Smith Barney's *Yield Book* and Perli and Sack [2003]. The time series for each of these volatilities is constructed as the product of daily rate volatility and the appropriate forward rate.[2]

The descriptive statistics for the variables and their definitions are described in Exhibit 1. Mortgage market convexity measures the convexity exposure in terms of the magnitude of swaps that would have to be traded to remain duration-neutral in the presence of a rate shock of 50 bp. We use weekly observations of the mortgage market convexity measure obtained from Perli and Sack to replicate their results. The remaining eight weekly series measure the implied volatility of both short- and long-term swaptions.

Exhibit 2 displays both short and long volatility series. While the Perli and Sack "long" implied volatility ($2y \times 10y$) series deviates from our series starting in 2001, the series generally agree with our pre-2001 data. Additional independent volatility data obtained from JP Morgan and LehmanLive.com more closely correspond to our series than the Perli and Sack series over 2001–2003, so we consider model estimates using both the Perli and Sack series and the Salomon Smith Barney series.

First, we replicate the Perli and Sack first-stage results using their series. These findings are given in the first row of Exhibit 3. Perli and Sack use the residuals from this first-stage regression (after adding the average squared volatility) as the dependent variable in their second-stage regression. Note that we can numerically replicate their coefficient estimates.

The first set of results highlighted in the first row of Exhibit 3 is obtained using Perli and Sack data, while all the other results are obtained using our implied volatility series. According to our series, the value of the coefficients and the R^2s are all significantly lower than those obtained by Perli and Sack.[3]

We then reestimate this first stage using the alternative data series for both short and long volatility. These results, given in the second row of Exhibit 3, match the signs of the Perli and Sack coefficient estimates, but differ materially in their size.

To evaluate the sensitivity of the Perli and Sack findings, we perform alternative first-stage regressions using different swaption structures. The third row varies the Perli and Sack experiment by basing the short volatility series on $6m \times 10y$ swaptions. The fourth row shortens the long volatility series to the $2y \times 5y$ swaption. The last row alters both series, taking the $6m \times 5y$ swaption volatility as the short series and the $2y \times 5y$ series as the long series. In these three cases, the sign and the size of the coefficients are similar to what we obtain in replication of the Perli and Sack model using our short and long series.

While there may be a difference between the results depending on the data sources, the first-stage regressions produce similar results across the range of swaption volatilities that we explore. This suggests that these Perli and Sack findings are not specific to the particular swaption contracts that they examine.

EXHIBIT 1

Variables in the Analysis: January 1997–May 2003*

Variable	Definition	Source	Frequency	Mean	Standard Deviation
Mortgage Market Convexity	Duration exposure to be covered as a result of a rate shock at any point in time	Data used in Perli and Sack [2003]	Weekly	-0.475	0.231
3m × 10y Implied Volatility	Annual basis points (bp) implied volatility.	Data used in Perli and Sack [2003]	Weekly	1.164	0.445
2y × 10y Implied Volatility	Annual basis points (bp) implied volatility.	Data used in Perli and Sack [2003]	Weekly	0.983	0.190
3m × 10y Implied Volatility	Annual basis points (bp) implied volatility.	Salomon Smith Barney series from YieldBook	Weekly	1.199	0.478
6m × 10y Implied Volatility	Annual basis points (bp) implied volatility.	Salomon Smith Barney series from YieldBook	Weekly	1.204	0.443
2y × 10y Implied Volatility	Annual basis points (bp) implied volatility.	Salomon Smith Barney series from YieldBook	Weekly	1.150	0.328
2y × 5y Implied Volatility	Annual basis points (bp) implied volatility.	Salomon Smith Barney series from YieldBook	Weekly	1.247	0.375
3m × 5y Implied Volatility	Annual basis points (bp) implied volatility.	Salomon Smith Barney series from YieldBook	Weekly	1.235	0.537
6m × 5y Implied Volatility	Annual basis points (bp) implied volatility.	Salomon Smith Barney series from YieldBook	Weekly	1.253	0.512

Note: The sample period begins with the week ending January 1997 and ends with the last week of May 2003, providing a total of 328 observations.

