TIPS, Break-Even Inflation, and Inflation Forecasts

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The research on inflation-indexed government bonds, also referred to as index-linked government bonds, often yields inconclusive results, which could imply that this market may be inefficient. With the elimination of default risk and inflation risk, one could expect a fairly limited set of risk factors will influence the price process. Adjusting the payout of the bond for inflation keeps the real yield constant, which at the same time makes the nominal yield change with inflation. Consequently, as the nominal value of an index-linked bond depends on inflation realized over its life, financial markets could be expected to anticipate future realizations of inflation—and hence the future nominal cash flow to the bondholders—by factoring this assessment into the pricing.

Several studies show that the real yield implied by the financial markets, i.e., the nominal yield less inflation, could change over the lifetime of the bond. Thus, while index-linked bonds eliminate the risk of inflation, they seem to expose the investor to changes in the real rate as a result of changing expectations of future inflation throughout the remaining life of the bond. Therefore, to understand the price process of index-linked bonds, we must take into account its influencing factors, notably expectations of future inflation.

By construction, both expected future inflation implied in the pricing of index-linked government bonds, commonly referred to as *break-even inflation,* and other variables fundamentally important to the price process, such as the implied volatility of the short rate, should be stable over time. Brown and Schaefer [1993], however, show this is not necessarily the case. The fact that prices and break-even inflation of index-linked bonds change over time shows that the market is uncertain about the future realization of some state variables such as inflation, which implies that not all relevant information about the future price process is revealed to the markets today.

Another view is that current information is not completely included in the pricing of the security. Barr and Campbell [1997] suggest the size of the inflation-indexed bond market relative to the nominal bond market, with its relative youth and local segregation, could be a source of market inefficiencies and inconsistent pricing (see also "Inflation-Linked Bonds for €-Based Investors" [2001].

This study is intended to provide some insight into the price process of index-linked bonds. We examine the potential for using some of the price process drivers to construct signals indicating the future price process. Because the individual markets for inflation-indexed bonds seem to be driven mainly by domestic players, inconsistencies in the pricing of inflation-indexed bonds across markets are very likely to be observed.

As the inflation adjustment is one of the factors most directly affecting the cash flow of the otherwise risk-free inflation-indexed bond, it seems reasonable to start with investigating relevant information revealed today and the extent to which this information is reflected in pricing in a single market. If the assessment of future inflation implied earlier by the market differs from the realized rate, the price of inflation-indexed bonds is expected to be adjusted accordingly, as its cash flow is directly related to the realized rate of inflation. If some investors could assess realized future inflation more accurately than the market, they could expect to be able to forecast the price movement of the inflation-indexed bond, which could be the basis for portfolio allocation in a certain market.[1]

Is it possible to forecast the future price movement of index-linked government bonds by explicitly including pieces of information, such as expected future inflation other than that available in the index-linked bond market? The implied volatility of the short rate could be useful in modeling the future price process of bonds. We seek to determine whether the market for index-linked government bonds is efficient in incorporating relevant information into its pricing by testing the effectiveness of informed trades using an explicit inflation forecast.

First, we test several trading strategies using information signals in market environments that include and exclude market frictions for their capacity to generate returns in excess of a buy-and-hold strategy. The sample covers the largest index-linked bond market, the U.S. Treasury Inflation-Protected Securities, TIPS. The models forecast inflation from one to three years. We compare the inflation forecasts generated by the models with break-even inflation, and use the resulting differential to guide allocation decisions. The value of the information signal is backtested using several combinations of trading strategies, holding periods, and market frictions.

I. LITERATURE REVIEW

Index-linked government bonds (ILB) are government bonds that pay either an inflation-adjusted coupon or principal (in the rare case of zeros) or both, depending on the increase in an index relative to its base period over time. In practice, there is a time lag of three to eight months between the fixing of the payment adjustment and the actual cash flow.

Research on ILBs focuses mainly on the derivation of a term structure model of real interest rates and inflation expectations, the two factors perceived to be the most crucial in their dynamic price behavior. Significant market frictions make a clean analysis of this market very difficult, however. Most important, the smallness of the market is the cause of persistent market inefficiencies, such as illiquidity and the absence of short-selling possibilities. Hence, arbitrage mechanisms needed to restore prices are likely to take place with a time lag—if at all—which implies ample opportunities for an informed investor.

One of the earliest studies on ILBs is Woodward [1990]. Assuming a zero inflation risk premium, he tries to model inflation expectations via real rates generated by subtracting rates of ILBs from otherwise comparable nominal bonds matched by duration. Woodward uses the observed term structure of nominal yields applied to the payout structure of currently traded ILBs to derive the expected inflation. He finds the real yield curve is flat and far less volatile than its nominal equivalent.

A complementary result is that the inflation risk premium can be assumed to be fairly low, considering the consistent low value of the estimated measure including both expected inflation and its risk premium. By examining the effect of the hypothetical risk premium on the estimation of inflation expectations and the real rate, Woodward establishes that its impact declines with increasing maturity, which has the effect of increasing volatility at the short end of the real yield curve.

One major weakness of the Woodward [1990] research concerns the adjustment of yields to an imputed tax rate, which artificially smooths the yield curve and potentially masks information embedded in it.

The finding that index-linked securities have only slightly lower risk premiums than their non-indexed counterparts is contested by Gorton and Pennacchi [1991]. They note that indexed securities generally enhance market efficiency in the presence of imperfect foresight and both informed and uninformed investors.

Gorton and Pennacchi argue that the difference between expected and realized inflation, so-called *unexpected* inflation, affects all market participants in a random fashion. Accordingly, every investor is potentially affected by inflation risk, which usually is not correctly accounted for in the investment portfolio. Investors are imperfectly hedged and therefore bear some additional systematic risk. As Gorton and Pennacchi [1991] claim indexed securities can help overcome the random effects of unexpected inflation affecting every market participant, one could conclude that nominal bonds should have an inflation risk premium as compared to ILBs.

Other research questioning some of the findings in Woodward [1990] includes Brown and Schaefer [1993]. Fitting the Cox-Ingersoll-Ross [1985a, 1985b] model to ILB prices and thereby imputing the implied real yields, Brown and Schaefer find evidence that this framework is consistent with the observed real rates. Their fitted yield curve exhibits rich dynamics on both the short and the long end that change dynamically over time, contrary to Woodward. Some results are in line with Woodward [1990], however, such as the decline of the yield curve at the long end and high volatility at the short end. Brown and Schaefer [1993] also find that monthly inflation rates exhibit autocorrelation with significant seasonal patterns, which implies that approximating inflation expectations by a constant or simple extrapolation of past data will not properly capture its real dynamics.

Although their fitted model performs fairly well, Brown and Schaefer encounter persistent estimation errors over time and a high degree of parameter instability, suggesting strong negative correlation among the parameters. Their results for the speed of mean reversion, κ, support the assumption that the CIR model works in the context of the ILB market, as it assumes κ to be stable over time and maturity. Estimates of the risk premium, λ, are inconclusive, however, implying that κ is likely to be underestimated by the market, which could be exploited by a trading strategy if combined with an effective model of forecasting expected inflation. The high degree of parameter variability suggests that incorporation of time-dependent components in the estimation could significantly improve their model of the real yield curve.

Campbell and Shiller [1996] provide a general assessment of the ILB market and give strong conceptual support to the claims made by Gorton and Pennacchi [1991]. They suggest the presence of an inflation risk premium, implying that the rational expectations hypothesis holds for ILBs. Confirming other studies, they find significant short-term variation in prices of ILBs based on uncertainty regarding the real rate, which declines for longer maturities. Their approach to extracting the inflation risk premium from market data is consistent with Ross's [1976] arbitrage pricing theory and based on the covariance of the returns on inflation-sensitive but otherwise risk-free assets with the relevant state variable less the similarly calculated risk premium associated with an ILB. Campbell and Shiller estimate small but significant inflation risk premiums, contrary to Woodward [1990].

Barr and Campbell [1997] build on this finding to model the interaction of nominal and real yields with (expected) inflation. They estimate expected inflation and find that the dynamics of real and nominal yield curves forecast expected inflation better than real inflation.

Foresi, Penati, and Pennacchi [1997] attempt to estimate the change in real rates by means of a two-factor model including 1) correlation of the instantaneous expected rate of inflation with the real short rate, 2) the real short rate, and 3) a time-dependent intercept. Their model yields expected inflation that is half as volatile as real rates. Their further econometric analysis of the return structure of ILBs suggests these instruments have a significant idiosyncratic risk factor. ILBs seem to incorporate an indirect inflation risk premium, dependent on the dynamic relationship of real rates and inflation.[2]

Barone and Masera [1997] investigate this issue further. They use the equilibrium conditions evoked by the CIR model combined with several simplifying assumptions to derive an expression for the inflation risk premium of nominal interest rates, which they claim equals the instantaneous forward inflation rate. Although Barone and Masera structure a definition of the difference in return between ILB and comparable nominal bonds, they do not try to quantify this difference to check the reasonableness of their assumptions.[3]

Such an investigation is carried out by Kandel, Ofer, and Sarig [1997]. An innovative element of their study is definition of the price of ILBs in terms of 1) the ex ante real rate, 2) expected inflation over a certain *past* period that represents the indexation lag, and 3) the nominal forward interest (thereby avoiding the definition of expected future inflation). Their most important findings are, first, that expected inflation and real rates are negatively correlated, which confirms Foresi, Penati, and Pennacchi [1997], and, second, that the inflation risk premium increases with the volatility of inflation.

