The Term Structure of Sovereign Spreads in Emerging Markets

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The strong performance of investment returns in emerging markets during recent years seems to have confirmed their role in international investment portfolios, despite the crises occurring during the second half of the 1990s. As noted in the "Global Financial Stability Report" published by the International Monetary Fund in 2004, strong risk-adjusted returns in emerging securities, especially in sovereign bonds, have led many institutional investors to make strategic portfolio allocations in emerging markets.

Reallocations have been driven by improved emerging market fundamentals and also by the exceptionally low short-term interest rates in the major financial centers. These facts have created an excess of liquidity since 2001, especially in 2003, when the emerging market sovereign spread fell from historical high levels.

Exhibit 1 shows the total return and the annualized daily volatility of the EMBI Global, the S&P 500, and the GBI-US from January 1998 through July 2004.

Credit spread, defined as the yield difference between a risky and a riskless bond with similar characteristics, is related to the implied default probability and the credit risk of the issuer. Implied default probabilities are crucial for credit portfolio risk management and for pricing credit derivatives such as credit default swaps (CDS).[1]

There are two broad financial approaches to assess, price, and manage credit risk: structural models, and reduced-form models. Structural models, initially proposed by Merton [1974], adopt an option-based approach. The risky bond is modeled as a contingent claim over some measure related to the economic or financial condition of the debtor that triggers the default event—defined as when such a measure crosses a critical barrier. One makes assumptions about the recovery of capital and uses an interest rate model to derive the default probability endogenously along with the term structure of credit spread. Extensions of Merton's model include Black and Cox [1976], Leland [1994], Longstaff and Schwartz [1995], Saá-Requejo and Santa-Clara [1999], and Zhou [1997].

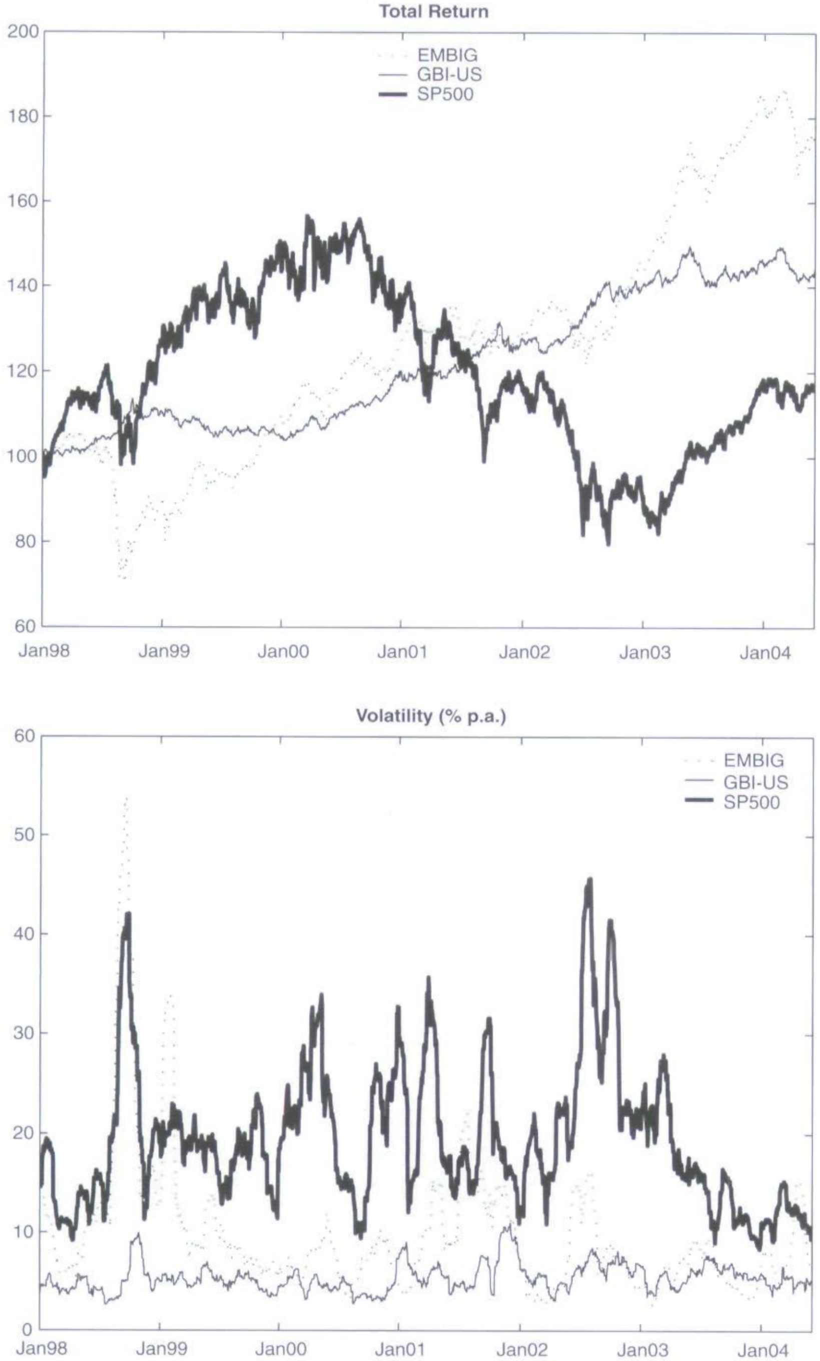
Reduced-form models, developed by Duffie and Singleton [1999] among others, take default as an unpredictable event governed by a hazard rate process, where the credit spread is not explicitly related to the financial state or economic condition of the bond issuer.

There is no agreement on the appropriate model to evaluate credit spreads, but the general opinion is that structural models are better for explaining and reduced-form models better for forecasting. Sarig and Warga [1989] empirically investigate the term structure of corporate credit spreads and find that they appear to conform to the theoretical predictions of Merton's model.