**Annual basis point (bp) implied volatility is divided by 100, consistent with Perli and Sack [2003]. Market convention, however, would express the implied volatility of a 3m × 10y swaption as 116.4 basis points.*

It is informative to examine a graph of the dependent variable created in the first-stage regression. Exhibit 4 plots the volatility series created by adding the residual from the first-stage regression to the average squared volatility for the short-term swaption.

Two features are striking in this graph. They are the spikes that occur in the latter half of 1998 and near the end of 2001, associated with the Long-Term Capital Management and the September 11 events. The LTCM extremum is approximately a 6.54 standard deviation event, and September 11 is a 3.77 standard deviation event. Since both least squares and maximum-likelihood methodologies can be highly sensitive to outlier observations, this graph motivates our exploration of the sensitivity of the Perli and Sack findings to these stressful episodes.

Given the possibility of model error and parameter instability, the issue might remain as to whether any variability needs to be explained, but our regressions (R^2s ranging from 0.78 to 0.91) indicate that 9% to 22% of the variability remains a question. To analyze this issue, we use a proxy for hedging activity.[4]

The interest rate risk in the universe of mortgage debt is measured by the total mortgage market duration and convexity. Starting from a duration-neutral position for a given portfolio, the measure of hedging intensity will depend on changes in the level of duration of mortgage debt. This change in duration will depend, in turn, on the degree of convexity, as convexity is a measure of the rate of change of duration. Convexity is a reasonable proxy for changes in duration for investors who choose to hedge changes in duration dynamically.

While dynamic hedging is a reactive hedging protocol, most firms use a mixture of static and dynamic hedging. When rate changes alter the duration of assets, a