Kothari and Shanken [2002] underscore the potential for risk reduction of ILBs if they are assessed by their return volatility as well as their correlation with other asset classes. Unlike Campbell and Shiller [1996], Kothari and Shanken [2002] find evidence of a negative inflation premium in ILBs when they subtract real yields and assess expected inflation according to nominal yields. As a consequence of the mean-reverting behavior of inflation rates, it is impossible to forecast changes in expected inflation beyond three years, which rejects the random walk hypothesis. Kothari and Shanken find strong evidence for a negative relationship of ILB prices with both realized and expected inflation.

II. DATA

Although inflation-indexed bonds have gained attention only recently, they are not a new financial innovation. As early as 1780, the state of Massachusetts issued debt instruments that were linked to the price development of a selection of consumption goods, which can be understood as an approximation of a consumer price index (Campbell and Shiller [1996]). It took about 200 years, however, until the governments of industrialized countries seriously considered index-linked bonds as a possible way to raise funds.

Today an increasing number of countries use inflation-indexed debt instruments as one way to raise funds, but the individual national markets are segregated and as a result relatively small and fairly illiquid. The lack of liquidity is a major constraint on investment strategies other than buy-and-hold, one reason secondary government debt market investors outside the U.S., the U.K., and France have taken little notice of these instruments. Governments tried several structures before one form emerged as the standard for inflation-indexed government debt, the so-called *Canadian structure.*[4]

The U.S., U.K., and French markets make up about 90% of the total inflation-linked debt outstanding (see Cross [2001]). As these governments commit themselves to further regular issues, the liquidity of the index-linked bond markets is likely to be further enhanced, to make this instrument increasingly attractive to a broader investor community.

Because official rates of inflation over past periods are available only with a time lag—e.g., the rate of inflation in June is available in August—there is an *indexation lag* of inflation that has occurred but that is not yet accounted for by the inflation adjustment of the bond. To avoid sudden price jumps every time the inflation adjustment takes place, some issuers, such as France, publish daily updates in the inflation adjustment between the official releases of the monthly inflation rate using an extrapolated inflation adjustment.[5] The standard for an indexation lag is three months, which is usually the first full month after the official publication of the inflation rates. Some issues also include insurance against deflation over the time until maturity; i.e., the investor gets back at least the value of the unadjusted principal.

The market for inflation-protected government debt issued by the U.S. government is linked to the Consumer Price Index for All Urban Consumers, CPI-U. The U.S. market is the world's largest, with a value of around USD 150 trillion outstanding. The features of the Treasury Inflation-Protected Securities, or TIPS, are considered standard for inflation-indexed government debt. They have a Canadian structure with a three month indexation lag; pay a semiannual coupon; include deflation protection of the principal; are issued in regular intervals with maturities of either 10 or 30 years; and have closely matching nominal bonds for every issue.

The inflation data we use are obtained from various sources. We use monthly data for the reference indexes for the U.S., the CPI-U, provided by the U.S. Census and checked for consistency with the time series retrieved from Fact Set. The monthly inflation is obtained by taking the natural logarithm of the period's realization of the inflation index divided by its value one year earlier. Our monthly inflation data cover the last 20 years.

III. ECONOMETRIC FRAMEWORK

The concept of break-even inflation is of crucial importance to the valuation of inflation-linked bonds. We use it to develop inflation forecasting models on which to base trading strategies.

Break-Even Inflation

The relationship of index-linked bond price, inflation, and price can be expressed by:

$$\text{Price}_t^{ILB} = 100e^{-TYTM+\int_0^T i_x dx} + c\int_0^T \frac{e^{i_x}}{e^{YTM_x}}dx \qquad (1)$$

where price_t^{ILB} is the price of the index-linked bond (the subscript denotes the time; i_t is inflation today at time t); T denotes time to maturity; YTM denotes the real yield to maturity; and c is the coupon at the time the bond is issued.

Note that inflation from issuance of the bond at time t = 0 until today is known, while future inflation until maturity at T is not known today. Therefore, one can extract a time-averaged consensus estimate of market participants as to the future inflation rate, commonly referred to as the break-even inflation (BEI), from market prices of the inflation-indexed bond.

Making several assumptions, we can calculate the market consensus forecast of inflation, given market data for an index-linked bond. Assuming the inflation risk premium equals zero and the yield to maturity equals the ex post return, the break-even inflation rate indicates at any moment the average level of inflation over the remaining maturity of the inflation-indexed bond necessary to equalize the payoff of the index-linked bond and the payoff of a comparable nominal bond.

Therefore, with no arbitrage, this implies that:

EXHIBIT 1
Bond Matching

Index-Linked Bond		Nominal Matching Bond		Time Series	
Name	Maturity	Name	Maturity	From	Until
TIPS	01/15/2007	STRIPS	02/15/2007	01/12/1997	01/31/2003
TIPS	01/15/2008	STRIPS	02/15/2008	01/12/1998	01/31/2003
TIPS	01/15/2009	STRIPS	02/15/2009	10/07/1999	01/31/2003
TIPS	01/15/2010	STRIPS	02/15/2010	01/18/2000	01/31/2003
TIPS	01/15/2011	STRIPS	02/15/2011	01/16/2001	01/31/2003
TIPS	01/15/2012	STRIPS	02/15/2012	01/10/2002	01/31/2003
TIPS	07/15/2012	STRIPS	08/15/2012	07/11/2002	01/31/2003
TIPS	04/15/2028	STRIPS	08/15/2028	04/14/1998	01/31/2003
TIPS	04/15/2029	STRIPS	02/15/2029	04/08/1999	01/31/2003
TIPS	04/15/2032	STRIPS	02/15/2031	10/17/2001	01/31/2003

$$\frac{1}{T}\int_t^T e^{i_x}dx = \bar{i} = BEI_t = \frac{1}{T}\int_t^T e^{E[i_x]}dx \qquad (2)$$

where $\bar{i}$ is the average inflation from time t to maturity T, and BEI_t denotes break-even inflation at date t. If the last equality always holds, there would be no relevant information revealed in the future that is not already known today. The last term has only weak empirical support, however, because index-linked bond prices as well as the individual BEI fluctuate over time.

In practice, the first difficulty is finding a comparable nominal bond in order to estimate break-even inflation. First, the types of bonds differ in their cash flow profiles; the nominal bond pays the same amount every period, while the index-linked bond generally pays a coupon that increases over time. As this indicates that index-linked bonds have a greater duration than otherwise similar nominal bonds, early studies on index-linked bonds matched them with nominal bonds on the basis of duration.

Then studies comparing the price volatility of both nominal and index-linked bonds led to changes in this practice. It turned out that the beta of an inflation-indexed bond typically lies around 0.5, which implies that index-linked bonds are less volatile than comparable nominal bonds (see Hunter and Simon [2002]).[6] Hence, both types of bonds cannot be compared on the basis of duration, as is usually the case for nominal bonds, but are commonly matched by maturity (Spiegel [1998]). We accordingly match bonds as in Exhibit 1.

While break-even inflation and a short-term forecast of inflation are hardly related, conceptually one can find some empirical support for supposing that current inflation influences the level of BEI. The typically observed temporal comovement of both series could suggest that realized current inflation is influencing the current realization of BEI, and hence the price of an inflation-indexed bond. In this case, predictions that are more precise than the currently observed BEI could significantly improve a pricing model of inflation-indexed bonds by providing a means of forecasting future movements in break-even inflation.

Inflation Forecasting in the Short Term

The model we use to forecast next-period inflation, for a time horizon within one year, is based on McCulloch and Stec [2000]. As our purpose is to derive an inflation forecast alternative to break-even inflation, the two must be conceptually as close as possible to cover expected inflation over the entire life of the bond. Inflation typically appears to be a trending series, but it might also fluctuate around a time-varying mean, or the trend itself might be time-varying.

In their study of the post-war U.S. inflation series, McCulloch and Stec [2000] correctly point out that monthly inflation does not have to be a stationary process. Additionally, standard unit root tests are not powerful enough to distinguish between a series with a near unit root or a unit root. Therefore, first-differencing can lead to inefficient parameter estimates if the underlying series does not have a unit root.

In the McCulloch and Stec autoregressive model, the only state variable of interest is the past rate of inflation itself. The standard AR(p) model reads:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \ldots + \beta_p y_{t-p} + \varepsilon_t \qquad (3)$$

or equivalently:

$$y_t = \beta_0 + \alpha y_{t-1} + \varphi_1 \Delta y_{t-1} + \ldots + \varphi_{p-1}\Delta y_{t-p+1} + \varepsilon_t \qquad (4)$$

where the intersection β_0 denotes a time-independent base level of the series; α is the coefficient accounting for the time-dependent base level of the series, corresponding to the common explanatory power of all the lags on y_t expressed by β_1 until β_p; Δy_{t-p+1} denotes the difference in level of y from period y_{t-p} to period y_{t-p+1}; and φ_{p-1} is the corresponding coefficient accounting for the effect each individual time lag has on the current realization (Fuller [1996]).

McCulloch and Stec find a significant relationship between current inflation and its past lags for up to 45 months. As a regression equation with 45 independent variables would significantly reduce the degrees of freedom and hence be meaningless for forecasting, they average the values after the sixth time lag to balance the need for inclusion of past information and enough degrees of freedom for meaningful estimation.