Collin-Dufresne, Goldstein, and Martin [2001] suggest that liquidity proxies drive credit

Exhibit 1

Index Performance



spread changes more than the structural variables. Huang and Huang [2002] show that structural models explain about 60% to 80% of the spreads on corporate bonds rated by Moody's as high-risk obligations (Ba), and roughly 100% of those rated as B, or speculative-grade. Hund [2002] points out the difficulty of reconciling the behavior implied by the structural models with the realities observed in the credit spread market, but Delianedis and Geske [2002] attribute this empirical finding to market incompleteness.

The literature on credit risk models applied to sovereign risk is not straightforward. Cantor and Packer [1996] find sovereign ratings are broadly consistent with macroeconomic fundamentals and spreads. Martins [1997] uses the default risk premium implied from U.S. speculative long-term corporate bonds to price Brady bonds. Lehrbass [1999] develops a structural model based on an equity index of the borrower country expressed in the lender's foreign currency to analyze DM-Eurobonds issued by emerging economies. Hui and Lo [2003] present a structural model based on foreign exchange rates to explain sovereign spreads in South Korea and Brazil. Duffie, Pedersen, and Singleton [2003] construct a reduced-form model for pricing sovereign debt in Russia. Xu and Ghezzi [2002] develop a model that relates the term structure of sovereign spreads in emerging markets to a country's fiscal dynamics, and Moreira and Rocha [2004] introduce a two-factor structural model based on macroeconomic fundamentals and time-varying risk premiums to forecast Brazilian sovereign risk.

We implement a simple calibrated structural model to estimate the term structure of sovereign spreads and the implied default probabilities of a select group of emerging countries representing more than 50% of the EMBIG index. The indicator triggering default is considered to be the real exchange rate of each sovereign with respect to the U.S. dollar.

Although the real exchange rate does not directly represent a country's solvency or liquidity, it has the advantage of being a daily market variable that promptly reflects and captures changes in daily market spreads, unlike the lower-frequency fundamentals (monthly or even quarterly) usually seen in structural models.

The assumption that the real exchange rate is interpreted as an indicator of default depends on the reasoning that depreciation in the domestic exchange rate of a sovereign issuer against the denominated sovereign bond currency (usually the U.S. dollar) puts pressure on the ability of the sovereign issuer to pay its liabilities, increasing country risk. Such an argument is consistent with Reinhart's [2002] results, where 84% of emerging market defaults are associated with currency crises, mainly because of the considerable dollar-denominated debt in such economies.

Therefore, constant currency depreciation may exacerbate fiscal problems when an economy has an open capital account but a relatively small tradable sector. Moreover, according to Kaminsky, Lizondo, and Reinhart [1998], real exchange rates are one particularly useful indicator in anticipating currency crises.

I. MODEL

Let S be the dynamics of the real exchange rate in the martingale equivalent measure, described by the stochastic process of Equation (1), where λ_t is the risk-neutral time-varying drift, σ_t is the volatility parameter, and dz is a Wiener increment:

$$\frac{dS}{S} = \lambda_t dt + \sigma_t dz^* \tag{1}$$

In the absence of arbitrage opportunities, the complete markets assumption implies that the risk-neutral drift in Equation (1) equals the short-term interest rate differential between the sovereign issuer and the U.S. dollar market (see Neftci [2000]). This relation, known as *covered interest rate parity* in international finance, is quite accurate for developed countries, according to Frankel [1993]. Empirical evidence indicates it fails for emerging economies, however, because of country risk that cannot be hedged. Therefore, we assume market incompleteness in emerging economies and estimate the risk-neutral time-varying drift parameter λ_t through calibration with market data. Bates [1991] uses a similar approach in order to explain the 1987 stock market crash.

Default is triggered the first time the real exchange rate variable S crosses the default barrier α, that is, when the exchange rate reaches a value that makes debt repayment unlikely. The moment of default is uncertain and has a probability distribution function (first hitting time) shown in the appendix.

The price of a default-risky zero-coupon bond $B(t, T)$ with a principal of $1 maturing at time T is given by Equation (2), where $P(t, T)$ is the price of a default-riskless zero-coupon bond with the same characteristics; the expectation is taken with respect to the equivalent martingale measure Q; w is the write-down in the case of default; $1_{\{\tau<T\}}$ is an indicator function if a default event occurs prior to maturity; and r is the default-riskless instantaneous rate:[2]

$$B(t,T) = P(t,T) - E_t^Q\left[w 1_{\{\tau<T\}} e^{-\int_t^T r(u)du}\right] \tag{2}$$

Equation (2) can be reexpressed as Equation (3), where $F_t(\tau < T)$ is the risk-neutral default cumulative probability function of a default event occurring before time T:

$$B(t,T) = P(t,T)\left[1 - wF_t(\tau < T)\right] \tag{3}$$

Let the spread $s(t,T)$ be the difference in yield between the risky and riskless bond. Hence, the sovereign spread is given by Equation (4):

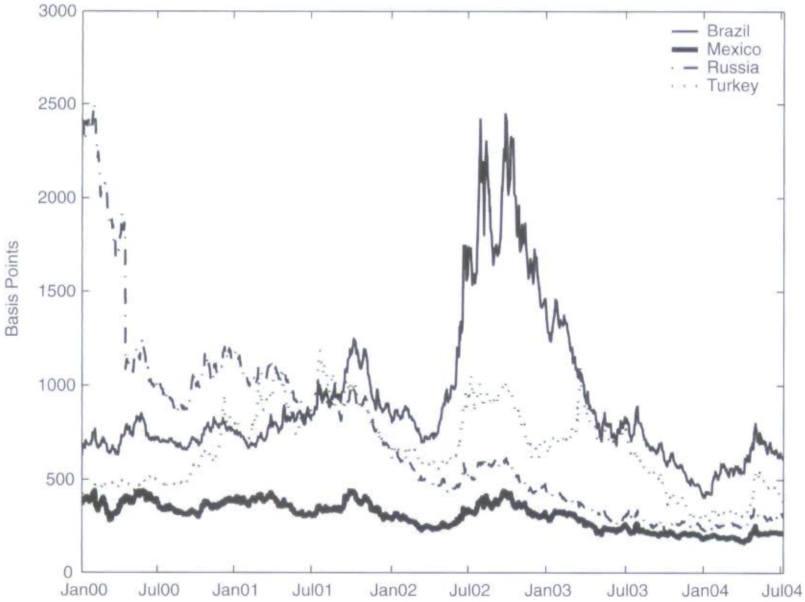
$$\begin{aligned} s(t,T) &= -\frac{1}{(T-t)}\ln\left(\frac{B(t,T)}{P(T,t)}\right) \\ &= -\frac{1}{(T-t)}\ln\left(1 - w.F_t(\tau < T)\right) \end{aligned} \tag{4}$$