EXHIBIT 2
Time Series of Perli and Sack and SSB Implied Volatility

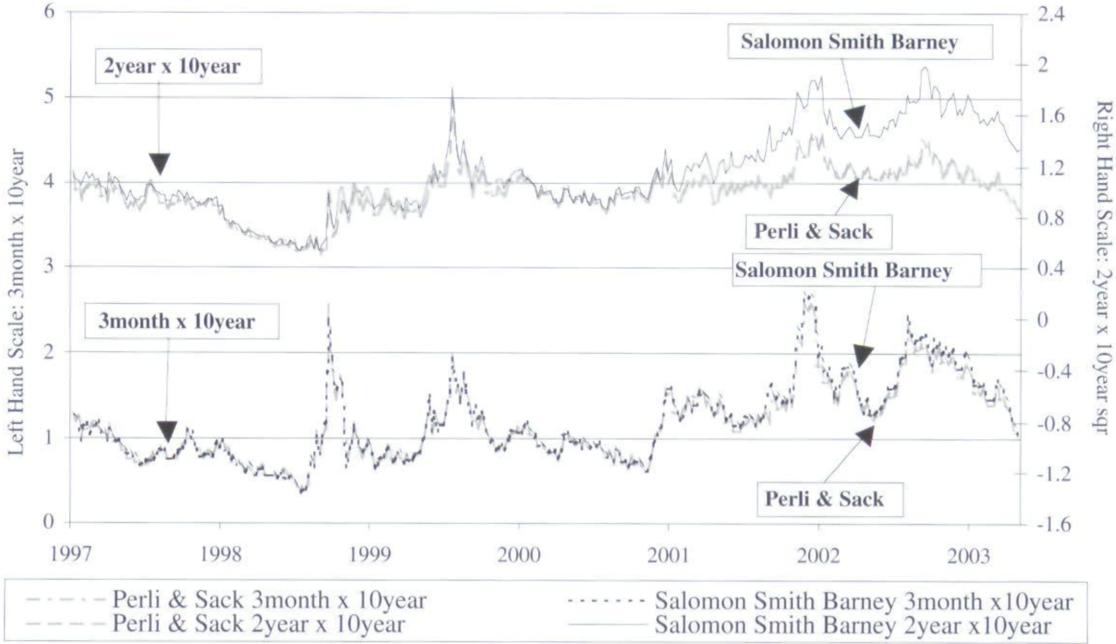


EXHIBIT 3
Regression Results for Short-Term on Long-Term Implied Variance

Dependent Variable	Independent Variable	Time Period	R-Square	Estimated Parameters*	
				ψ_0	ψ_1
3m × 10y	**2y × 10y**	**1997:01 – 2003:05**	**0.6000**	**-0.631**	**1.827**
3m × 10y	2y × 10y	1997:01 – 2003:05	0.7795	-0.2811	1.2876
6m × 10y	2y × 10y	1997:01 – 2003:05	0.8892	-0.2633	1.2765
3m × 5y	2y × 5y	1997:01 – 2003:05	0.8295	-0.3891	1.3022
6m × 5y	2y × 5y	1997:01 – 2003:05	0.9143	-0.3743	1.3051

**All coefficient estimates are significant at 99% level.*

portfolio can be rebalanced to a duration-neutral position by changing the duration of liabilities such as through swaps. To the extent that firms anticipate hedging needs and eliminate some or all of their convexity exposure through the use of swaptions to offset the negative convexity inherent in the mortgage universe, the residual negative convexity would be the appropriate measure of hedging intensity. This is an unobservable measure, so we use mortgage market convexity. If firms change their mix of dynamic and static hedging during the sample period, though, this could diminish the predictiveness of this regressor.

We use the same series for convexity as Perli and Sack. It provides a weekly measure of ten-year swap equivalents needed to swap out the changes in duration resulting from changes in rates during the period.[5]

III. IMPACT OF MORTGAGE HEDGING ON VOLATILITY

Perli and Sack measure the impact of mortgage hedging on volatility by regressing the constructed residual short-term volatility variable on a proxy for mortgage