Therefore, Equation (4) becomes:

$$\begin{aligned} y_t &= \beta_0 + \alpha_t y_{t-1} + \sum_{i=1}^{5} \varphi_i \Delta y_{t-i} + \\ &\varphi_6\left(y_{t-6} - \frac{1}{12}\sum_{i=1}^{12} y_{t-6-i} \right) + \\ &\varphi_7\left(\frac{1}{12}\left(\sum_{i=1}^{12} y_{t-6-i} - y_{t-18-i} \right)\right) + \\ &\varphi_8\left(\frac{1}{12}\left(\sum_{i=1}^{12} y_{t-18-i} - y_{t-30-i} \right)\right) + \varepsilon_t \end{aligned} \tag{5}$$

As one might expect, the estimation of α plays a crucial role in this setting as it determines the absolute size of the estimate. Therefore, it is of importance to rectify biases in its first estimation. As McCulloch and Stec [2000] point out, if y_t is highly persistent, ordinary least squares regression estimates are not sensitive enough to detect slight but significant departures from the simplifying unit root assumption (see Valkanov [1998]), leading to biased estimates of α.[7]

To estimate the coefficients of Equation (5) on past data, certain correcting calculations are necessary in order to construct an unbiased estimate of α (and hence of the entire forecasted inflation series). For the forecast of a time series based on past data to be meaningful, we have to create a stationary series.[8] Conversely, according to the Wold decomposition, we can express every stationary time series as the sum of two time series: a singular time series, i.e., one with exactly one element, and an infinitely moving average time series (Fuller [1996]). In the case at hand, the first component is represented by α, and the second by the expressions involving φ.

We can assume inflation to be stationary, as there is no reason to assume it to grow or decline indefinitely rather than fluctuate within certain bounds. This can be expressed as:

$$\begin{aligned} &y_t = \left[\alpha_t y_{t-1}\right] + \left[\sum_{i=1}^{8} \varphi_i \Delta y_{t-i}\right] \\ &\text{for } \Delta y_{t-i} \sim N(0, \sigma^2) \\ &\text{and Corr } \left(\Delta y_{t-i}, \Delta y_{t-j}\right) = 0 \text{ if } j \neq i \end{aligned} \tag{6}$$

where the terms indexed by i > 5 represent the three multiperiod average lags in Equation (5).

Equation (6) shows Wold's decomposition, with the singular time series represented by the first term in brackets and the moving average by the second. As shown in Equation (4), α expresses the relevant time-dependence of the lags over time.

As the series is assumed to be stationary, we can express the time series backward or forward:

$$\begin{aligned} &y_t = \left(\sum_{i=1}^{p} \beta_i y_{t-i}\right) + \varepsilon_t^{fwd} \\ &\text{or } \ y_t = \left(\sum_{i=1}^{p} \beta_i y_{t+i}\right) + \varepsilon_t^{bwd} \\ &\text{for } \varepsilon_t^{fwd} \sim \varepsilon_t^{bwd} \sim N(0, \sigma^2) \\ &\text{and Corr}\left(\varepsilon_{t+n}^{fwd}, \varepsilon_{t+m}^{fwd}\right) = \text{Corr}\left(\varepsilon_{t+n}^{bwd}, \varepsilon_{t+m}^{bwd}\right) = \forall m \neq n \end{aligned} \tag{7}$$

where the constant has been suppressed, p indicates the level of autoregression, ε^{fwd} denotes the error term of the forward representation, and ε^{bwd} represents the error term of the backward representation (Fuller [1996]).

Conversely, Equation (7) can be written as

$$\begin{aligned} &y_t - \left(\sum_{i=1}^{p} \beta_i y_{t-i}\right) = \varepsilon_t^{fwd} \\ &\text{or } \ y_t - \left(\sum_{i=1}^{p} \beta_i y_{t+i}\right) = \varepsilon_t^{bwd} \end{aligned} \tag{8}$$

The efficient estimation of the β coefficients describing the relationship between the current realization of the time series and its later or earlier realizations is therefore the one that minimizes both error terms simultaneously. The efficiency of time series estimates is commonly measured by the squared error of the prediction against the realization, the so-called mean squared error, MSE. Therefore, the efficient regression estimate of α is the one minimizing the expression (adapted from Fuller [1996]):

$$\varepsilon_{(\alpha)} = \sum_{t=p+1}^{n} w_t \left[\psi_\tau - \left(\sum_{i=1}^{\pi} b_\iota \psi_{\tau-\iota} \right) \right]^2 +$$

$$\sum_{t=1}^{n-p} (1 - w_t) \left[\psi_\tau - \left(\sum_{\iota=1}^{\pi} b_\iota \psi_{\tau+\iota} \right) \right]^2 \text{ for } 0 \le w_t \le 1 \qquad (9)$$

where $\varepsilon_{(\alpha)}$ indicates the error to be minimized as a function of α; p is the level of autoregression; n is the size of the sample the minimization is performed on; and w_t represents a weight to be used to balance the effect of ε^{fwd} and ε^{bwd} on the overall error $\varepsilon_{(\alpha)}$. As Fuller [1996] points out, the choice of w_t is crucial and dependent on p, n, and the position of the time vector t [1, n] within the sample.

If w_t is constant for all t and chosen to be 0.5, one gets an equally weighted estimator of α that is biased toward zero for stationary series, however. Hence, one can improve the statistical fit of α to its true measure by choosing w_t such that

$$w_t = \begin{cases} 0 & \text{if } t = 1, 2, \ldots, p \\ \left(\dfrac{t - p}{(n-2)(p+2)} \right) & \text{if } t = p+1, p+2, \ldots, n-p+1 \\ 1 & \text{if } t = n-p+2, n-p+3, \ldots, n \end{cases} \qquad (10)$$

And estimate α using

$$\begin{array}{cccccc} w_{p+1} & y_{p+1} & y_p & y_{p-1} & \cdots & y_1 \\ w_{p+2} & y_{p+2} & y_{p+1} & y_p & \cdots & y_2 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ w_n & y_n & y_{n-1} & y_{n-2} & \cdots & y_{n-p} \\ (1 - w_{n-p+1}) & y_{n-p} & y_{n-p+1} & y_{n-p+2} & \cdots & y_n \\ (1 - w_{n-p}) & y_{n-p-1} & y_{n-p} & y_{n-p+1} & \cdots & y_{n-1} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ (1 - w_2) & y_1 & y_2 & y_3 & \cdots & y_{p+1} \end{array} \qquad (11)$$

where the first column indicates the weights, which are defined in Equation (8) and have to be applied to the individual values of the entire row; the second column indicates the dependent variables with their coefficients; and the following columns indicate the independent variables.

Applying the framework specified in Equations (10) and (11) to Equation (5) we get estimates of α_t that are still deemed to be biased toward zero. An adjustment must be made to the estimated coefficients in order to derive an unbiased α for each regression, which is given by:

$$\alpha_t^{\text{unbiased}} = \alpha_t + c_{(\tau_t)} s_t$$

$$\text{with} \quad c_{(\tau_t)} \begin{cases} t_t \pm & \text{if } t_t > -1.2 \\ 0.0325672(t_t + 7)^2 & \text{if } -7 < t_t < -1.2 \\ 0 & \text{if } t_t < -7 \end{cases} \qquad (12)$$

where α_t is the estimate of α at time t, τ_t denotes the t-value of α_t in the estimation according to Equations (10) and (11), and s_t is the standard error of α_t (see McCulloch and Stec [2000]).

In order to apply Wold's decomposition as shown in Equation (6), a stationary series is derived to generate the φ-coefficients corresponding to the first part of the right-hand side of Equation (6). McCulloch and Stec [2000] do so by creating a so-called pseudo-differenced time series process, $\{X_t\}$, defined as

$$X_t \equiv y_t - \left(\alpha_{t-1}^{\text{unbiased}}\right) y_{t-1} \qquad (13)$$

If this process is truly stationary according to Equation (6), $\{X_t\}$ should be distributed according to ε in Equation (7) and hence fully represent the second term in brackets in Equation (6). The $\{X_t\}$ process is then used to estimate the variables φ_p via ordinary least squares regression, where the intersection is suppressed to construct the coefficients equivalent to the stationary series of Equation (7). Following McCulloch and Stec [2000], the values of φ_p are estimated up to a level equal to 9. That is, for every level of integration, the coefficients φ_p are estimated anew.

The model indicates the most appropriate level of integration per forecast horizon t, and thus the most efficient model for our purposes. Once the model is chosen and estimated, forecasts are generated for the particular forecasting horizons. Subsamples were chosen to include 36 monthly observations. The objective is to balance a statistically sufficiently large subsample with the need to account for the possibility of changes in the estimated coefficient over time (which would call for a smaller sample). With inclusion of the coefficients to produce the weighted symmetric estimator according to Equation (11), we have 54 observations in each regression and 206 months in the entire estimation horizon. This framework generates 12 times 134 estimated monthly inflation rates per forecast horizon of one year.

Inflation Forecasting in the Medium Term

The next step is to determine an inflation forecast going from one to three years into the future. As break-even inflation covers expectations of the average rate of inflation over the entire life of the bond, longer-period forecasts could be useful for this study.

We use an estimation technique derived from Kothari and Shanken [2002] to forecast future inflation. Kothari and Shanken note that a reliable relationship between current state variables and realized future inflation is observable only for a horizon of at most three years. An educated guess about inflation beyond three years would be to assume it is constant at the level of the third year. Hence, the accumulated inflation (and hence accumulated cash flow adjustments) over the life of a bond using the Kothari-Shanken forecast model can be expressed as

$$\text{Inflation}_{0,T} = \left[\left(1+\hat{i}_t\right)\left(1+\hat{i}_t+\hat{i}_t\Delta_{t+1}\right)\left(1+\hat{i}_t+\hat{i}_t\Delta_{t+2}\right)\left(1+\hat{i}_t+\hat{i}_t\Delta_{t+2}\right)^{T-3}\right]-1 \tag{14}$$

where Δ_{t+1} and Δ_{t+2} denote the one-year change of inflation, respectively, from next year to two years in the future and from two to three years in the future, T is the maturity of the inflation-indexed bond, and i the estimated one-year-ahead inflation.

