

Credit Spread Modeling with Regime-Switching Techniques

ANDREW DAVIES

ANDREW DAVIES

is a research analyst at Man Investments Limited in London and a visiting research fellow at the department of economics, University of Strathclyde.
andrewldavies@hotmail.com

This research considers the determinants of the excess yield earned on corporate bonds over equivalent-duration U.S. Treasury securities. Some empirical studies have reported results that are contrary to the theoretical predictions of the pricing of corporate debt. Merton [1974] argues that theoretically the impact of a higher risk-free rate should be to tighten the corporate spread because higher risk-free rates increase the value of corporate debt. Empirical studies such as Morris, Neale, and Rolph [1998] and Bevan and Garzarelli [2000] using data stretching back to the 1960s find the opposite relationship. Collin-Dufresne, Goldstein, and Martin [2001] use a much more recent data history and find in favor of the theoretical conjecture.

I develop the empirical line of inquiry using a recent data history, and apply a new set of regime-switching estimators that have recently gained in popularity. Key findings are that the corporate spread is inversely related to the risk-free rate in the long run, a result at odds with the finding of Morris, Neale, and Rolph [1998] and Bevan and Garzarelli [2000].

I further find there is significant merit in allowing for distinct regimes in terms both of an increase in explanatory power and identification of interesting economic relationships. Specifically, the analysis identifies the presence of high- and low-volatility regimes. The low-volatility regime is associated with an inverse relationship between the risk-free rate and corporate spreads in the short run. In the

high-volatility regime, the inverse relationship disappears. Under more turbulent market conditions, the corporate spread is determined by changes in yield curve slope, industrial production growth, and equity market returns. All these findings have plausible economic interpretations.

Analysis of the corporate bond market is important for several reasons. In recent years, developments in the sovereign debt market have caused increasing numbers of fixed-income investors to view corporate debt as a major asset class. Globally, shrinking budget deficits have reduced the issuance of sovereign debt relative to corporate issuance. The European Monetary Union has curtailed the opportunity for portfolio diversification at the same time as increasing numbers of firms have sought to finance expansion through debt issuance in preference to alternative sources of funds. Overall this has led to growing interest in corporate debt. Finally, corporate spreads are of interest in a wider sense, in that they have been shown to embody useful information in locating turning points for the macroeconomic business cycle, as in Gertler and Lown [1999], and are commonly used as key components in leading economic indicators.

I. THEORY ON CREDIT SPREADS AND EMPIRICAL FINDINGS

The literature on the determinants of credit spreads can be separated into two broad

categories. The first line of inquiry, with the longer history, has become known as the structural pricing of risky debt literature as developed by Merton [1974], and extended by Longstaff and Schwartz [1995], Leland and Toft [1996], and Duffee [1998]. A second more empirically based approach uses a set of appealing macroeconomic and financial factors to construct statistical models of corporate spreads. This second empirical tradition is typified by the work of Morris, Neale, and Rolph [1998], Bevan and Garzarelli [2000], and Collin-Dufresne, Goldstein, and Martin [2001].

An interesting dichotomy has developed across these two lines of inquiry regarding the risk-free rate and its relationship to the credit spread. The theoretical literature argues in favor of an inverse relationship where increases in the risk-free rate induce a tightening of credit spreads. Morris, Neale, and Rolph [1998] and Bevan and Garzarelli [2000] find the opposite; increases in the risk-free rate induce a widening of credit spreads.

My analysis by considering a more recent data history, as in Collin-Dufresne, Goldstein, and Martin [2001], and by applying recent developments in econometric methodology, finds empirical evidence to support the theoretical predictions of Merton [1974] and similar researchers. In the short run, this relationship is found to depend on regime volatility.

Merton [1974] demonstrates that options pricing theory can be extended to value risky corporate debt. He argues that the purchase of a risky corporate bond is equivalent to being long a risk-free bond and short a put option on the value of the firm with a strike price equivalent to the value of the corporate bond.

Consider a firm owned by shareholders who issue a zero-coupon bond. If for some reason the value of the firm on maturity of that bond is less than the value of the bond, the firm defaults on its debt. Shareholders walk away from the firm by virtue of limited liability, and bondholders receive the remaining value of the firm. The payoff to a corporate bond investor is therefore the face value of the bond less a put option on the value of the firm with a strike price equal to the face value of that bond.

Longstaff and Schwartz [1995] argue that the effect of a higher risk-free rate is to increase the risk-neutral drift of the firm value process. Increases in the extent of that drift process imply a reduced likelihood that the process will breach a given default threshold. A reduced probability of debt default implies a higher price for that debt and a consequent narrowing in the spread over the risk-free rate. Hence, for an increase in the risk-free rate,

the corporate bond investor, in being short a put option, observes an increase in the value of the corporate bond. The corporate yield declines, and the credit spread tightens. From a theoretical standpoint then, the credit spread and the risk-free rate are inversely related.

Merton [1974] also demonstrates that the value of corporate bonds subject to default risk is a function of several other factors in addition to the risk-free rate: the underlying value of the firm, the face value of the debt, the volatility of the value of the firm's underlying assets, and the time to maturity of the bond. Hence, the underlying value of the firm and the volatility of that firm value are also key determinants of the credit spread.

In the empirical literature, Collin-Dufresne, Goldstein, and Martin [2001] and Bevan and Garzarelli [2000] both include an additional set of factors designed to take account of the possibility of firm default.

The empirical study of Morris, Neale, and Rolph [1998], however, focuses on the role of the risk-free rate in isolation. The authors take advantage of recent advances in the time series literature, and use a Johansen [1995] estimator to analyze both long- and short-run relationships between the credit spread and the risk-free rate. Using monthly Moody's credit data for 1960–1997, they find a significant positive relationship between the risk-free rate and the credit spread, a result at odds with the theoretical literature.

Bevan and Garzarelli [2000] also employ cointegration techniques using quarterly Moody's data from 1960 through 1999, but in addition to the risk-free rate apply a variety of additional explanatory variables to capture the risk of default and the overall level of corporate issuance. They find a marginal positive long-run relationship between corporate spreads and the risk-free rate that is not particularly significant. They do, however, find that key determinants of the credit spread are stock market volatility, GDP growth, and a financing gap variable based upon the difference between corporate expenditures and internal revenue.

Collin-Dufresne, Goldstein, and Martin [2001] use Lehman Brothers data at a disaggregated level to examine the changes in corporate spreads from 1988 through 1997 at a monthly frequency. They find evidence to support the theoretical role of the risk-free rate and also find additional explanatory power in aggregate equity returns, equity market volatility, and a proxy for individual firm leverage.

My work is most closely associated with the empirical work of Bevan and Garzarelli [2000] from an econometric perspective and with Collin-Dufresne, Goldstein,

and Martin [2001] as regards the sample period under consideration. The analysis uses an error correction methodology in the spirit of Johansen [1995] that includes several explanatory variables in addition to the risk-free rate. The explanatory variables are included in an attempt to capture the possibility of default and the degree of firm leverage over time. As an extension to the empirical literature, the analysis considers the use of recently developed regime-switching estimation techniques in an attempt to discern alternative phases in the data history.

I argue in favor of the shorter data history from the standpoint of data availability and market efficiency considerations. The first point regarding data availability is relatively straightforward. One of the key findings of this analysis is that market measures of market volatility play a key role in determining credit spread over Treasury yields.