EXHIBIT 4

Time Series Comparison of Perli and Sack and SSB Dependent Variable

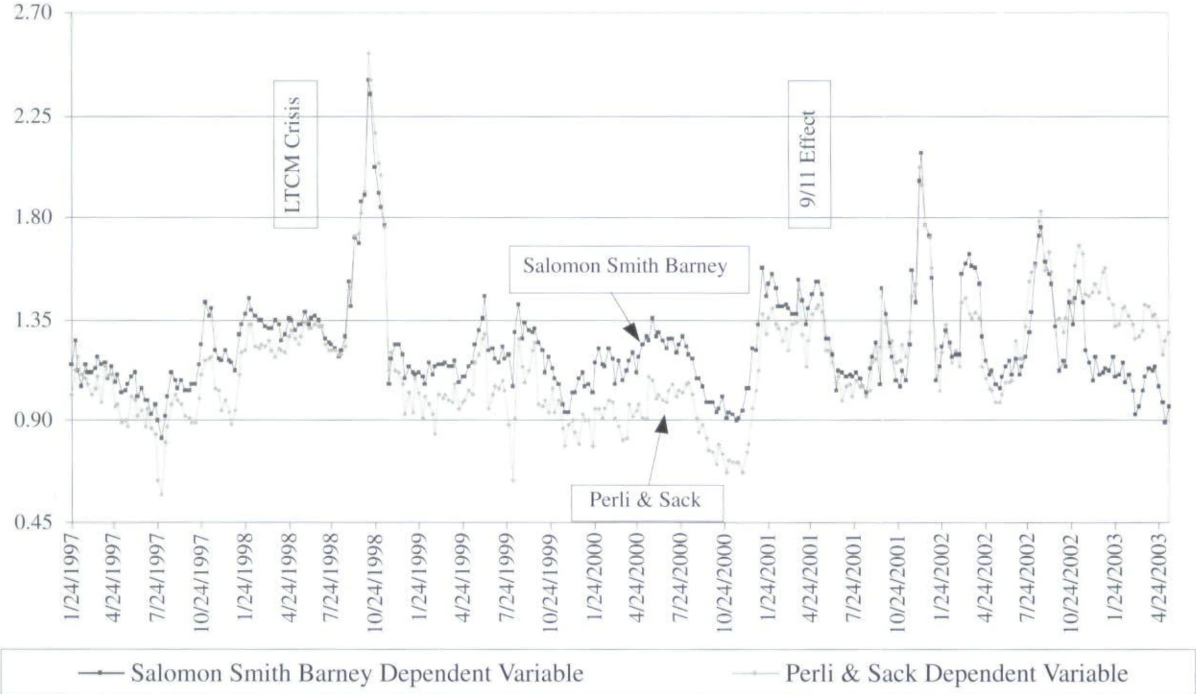


EXHIBIT 5

Regression Results for Orthogonalized Implied Variance

Dependent Variable	Time Period	Mortgage Market Convexity as Independent Variable		
		Estimated Parameters (p-values in parentheses)		
		α_0	α_1	$\alpha_0\beta$
3m × 10y	1997:01 – 2003:05	0.1000 (0.00)	0.8856 (0.00)	-0.0846 (0.00)
6m × 10y	1997:01 – 2003:05	0.1511 (0.00)	0.8684 (0.00)	-0.0146 (0.41)
3m × 5y	1997:01 – 2003:05	0.1395 (0.00)	0.8746 (0.00)	-0.0314 (0.22)
6m × 5y	1997:01 – 2003:05	0.1646 (0.00)	0.8597 (0.00)	-0.0228 (0.22)

market hedging, the total mortgage market convexity. First, we replicate the Perli and Sack second-stage analysis, obtaining the identical coefficients as presented in the first row of Exhibit 5. We then extend the second-stage analysis to different swaption structures.

Using three other swaption volatilities for the same sample period ($6m \times 10y$, $3m \times 5y$, and $6m \times 5y$), we can obtain estimates of the coefficients α_0 and α_1 that are nearly identical and significant. For the parameter of interest, $\alpha_0\beta$, the sign is the same as in Perli and Sack for all the swaption structures, but the size is substantially smaller. The estimates do not appear significant, but note that the reliability of any inferences drawn is questionable, because the standard errors are not adjusted for heteroscedasticity, and the confidence intervals do not acknowledge the possible cointegration of these volatility series.

We move next to estimation of the parameters using maximum-likelihood estimation with the specification as described in Equation (8). The MLE is efficient and provides a basis for asymptotically valid inference. The results appear in Exhibit 6. In the MLE estimation, we estimate four parameters: α_0, α_1, β, and υ; the parameter of interest is β, the amplification parameter. For all the swaption structures in this sample period, the coefficients are all highly significant and closely approximate the results of Perli and Sack.[6]

One important test of a model's validity is parameter stability. We know there may be structural differences between periods with little hedging activity as compared with periods of considerable hedging activity. When there is little demand for hedging, risk managers may increase the proportion of hedging accomplished through static hedges.

We examine this issue by splitting the sample period into times the Mortgage Bankers Association Refi-index was low, with expected low levels of hedging activity, and

EXHIBIT 6
MLE Results for Orthogonalized Implied Variance

Dependent Variable	Time Period	Mortgage Market Convexity as Independent Variable — Estimated Parameters (p-values in parentheses)			
		α_0	α_1	β	υ
3m × 10y	1997:01 – 2003:05	0.0792 (0.00)	0.908 (0.00)	-0.834 (0.00)	0.0058 (0.00)
6m × 10y	1997:01 – 2003:05	0.1093 (0.00)	0.892 (0.00)	-0.393 (0.01)	0.0037 (0.00)
3m × 5y	1997:01 – 2003:05	0.0816 (0.00)	0.906 (0.00)	-0.917 (0.00)	0.0053 (0.00)
6m × 5y	1997:01 – 2003:05	0.1273 (0.00)	0.879 (0.00)	-0.399 (0.00)	0.0039 (0.00)

times it was high and volatile, implying greater hedging pressures: 1997:01–2000:12, and 2001:01–2003:05. Then we reestimate the model for these two subperiods. During the first period, the mean value of the weekly Refi-index was 837 with a standard deviation of 651. During the second period, the mean value was 3,467 with a standard deviation of 2,076. The results are in Exhibit 7.