Kothari and Shanken [2002] examine the explanatory power for future inflation of a selection of independent variables by conducting a regression of realized future inflation on a set of forecasting variables. Our intention is to use the current realization of the state variables to conduct a forecast of future inflation. Therefore, we regress current inflation on past state variables to determine the coefficients to estimate future inflation using current realizations of the state variables. The approach relies on the assumption that the coefficients either stay constant or change slowly over time in the markets under investigation.

Monthly data points are converted into weekly inflation by dividing the year-on-year increase in the relevant inflation index by the number of days in that month and multiplying this figure by seven. Weeks falling in between two months are the sum of the fraction of the days of the week in this month times inflation divided by the number of days in this month and the same expression for the other month in question. The weekly data are converted into year-on-year inflation by multiplying the previous weeks' index number by e raised to the power of the weekly inflation increase. Taking the natural logarithm of the quotient of two index numbers 52 weeks apart, we get the year-on-year inflation converted into weekly values.

The state variables of interest are the realized one-year lag of inflation, the one-year spot rate, the forward rate for a one-year deposit starting in 12 months (or 12/24 FRA for short), the 24/36 FRA, the five-year nominal yield on a five-year zero-coupon bond, and the monthly realized nominal return over the last year (Kothari and Shanken [2002]). Almost all these data are generally available, with the exception of the last one. Hence, we calculate the realized nominal return as the sum of the capital gain and the promised return.

Consequently, the sum of the realized nominal monthly returns on a one-year government security realized over one year at time t is given by:

$$\text{return}^{i}_{\text{realizedmonthlyreturnoveroneyear}} = \left(\text{YTM}^{i}_{\text{lastyear}}\right) + \sum_{t=1}^{12}\left(\left(\frac{1+\text{YTM}^{i}_{\text{beginningofmonthlyperiodt}}}{1+\text{YTM}^{i}_{\text{beginningofmonthlyperiodt+1}}}\right)^{1/12}-1\right) \tag{15}$$

where YTM indicates the yield to maturity on a one-year deposit, and i denotes the country. The daily values of one-year constant-maturity zero yield curves are taken. The monthly return is determined as the quotient of the quoted yield to maturity and the yield to maturity on the same day of the month but one month later.

To construct x/y forward rates unavailable in the market, the yield on an x-period zero-coupon bond is subtracted from the yield on a y-period zero-coupon bond.[9] These synthetic forward rates are constructed using daily constant-maturity Treasury yield data for various maturities, provided by the Federal Reserve Board. Realized inflation during a particular week is assumed to accumulate in a constant fashion over the five working days in order to avoid unnecessary complexity of the data conversion. Finally, the daily inflation data are regressed on the daily data of the state variables.

For this model to be valuable for our purposes, we go one step further to assess the information value from generating coefficients by regression on past time series, and then use these with the current realizations of the same state variables to forecast future inflation. If the coefficients are stable over time, the length of the time window of the regression would not matter, so for the sake of statistical validity one would opt for the longer sample. If the coefficients vary over time, a shorter time horizon of the regression would be more appropriate, as we would get more timely estimation of the coefficients to be used for the forecast.

Therefore, we form subperiods of two weeks (10 daily observations), four weeks (20 daily observations), three months (64 daily observations), six months (129 daily observations), and one year (261 daily observations) in order to determine the right trade-off between statistical robustness and timeliness of the estimate. Starting from February 1, 1994, we regress daily realizations of the dependent variable on realizations of the independent variables. Regressions are intended to determine the coefficients per forecasting horizon for regression horizons of two and four weeks, three and six months, and one year. The coefficients are taken to forecast inflation over the next one, two, or three years using the current realizations of the independent variables. The estimate is then compared with realized inflation to get an indication of the accuracy of the model.

The regressions are:

$$\text{Inflation}^{i}_{\text{today}} = \beta_0 + \beta_1(1\text{YrS}_{t-1})^{i} + \beta_2(5-1\text{Spr}_{t-1})^{i} + \beta_3(\text{Inf}-\text{LY}_{t-1})^{i} + \beta_4(\text{RR}_{t-1,t})^{i} + \varepsilon^{i}_{t} \tag{16}$$

$$\text{Inflation}^{i}_{\text{today-lastyear}} = \beta_0 + \beta_1(1, 2\text{FwdSpr}_{t-2})^{i} + \beta_2(\text{Inf}-\text{LY}_{t-2})^{i} + \beta_3(\text{RR}_{t-2})^{i} + \varepsilon^{i}_{t} \tag{17}$$

$$\text{Inflation}^{i}_{\text{today-lastyear}} = \beta_0 + \beta_1(2, 3\text{FwdSpr}_{t-3})^{i} + \beta_2(\text{Inf}-\text{LY}_{t-3})^{i} + \beta_3(\text{RR}_{t-3})^{i} + \varepsilon^{i}_{t} \tag{18}$$

where Equation (16) is used to derive the coefficients to forecast the year-ahead inflation; Equation (17) is used to derive the coefficients to forecast the change in inflation from next year to the subsequent year; and Equation (18) is used to derive the coefficients for the change in inflation from two years hence to three years. In Equations (16)–(18), i indicates the time window of the regression; t indicates the time in years; and 1YrS, 5-1Spr, Inf-LY, $RR_{t-1,t}$, 1, 2FwdSpr, and 2, 3FwdSpr refer, respectively, to the coefficients of the independent variables of the one-year spot rate; the yield spread of five-year zero-coupon government bonds over one-year zero bills; the one-year lag of inflation; the sum of realized monthly real returns over the last year; the spread of the forward rate of a one-year deposit in one year over the current spot rate; and the spread of the forward rates for a one-year deposit made in two years over a deposit made in one year.

Construction of the Signal and Trading Strategies

The relationship of observed break-even inflation and forecasted inflation over several time periods serves as a signal for upward and downward movements of break-even inflation. We generate forecasts for different time horizons from the adapted Kothari and Shanken [2002] and McCulloch and Stec [2000] models, lagged by two months due to the time difference between when inflation is realized and when the actual figures are published, and compare them with the observed daily values of break-even inflation.

A variety of calculations are performed. We use the differential between break-even inflation and the inflation forecast as an indicator of the expected future change in the observed yield:

$$E_t[\Delta r] = \frac{\text{Forecast}^{i}_{t} - \text{BEI}^{j}_{t}}{\text{BEI}^{j}_{t}} \tag{19}$$

or as an indicator of the expected future change in break-even inflation:

$$E_t[\Delta \text{BEI}] = \frac{\text{Forecast}^{i}_{t} - \text{BEI}^{j}_{t}}{\text{BEI}^{j}_{t}} \tag{20}$$

where t refers to the time the ratio is derived based on the information available at time t; i refers to the inflation forecast using model i; BEI denotes the break-even inflation; and j indicates bond j. Depending on the trading strategy used, Equation (19) or Equation (20) is used as a signal to guide the resource allocation process over time. In particular, it is either interpreted as the expected change of 1) the observed yield over time, or 2) the future break-even inflation:

$$E_{t+\Delta t}[r] = r_t + E_t[\Delta r] \tag{21}$$

or

$$E_{t+\Delta t}[\text{BEI}] = \text{BEI}_t + E_t[\Delta \text{BEI}] \tag{22}$$

where Δ_t denotes the holding period; i.e., the index $t + \Delta_t$ indicates the expectations at the end of the holding period.

In the next step, the signal serves as an input for different investment strategies. To test for market efficiency, i.e., the information value-added of the signals in the index-linked bond market, we use a *long-short strategy*.[10]

That is, if the expected change in yield indicates a decline in bond prices, a short position is taken, and a long position is assumed in the opposite case. The equation is:

$$P_{Estimated}^{t+\Delta t} = e^{-r_{Estimated}^{t+\Delta t}} = e^{-E_{t,\Delta t}[r]} \tag{23}$$

where P^t denotes the bond price at time t.

There are two trading strategies generally used in the index-linked bond market, so-called *break-even trades* and *carry trades*, which we use to test the potential for an information advantage using the various signals. Break-even trades aim to capture the change in break-even inflation over time exclusively, and thereby try to isolate the price process of a position of inflation-indexed bonds from the change in other factors over time, most notably the real yield. This is achieved by taking a position in index-linked bonds and a position with the opposite sign but as many securities in nominal bonds, which is usually the matching bond used to calculate break-even inflation. For instance, if one expects inflation to rise above the observed break-even inflation, one takes a long position in index-linked bonds and a short position in nominal bonds.

The carry on an index-linked bond corresponds to the expected performance of a bond under the assumption of constant real yield, which at time t = 0 is calculated as follows (see "European Interest Rate Products" [2003]):

$$\begin{aligned} Carry_{t=0,t=1} &= P_{t=0}\left(IC_{t=1} - IC_{t=0}\right) + \\ & IC_{t=1}\left(P_{t=1,r_0} - P_{t=0}\right) - repoP_{t=0}IC_{t=0}\frac{n}{360} \end{aligned} \tag{24}$$

where the subscripts indicate the time (t = 0 means today, and t = 1 indicates the next period over which the carry is calculated); P corresponds to the value of the bond unadjusted for inflation; IC indicates the inflation index-coefficient; r_0 denotes the real yield of the index-linked bond at time 0; repo denotes the repo rate; and n is the number of days between t = 0 and t = 1.[11]

On the right-hand side of Equation (24), the first expression corresponds to the expected change in value of the bond due to changes in the relevant inflation index. The second expression corresponds to the expected change in the value due to factors other than changes in inflation and real yield, e.g., accrued interest. The third expression indicates the financing costs at spot.

The carry can thus be interpreted as the term premium, assuming the real rate will not change over time, and inflation until the relevant date for $IC_{t=1}$ is known. In other words, it indicates the expected return on an index-linked bond investment financed at spot, assuming the real rate will not move in an adverse direction and that inflation at time t = 1 will be realized as assumed. Note that the signal used with carry trades explicitly takes into account the financing costs associated with a trade, and therefore could convey additional information to the investor. If the carry determined in Equation (24) is positive, a long position in index-linked bonds and a short in nominal bonds is taken; the opposite position is assumed in case of a negative carry. The time series taken for the repo rates of the U.S. is the U.S. repo rate.