II. DATA

An important benchmark for the analysis of risk and returns of worldwide emerging markets appeared with establishment of the J.P. Morgan Emerging Markets Bond Index (EMBI). Successor indexes are the EMBI Plus (EMBI+) and the EMBI Global (EMBIG). The J.P. Morgan EMBIG covers U.S. dollar-denominated Brady bonds, eurobonds, traded loans, and local market debt instruments issued by sovereign and quasi-sovereign entities, with different criteria for inclusion of eligible countries and admitting less liquid instruments than its predecessor EMBI+.

For consistency, we use the sovereign spread and duration implicit in the EMBIG of each country. Sovereign spread is the yield (stripped yield) difference in basis points between a risky and a risk-free instrument with similar characteristics, where the present value of the flows from the collateral is not a factor because the collateral is not subject to sovereign risk. Sovereign spread duration is defined as the percentage price change per basis point

EXHIBIT 2
Sovereign Spreads



The average defaulted debt recovery rate for sovereign bonds is taken from "Moody's Special Comment" [2003], as roughly 40%. Hence, the corresponding write-down value w is 60%.

Nominal exchange rate data for each emerging country are converted into real exchange rates with the consumer price indexes available from the IMF's International Finance Statistics. Exhibit 4 shows the real exchange rate of the four countries since January 1995 in U.S. dollars of July 2004 and the highest rates achieved. Exhibit 5 shows the historical annualized daily volatility parameter σ_t of Equation (1), estimated in moving windows of 60 days since January 2003.

change in the sovereign spread; it can also be interpreted as the average maturity of the index (without collateral).

We choose for our analysis the EMBIG for Brazil, Mexico, Russia, and Turkey, countries that represented roughly 58% of the EMBIG composite on July 15, 2004. Exhibit 2 shows the sovereign spreads of these emerging economies since January 2000. Composition of the EMBIG by country as of July 15, 2004, is detailed in Exhibit 3.

III. RESULTS

Our calibration process is consistent with market practice. The risk-neutral time-varying drift parameter λ_t is calibrated using the most liquid instrument (in our case the benchmark EMBIG of each country) and then used as an input for pricing the less liquid ones (the real instruments included in the country's EMBIG). By constantly updating the calibrated parameter, we can incorporate all market information available to date.

The study period begins in September 2003 and ends in July 2004. The time series of the EMBIG sovereign spread and duration are available on a daily basis, but the real instru-

EXHIBIT 3
EMBIG Index Country Weights (%)