Here we find a result that is counter-intuitive. When the refincentives were low and we might have expected to find a low amplification parameter β, we instead find it high not only for the $3m \times 10y$ swaption structure, but also for all the others. Furthermore, the estimates all have high p-values and t-statistics.

Equally counter-intuitive, for the period 2001:01–2003:05, when the MBA refi-index was high and hedging pressures were great, we find the amplification parameter β implies that higher levels of hedging actually reduced volatility. These results are highly statistically significant. Were the original Perli and Sack specification correct, we would be unlikely to observe this degree of parameter instability.

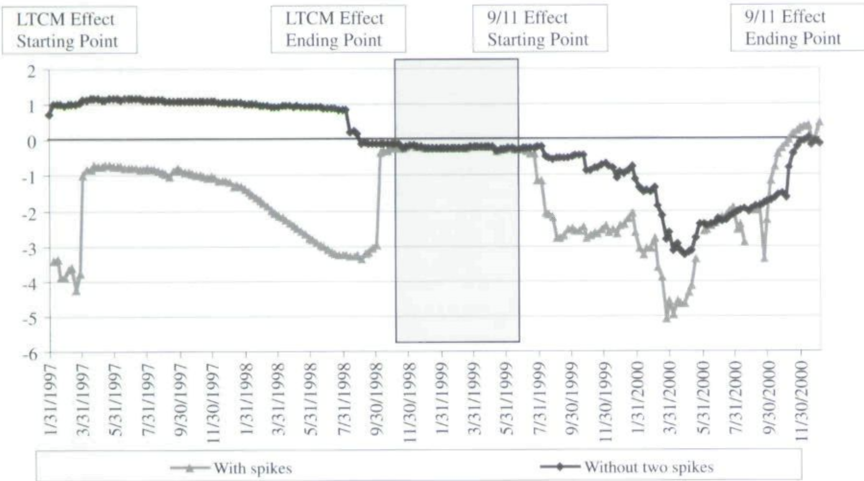
To further refine our understanding of the parameter instability in the Perli and Sack model, we examine the parameter estimates of a rolling window. We take 123-week intervals, starting with the first week of 1997, and estimate the model parameters for each data window. If the model is correctly specified, these estimates would be constant, except for sampling error. Or, if the stability is caused by an outlier in the data, we would see substantial movement in the parameter estimates as the outlier data points move into or out of the sample window.

Exhibit 8 plots the amplification parameter estimate β as a function of the start date of the 123-week window. This graph exhibits two discontinuity points, one at about March 31, 1997, and the other at about September 30, 1998. These two points correspond to times the market disruptions brought about by LTCM enter and exit the sample window. Removing the few data observations associated with the LTCM market disruption and reestimating produces dramatically different coefficient estimates. The

EXHIBIT 7
Subperiod Results for Orthogonalized Implied Variance

Dependent Variable	Time Period	Mortgage Market Convexity as Independent Variable — Estimated Parameters (p-values in parentheses)			
		α_0	α_1	β	υ
3m × 10y	1997:01 – 2000:12	0.0529 (0.05)	0.930 (0.00)	-1.794 (0.00)	0.0038 (0.00)
	2001:01 – 2003:05	0.2958 (0.05)	0.838 (0.00)	0.476 (0.00)	0.0294 (0.00)
6m × 10y	1997:01 – 2000:12	0.0671 (0.01)	0.928 (0.00)	-0.878 (0.01)	0.0025 (0.00)
	2001:01 – 2003:05	0.3319 (0.01)	0.784 (0.00)	0.292 (0.02)	0.0107 (0.00)
3m × 5y	1997:01 – 2000:12	0.0707 (0.01)	0.917 (0.00)	-1.334 (0.00)	0.0039 (0.00)
	2001:01 – 2003:05	0.2966 (0.04)	0.870 (0.00)	0.620 (0.00)	0.0467 (0.00)
6m × 5y	1997:01 – 2000:12	0.0926 (0.00)	0.907 (0.00)	-0.714 (0.00)	0.0027 (0.00)
	2001:01 – 2003:05	0.3574 (0.00)	0.793 (0.00)	0.364 (0.01)	0.0135 (0.00)

Exhibit 8
Changing MLE Estimates of Sensitivity Parameter



results suggest that hedging adds to market stability in the pre-LTCM period.