The most commonly followed forward-looking measure of market volatility is derived from the options market of the S&P 500 equity index. The measure, known as the VIX, is constructed on a daily basis by the Chicago Board Options Exchange. The history for this measure stretches back to January 1986, long enough to cover several market cycles. Other empirical work based on a longer data history, such as Bevan and Garzarelli [2000], also use measures of volatility, but these variables tend to be constructed from a historical perspective.¹

Campbell and Taksler [2003] note more recently that commonly used volatility measures based on market index data fail to capture much of the recent movement in corporate spreads. In using the VIX, my analysis employs a variable that attempts to gauge market expectations of future equity market volatility. Given that the VIX is found to behave as a useful regime differentiation variable, the analysis is based on a sample period beginning January 1986.

Another reason to concentrate on recent data has to do with a market efficiency argument. My findings show that the risk-free rate conforms to theoretical predictions, but is contrary to most empirical work that uses a longer history. As markets have likely become more efficient over recent years, formal theoretical explanations of the risk-free rate are likely to be increasingly meaningful over time.

Interest in the corporate bond market at an institutional investor level has developed only over the last 10 to 15 years or so, as yields and the level of issuance of sovereign debt have fallen. When Bevan and Garzarelli [2000] use quarterly data that extend as far back as 1960, it is more likely they have studied market dynamics that are less likely to have been derived from a truly efficient

market. As my original aim is to develop an asset allocation tool for modern fixed-income markets, it would seem that the more recent data history is more relevant for our purposes. Taken together with the fact that previous empirical work will have included the extremely volatile events in the U.S. bond market of the late 1970s and early 1980s, this point implies that the use of the more recent data history is appropriate.²

II. DATA

Corporate bond data are obtained from Moody's Investors Services for the yields derived from AAA and BAA indexes for non-financial seasoned corporate bonds. Moody's indexes are constructed from an equally weighted sample of yields of up to 100 bonds issued by large non-financial corporations with a face value in excess of \$100 million. To calculate the spread over Treasuries, I subtract the yield on the ten-year constant-maturity Treasury yield obtained from the U.S. Federal Reserve data archive.

The average time to maturity for bonds to be included in the Moody's data set is in excess of 20 years. We use a 10-year Treasury rate in an attempt to match duration as opposed to maturity. Corporate bonds, being subject to default risk, have shorter durations than Treasuries of a similar maturity; hence, we would expect the average duration to be relatively close to that of a 10-year Treasury bond. We use a data history spanning January 1986 to February 2003 at monthly frequencies.

Following most empirical and theoretical work, I model corporate bonds as a function of the risk-free rate and a set of factors designed to capture the risk of default and underlying corporate value. I therefore consider the relationship:

$$\text{Credit Spread} = \{\text{Risk-Free Rate, Yield Slope, S\&P 500, Industrial Production}\} \quad (1)$$

The risk-free rate is included because theoretical models of corporate debt suggest that increases in the risk-free rate should reduce the corporate spread over Treasuries. The analysis uses the ten-year constant-maturity yield on U.S. Treasuries, and we expect to find a negative coefficient if the theoretical literature is correct.

The yield curve slope is included as a proxy for future short interest rate movements. A steep yield curve can be taken to imply future increases in the short rate, so increases in the yield curve slope might be expected to induce credit spread tightening. At the same time, a

decline in the yield curve slope could be interpreted as an indication of a deterioration in the wider macroeconomic outlook. Since the risk of default and, in the event of that default, the recovery rate are inversely related to the overall economic environment, again we expect to find an inverse relationship between yield curve slope and the credit spread. Yield curve slope is measured as the difference between ten- and two-year constant-maturity Treasury yields, again obtained from the U.S. Federal Reserve.

A primary determinant of the probability of financial distress is a firm's leverage. The book value of a firm's outstanding debt is likely to be considerably less volatile than the market value of its equity. Kwan [1996] finds a negative relationship between changes in stock prices and the future change in bond yields at the level of the individual firm. Hence, we would expect a positive stock market return to be associated with a reduction in leverage and thus a lower probability of financial distress. We use data from the S&P 500 equity index as a proxy for the degree of overall leverage, and expect it to be inversely related to the credit spread.

The final core variable in the analysis is the level of industrial production, also obtained from the Federal Reserve data archive. Increases in industrial production will improve corporate profitability, thereby reducing the probability of default. Hence, industrial production is also expected to enter the analysis with a negative sign.

In the initial single-regime analysis, I also considered the use of volatility measures as central determinants of corporate bond spreads, but in the multivariate setting they were found to add little. The specific measure used here for volatility is obtained from the Chicago Board Options Exchange and is known as the VIX. It is constructed as a weighted average of the implied volatilities of eight options on the S&P 500 index, each with 30 days to maturity. The VIX is widely followed by market participants because it is a measure of expected future volatility, and thus represents a more forward-looking volatility variable than simple adaptively derived volatility measures.

III. ESTIMATION

I use several time series estimation techniques that have been developed recently to model non-stationary $I(1)$ variables. The results demonstrate that every series I consider is integrated of order 1, so the data must be treated with care.

Initial estimation results are based upon the estimation technique developed by Johansen in a series of arti-

cles in the early 1990s and summarized in Johansen [1995]. Its appeal is that it is a system-based estimator and thus generates models that can be used relatively easily for simulation and forecasting purposes. As the analysis stems from a desire to develop fixed-income asset allocation tools the development of forecasting and simulation results is an obvious requirement.

I generalize the models by considering the possibility of distinct regimes in the data history. Recent advances in the time series literature, represented by Krolzig [1997] in the case of Markov regime techniques and summarized by Franses and van Dijk [2000] and Franses, van Dijk, and Teasvirta [2000] in the case of deterministic regimes, have prompted many empirical applications based on the concept of a regime. Many financial market practitioners have long held that there are distinct phases within financial market history. Those phases might be based on low versus high volatility, deflation versus inflation, growth versus recession, bull versus bear markets, technical factors versus fundamentals, and so on. Whatever the specific nature of the regime-generating process, the point is that advances in the time series literature have provided tools to test for distinct regimes within a data history.

To test for separate regimes in the U.S. corporate bond market, I use Markov switching vector autoregression (VAR) estimation techniques developed by Krolzig [1997] and a self extracting threshold autoregressive (SETAR) model applied to the vector error correction model (VECM) of Johansen as detailed by Franses and van Dijk [2000].

The basic form of the Johansen estimator is based on the vector error correction form of a standard VAR:

$$\Delta z_t = \Gamma_1 \Delta z_{t-1} + \dots + \Gamma_{k-1} \Delta z_{t-k+1} + \Pi z_{t-k} + u_t \quad (2)$$

where z_t is an $(n \times 1)$ vector of endogenous variables, in this case the credit spread, the risk-free rate, the yield curve slope, the S&P 500 index, and the industrial production index, all in levels form. The vector Δz_t is the first difference transformation of z_t . Written in this form, the model includes information on the long-run behavior of the system (in the Π matrix of coefficients) and short-run dynamic components (in the Γ_j matrices).

Estimation of this model proceeds in two steps. Under the assumption that the levels data in z_t are $I(1)$, it follows that the data in Δz_t must be $I(0)$. Hence, for the residual vector u_t to be white noise, it follows that Πz_{t-k} must be $I(0)$. The first step then in the Johansen proce-

ture is to estimate a matrix Π that yields $\Pi z_{t-k} \sim I(0)$.