Finally, as Mulvey and Zenios [1994] show, a *growth-optimal strategy* can be a very useful allocation procedure in fixed-income markets. According to this specification, the utility derived from a position in a certain asset is based on the power utility function

$$U(w_t) = \frac{w_t^{\gamma}}{\gamma} \tag{25}$$

where U denotes the utility, w the wealth at time t, and γ is the parameter specifying the risk aversion of the agent. Setting γ equal to zero, one obtains a logarithmic utility function with a constant factor of absolute risk aversion (the ratio of the first and the second derivative of the utility function), the so-called growth-optimal utility (Campbell, Lo, and MacKinlay [1996]).

Asset allocation maximizing the utility is implemented for every day covered by the bond specific time series, whereby future wealth is calculated according to Equation (23), which results in:

$$\begin{aligned} &\max_{\{w_i\}_t}\sum_{i=1}^{I} w_i E\left[U(W^i)_T\right] \\ &U(W)_T = \left(\frac{W^{\gamma}}{\gamma}\right)_T \text{ and } \lim_{\gamma\to 0}\frac{W^{\gamma}-1}{\gamma} = \ln(W) \\ &\text{with } W_t = \left(\sum_{i=i}^{I} w_i{*}P_i\right)_t - tr \text{ and } P = e^{-r*\Delta t} \end{aligned} \tag{26}$$

where w is the portfolio weight; i indicates the individual bond; I denotes the number of bonds in a portfolio; subscript t is the time the calculation is done; W is the expected future wealth; T is the horizon over which the expectations are taken, which corresponds to the forecasting horizon of the model; (P_i) is the price of bond i at time t; and tr denotes trading costs, if they are a consideration (for derivation of the second line of Equation (26) see Ingersoll [1987]).

The difference between the forecast and observed break-even inflation serves as an indicator for this trading strategy. Accordingly, a long position is taken if the expected future price of an index-linked bond is higher than the current price, and a short position is taken in the opposite case.

Using the signal, holding periods of one, two, three, six, nine and twelve months are applied. For each holding period, the daily proceeds are computed and used to derive the holding-period returns. Taking the example of a break-even trade, the return at time t over holding period hp is calculated as follows:

$$\theta_t\left(P_t^{\text{nominal}} - P_t^{\text{index-linked}}\right) + \theta_\tau\left(P_{t+hp}^{\text{index-linked}} - P_{t+hp}^{\text{nominal}}\right)$$
$$= \text{net proceeds} \tag{27}$$

where θ denotes the number of securities, hp indicates the holding period, and the superscripts refer to the respective bond series. For index-linked bonds with several matching nominal equivalents, the matching is achieved according to Exhibit 1, and the outstanding positions in the first nominal bond are cleared before a position in the second nominal bond is taken. Subsequently, the proceeds are divided by the initial net investment at the beginning of the holding period to determine the return per holding period.

IV. EMPIRICAL RESULTS

We describe results in terms of inflation forecasts and the performance of portfolio strategies.

Inflation Forecasts

Our results suggest that the data support the McCulloch and Stec [2000] short-term forecasting model. As it is a core assumption that the estimates are theoretically appropriate estimators of future inflation, the assumption of a stationary $\{X_t\}$ is tested. The results are shown in Exhibit 2. Although the relatively high t-statistic could indicate that the pseudo-stationary series created is not necessarily stationary with mean 0, it is not high enough to reject the null hypothesis.

The usefulness of the model can also be confirmed by looking at the time series of the estimated unbiased α-coefficients (results not tabulated). The time series suggests that the unit root assumption, i.e., assuming α to be equal to unity, seems to be overly simplifying, as McCulloch and Stec [2000] also observe.

Next we generate forecasts of future inflation rates to compare with the actual realization. Adding past information via a higher degree of integration seems to add information to the forecast. In fact, it seems to be possible to construct a quite accurate inflation forecast.

The McCulloch and Stec [2000] model seems to be useful. Extension of the approach to a multiperiod setting proves successful, making the model a promising element of the framework developed. The model peforms better than a simple forecasting model, and is therefore helpful in determining future inflation.

To generate longer-term forecasts, we follow the approach proposed by Kothari and Shanken [2002] to construct the x/y forward rates by subtracting the yield on an x-period zero-coupon bond from the yield on a y-period zero-coupon bond, using daily constant-maturity Treasury yield data for various maturities provided by the Federal Reserve Board. Forecasts based on different-length past regression periods are very similar, although, the shorter the regression horizon, the more volatile the series becomes, which leads to greater misspecification, particularly for the two- and four-week regression window series.

The intertemporal behavior of the regression coefficients indicates a negative relationship between the length of the estimation horizon and the variability of the series. Generally, the longer the time period, the more stable the coefficients are around some central value. The shape of the time series could indicate some degree of mean reversion, which suggests that the variability is of a rather seasonal nature. This also implies that very short time horizons do not necessarily add in terms of preciseness, if the coefficient fluctuates around some mean rather than following a long-term time trend, which would justify more recent estimates.

Yet seasonality suggests that a regression period of one year is too long, as it might cut through these periods and hence produce an imprecise estimate. Since a regression period of longer than three months seems to explain the past realization better than a short regression horizon, one could reasonably expect to obtain more precise future estimates using longer time horizons for the regression. Although the forecast and the realization seem to differ in their intertemporal behavior, the estimation does not seem to be too far off the true realization, given a small estimation error.

EXHIBIT 2

Stationarity of Inflation Forecast

CPI-U						
Observations	σ_	AE	AAE	t-value	Upper Bound	Lower Bound
171	0.017%	0.025%	0.172%	1.4838	0.0574%	-0.0079%

AE: Average forecasting error. AAE: Average absolute forecasting error.

As all regression periods seem to perform equally well as to how precisely they estimate the future realization, we must use additional factors to make a final choice as to the most optimal regression period. Inflation exhibits strong seasonality, so it seems to be better not to take an annual estimation period for the coefficients. Very short periods between two and four weeks might suffer from small-sample bias, which can lead to fairly extreme coefficient estimates. A three-month period to estimate the regression coefficients is chosen to forecast next year's absolute level of inflation, which seems to sufficiently balance both effects.

If we look at longer forecasting horizons, similar conclusions can be drawn. The estimates generated from different-length estimation windows move together fairly closely. Lengthening of the forecasting horizon leads to greater differences between the estimate and the realization, and produces more extreme outliers. The two-week window exhibits a smaller confidence interval for the estimation error and thus would seem to deliver the best forecast. It also delivers the most volatile estimates, however, and should therefore be disregarded.

To test the usefulness of the forecasting model, we use the coefficients as estimated above to forecast average inflation over several time horizons. As in the single-period case, the forecasting error with a three-month window is fairly low and stable, which could make the model useful for investment decisions.

As break-even inflation is readily observable in the marketplace, the idea behind using another inflation forecast is to decide whether realized inflation will be above or below the market consensus as expressed in break-even inflation. If realized inflation turns out to be higher than the break-even inflation anticipated, the price of an inflation-linked bond will react positively while the opposite would be expected for negative inflation surprises. Therefore, both the level and the sign of the forecast compared to the realization are of importance, which can be expressed by the equation:

$$\begin{aligned}&\left(\text{Realized Inflation}_{t,t+n} - \text{BEI}_t\right)\\ &\quad = \beta^n\left(\text{Forecasted Inflation}_{t,t+n} - \text{BEI}_t\right) + \varepsilon_t \end{aligned} \tag{28}$$

where t is a time index, and n indicates the future period over which the preciseness of average inflation forecasts is tested, i.e., one, two, three, or four years. For an inflation model to provide useful information to an investor, the coefficient β^n should be significantly positive.

In a regression performed per time window n, apart from the one-year-ahead forecast, β^n is consistently negative, suggesting that an investor cannot necessarily benefit from relying on the forecast to determine whether inflation-linked bonds will increase in value or not. The one-year-ahead window, however, provides more positive results (results not tabulated). The coefficient seems to be positive for almost all inflation-indexed bonds. A positive coefficient indicates that the model could be profitably used by an investor if both the holding period is shorter than one year and the change in one-year-ahead inflation significantly affects the price of an inflation-indexed bond.

Performance Measurement—No Market Frictions

We test the information value of the signal by using it as an input for trading strategies for various holding periods. To assess whether the signal constructed yields valuable additional information, we compare the net return on the index-linked bond to the net return of its so-called long-only equivalent. This simple investment strategy always takes a long position in the index-linked bond under investigation and keeps it until the end of the holding period, when it is sold. The exact structure of the position depends on the trading strategy it is compared to.

The simple long-short strategy, the utility-maximizing trading strategy, and the carry trades are compared to the return generated over the same holding period by a position consisting of one index-linked bond bought at the beginning of the period and sold at the end. The break-even trade strategy is compared to a position long in index-linked bonds and short in the matching nominal bonds. This difference in benchmark for the individual bond return series is deemed necessary because of the different risk profiles of the various trading strategies.

Therefore, the main difference between the trading strategies investigated and the long-only series as a control is in the characteristic of the latter to always take a long position in index-linked bonds and keep this position until the end of the holding period. The differential in average return generated by the long-only strategy and a particular trading strategy is henceforth referred to as *average excess return*.

To check for time-variability and to better compare the results for individual bonds, the return series is broken into subperiods starting with the issuance of a new bond in the market and ending one day before the next bond is issued. The simple average returns over these periods per strategy, signal, and holding period are presented in Exhibit 3. The results are reported for the entire period, from January 12, 1998, until March 15, 2003.