IV. CONCLUSION

The possibility that risk mitigation strategies could have the unintended consequence of amplifying exogenous shocks to financial markets raises important policy questions. Understanding volatility and its determinants is an important issue relating to the implications of mortgage risk management for interest rate volatility.

Perli and Sack [2003] focus on explaining the component of short-term volatility unrelated to movements in long-term volatility. While their empirical findings that link mortgage hedging to increased volatility are an artifact of several outlier observations in the data, modifying their analysis by dropping periods associated with the LTCM crisis and September 11, 2001, generates estimates that suggest there is little impact of mortgage hedging on interest rate volatility. Indeed, over some periods the proxies for hedging activity appear to mitigate some of the market volatility.

While this analysis represents a step toward a better understanding of the linkage between hedging activities and financial stability, we believe a more structural approach to this problem is needed. The term structure of volatility, for example, would suggest an alternative empirical approach to obtaining information about short-term volatility unrelated to long-term volatility movements from that taken in the Perli and Sack first-stage regression. Further, the component of short-term volatility not explained by movement in long-term volatility is minor both absolutely and relatively.

Any theory that has policy relevance must explain material movements in market volatility, ideally across the entire volatility surface. Moreover, if market dynamics are different during periods of stress, a modeling strategy capturing the differential impacts of hedging may be warranted.

APPENDIX

Likelihood Function

For a variable x that follows a normal distribution $N(\mu_x, \sigma_x^2)$, with mean μ_x and variance σ_x^2, the likelihood function, or the density function, is as follows:

$$L = f(x) = \frac{1}{(2\pi)^{\frac{1}{2}}\sigma_x}\exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^{\ 2}}\right)$$

The log-likelihood function is the result of taking the natural log on both sides of the equation above:

$$\log L = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln\sigma_x^{\ 2} - \frac{1}{2\sigma_x^{\ 2}}(x-\mu_x)^2$$

To maximize log L is to maximize

$$-\frac{1}{2}\ln\sigma_x^{\ 2} - \frac{1}{2\sigma_x^{\ 2}}(x-\mu_x)^2$$

From Equation (7), we have:

$$\sigma^2_{\Delta r,t} = \alpha_0 + \alpha_0\beta X_{t-1} + \alpha_1\sigma^2_{\Delta r,t-1} + (1+\beta X_{t-1})u_t$$

We can use ε_t to denote the amplified error term, with $\varepsilon_t = (1+\beta X_{t-1})u_t$. ε_t follows a normal distribution $N(0, V(1+\beta X_{t-1})^2)$.

Our x variable is $\sigma^2_{\Delta r,t}$, with $\sigma_x^2 = V(1+\beta X_{t-1})^2$ and $\mu_x = \alpha_0(1+\beta X_{t-1}) + \alpha_1\sigma^2_{\Delta r,t-1}$.

Substituting these terms into the log-likelihood function, we have:

$$L(\alpha_0, \alpha_1, \beta, \upsilon) = -\frac{1}{2}\ln\left(\upsilon(1+\beta X_{t-1})^2\right) - \frac{1}{2\upsilon(1+\beta X_{t-1})^2}\left(\sigma^2_{\Delta r,t} - \alpha_0(1+\beta X_{t-1}) - \alpha_1\sigma^2_{\Delta r,t-1}\right)^2$$

instead of

$$L(\alpha_0, \alpha_1, \beta, \upsilon) = -\frac{1}{2}\ln\left(\upsilon(1 + \beta X_{t-1})^2\right) - \frac{1}{2\upsilon}\left(\sigma^2_{\Delta r,t} - \alpha_0(1 + \beta X_{t-1}) - \alpha_1\sigma^2_{\Delta r,t-1}\right)^2$$

as in Equation (8).