Essentially the estimator searches for combinations of the levels $I(1)$ data that have high canonical correlations with $I(0)$ first-difference data. The size of those correlations determines the degree of cointegration across the levels series. With a matrix of long-run coefficients, the remaining dynamic coefficients in the Γ_i matrices can be estimated by a straightforward application of ordinary least squares. Greater detail regarding the estimation technique can be found in Johansen [1995].

The second estimator generalizes the Johansen model by assuming a regime generation process that is not directly observable but is instead determined by a first-order Markov process s_t .³ A model based on two regimes takes the form:

$$\Delta z_t = \begin{cases} \Gamma_{0,1} + \Gamma_{1,1}\Delta z_{t-1} + \dots + \Gamma_{k,1}\Delta z_{t-k+1} + \alpha_1 \beta' z_{t-k} + u_t & \text{if } s_t = 1 \\ \Gamma_{0,2} + \Gamma_{1,2}\Delta z_{t-1} + \dots + \Gamma_{k,2}\Delta z_{t-k+1} + \alpha_2 \beta' z_{t-k} + u_t & \text{if } s_t = 2 \end{cases} \quad (3)$$

Hence, the corporate spread equation is expressed as:

$$\Delta Spread_t = \begin{cases} \phi_{0,1} + \phi_{1,1}\Delta Spread_{t-1} + \phi_{2,1}\Delta 10YearYield_{t-1} + \phi_{3,1}\Delta Slope_{t-1} + \\ \phi_{4,1}\Delta Log[S\&P500]_{t-1} + \phi_{5,1}\Delta Log[IndustrialProduction]_{t-1} + \\ \phi_{6,1}ErrorCorrectionTerm_{t-1} + \varepsilon_t & \text{if } s_t = 1 \\ \phi_{0,2} + \phi_{1,2}\Delta Spread_{t-1} + \phi_{2,2}\Delta 10YearYield_{t-1} + \phi_{3,2}\Delta Slope_{t-1} + \\ \phi_{4,2}\Delta Log[S\&P500]_{t-1} + \phi_{5,2}\Delta Log[IndustrialProduction]_{t-1} + \\ \phi_{6,2}ErrorCorrectionTerm_{t-1} + \varepsilon_t & \text{if } s_t = 2 \end{cases} \quad (4)$$

where the error correction term is derived from the residuals obtained in the first step of the Johansen procedure.

Rewriting the system specification yields:

$$\Delta z_t = \Gamma_{0,s_t} + \Gamma_{1,s_t}\Delta z_{t-1} + \dots + \Gamma_{k,s_t}\Delta z_{t-k+1} + \alpha_{s_t} \beta' z_{t-k} + u_t \quad (5)$$

where the residuals are conditionally Gaussian as: $u_t | s_t \sim \text{NID}[0, \Sigma(s_t)]$

The estimation problem is to obtain $\theta = (\phi_1, \phi_2, \rho_{11}, \rho_{22}, \sigma_2)$, where the ϕ_i are the autoregressive coefficients for regime i , i.e., $\phi_i = (\Gamma_{0,i}, \Gamma_{1,i}, \dots, \Gamma_{k,i}, \alpha_i)$; ρ_{ij} is the transition probability of switching from regime i to j ; and σ^2 is the residual variance. More detail regarding the Markov switching estimator is provided in Appendix A.

The estimator is an application of the expectations maximization (EM) algorithm developed by Dempster, Laird, and Rubin [1977]. Hamilton [1990] demonstrates

that the algorithm increases the value of the log-likelihood function at each step and converges asymptotically to the global maximum.

The third and final estimator is a self-extracting threshold autoregressive (SETAR) model applied to the Johansen VECM to search for a regime generation variable. The spread equation takes the form:

$$\Delta Spread_t = \begin{cases} \phi_{0,1} + \phi_{1,1}\Delta Spread_{t-1} + \phi_{2,1}\Delta TBill_{t-1} + \phi_{3,1}\Delta Slope_{t-1} + \\ \phi_{4,1}\Delta Log[S\&P500]_{t-1} + \phi_{5,1}\Delta Log[VixIndex]_{t-1} + \\ \phi_{6,1}\Delta Log[IndustrialProduction]_{t-1} + \\ \phi_{7,1}ErrorCorrectionTerm_{t-1} + \varepsilon_t & \text{if } Log[VixIndex]_{t-1} \leq c \\ \phi_{0,2} + \phi_{1,2}\Delta Spread_{t-1} + \phi_{2,2}\Delta TBill_{t-1} + \phi_{3,2}\Delta Slope_{t-1} + \\ \phi_{4,2}\Delta Log[S\&P500]_{t-1} + \phi_{5,2}\Delta Log[VixIndex]_{t-1} + \\ \phi_{6,2}\Delta Log[IndustrialProduction]_{t-1} + \\ \phi_{7,2}ErrorCorrectionTerm_{t-1} + \varepsilon_t & \text{if } Log[VixIndex]_{t-1} > c \end{cases} \quad (6)$$

In vector error correction form, we can write:

$$\Delta z_t = (\phi_{0,1} + \Gamma_{1,1}\Delta z_{t-1} + \dots + \Gamma_{k,1}\Delta z_{t-k+1} + \alpha_1 \beta' z_{t-k})I[y_{t-1} \leq c] + (\phi_{0,2} + \Gamma_{1,2}\Delta z_{t-1} + \dots + \Gamma_{k,2}\Delta z_{t-k+1} + \alpha_2 \beta' z_{t-k})I[y_{t-1} > c] + \varepsilon_t \quad (7)$$

And more compactly as:

$$\Delta z_t = \phi_1 \Delta z_{t-p} I[y_{t-1} \leq c] + \phi_2 \Delta z_{t-p} I[y_{t-1} > c] + \varepsilon_t \quad (8)$$

where $\phi_j = [\phi_{0,j}, \Gamma_{j,1}, \dots, \Gamma_{j,k-1}, \alpha_j]$, $j = 1, 2$, and $\Delta z_{t-p} = [1, \Delta z_{t-1}, \dots, \Delta z_{t-k+1}, \beta' z_{t-k}]'$.

Appendix B provides further detail regarding this estimator. The procedure involves searching for a value of the regime-generating variable that minimizes the residual variance.

IV. RESULTS

Preliminary results involve formally testing the time series properties of all the data series in the analysis. Casual inspection of each series through time indicates that every series appears to be non-stationary, or, more formally, integrated of order 1, $I(1)$, in levels form. This is confirmed when we look at the results in Exhibit 1. Aug-

EXHIBIT 1

Unit Root—Augmented Dickey-Fuller Tests

Variable	ADF Statistic with Constant {Number of Lags in Braces}	ADF Statistic with Trend {Number of Lags in Braces}	Conclusion
AAA Spread	-1.7642 {2}	-2.9614 {2}	I[1]
Δ AAA Spread	-8.1921** {3}	-8.2583** {3}	I[0]
BAA Spread	-1.0742 {6}	-1.7738 {6}	I[1]
Δ BAA Spread	-7.9750** {3}	-8.1146** {3}	I[0]
BAA – AAA Spread	-2.0915 {5}	-1.7212 {5}	I[1]
Δ BAA – AAA Spread	-5.8612** {7}	-6.1065** {7}	I[0]
Yield Curve Slope	-2.3882 {3}	-2.3993 {3}	I[1]
Δ Yield Curve Slope	-6.3614** {2}	-6.3415** {2}	I[0]
Log S&P 500 Index	-1.7103 {0}	-0.3859 {0}	I[1]
Δ Log S&P 500 Index	-14.3136** {0}	-14.1143** {0}	I[0]
Log Vix Index	-1.6449 {4}	-1.9039 {4}	I[1]
Δ Log Vix Index	-7.4579** {6}	-7.5056** {6}	I[0]
Log Industrial Production Index	-0.9429 {3}	-1.4303 {3}	I[1]
Δ Log Industrial Production Index	-4.9855** {2}	-5.0049** {2}	I[0]

5% and 1% critical Dickey-Fuller statistics are -2.88 and -3.46 for the constant model and -3.43 and -3.99 for the trend model. Statistical significance denoted by * and ** at the 5 and 1% levels, respectively. Statistical significance in bold.