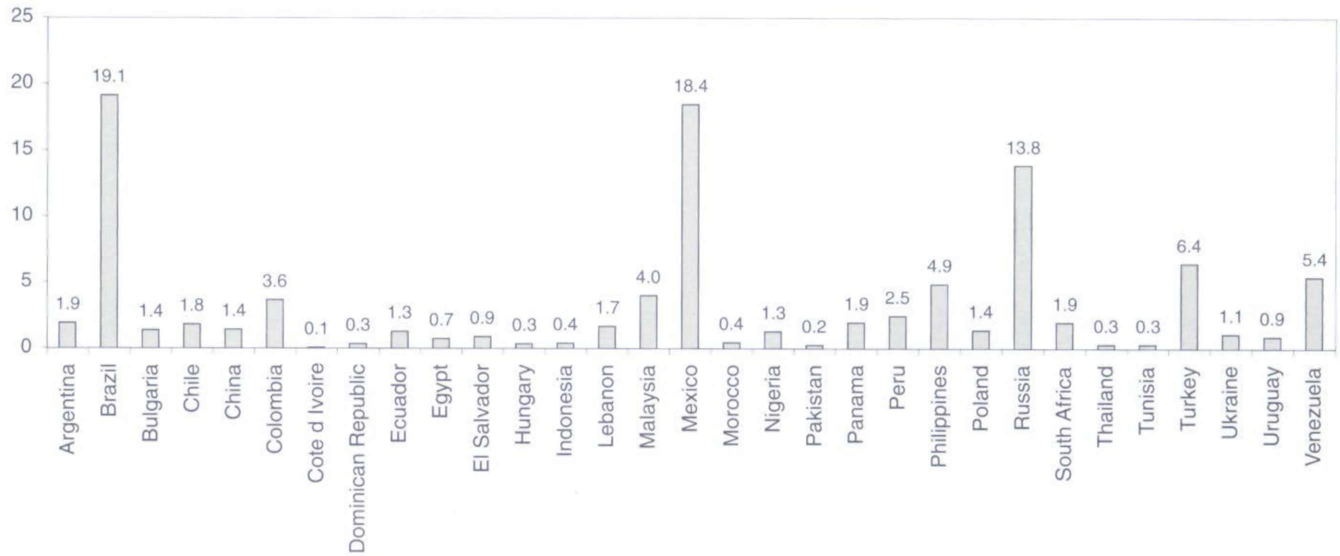
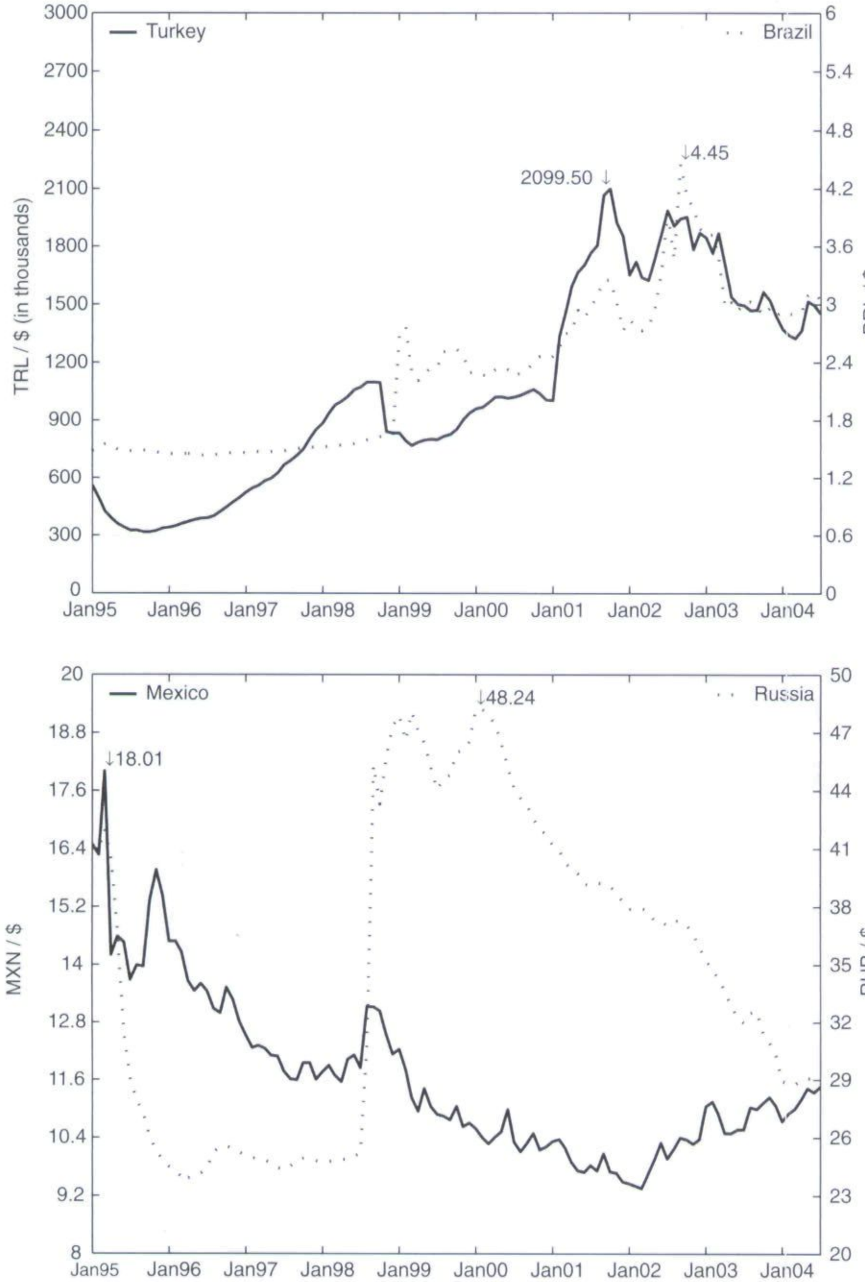


EXHIBIT 4
Real Exchange Rates

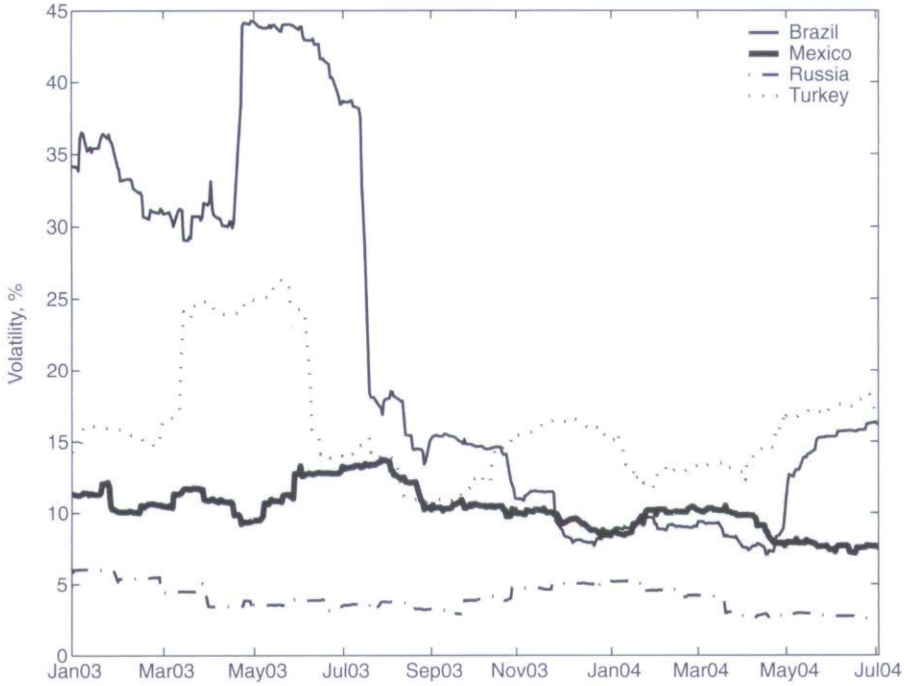


ments used for estimation of the default barrier are taken on a weekly basis because of data access limitations.

The default barrier is estimated by minimizing the mean squared error between the sovereign spreads generated by the model and spreads of the real instruments. While the estimated barrier is updated weekly and kept constant over the next week, the risk-neutral time-varying drift is calibrated using the daily EMBIG sovereign spread and duration for every country. The calibrated parameter is used as an input in Equation (4) to calculate the term structure of sovereign spreads, which we compare to the term sentence observed in the market.