ENDNOTES

The authors are grateful to James Berkovec, Mustafa Chowdhury, Devajyoti Ghose, Edward Golding, Frank Nothaft, Mitchell Post, Jorge Reis, and Peter Zorn for their suggestions and insights. An earlier draft of this article was presented at the Federal Reserve Bank of New York and the 2004 International AREUEA meetings in Fredericton, New Brunswick. The authors thank participants at those meetings for their suggestions. Robert Winata and Murat Karakurt helped in construction of the data set. The views presented are those of the authors and not of Freddie Mac.

[1]Swaptions are options contracts on a swap. They provide the holder with an option to enter into a swap at some future time. As is the practice in these markets, the quoted prices of these contracts are in volatility units, the market-determined value of ex ante volatility as implied by Black's model.

[2]Basis point volatility for the three-month × ten-year swaption equals the product of the rate volatility for the three-month × ten-year and the three-month forward ten-year rates.

[3]Note also that after replicating the Perli and Sack first-stage results, the R^2 of 0.60 differs from the 0.95 value reported in Perli and Sack. This residual volatility is small both relatively and absolutely. The average squared volatility is 1.07, of which the unexplained component is 0.21, and the coefficient of variation is a low 0.19. In fact, it might be argued that this effect (of residual volatility) is so slight that it is not of great consequence to any policy issues that relate to stability in financial markets.

[4]Of three proxies for hedging, Perli and Sack indicate that convexity is the most robust and most indicative measure of hedging intensity, so we confine our attention to this proxy.

[5]Ten-year swap equivalents are a standard measure of the number of dollars of this contract that must be used to eliminate a duration exposure.

[6]We identify one problem with specification of the likelihood function in the original Perli and Sack analysis, and correct it for the estimates provided in Exhibit 6. The corrected likelihood estimates do not differ materially. A derivation of the corrected likelihood function appears in the appendix.

REFERENCES

Black, F. "The Pricing of Commodity Contracts." *Journal of Financial Economics*, 3 (1976), pp. 167-179.

De Long, B., A. Shleifer, L.H. Summers, and R.J. Waldmann. "Positive Investment Feedback Strategies and Destabilizing Rational Speculation." *Journal of Finance*, 45 (2) (1990), pp. 379-395.

Dunbar, N. *Inventing Money: The Story of Long-Term Capital Management and the Legends Behind It.* New York: John Wiley & Sons, 2000.

Friedman, M. *Essays in Positive Economics.* Chicago: University of Chicago Press, 1953.

González-Rivera, G. "Linkages Between Secondary and Primary Markets for Mortgages: The Role of Retained Portfolio Investments of the Goverment-Sponsored Enterprises." *The Journal of Fixed Income*, June 2001, pp. 29-36.

Hart, O.D., and D.M. Kreps, "Price Destabilizing Speculation." *Journal of Political Economy*, 94 (5) (1986), pp. 927-952.

Jorion, P. "Fallacies in the Effects of Market Risk Management Systems." Working paper, University of California, Irvine, July 2002.

Kambhu, J., and P. C. Mosser. "The Effect of Interest Rate Options Hedging on Term-Structure Dynamics." *Federal Reserve Bank of New York Quarterly Review*, December 2001.

Miles, D. "The UK Mortgage Market: Taking a Longer Term View." Report for Her Majesty's Treasury, 2004.

Naranjo, A., and A. Toevs. "The Effects of Purchases of Mortgages and Securitization by Government Sponsored Enterprises on Mortgage Yield Spreads and Volatility." *Journal of Real Estate Finance and Economics*, 25 (2/3) (2002), pp. 173-195.

Perli, R., and B. Sack. "Does Mortgage Hedging Amplify Movements in Long-Term Interest Rates?" *The Journal of Fixed Income*, 13 (3) (2003), pp. 7-17.

Persaud, A. "Sending the Herd off the Cliff Edge: The Disturbing Interaction Between Herding and Market Sensitive Risk Management Practices." *Journal of Risk Finance*, 2 (2000), pp. 59-65.

"Report of the Presidential Task Force on Market Mechanisms." The Brady Commission, 1988.