EXHIBIT 2

Johansen Cointegration Diagnostics

AAA System			BAA System		
Eigenvalue	λ -max	Trace	Eigenvalue	λ -max	Trace
0.2024	46.14*	82.73**	0.1561	34.62	74.87*
0.0926	19.82	36.60	0.0962	20.64	40.25
0.0521	10.91	16.77	0.0576	12.11	19.61
0.0238	4.92	5.87	0.0280	5.80	7.51
0.0047	0.95	0.95	0.0083	1.71	1.71

** and * Significant at the 1 and 5% levels, respectively, based on Osterwald-Lenum [1992] test statistics adjusted by Reimers [1992] small-sample correction factor. Statistical significance in bold.

mented Dickey-Fuller tests demonstrate that the data in levels form are indeed non-stationary, but first-difference transformations yield series that have desirable stationary I(0) properties.

Given the results in Exhibit 1, it seems logical to continue by estimating a vector error correction model (VECM) using techniques suggested by Johansen [1988], Johansen and Juselius [1990, 1994], and Johansen [1995]. Results in Exhibit 2 provide details of the Johansen test for reduced rank r of the Π matrix relating to the levels relationship in the VECM.⁴

Results for each system give the five highest eigenvalues associated with each eigenvector estimated by the Johansen procedure. The eigenvalues are then converted into the λ -max and trace statistics, and these statistics are compared to critical values tabulated by Osterwald-Lenum [1992], which have exhibited a biased tendency toward

finding cointegration; see, for example, Reimers [1992] and Cheung and Lai [1993].⁵

We therefore make a small sample correction to the critical values as suggested by Cheung and Lai [1993], and find that for each system there is at most one significant cointegrating relationship across the levels series in each system.

Results in Exhibit 3 provide the estimated long-run cointegrating coefficients on the assumption that there is one cointegrating relationship in each system. Two separate relationships are presented, one general and one restricted. In the general specification, all five variables enter with unrestricted coefficients in the long-run specification.

The estimates demonstrate that the risk-free rate, the yield curve slope, and industrial production are inversely related, while the equity variable is positively related to the credit spread variable in the levels long-run relationship. The expectation was that equity, industrial production, and slope should all have a negative effect on the spread variable of interest. The equity variable therefore enters the long-run specification with the incorrect sign. The yield curve slope, equity, and industrial production variables do, however,

appear to be jointly insignificant as demonstrated by the results of a χ^2 test for coefficient restrictions.

Of more interest is that the risk-free rate enters the long-run specification with a negative sign. For the data history considered here, then, an increase in the risk-free rate is expected to induce a tightening of the credit spread over the long run. This is contrary to the findings in empirical work of Morris, Neale, and Rolph [1998] and Bevan and Garzarelli [2000]. This finding is almost certainly the result of considering the more recent data history and ignoring the sharp movements in interest rates witnessed in the late 1970s and early 1980s.

As a primary aim is to consider the more recent data history in isolation, and to ignore the earlier more volatile data covered in other studies, this result should not be too surprising. The result is important, though, because it lends support to the theoretical asset pricing literature of

EXHIBIT 3

Long-Run Cointegrating Relationships

AAA System	α	β	δ	γ
General	-0.259 [-0.056]	-0.022 [-0.059]	0.097 [0.280]	-0.809 [-1.114]
	Restriction			
Restricted	-0.217 [-0.025]	$\chi^2_{[3]} : \beta = \delta = \gamma = 0$ 0.84 p-value = 0.84		
BAA System	α	β	δ	γ
General	-0.359 [-0.123]	-0.133 [-0.131]	0.326 [0.617]	-3.166 [-2.459]
	Restriction			
Restricted	-0.079 [-0.017]	$\chi^2_{[3]} : \beta = \delta = \gamma = 0$ 2.31 p-value = 0.51		

Standard errors in brackets.

EXHIBIT 4

Short-Run Dynamics

AAA System	Constant ϕ_0	Spread _{t-1} ϕ_1	10 Year _{t-1} ϕ_2	Slope _{t-1} ϕ_3	S&P _{t-1} ϕ_4	Ind P _{t-1} ϕ_5	ECT _{t-1} ϕ_6
$\Delta \text{Spread}_t =$	0.324 {3.857}	0.364 {3.903}	-0.008 {-0.177}	-0.109 {-1.880}	-0.765 {-4.746}	-3.981 {-2.384}	-0.115 {-3.729}
Diagnostics	$\sigma_e^2 = 0.1042$ SK = -0.2004 ARCH[1-7] = 1.2337 [0.2826]		$\bar{R}^2 = 0.256$ EK = 2.3141 AR[1-7] = 1.5680 [0.1469]		DW = 1.96 Norm = 33.102 [0.0000]** AIC = -14.9540		
BAA System	Constant ϕ_0	Spread _{t-1} ϕ_1	10 Year _{t-1} ϕ_2	Slope _{t-1} ϕ_3	S&P _{t-1} ϕ_4	Ind P _{t-1} ϕ_5	ECT _{t-1} ϕ_6
$\Delta \text{Spread}_t =$	0.226 {3.552}	0.320 {3.740}	-0.027 {-0.570}	-0.145 {-2.294}	-0.995 {-5.595}	-4.458 {-2.473}	-0.064 {-3.349}
Diagnostics	$\sigma_e^2 = 0.1156$ SK = 0.1689 ARCH[1-7] = 0.8388 [0.5564]		$\bar{R}^2 = 0.278$ EK = 0.6084 AR[1-7] = 2.9479 [0.0059]**		DW = 1.93 Norm = 4.6753 [0.0966] AIC = -14.5300		

T-statistics in braces. Statistical significance in bold.

σ_e^2 refers to the residual variance. DW is the Durbin-Watson statistic. SK is skewness. EK is excess kurtosis. Norm is the Doornik and Hansen [1994] small-sample corrected test for normality of the residuals. ARCH is the autoregressive conditional heteroscedasticity test of Engle [1982]. AR is the portmanteau test for residual autocorrelation as in Godfrey [1978]. Statistical significance denoted by * at the 1% level.