To show how the model fits the actual market data, Exhibit 6 plots the term structure of sovereign spreads generated by the model versus the term structure observed in the market for the last two months of the study. The

EXHIBIT 5
Annual Volatility



Xs in the graphs represent the country's EMBIG used in the sovereign term structure calculation.

The term structure of the sovereign spreads we obtain is consistent with empirical findings on corporate credit spreads, suggesting a hump-shaped term structure for speculative-grade sovereigns (Brazil and Turkey) and an upward-sloping one for investment-grade sovereigns (Mexico and Russia).

Exhibit 7 shows the average of absolute deviations in basis points of the model compared to market data over time. This measure is quite stable during the study period and indicates a more accurate fit for Mexico, Russia, and Turkey than for Brazil, probably because Brazil accounts for roughly 60% of the trading in the EMBIG index, causing extra volatility that cannot be completely explained by the model.

Considering the deviations for the short-, medium-, and long-term sovereign spreads, set to less than three years, from three to five years, and over five years, respectively, we find no strong evidence of a systematic underpricing bias in the short-term spreads generated by the model. Deviations are on average less than 30 basis points for Mexico, Russia, and Turkey, and less than 60 basis points for Brazil, providing evidence of the model's consistency. This evidence is strengthened if we assume that absolute deviations below 50 basis points fall within the model's confidence interval and are therefore not significant for the identification of under- or overpriced spreads.

The average percentage of instruments over the study period with deviations outside this range is shown in Exhibit 8. On average, nearly 28% of the instruments are overpriced for Brazil, 6% for Mexico, and roughly 14% for Russia and 10% for Turkey. If we compute the overpriced spreads over 100 basis points, only the results for Brazil appear to be meaningful (13%), which indicates that the market tends to overprice Brazilian spreads compared to spreads in the three other emerging economies.

When overpricing is defined as the difference between the model versus market spread of more than 50 basis points, Brazil shows an average deviation of 120 basis points. In the case of underpricing, only Brazil appears to be significant with 12% for deviations lower than –50 and 4% for –100 basis points, but the underpricing is only half the level of the overpricing. We find no evidence of a systematic underpricing spread bias for the other countries analyzed.

The average positive deviations over time for all countries are graphed in Exhibit 9.

Exhibit 10 shows the daily implied default probability for three and five years. The implied default probability distribution declines sharply in autumn 2003, after the turmoil of the Argentine crisis in 2002 and the Brazilian elections in October 2002, performance achieved through