Our approach of combining inflation forecast with portfolio allocation decisions seems to perform fairly well in the markets for TIPS. The most promising holding periods seem to be two to three months and nine months.

The allocation procedure using forecasts that yields mostly positive returns above the control series is the break-even strategy. Its performance seems to be best if it is combined with a one-year inflation forecast and implemented at the short to medium end of the real yield curve; i.e., bonds with maturities before July 2012 seem to perform better than longer-maturity bonds if trading decisions are combined with inflation forecasts. Surprisingly, investments in longer-maturity TIPS, i.e., with maturities beyond April 2028, seem to perform better if these are guided by inflation forecasts over a shorter time horizon (six to nine months). The combination performing best is a break-even trade strategy investing in the short to medium end of the yield curve over a holding period of nine months and based on an inflation forecast of one to two years.

Although the optimal forecasting horizon seems to vary over time, the forecasting horizon performing well most consistently turns out to be a one-year-ahead forecast. Also, apart from some exceptions during the early days of the issue, e.g., in the period from April 15, 1998, to June 1, 1999, there are no significant differences in the best-performing forecasting horizon across the yield curve.

The market for TIPS thus indicates investors might improve performance significantly if the allocation decision is combined with an inflation forecast over one to two years. Note this result should be interpreted with caution, however, as we have yet to account for market frictions.

All the same, we obtain several interesting results in the scenario without market frictions. It seems to be possible to enhance returns on TIPS if the investment decision is combined with inflation forecasts. This result could be an indication of inefficiencies in the market; our analysis including market frictions will provide an indication of whether market participants can effectively exploit these findings.

Considering the successful information signals, one can identify several interesting patterns. The fact that holding periods of three months and one year perform well indicates the potential of the framework to inform opportunistic bets and longer-term allocation strategies.

Performance Measurement—Market Frictions

In additional trials we generate new return series that include market frictions in the determination of the allocation signal and the return calculation. Short-selling restrictions and trading costs are included separately in order to isolate the impact of these factors on returns individually and in combination, and to gain information about market characteristics.

Short-selling constraints mean that the position taken could be only a positive one, i.e., a long index-linked bond and a short nominal bond in the case of break-even trades. If a negative return over the holding period is indicated by the signal, no trade is executed, and the resulting position and holding-period return are set to zero.

Discussions with a senior fund manager reveal that the common trading costs for index-linked bonds are typically around 9 to 13 basis points of the value of the transaction. We therefore set these costs at the midpoint range of 11 basis points.

Some trading strategies had to be adapted in order to properly reflect this additional environmental variable. In the trading strategy maximizing the expected utility, expected trading costs are calculated using the expected security price, which reduces the expected utility derived from the future value of the index-linked bonds. If trading costs affect the expected return so that it would become negative, the resulting investment is contingent on the presence of short-selling constraints. In the latter case, no action

Exhibit 3

Performance of Trading Strategies—No Market Frictions

Panel A: 1-Month Holding Period

		Long-Only					
Name of Bond	Statistic	LO	BEI	Long-Short	Break-Even	Growth Optimal	Carry Trades
TIPS (01/15/2007)	Return	0.10	-0.41	0.06	0.19	0.06	
	Difference: LO / BEI			-0.04	**0.60**	-0.04	-0.03
	P-value			0.07	0.00	0.07	
	Model			M&S 9 Ms.	**K&S 1 Yr.**	M&S 9 Ms.	M&S 9 Ms.
TIPS (01/15/2008)	Return	0.16	-0.30	0.10	0.02	0.10	
	Difference: LO / BEI			-0.06	**0.32**	-0.06	-0.06
	P-value			0.05	0.00	0.05	
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2009)	Return	0.21	-0.19	0.01	-0.03	0.01	
	Difference: LO / BEI			-0.21	**0.16**	-0.21	-0.20
	P-value			0.00	0.00	0.00	
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2010)	Return	0.39	-0.27	0.13	-0.11	0.13	
	Difference: LO / BEI			-0.26	**0.15**	-0.26	-0.26
	P-value			0.00	0.00	0.00	
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2011)	Return	0.40	-0.13	-0.06	0.05	-0.06	-0.06
	Difference: LO / BEI			-0.46	**0.18**	-0.46	-0.46
	P-value			0.00	0.01	0.00	
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2012)	Return	0.79	0.06	-0.53	0.22	-0.53	-0.53
	Difference: LO / BEI			-1.32	**0.16**	-1.32	-1.32
	P-value			0.00	0.05	0.00	
	Model			M&S 3 Ms.	**M&S 3 Ms.**	M&S 3 Ms.	M&S 3 Ms.
TIPS (04/15/2028)	Return	0.30	0.07	0.07	0.30	0.07	
	Difference: LO / BEI			-0.23	**0.23**	-0.23	-0.21
	P-value			0.00	0.01	0.00	
	Model			K&S 1 Yr.	**M&S 6 Ms.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (04/15/2032)	Return	1.22	0.26	-0.31	0.37	-0.31	-0.31
	Difference: LO / BEI			-1.54	**0.11**	-1.54	-1.54
	P-value			0.00	0.29	0.00	
	Model			M&S 9 Ms.	**M&S 9 Ms.**	M&S 9 Ms.	M&S 9 Ms.

is taken; i.e., the position and the related holding-period return are set equal to zero. Note that non-invested funds are kept as cash and thus generate no interest revenue.

Combinations of signal and holding period leading to positive average excess returns in at least one subperiod of a bond in each individual market are used in the return calculations. The results with short-selling constraints and trading costs are in general similar to those in the unconstrained case, but less pronounced. If both constraints are introduced individually, the model still seems to provide a useful signal for a mean-variance-efficient investor for the short end of the TIPS series. It is interesting to see higher average excess returns when we include short-selling constraints than when we include trading costs.

For the medium-maturity TIPS spanning maturities from 2010 to 2012, interestingly, fewer securities and trading strategies lead to a positive average excess return than for short-maturity TIPS. The break-even trading strategy is the most profitable one and also the only profitable strategy under market frictions. Market frictions have a strong negative impact on the usefulness of information about likely future inflation in portfolio allocation decisions in the U.S. index-linked bond market. And some trends that began to be observable in moving from short- to medium-maturity TIPS continue in the case of long-maturity TIPS. The only profitable trading strategy is break-even trading.

EXHIBIT 3 (continued)
Performance of Trading Strategies—No Market Frictions

Panel B: 2-Month Holding Period

Name of Bond	Statistic	Long-Only LO	Long-Only BEI	Long-Short	Break-Even	Growth Optimal	Carry Trades
TIPS (01/15/2007)	Return	0.22	-0.85	0.14	0.38	0.14	0.17
	Difference: LO / BEI			-0.08	**1.23**	-0.08	-0.05
	P-value			0.03	0.00	0.03	0.13
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2008)	Return	0.32	-0.64	0.16	0.09	0.16	0.16
	Difference: LO / BEI			-0.16	**0.73**	-0.16	-0.16
	P-value			0.00	0.00	0.00	0.00
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2009)	Return	0.44	-0.42	0.05	-0.12	0.05	0.08
	Difference: LO / BEI			-0.39	**0.30**	-0.39	-0.36
	P-value			0.00	0.00	0.00	0.00
	Model			M&S 1 Yr.	**K&S 1 Yr.**	M&S 1 Yr.	M&S 1 Yr.
TIPS (01/15/2010)	Return	0.77	-0.53	0.10	-0.22	0.10	0.10
	Difference: LO / BEI			-0.67	**0.30**	-0.67	-0.67
	P-value			0.00	0.00	0.00	0.00
	Model			M&S 3 Ms.	**K&S 1 Yr.**	M&S 3 Ms.	M&S 3 Ms.
TIPS (01/15/2011)	Return	0.74	-0.33	-0.40	0.08	-0.40	-0.40
	Difference: LO / BEI			-1.15	**0.41**	-1.15	-1.14
	P-value			0.00	0.00	0.00	0.00
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2012)	Return	1.57	-0.06	-1.21	0.69	-1.21	-1.20
	Difference: LO / BEI			-2.78	**0.75**	-2.78	-2.78
	P-value			0.00	0.00	0.00	0.00
	Model			M&S 3 Ms.	**K&S 1 Yr.**	M&S 3 Ms.	M&S 3 Ms.
TIPS (07/15/2012)	Return	1.04	-0.30	-0.47	0.30	-0.47	-0.44
	Difference: LO / BEI			-1.51	**0.61**	-1.51	-1.47
	P-value			0.00	0.00	0.00	0.00
	Model			M&S 1 Yr.	**K&S 1 Yr.**	M&S 1 Yr.	M&S 1 Yr.
TIPS (04/15/2028)	Return	0.58	0.14	0.27	0.36	0.27	0.34
	Difference: LO / BEI			-0.30	**0.22**	-0.30	-0.23
	P-value			0.00	0.04	0.00	0.01
	Model			K&S 1 Yr.	**M&S 6 Ms.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (04/15/2032)	Return	1.63	0.23	-0.70	0.37	-0.70	-0.70
	Difference: LO / BEI			-2.34	**0.14**	-2.34	-2.34
	P-value			0.00	0.32	0.00	0.00
	Model			M&S 9 Ms.	**M&S 9 Ms.**	M&S 9 Ms.	M&S 9 Ms.

Therefore, the performance of the different trading strategies regarding returns and the most useful signal is fairly consistent across the term structure. Short-term bonds have similar returns as and more useful signals than medium- and long-maturity TIPS. The return volatility for trading strategies using inflation forecasts increases with maturity.