Merton [1974] and Longstaff and Schwartz [1995], who argue that the spread variable should be inversely related to the risk-free rate.⁶

The second set of results for each system in Exhibit 3 is for a restricted set of coefficient estimates obtained by imposing zero coefficients on each of the yield curve slope, equity, and industrial production variables. A χ^2 test for each system shows that these restrictions are valid for each system, demonstrating that the key underlying

long-run relationship is a strong inverse relationship between the spread and risk-free rates. The relationship appears to be weaker for the lower-grade spread variable, lending support to Duffie [1998], who argues that the relationship should be stronger for investment-grade credit.

Exhibit 4 provides coefficient estimates for the short-run part of the system, focusing on the spread variable equation in first differences. Here the results are in line with theoretical priors. Equity return and industrial production growth, and the yield curve slope in the case of the BAA spread equation, enter significantly with negative coefficients. Increases in all these variables in the short run induce a tightening of the credit spread over Treasuries.

There also appears to be a strong degree of persistence in the spread variable itself since there is a large positive coefficient on the lagged spread variable. The error correction terms enter each specification with the anticipated negative sign, indicating that the spread variable is expected to converge toward its estimated equilibrium level. Overall the residuals for each equation appear to be relatively well behaved, and the level of explanatory power appears to be high for a financial market forecasting equation.

Perhaps the most interesting result in Exhibit 4, though, is that the risk-free rate appears to have no explanatory power whatsoever over the short-run movement of the spread variable. This is in contrast to all previous empirical work with either a longer data history or less frequent observations.

Exhibit 5 shows the results of estimating a Markov switching vector error correction model (MS-VECM) that allows for regime shifts in autoregressive coefficients and residual variance.⁷ The graphs in Exhibit 6 provide regime classification information.

Exhibit 5 shows the benefits of allowing for two regimes in the analysis. The log-likelihood value rises markedly, and the likelihood ratio statistic is statistically significant at the 1% level. The graphs in Exhibit 6 demonstrate that regime 1 generally corresponds to periods of low

EXHIBIT 5

Regime-Switching Model—Short-Run Dynamics

AAA System	Constant	Spread _{t-1}	10 Year _{t-1}	Slope _{t-1}	S&P500 _{t-1}	Ind Prod _{t-1}	ECT _{t-1}
	$\phi_{0,1}$	$\phi_{1,1}$	$\phi_{2,1}$	$\phi_{3,1}$	$\phi_{4,1}$	$\phi_{5,1}$	$\phi_{6,1}$
$\Delta \text{Spread}_t =$	0.264	0.140	-0.036	0.004	-0.188	-2.151	-0.103
	{4.171}	{1.515}	{-0.892}	{0.059}	{-1.132}	{-1.216}	{-4.284}
	$\phi_{0,2}$	$\phi_{1,2}$	$\phi_{2,2}$	$\phi_{3,2}$	$\phi_{4,2}$	$\phi_{5,2}$	$\phi_{6,2}$
	0.264	0.509	0.019	-0.232	-1.080	-4.013	-0.088
	{4.171}	{2.966}	{0.214}	{-2.421}	{-4.002}	{-1.420}	{-3.781}
Regime Properties	T Matrix	Regime 1	Regime 2	Observations	Probability	Duration	
	Regime 1	0.8076	0.1924	105.6	0.5222	5.20	
	Regime 2	0.2013	0.7897	98.4	0.4778	4.75	
Regime Diagnostics	Log Like ^h	AIC	Like ^h Ratio			Davies Test	
	Linear	1580.31	-14.9540	Linearity	$\chi^2_{[57]}$		
	2 - Regime	1649.37	-15.1213	138.127	[0.0000]**	[0.0000]**	
Equation	$\sigma_\varepsilon^2 = 0.1841$		$\bar{R}^2 = 0.276$			DW = 1.93	
Diagnostics	SK = 0.7281		EK = 0.7687			Norm = 18.100 [0.0001]**	
	ARCH[1-7] = 3.1534 [0.0035]**		AR[1-7] = 1.4414 [0.1909]			AIC = -15.1213	
BAA System	Constant	Spread _{t-1}	10 Year _{t-1}	Slope _{t-1}	S&P500 _{t-1}	Ind Prod _{t-1}	ECT _{t-1}
	$\phi_{0,1}$	$\phi_{1,1}$	$\phi_{2,1}$	$\phi_{3,1}$	$\phi_{4,1}$	$\phi_{5,1}$	$\phi_{6,1}$
$\Delta \text{Spread}_t =$	0.244	-0.105	-0.166	-0.034	-0.064	-3.528	-0.084
	{4.803}	{-0.949}	{-3.808}	{-0.64}	{-0.307}	{-2.274}	{-4.992}
	$\phi_{0,2}$	$\phi_{1,2}$	$\phi_{2,2}$	$\phi_{3,2}$	$\phi_{4,2}$	$\phi_{5,2}$	$\phi_{6,2}$
	0.244	0.412	0.042	-0.252	-1.250	-4.954	-0.064
	{4.803}	{3.547}	{0.574}	{-2.388}	{-5.029}	{-1.717}	{-4.191}
Regime Properties	T Matrix	Regime 1	Regime 2	Observations	Probability	Duration	
	Regime 1	0.8788	0.1212	92.6	0.4686	8.25	
	Regime 2	0.1069	0.8931	111.4	0.5314	9.36	
Regime Diagnostics	Log Like ^h	AIC	Like ^h Ratio			Davies Test	
	Linear	1537.06	-14.530	Linearity	$\chi^2_{[57]}$		
	2 - Regime	1620.42	-14.837	166.719	[0.0000]**	[0.0000]**	
Equation	$\sigma_\varepsilon^2 = 0.1091$		$\bar{R}^2 = 0.3101$			DW = 1.9000	
Diagnostics	SK = -0.0446		EK = 0.8656			Norm = 7.7499 [0.0208]*	
	ARCH[1-7] = 3.8740 [0.0006]**		AR[1-7] = 1.4724 [0.1793]			AIC = -14.837	

T-statistics in braces. Statistical significance in bold. Like^h refers to likelihood.

volatility and regime 2 generally to more volatile times.

What is particularly interesting is that the key variables appear to be more important in the high-volatility regime. In tranquil periods, the error correction term is the only significant variable for the AAA spread model, but for the BAA model industrial production appears to be important in quiet periods. Contrast these results with the volatile regime. Under the higher residual variance regime, yield curve slope, equity return, and the previous slope itself appear to be the key drivers besides the error correction term.

The most interesting result of the MS-VECM analysis is in the higher-risk BAA spread equation. Note that

the risk-free rate enters the low-volatility regime equation with a significantly negative sign. Results in Exhibit 7 that relate to a single regime mask this effect, since the risk-free rate entered with an insignificant coefficient. The benefit of generalizing the model to allow for distinct regimes has been to uncover a significant relationship that applies to the low-volatility regime.

Further points worth noting in the MS-VECM results are the increase in explanatory power as demonstrated in the higher R^2 statistics as well as a marginal improvement in the behavior of the residuals as reported in the normality statistics. A final point worth noting is the importance of the error correction term, regardless of regime, and the fact that deviations from equilibrium can be expected to reverse over time, given the negative coefficient on the error correction component.