Exhibit 6

Model versus Market Spread

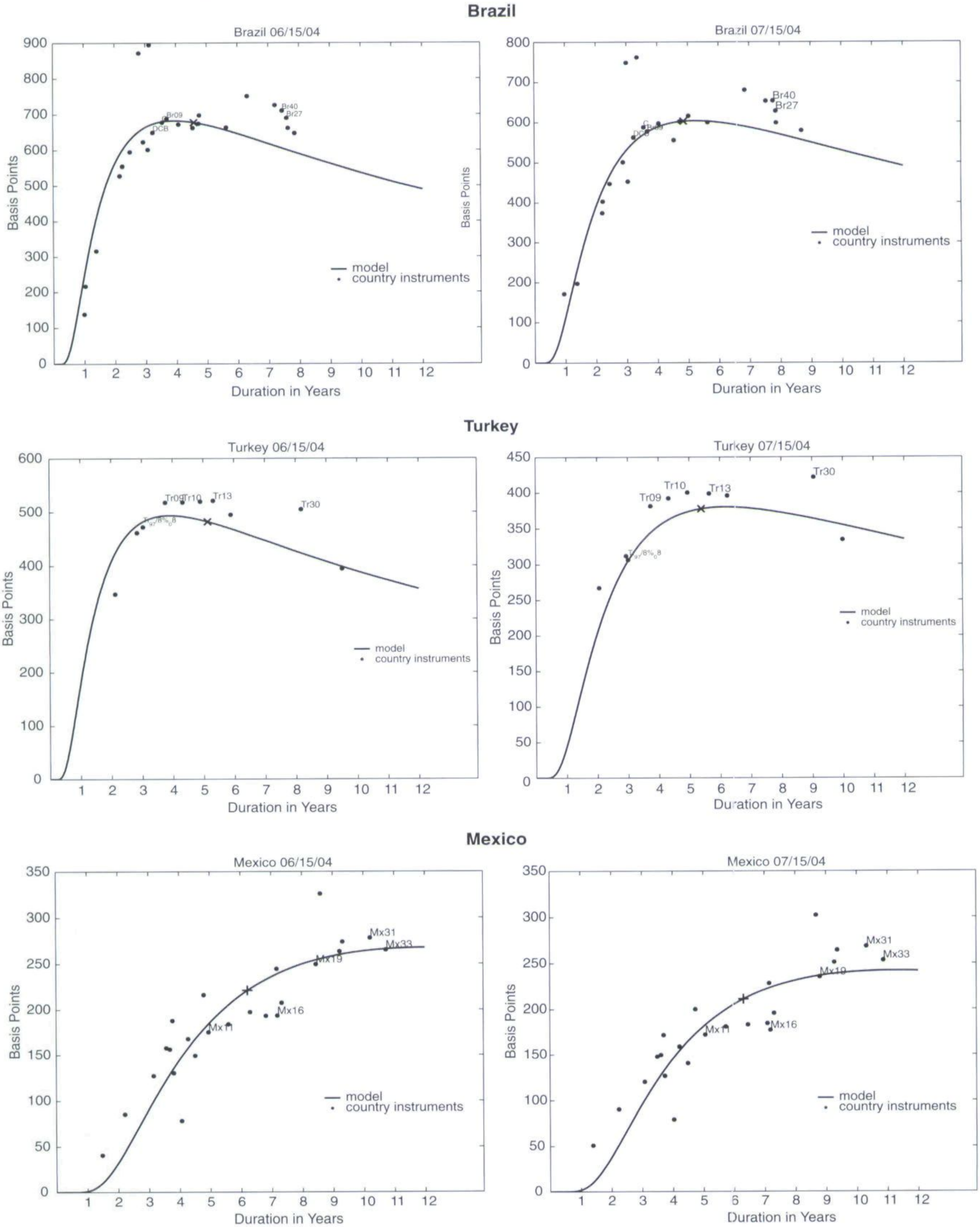


EXHIBIT 6 (continued)
Model versus Market Spread

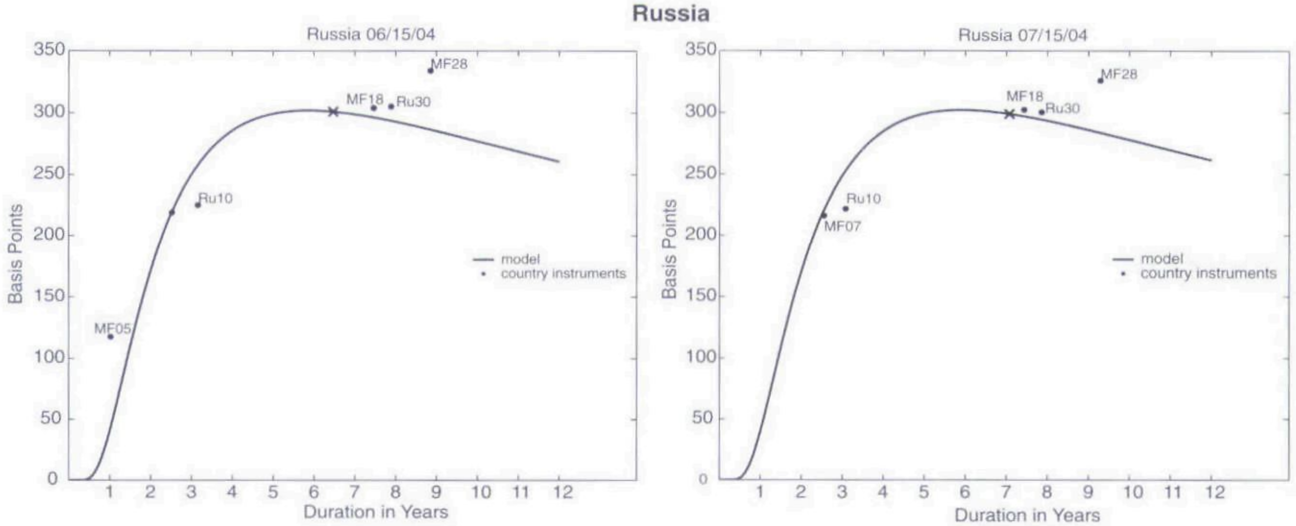
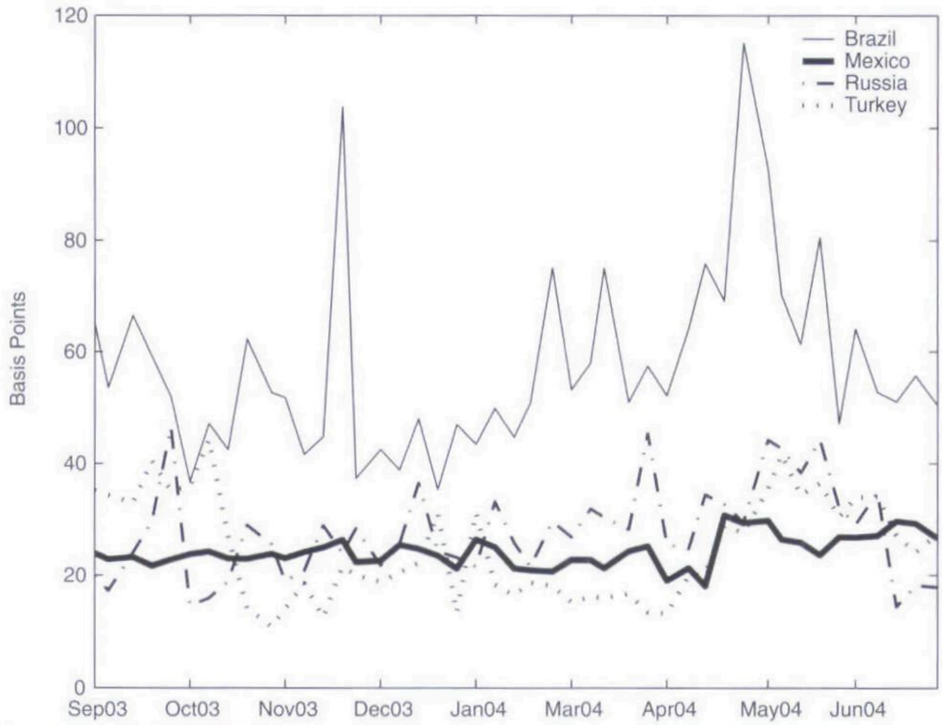


EXHIBIT 7
Average Absolute Deviations



improvements in country-specific fundamentals combined with high global liquidity in 2003. This favorable external environment changed in the beginning of 2004 with the expectation of an increase in U.S. interest rates and rising oil prices, especially in May 2004.

Exhibit 11 compares the estimates of the implied (risk-neutral) cumulative default probability for the last day in the sample (July 15, 2004) and the issuer-weighted cumulative sovereign default rates available in "Moody's Special Comment" [2003]. As expected, the implied (risk-neutral) default probabilities are higher than the historical ones estimated by Moody's.