We can draw several general conclusions from the individual introduction of market frictions. First, transaction costs are likely to have the most pronounced effect on realized returns. Second, the simple model using inflation forecasts of about one year seems to offer the most useful information content to the user. If this approach is used to support break-even trades for holding periods of around three months, performance can generally be even further enhanced. The short end of the TIPS yield curve turns out to represent the segment of the index-linked government bond market where these specifications could be most profitably implemented.

EXHIBIT 3 (continued)
Performance of Trading Strategies—No Market Frictions

Panel C: 3-Month Holding Period

Name of Bond	Statistic	Long-only LO	Long-only BEI	Long-Short	Break-Even	Growth Optimal	Carry Trades
TIPS (01/15/2007)	Return	0.35	-1.29	0.25	0.58	0.25	0.31
	Difference: LO / BEI			-0.10	**1.87**	-0.10	-0.04
	P-value			0.02	0.00	0.02	0.20
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2008)	Return	0.47	-0.98	0.31	0.09	0.31	0.32
	Difference: LO / BEI			-0.17	**1.07**	-0.17	-0.15
	P-value			0.00	0.00	0.00	0.01
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2009)	Return	0.68	-0.65	0.27	-0.18	0.27	0.30
	Difference: LO / BEI			-0.41	**0.47**	-0.41	-0.38
	P-value			0.00	0.00	0.00	0.00
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2010)	Return	1.11	-0.78	0.06	-0.27	0.06	0.08
	Difference: LO / BEI			-1.05	**0.52**	-1.05	-1.04
	P-value			0.00	0.00	0.00	0.00
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	M&S 3 Ms.
TIPS (01/15/2011)	Return	1.08	-0.51	-0.74	0.24	-0.74	-0.72
	Difference: LO / BEI			-1.82	**0.75**	-1.82	-1.80
	P-value			0.00	0.00	0.00	0.00
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2012)	Return	2.45	-0.36	-1.81	1.03	-1.81	-1.76
	Difference: LO / BEI			-4.26	**1.40**	-4.26	-4.21
	P-value			0.00	0.00	0.00	0.00
	Model			M&S 3 Ms.	**K&S 1 Yr.**	M&S 3 Ms.	M&S 3 Ms.
TIPS (07/15/2012)	Return	4.81	0.04	-0.12	0.59	-0.12	-0.12
	Difference: LO / BEI			-4.93	**0.55**	-4.93	-4.93
	P-value			0.01	0.00	0.01	0.01
	Model			M&S 1 Yr.	**K&S 2 Yrs.**	M&S 1 Yr.	M&S 1 Yr.
TIPS (04/15/2028)	Return	0.58	0.14	0.27	0.36	0.27	0.34
	Difference: LO / BEI			-0.30	**0.22**	-0.30	-0.23
	P-value			0.00	0.04	0.00	0.01
	Model			K&S 1 Yr.	**M&S 6 Ms.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (04/15/2032)	Return	0.89	-0.22	-0.89	0.22	-0.89	-0.89
	Difference: LO / BEI			-1.78	**0.44**	-1.78	-1.78
	P-value			0.00	0.13	0.00	0.00
	Model			K&S 2 Yrs.	**K&S 2 Yrs.**	K&S 2 Yrs.	K&S 2 Yrs.

Portfolio Setting

Institutional investors active in the index-linked bond market, who are the main holders of index-linked bond portfolios, mostly follow particular market indicators in their asset allocation. Selective bets on single securities are usually made via relative under- and overweighting with respect to a certain benchmark. Hence, we calculate the portfolio return over a specific holding period as the weighted-average return on the single securities including transaction costs but excluding short-selling constraints. By this means, we hope to track as closely as possible the weighting of the relevant market value-weighted index-linked bond indexes that institutional investors use as a benchmark for portfolio weights and returns.

Our results so far consider the effect of using inflation forecasts for investment in individual securities. To complete our analysis, we want to examine the effect of the approach on portfolio returns. A full-fledged mean-variance optimization is difficult to implement, as cross-correlations are hard to obtain for fixed-income instruments,

EXHIBIT 3 (continued)
Performance of Trading Strategies—No Market Frictions

Panel D: 6-Month Holding Period

Name of Bond	Statistic	Long-only LO	Long-only BEI	Long-Short	Break-Even	Growth Optimal	Carry Trades
TIPS (01/15/2007)	Return	0.79	-2.52	0.66	0.83	0.66	0.74
	Difference: LO / BEI			-0.13	**3.35**	-0.13	-0.05
	P-value			0.05	0.00	0.05	0.25
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2008)	Return	1.02	-1.95	0.77	0.24	0.77	0.80
	Difference: LO / BEI			-0.25	**2.19**	-0.25	-0.21
	P-value			0.00	0.00	0.00	0.00
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2009)	Return	1.34	-1.42	0.75	-0.36	0.75	0.89
	Difference: LO / BEI			-0.59	**1.06**	-0.59	-0.44
	P-value			0.00	0.00	0.00	0.00
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2010)	Return	2.06	-1.58	0.53	-0.80	0.53	0.42
	Difference: LO / BEI			-1.53	**0.78**	-1.53	-1.64
	P-value			0.00	0.00	0.00	0.00
	Model			K&S 1 Yr.	**K&S 2 Yrs.**	K&S 1 Yr.	M&S 9 Ms.
TIPS (01/15/2011)	Return	2.38	-1.30	-1.14	-0.12	-1.14	-1.09
	Difference: LO / BEI			-3.52	**1.17**	-3.52	-3.47
	P-value			0.00	0.00	0.00	0.00
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (01/15/2012)	Return	6.15	-1.08	-3.36	1.48	-3.36	-3.18
	Difference: LO / BEI			-9.52	**2.56**	-9.52	-9.34
	P-value			0.00	0.00	0.00	0.00
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (07/15/2012)	Return	4.81	0.04	0.13	0.39	0.13	0.13
	Difference: LO / BEI			-4.68	**0.35**	-4.68	-4.68
	P-value			0.00	0.00	0.00	0.00
	Model			M&S 3 Ms.	**M&S 3 Ms.**	M&S 3 Ms.	M&S 3 Ms.

so we base our analysis merely on the returns generated by using portfolios constructed as described above. Because the benchmark used to calculate the average excess return is the index-linked government bond index return of the investor's home country, the excess return reported is probably of more information value for practitioners tracking index-linked bond indexes of the markets they are active in.

We construct a portfolio for the U.S. index-linked bond market to investigate whether investment in an entire index-linked bond market, if guided by the differential of break-even inflation and inflation forecasts, can lead to profitable results. If positive results can be obtained, this might simplify implementation of our approach, as we could ignore the dependence on selective parts of the entire bond issue and the risk of this dependence to change over time.

Exhibit 4 indicates that break-even trades deliver a consistently positive return for a portfolio of TIPS if combined with an inflation forecast going one to two years into the future. Thus long-term holding periods seem to lead to better results than shorter ones, confirming our earlier findings. Carry trades and the utility maximization strategy also generate positive excess returns for almost every holding period if the simple signal is used.

The results in Exhibit 4 suggest that investors in TIPS had consistently positive average excess returns if they closely tracked the index and increased or reduced positions in the individual bonds according to the signal given by our model. This result holds regardless of the holding period chosen for the break-even trades and for almost all holding periods for carry trades and the utility maximization strategy. Although the other trading strategies seem to produce higher excess returns, their higher return volatility suggests more pronounced risk.

EXHIBIT 3 (continued)
Performance of Trading Strategies—No Market Frictions

Panel E: 12-Month Holding Period

Name of Bond	Statistic	Long-only LO	Long-only BEI	Long-Short	Break-Even	Growth Optimal	Carry Trades
TIPS (01/15/2007)	Return	1.42	-4.74	2.08	0.50	2.08	2.23
	Difference: LO / BEI			**0.66**	**5.23**	**0.66**	**0.81**
	P-value			0.00	0.00	0.00	0.00
	Model			**K&S 2 Yrs.**	**K&S 1 Yr.**	**K&S 2 Yrs.**	**M&S 6 Ms.**
TIPS (01/15/2008)	Return	1.81	-3.36	2.75	-0.94	2.75	2.77
	Difference: LO / BEI			**0.93**	**2.42**	**0.93**	**0.95**
	P-value			0.00	0.00	0.00	0.00
	Model			**K&S 2 Yrs.**	**K&S 1 Yr.**	**K&S 2 Yrs.**	**K&S 2 Yrs.**
TIPS (01/15/2009)	Return	2.86	-3.26	3.01	-1.25	3.01	3.03
	Difference: LO / BEI			**0.15**	**2.01**	0.15	0.17
	P-value			0.18	0.00	0.18	0.14
	Model			**M&S 9 Ms.**	**K&S 1 Yr.**	M&S 9 Ms.	K&S 2 Yrs.
TIPS (01/15/2010)	Return	4.08	-3.14	4.03	-2.18	4.03	3.81
	Difference: LO / BEI			-0.05	**0.95**	-0.05	-0.27
	P-value			0.38	0.00	0.38	0.06
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	M&S 3 Ms.
TIPS (01/15/2011)	Return	4.15	-1.90	4.03	-0.70	4.03	4.15
	Difference: LO / BEI			-0.12	**1.20**	-0.12	0.00
	P-value			0.33	0.00	0.33	0.50
	Model			K&S 1 Yr.	**K&S 1 Yr.**	K&S 1 Yr.	K&S 1 Yr.
TIPS (04/15/2028)	Return	3.09	2.43	4.55	0.94	4.55	4.35
	Difference: LO / BEI			**1.45**	-1.49	**1.45**	**1.25**
	P-value			0.00	0.00	0.00	0.00
	Model			**K&S 1 Yr.**	M&S 1 Yr.	**K&S 1 Yr.**	**M&S 9 Ms.**
TIPS (04/15/2032)	Return	7.50	2.33	10.99	5.76	10.99	8.27
	Difference: LO / BEI			**3.50**	**3.43**	**3.50**	**0.78**
	P-value			0.07	0.07	0.07	0.00
	Model			**K&S 1 Yr.**	**K&S 1 Yr.**	**K&S 1 Yr.**	**K&S 1 Yr.**

LO: Long only. BEI: Break-even inflation.