With the obvious merits of modeling credit spreads separately across different regimes, one intuitively appealing extension is to model the regime-generating process explicitly by

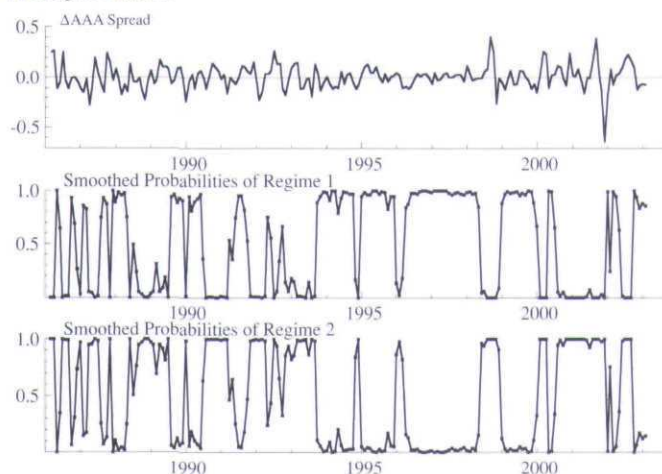
searching for a regime-generating variable. The MS-VECM results appear to be based on regimes that are governed by some kind of volatility switch within the data-generating process. An obvious candidate variable for regime generation would therefore be some measure of volatility.

A particularly appealing proxy for market volatility is thus the CBOE's VIX. The VIX is a measure of expected volatility derived from the options market of the S&P 500 index. It is widely followed by participants in the financial markets as an indicator of the market perception of future volatility. From an empirical standpoint, it is useful as it has a relatively long data history stretching back to 1986 and therefore covers several market cycles.

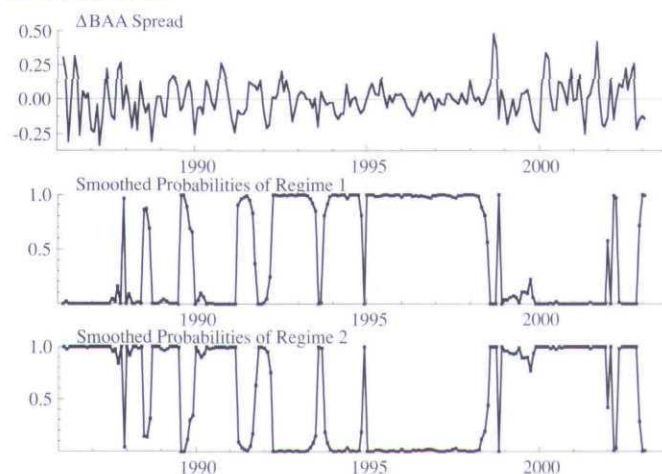
EXHIBIT 6

Regime Classification

AAA Spread Model



BAA Spread Model



Results in Exhibit (7) detail a self-extracting threshold autoregressive (SETAR) model based on the log of the VIX as the regime-differentiating variable. The graphs in Exhibit 8 provide a similar summary to that in Exhibit 6. Overall, the benefits of differentiating regimes based upon VIX volatility are superior to those provided by the basic single-regime Johansen results, but the overall fit is not as good as that obtained using the Markov switching regime modeling technique.

Both models define the threshold for regime differentiation as a level of 2.7881 for the log of the VIX. This distinguishes an extremely low-volatility regime that applied during the mid-1990s for a period of approximately four years and a higher-volatility regime that applied for the rest of the sample period.

For the low-volatility regime, the only significant

driver of the change in the credit spread variable is the change in the risk-free rate, entering with a significantly negative coefficient. In the higher-volatility regime, the previous spread itself, the yield curve slope, equity return, and industrial production are all significant determinants of the credit spread.

The interesting point is that differentiating separate models for separate regimes has revealed the significance of the risk-free rate in the lower-volatility regime, a result that was effectively hidden in the single-regime VECM analysis.

V. CONCLUSION

It would seem that considering the comparatively recent data history of the last 16 to 18 years has a marked effect upon the role of the risk-free rate in the credit spread variable. Results presented for the Johansen VECM analysis show there is a strongly significant inverse relationship between these two variables over the long run. This result is contrary to earlier empirical findings based on a longer data history or less frequent observation.

Results of two forms of regime-switching analysis demonstrate that the risk-free rate has a role to play in explaining shorter-term dynamic behavior of the credit spread as well. The risk-free rate has a negative effect on the credit spread during periods of low volatility but seems to be redundant during more volatile times.

This result is important because most empirical studies argue the opposite. Our results do, however, support the theoretical corporate bond pricing models of Merton [1974], Duffie and Singleton [1995], and others who argue that increases in the risk-free rate make corporate debt more attractive, thereby leading to a tightening of the credit spread. The empirical findings presented depend on a combination of relatively new developments in time series estimation and a shorter, more recent, data history.

Other interesting results in the analysis are the fact that equity return, the yield curve slope, and industrial production growth all have significant explanatory power for the short-term dynamic path of the corporate bond spread. This is particularly so during periods of relatively high volatility, although this general finding also applies for the single-regime VECM Johansen results.

One obvious extension to the analysis is the application of more sophisticated regime-switching techniques. The SETAR model is relatively unsophisticated, in that it searches for a specific level in the regime-generating variable to differentiate regimes. More subtle alternatives

EXHIBIT 7

SETAR Model—Short-Run Dynamics

AAA System	Constant	Spread _{t-1}	10 Year _{t-1}	Slope _{t-1}	S&P500 _{t-1}	Ind Prod _{t-1}	ECT _{t-1}
Δ Spread _t =	$\phi_{0,1}$	$\phi_{1,1}$	$\phi_{2,1}$	$\phi_{3,1}$	$\phi_{4,1}$	$\phi_{5,1}$	$\phi_{6,1}$
	0.257	-0.027	-0.189	0.017	0.042	-2.448	-0.097
	{1.156}	{-0.125}	{-2.184}	{0.182}	{0.069}	{-0.831}	{-1.116}
	$\phi_{0,2}$	$\phi_{1,2}$	$\phi_{2,2}$	$\phi_{3,2}$	$\phi_{4,2}$	$\phi_{5,2}$	$\phi_{6,2}$
	0.325	0.431	0.042	-0.155	-0.825	-4.445	-0.114
	{3.392}	{4.158}	{0.822}	{-2.166}	{-4.806}	{2.312}	{-3.272}
Regime Properties	Observations		Probability		Threshold Variable		
	Regime 1	47	0.2304		Log Vix _{t-1} = 2.7881		
	Regime 2	157	0.7696				
Regime Diagnostics	Log Like ^h		AIC	Like ^h Ratio	Davies		
	Linear	1580.31	-14.9540	Linearity	$\chi^2_{[57]}$	Test	
	2 – Regime	1644.58	-15.0351	128.54	[0.0000]**	[0.0000]**	
Equation	$\sigma^2_{\varepsilon} = 0.1022$		$\bar{R}^2 = 0.2270$		DW = 1.99		
Diagnostics	SK = -0.2197		EK = 2.4877		Norm = 36.549 [0.0000]**		
	ARCH[1-7] = 1.3763 [0.2172]		AR[1-7] = 1.4082 [0.2041]		AIC = -15.0351		
BAA System	Constant	Spread _{t-1}	10 Year _{t-1}	Slope _{t-1}	S&P500 _{t-1}	Ind Prod _{t-1}	ECT _{t-1}
Δ Spread _t =	$\phi_{0,1}$	$\phi_{1,1}$	$\phi_{2,1}$	$\phi_{3,1}$	$\phi_{4,1}$	$\phi_{5,1}$	$\phi_{6,1}$
	0.131	-0.255	-0.241	0.094	-0.241	-1.695	-0.043
	{0.883}	{-1.256}	{-3.477}	{1.133}	{-0.495}	{-0.731}	{-0.846}
	$\phi_{0,2}$	$\phi_{1,2}$	$\phi_{2,2}$	$\phi_{3,2}$	$\phi_{4,2}$	$\phi_{5,2}$	$\phi_{6,2}$
	0.249	0.367	0.019	-0.213	-1.072	-5.355	-0.069
	{3.214}	{3.811}	{0.351}	{-2.632}	{-5.454}	{-2.490}	{-3.027}
Regime Properties	Observations		Probability		Threshold Variable		
	Regime 1	47	0.2304		Log Vix _{t-1} = 2.7881		
	Regime 2	157	0.7696				
Regime Diagnostics	Log Like ^h		AIC	Like ^h Ratio	Davies		
	Linear	1537.06	-14.5300	Linearity	$\chi^2_{[57]}$	Test	
	2 – Regime	1613.64	-14.7318	153.1549	[0.0000]**	[0.0000]**	
Equation	$\sigma^2_{\varepsilon} = 0.1130$		$\bar{R}^2 = 0.310$		DW = 1.95		
Diagnostics	SK = 0.1318		EK = 0.6961		Norm = 5.5839 [0.0613]		
	ARCH[1-7] = 3.3492 [0.0021]**		AR[1-7] = 1.1272 [0.3477]		AIC = -14.7318		