EXHIBIT 8
Overpricing versus Underpricing

Average Percentage of Instruments in the Range				
	≥ 50 bp	≤ – 50 bp	≥ 100 bp	≤ – 100 bp
Brazil	28.17%	12.04%	12.86%	4.16%
Mexico	5.89%	1.02%	0.00%	0.00%
Russia	14.44%	4.07%	1.48%	0.00%
Turkey	9.95%	2.40%	0.49%	0.49%

IV. SUMMARY AND CONCLUSIONS

We have proposed a structural model for estimating the term structure of sovereign spreads and the implied default probabilities in emerging countries that account for more than 50% of the EMBIG index. The real exchange rate is assumed to trigger the default event, and as a daily market variable it captures changes in daily spread sooner than low-frequency fundamentals (monthly or quarterly).

Once the model is calibrated, the predicted term structure of sovereign spreads agrees with empirical findings, which supports the structural approach. The term structure is hump-shaped for speculative-grade sovereigns (Brazil and Turkey) and upward-sloping for investment-grade countries (Mexico and Russia).

The fit of the model over time indicates robustness, with greater accuracy for Mexico, Russia, and Turkey than for Brazil, probably because of the higher volatility of Brazil's currency. The low average absolute deviations over time of under 30 basis points for Mexico, Russia, and Turkey, and under 60 basis points for Brazil are evidence in favor of the model.

According to the model, the market tends to overprice the spreads for Brazil by 120 basis points on average, but it replicates market behavior in Mexico, Russia, and Turkey. These results are supported by credit rating agency upgrades of the rating for Brazil in September-October 2004, just after our study period. As expected, implied risk-neutral default probabilities are higher than the historical ones available from Moody's.

EXHIBIT 9
Positive Deviations

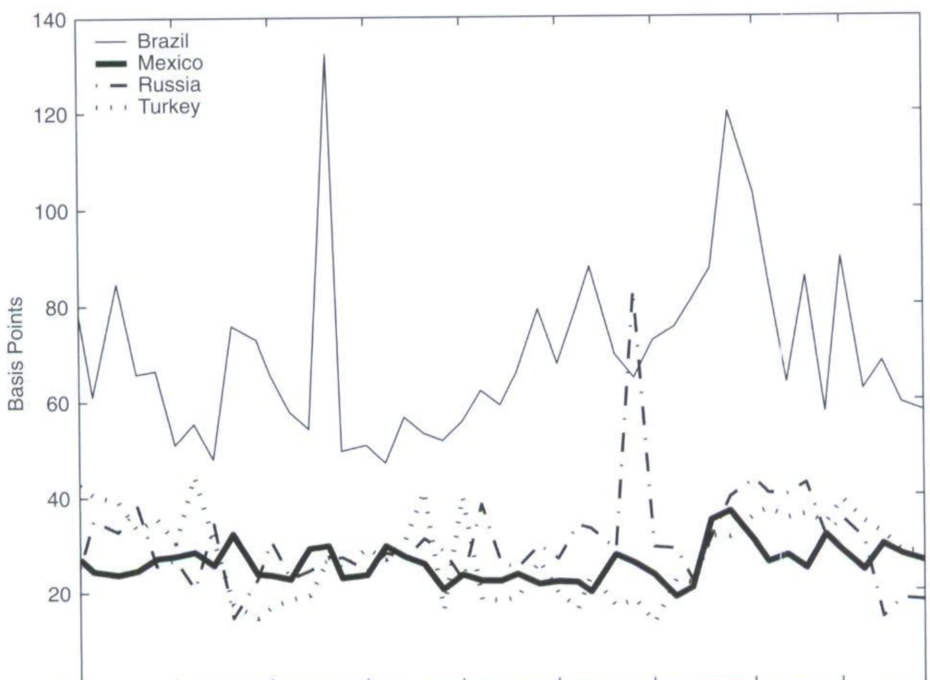
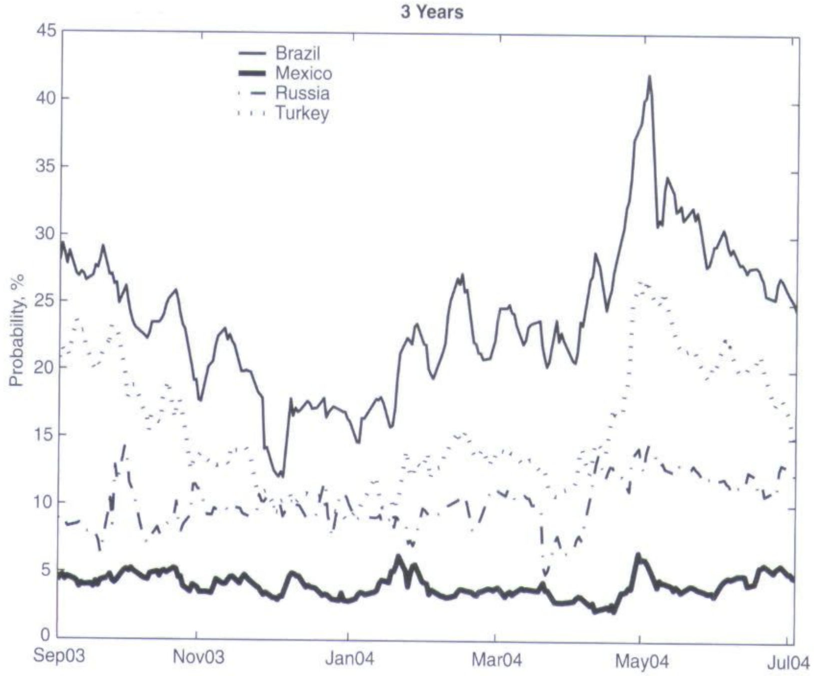