T-statistics in braces. Statistical significance in bold. Like^h refers to likelihood.

such as those of the smooth transition family may provide further insights.

The analysis presented here also focuses on the U.S. experience. There are advanced corporate debt markets in most developed economies, and it should be interesting to examine the determinants of corporate spreads using alternative data sets.

APPENDIX A

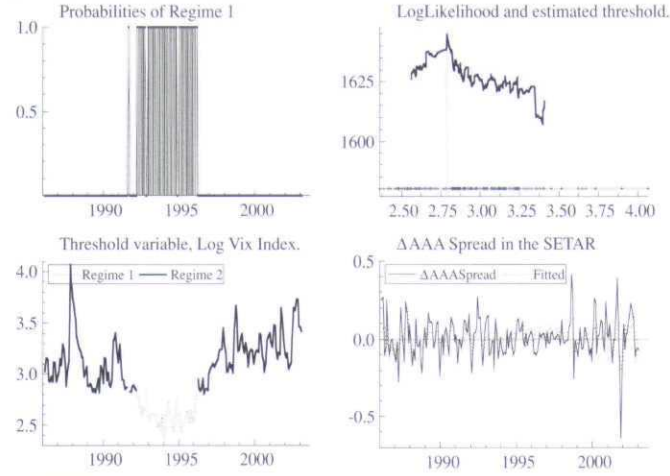
Markov Switching Estimation

A model based upon two regimes takes the form:

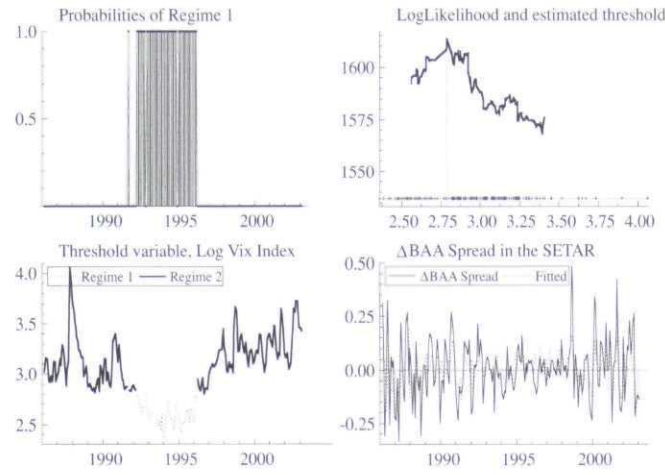
$$\Delta z_t = \begin{cases} \Gamma_{0,1} + \Gamma_{1,1} \Delta z_{t-1} + \dots + \Gamma_{k,1} \Delta z_{t-k+1} + \alpha_1 \beta' z_{t-k} + u_t & \text{if } s_t = 1 \\ \Gamma_{0,2} + \Gamma_{1,2} \Delta z_{t-1} + \dots + \Gamma_{k,2} \Delta z_{t-k+1} + \alpha_2 \beta' z_{t-k} + u_t & \text{if } s_t = 2 \end{cases} \quad (\text{A-1})$$

EXHIBIT 8 Regime Classification

SETAR AAA Spread Model



SETAR BAA Spread Model



Hence, the corporate spread equation takes the form:

$$\Delta Spread_t = \begin{cases} \phi_{0,1} + \phi_{1,1}\Delta Spread_{t-1} + \phi_{2,1}\Delta 10YearYield_{t-1} + \phi_{3,1}\Delta Slope_{t-1} + \phi_{4,1}\Delta \log[S\&P500]_{t-1} + \phi_{5,1}\Delta \log[IndustrialProduction]_{t-1} + \phi_{6,1}ErrorCorrectionTerm_{t-1} + \varepsilon_t & \text{if } s_t = 1 \\ \phi_{0,2} + \phi_{1,2}\Delta Spread_{t-1} + \phi_{2,2}\Delta 10YearYield_{t-1} + \phi_{3,2}\Delta Slope_{t-1} + \phi_{4,2}\Delta \log[S\&P500]_{t-1} + \phi_{5,2}\Delta \log[IndustrialProduction]_{t-1} + \phi_{6,2}ErrorCorrectionTerm_{t-1} + \varepsilon_t & \text{if } s_t = 2 \end{cases} \quad (A-2)$$

Rewriting the system specification yields:

$$\Delta z_t = \Gamma_{0,st} + \Gamma_{1,st}\Delta z_{t-1} + \dots + \Gamma_{k,st}\Delta z_{t-k+1} + \alpha_{st}\beta'z_{t-k} + u_t \quad (A-3)$$

where the residuals are conditionally Gaussian as: $u_t | s_t \sim \text{NID}[0, \Sigma(s_t)]$

The estimation problem is to obtain $\theta = (\phi_1, \phi_2, \rho_{11}, \rho_{22}, \sigma^2)$, where the ϕ_i are the autoregressive coefficients for regime i , i.e., $\phi_i = (\Gamma_{0,i}, \Gamma_{1,i}, \dots, \Gamma_{k,i}, \alpha_i)$; ρ_{ij} is the transition probability of switching from regime i to j ; and σ^2 is the residual variance. The estimator uses the forecast, smoothed, and inference conditional regime probability vectors as follows:

$$\text{Forecast: } \hat{\varepsilon}_{t+1|t} = P\hat{\varepsilon}_{t|t} \quad (A)$$

$$\text{Inference: } \hat{\varepsilon}_{t|t} = \frac{\hat{\varepsilon}_{t|t-1} \otimes f_t}{1'(\hat{\varepsilon}_{t|t-1} \otimes f_t)} \quad (B)$$

$$\text{Smoothed: } \hat{\varepsilon}_{t|n} = \hat{\varepsilon}_{t|t} \otimes \left(P[\hat{\varepsilon}_{t+1|n} + \hat{\varepsilon}_{t+1|t}] \right) \quad (C)$$

where, for example, $\hat{\varepsilon}_{t|n}$ represents the 2×1 vector of the smoothed conditional regime probabilities as:

$$\hat{\varepsilon}_{t|n} = \begin{bmatrix} P(s_t = 1 | \Omega_n; \theta) \\ P(s_t = 2 | \Omega_n; \theta) \end{bmatrix} \quad (A-4)$$

P is defined as the matrix including the regime transition probabilities, and f_t is the density of Δz_t conditional on regime s_t :

$$f(\Delta z_t | s_t = j, \Omega_{t-1}; \theta) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left\{ -\frac{(\Delta z_t - \phi_j' \Delta z_{t-p})^2}{2\sigma^2} \right\} \quad (A-5)$$

with $\Delta z_{t-p} = (1, \Delta z_{t-1}, \dots, \Delta z_{t-k+1}, \beta'z_{t-k})'$

Beginning with some initial guess at θ , call it $\theta^{(0)}$, and $\hat{\varepsilon}_{1|0}$, we can use the inference and forecast regime probabilities to obtain forecast and inference regime probabilities for all t . Next, using the final $\hat{\varepsilon}_{n|n}$ obtained for Equation (B) we can obtain the smoothed regime probabilities for all t from Equation (C) by running backward through time beginning at time $t = n$, $t = n-1, \dots$, and finishing at time $t = 1$.

Having obtained the smoothed regime probabilities from Equation (C) denoted: $P(s_t = j | \Omega_n; \theta)$, we can use these together with the initial estimates of the transition probabilities $\rho_{ij}^{(0)}$ from $\theta^{(0)}$ to obtain new transition probabilities from:

$$\rho_{i,j} = \frac{\sum_{t=2}^n P(s_t = j, s_{t-1} = i | \Omega_n; \hat{\theta})}{\sum_{t=2}^n P(s_{t-1} = i | \Omega_n; \hat{\theta})} \quad (A-6)$$

Finally, new autoregressive coefficients and residual variances are obtained from:

$$\hat{\phi}_j = \left(\sum_{t=1}^n \Delta z_{t-p}(j) \Delta z_{t-p}(j)' \right)^{-1} \left(\sum_{t=1}^n \Delta z_{t-p}(j) \Delta z_t(j) \right)$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{t=1}^n \sum_{j=1}^2 \left(\Delta z_t - \hat{\phi}_j' \Delta z_{t-p} \right)^2 P(s_t = j | \Omega_n; \hat{\theta}) \quad (\text{A-7})$$

with

$$\Delta z_t(j) = \Delta z_t \sqrt{P(s_t = j | \Omega_n; \hat{\theta})}$$

$$\Delta z_{t-p}(j) = \Delta z_{t-p} \sqrt{P(s_t = j | \Omega_n; \hat{\theta})} \quad (\text{A-8})$$

Overall we obtain a new set of coefficient estimates $\theta^{(n)}$. The algorithm continues until we obtain a set of coefficient estimates that change by less than some prespecified amount from the previous recursion of the algorithm.

The estimator is an application of the expectations maximization (EM) algorithm developed by Dempster, Laird, and Rubin [1977]. Hamilton [1990] demonstrates that the algorithm increases the value of the log-likelihood function at each step and converges to the maximum over time.

APPENDIX B

SETAR Estimation

In vector error correction form, we can write:

$$\Delta z_t = (\phi_{0,1} + \Gamma_{1,1} \Delta z_{t-1} + \dots + \Gamma_{1,k-1} \Delta z_{t-k+1} + \alpha_1 \beta' z_{t-k}) \times I[y_{t-1} \leq c] +$$

$$(\phi_{0,2} + \Gamma_{2,1} \Delta z_{t-1} + \dots + \Gamma_{2,k-1} \Delta z_{t-k+1} + \alpha_2 \beta' z_{t-k}) \times I[y_{t-1} > c] + \varepsilon_t \quad (\text{B-1})$$

And more compactly as:

$$\Delta z_t = \phi_1 \Delta z_{t-p} I[y_{t-1} \leq c] + \phi_2 \Delta z_{t-p} I[y_{t-1} > c] + \varepsilon_t \quad (\text{B-2})$$

where $\phi_j = (\phi_{0,j}, \Gamma_{j,1}, \dots, \Gamma_{j,k-1}, \alpha_j)$, $j = 1, 2$, and $\Delta z_{t-p} = (1, \Delta z_{t-1}, \dots, \Delta z_{t-k+1}, \beta' z_{t-k})$.

With a fixed threshold c , estimates of the $\phi_{i,j}$ are obtained easily by ordinary least squares as:

$$\hat{\phi}(c) = \left(\sum_{t=1}^n \Delta z_{t-p}(c) \Delta z_{t-p}(c)' \right)^{-1} \left(\sum_{t=1}^n \Delta z_{t-p}(c) \Delta z_t \right) \quad (\text{B-3})$$

where: $\Delta z_{t-p}(c) = \{\Delta z_{t-p}' I(y_{t-1} < c), \Delta z_{t-p}' I(y_{t-1} > c)\}'$.

The residuals from this expression are denoted $\hat{\varepsilon}_t(c) = \Delta z_t - \hat{\phi}(c)' \Delta z_{t-p}(c)$, and residual variance is given by:

$$\hat{\sigma}^2(c) = \frac{1}{n} \sum_{t=1}^n \hat{\varepsilon}_t(c)^2$$

The least squares estimate of c can then be obtained by minimizing the residual variance as:

$$\hat{c} = \underset{c \in C}{\operatorname{argmin}} \hat{\sigma}^2(c) \quad (\text{B-4})$$

where C represents all possible threshold values.

ENDNOTES

This article was written while the author was a visiting research fellow in the economics department at the University of Strathclyde during the 2003–2004 academic year. It has benefited from discussions with Mark Benfold, James Binny, Denis Davies, Jeremy Hale, and David Hillier. The views expressed are the author's and should not be taken to represent those of his employer.

¹Bevan and Garzarelli [2000] use historical annualized standard deviations of daily log returns of the S&P 500 index.

²From a formal analysis of the Morris, Neale, and Rolph [1998] data set and methodology, I am able to estimate long-run cointegrating relationships across the corporate yield and risk-free rate of [1, -1.1028] and [1, -1.177] for the AAA and BAA data sets. This compares with the original Morris, Neale, and Rolph [1998] estimates of [1, -1.1028] and [1, -1.178]. At the same time, using the original data set, I find normality statistics for the individual corporate yield equations of 205.67 and 121.15 for the AAA and BAA bonds, respectively, indicating the presence of some extremely large outliers during the early 1980s data history.

³In assuming a first-order Markov process, I restrict the current regime s_t to be determined solely by the regime that prevailed one period ago, s_{t-1} .

⁴Hannan and Quinn diagnostic statistics indicate that a satisfactory lag length of 2 in levels is sufficient.

⁵Cheung and Lai [1993] also note the tendency of λ -max and trace tests to spuriously find cointegration when low-order VECM are estimated. Given that the analysis details a model based upon a single lag in the first-difference dynamics part of the model, I also estimate models based on more complex lag structures. Results based on systems estimated with lag structures up to the fourth order also suggest a single significant cointegrating relationship (available upon request).

⁶Bevan and Garzarelli [2000] refer to a barely significantly positive relationship between the spread and risk-free rates in their long-run estimation results.

⁷Estimation of models that allowed for more than two

regimes was found to add no appreciable gain in explanatory power.

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