EXHIBIT 10
Implied Default Probabilities



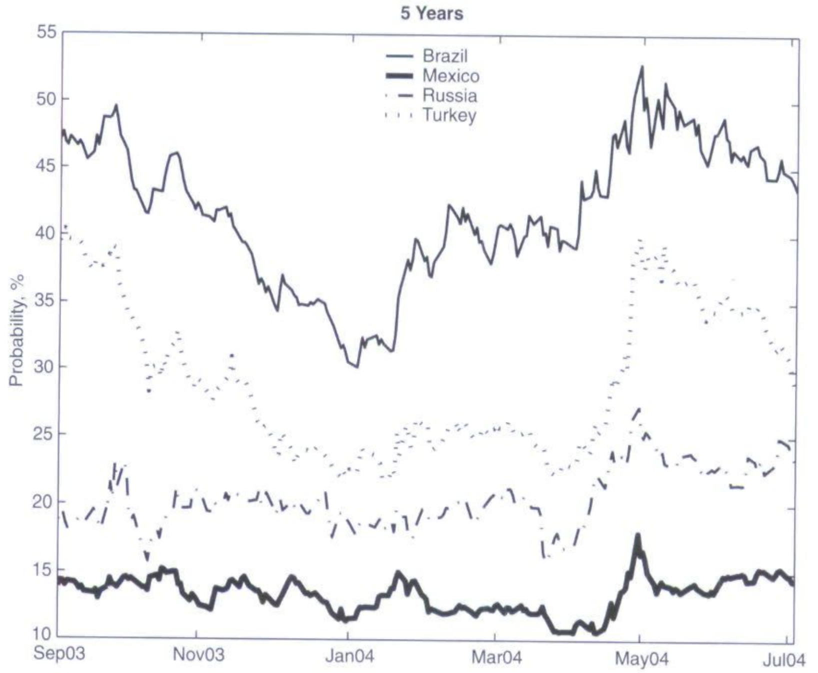


EXHIBIT 11
Cumulative Default Probabilities

	Implied Default Probabilities									
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Mexico Baa2	0.03%	1.26%	4.74%	9.43%	14.43%	19.32%	23.93%	28.19%	32.10%	35.68%
Russia Baa3	0.66%	5.58%	11.98%	17.96%	23.16%	27.64%	31.51%	34.88%	37.84%	40.46%
Turkey B1	0.75%	6.68%	14.57%	21.99%	28.45%	34.01%	38.80%	42.96%	46.60%	49.81%
Brazil B2*	1.88%	12.55%	24.58%	34.93%	43.43%	50.40%	56.18%	61.01%	65.10%	68.59%
Moody's data on Sovereign Default Rate (January 1985–December 2002)										
Baa	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
B	7.89%	14.25%	18.33%	18.33%	22.22%	27.08%	32.69%	38.81%	45.61%	53.38%
Investment Grade	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Speculative Grade	3.87%	7.87%	10.62%	14.19%	16.59%	19.74%	23.75%	28.67%	35.47%	45.39%

**Credit rating agencies upgraded Brazil from B2 to B1 in October 2004, after the conclusion of our study period.*

APPENDIX

First Hitting Time Distribution

Let Equation (A-1) be a stochastic process with dz as a Wiener process:

$$dx = \lambda_t dt + \sigma_t dz \tag{A-1}$$

Following Karatzas and Shreve [1991], the first-passage time density of x evaluated at $\tau > t$, i.e., $\tau = \inf\{t \geq 0, x(t) > 0\}$, is given by Equation (A-2):

$$\pi(\tau | x_t, \lambda_t, \sigma_t) = \frac{|x_t|}{\sigma_t \sqrt{2\pi(\tau - t)^3}} \exp\left[\frac{(x_t + \lambda_t(\tau - t))^2}{2.\sigma_t^2(\tau - t)}\right] \tag{A-2}$$

Through Itô's lemma, we have that if S follows Equation (1), then $x_t = \ln(S_t/\alpha)$ follows the stochastic differential equation:

$$dx = (\lambda_t - 0.5\sigma^2)dt + \sigma_t dz \tag{A-3}$$

Hence, the first-passage time density of S evaluated at $\tau > t$, i.e., $\tau = \inf\{t \geq 0, S(t) \geq \alpha\}$, is given by Equation (A-4):

$$\pi(\tau | S_t, \lambda_t, \sigma_t, \alpha) = \frac{|\ln(S_t/\alpha)|}{\sigma_t \sqrt{2\pi(\tau - t)^3}} \times \exp\left[-\frac{\left(\ln(S_t/\alpha) + \left(\lambda_t - 0.5\sigma_t^2\right).(\tau - t)\right)^2}{2.\sigma_t^2(\tau - t)}\right] \tag{A-4}$$

The cumulative distribution function is given by Equation (A-5), where $\phi(\cdot)$ is the cumulative normal distribution function:

$$F_t(\tau < T) = \left\{ 1 - \phi\left(\frac{\left|\ln\left(\frac{S_t}{\alpha}\right)\right| - \left(\lambda_t - 0.5\sigma_t^2\right).(T - t)}{\sigma_t \sqrt{T - t}}\right) + e^{\left(\frac{2.(\lambda_t - 0.5\sigma_t^2)\cdot\left|\ln\left(\frac{S_t}{\alpha}\right)\right|}{\sigma_t^2}\right)} \phi\left(\frac{-\left|\ln\left(\frac{S_t}{\alpha}\right)\right| - \left(\lambda_t - 0.5\sigma_t^2\right).(T - t)}{\sigma_t \sqrt{T - t}}\right) \right. \tag{A-5}$$

ENDNOTES

The authors thank Estêvão Kopschitz, Paulo Levy, Luis Martins, Ajax Moreira, and João Carlos Scandiuzzi for useful suggestions and discussion.

[1]According to the British Bankers' Association "Credit Derivatives Report (2001/2002)" [2004], the credit derivatives market is the fastest-growing segment of the over-the-counter derivatives market, especially since the Asian and Russian crises. This market has grown from a US$40 billion outstanding notional value in 1996 to an estimated US$4.8 trillion by the end of 2004. In 2002, CDS accounted for roughly 45% of the overall credit derivative market while sovereign CDS represented around 8% of the CDS market.

[2]Write-down is assumed to be constant. We use the recovery of treasury formulations as in Jarrow and Turnbull [1995].

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