K&S xYr: 3-, 2-, 1-year inflation forecast based on Kothari and Shanken [2002]. M&S xYr: 12-, 9-, 6-, 3-month forecast based on McCulloch and Stec [2000].

Only positive results shown for readability. Boldface indicates signal that leads to a positive excess return over the long-only benchmark.

V. SUMMARY AND CONCLUSIONS

Investors seem to be able to earn better returns than uninformed investors could, if they combine an investment strategy with inflation forecasts. This finding, which is quite pronounced in a market environment without market frictions, persists in a setting that includes market frictions, although the positive average excess returns are generally lower. The potential for better performance using one particularly dominant signal, i.e., a one-year-ahead inflation forecast, seems to be little affected by the type of constraint.

Average excess returns seem to be contingent on the type of market friction, however. The excess return over the long-only strategies is almost always higher under short-selling constraints than in the presence of trading costs. This implies that it is not necessarily the incompleteness of the index-linked bond market—that is, the inability to short this security—but its thinness expressed in the wide spread asked for every deal that inhibits full incorporation of information about future inflation in current prices. As trading costs seem to have the strongest impact on the return generated when used in combination with the information signal, they can be interpreted as the prime barrier to full use of information in the index-linked government bond market.

EXHIBIT 4
Average Excess Return on Market Value-Weighted Portfolio of TIPS

Signal			Trading Strategy		
Model	Forecast Horizon	Holding Period	Break-Even Trades	Growth Optimal	Carry Trades
Kothari and Shanken (2002)	1 Year	1 Month	0.002	0.088	0.087
Kothari and Shanken (2002)	1 Year	2 Months	0.004	0.027	0.024
Kothari and Shanken (2002)	1 Year	3 Months	0.005	0.010	0.006
Kothari and Shanken (2002)	1 Year	6 Months	0.007	-0.052	-0.088
Kothari and Shanken (2002)	1 Year	12 Months	**0.008**	**0.198**	**0.275**

The strategy maximizing expected utility, while not performing as well as the other strategies when we eliminate market frictions and include short-selling constraints, outperforms the other strategies once trading costs are considered. Since this trading strategy is the only one explicitly considering the effect of trading costs in establishing a position, this result shows the importance of transaction costs as a factor influencing the price structure of the index-linked government bond market.

As various governments commit themselves to further supply this market with securities, and as potential buyers such as pension funds seem to become more aware of this segment of the fixed-income market, one can expect the index-linked government debt markets to become increasingly liquid. This could cause trading costs to drop and hence make our approach more profitable.

Another result of including market frictions is that longer holding periods seem to be more favorable than shorter ones, which could mean that transaction costs penalize too-active traders. This finding improves the precision of the results in the unconstrained framework, where holding periods of three, six, and nine months are potentially profitable.

Confirming the results of the unconstrained scenario, break-even trades perform best when combined with information on inflation forecasts. The consistency of this performance shows that this trading strategy seems to be the most appropriate. One reason could be the portfolio structure resulting from break-even trades, which eliminates the risk of changes in the real rate impacting the returns on a position in index-linked bonds; this can be interpreted as confirming the findings of Foresi, Penati, and Pennacchi [1997].

This finding differs from the argument underlying common practice in trading strategies in the fixed-income market, where maximizing the expected utility of future wealth as in Mulvey and Zenios [1994] is typically the most optimal trading rule. This can be interpreted as an indication that the index-linked bond market is different from the (nominal) debt markets.

Moreover, this result gives an indication of the relationship between break-even inflation and the inflation rate itself. The combination of inflation forecast and trading strategy works best if the trading strategy aims at movements of break-even inflation itself.

One could interpret this finding as confirming that price movements in index-linked bonds can be accurately foreseen using inflation forecasts, and the visible comovements of the realized inflation rate and break-even inflation serve as a first hint of this relationship. The observed relationship of realized and break-even inflation can therefore be confirmed, and even be profitably exploited if one combines information from a sufficiently reliable inflation forecast with a portfolio that trades on movements in break-even inflation. Therefore, movements in break-even inflation seem to be forecastable by means of inflation forecasts.

As one might expect, we see similar results in a portfolio setting. TIPS perform fairly well across holding periods and trading strategies; break-even trades demonstrate the least variation in returns. The portfolio setting sharpens the results in the individual case regarding the most promising forecasting horizons and the type of model to use.

We have asked whether investors can obtain better investment returns in the market for index-linked government debt if they include some additional pieces of information in their asset allocation decision.

It turns out that forecasts over several periods up to one year based on McCulloch and Stec [2000] provide a fairly good fit to realized inflation for the markets considered in this study. Although the fit of forecasts deteriorates significantly beyond a one-year horizon, forecasts generally provide closer approximations of the future rate of inflation than break-even inflation could.

The differential of forecasted inflation and break-even inflation provides a guide for investment decisions, considering different combinations of trading strategies and holding periods. TIPS reveal the possibility of earning returns beyond a long-only strategy, although the profit potential diminishes with increasing maturity of the TIPS. A simpler framework combined with a one to two years inflation forecast used for break-even trades and a holding period of three months to one year proves the most promising combination, a welcome finding because the approach is straightforward to implement.

The effectiveness of trading systematically on price movements, which for index-linked bonds are independent of price movements for their nominal counterparts, shows that the index-linked bond market must be considered as distinct from the nominal debt markets. This finding, also supported by the better performance of break-even trades over the expected utility maximization strategy, shows the importance of including TIPS in a portfolio to more effectively shape exposure and reduce idiosyncratic portfolio risk. More important, however, the apparent success of combining a one-year-ahead inflation forecast with a break-even trade strategy in the market for TIPS confirms our research question.

The market for index-linked bonds seems to be not entirely efficient. This inefficiency can be exploited by including inflation forecasts in trades on break-even inflation. The most useful horizon of the inflation forecast is between one and two years, which indidates that break-even inflation seems to be a consensus estimate of average inflation rates over this period. This confirms the claim by Kothari and Shanken [2002] that credible forecasts over a much longer horizon are not possible, but also shows one can make more precise forecasts than the market does. As our inflation models are reasonably precise for up to one year, one can assume that any inflation model could provide a successful signal, if it is at least as accurate as the ones in our study.

ENDNOTES

[1]Cooper and Hodgson [2001] and "Inflation-Linked Bonds for €-Based Investors" [2001] note there are other factors that influence price behavior and hence break-even inflation, such as regulation, supply and demand factors, and liquidity, which are of concern in smaller markets.

[2]This could be the result of the negative correlation of unexpected changes in expected inflation and the prices of ILBs.

[3]They assume 1) state-independent, log-form utilities making expressions with the state variable Y equal to zero, and 2) independence of the (real) wealth level from the rate of inflation, which yields:

$$r_{\text{nominal}} = r_{real} + E[\pi] - \sigma^2_{pricelevel} = r_{real} + \tilde{E}[\pi]$$

where $\tilde{E}[\pi]$ denotes risk-adjusted expected inflation.

[4]The main characteristic of the Canadian structure is that the principal of the bond is indexed to the (inflation) index and adjusted accordingly every period. This inflation adjustment in the principal is not paid out until maturity. The periodic coupon takes the inflation adjustment automatically into account, however, as it is defined as a fraction of the current value of the adjusted principal rather than a fixed periodic payment (see "Inflation-Indexed Bonds" [1996]).

[5]The daily adjustment in the reference index is calculated as follows:

$$\text{Inflation adjustment}_t = \text{Inflation index}_{t-3} + \frac{\text{currentdayinmonth}_{t-1}}{\text{totalnumberofdays}_{t-2}}\left(\text{Inflation index}_{t-2} - \text{Inflation index}_{t-3}\right)$$

where t denotes the month (see "Le Meilleur de l'Euro—OATei" [2001]).

[6]The beta of an index-linked bond is the return variance of an index-linked bond divided by the return variance of a comparable nominal bond (Hunter and Simon [2002]).

[7]The unit root assumption in this context would imply that

$$y_t = \beta_1 y_{t-1} + \varepsilon_t \, or (y_t - y_{t-1}) = \varepsilon_t \text{ for } \varepsilon_t \sim N(0, \sigma)$$

[8]A time series is stationary if:

$$E[X_t] = \text{constant} \forall t$$

$$\text{Cov}(X_t, X_{t+i}) = E[X_t X_{t+i}] = \gamma_{(i)}$$

This implies that the covariance of the members of the time series is constant; i.e., it does not depend on the individual members of the series but only on the distance over time between two elements, indicated by i. As one is dealing with an autoregression, this can be expressed by $\gamma_{(i)}$, the autocovariance as a function of i (Fuller [1996]).

[9]More specifically, Treasury STRIPS are taken for the entire analysis in order to minimize price distortions attributable to reinvestment risk, and to keep the data coherent with the general term structure context, which operates with prices derived from zero-coupon securities.

[10]It might be difficult to implement a true long-short portfolio because of regulatory constraints, management practice, and an inability to go short in this market. We later redefine this strategy to a simple over- and underweighting with respect to the relevant benchmark.

[11]The relevant inflation indexation coefficient comes from the issuing government: See "TIPS Indexation Coefficients" [2003] for the coefficients of the series of TIPS.

REFERENCES

Barone, E., and R.S. Masera. "Index-Linked Bonds from an Academic, Market and Policy-Making Standpoint." In *Managing Public Debt*. Cheltenham: Edward Elgar, 1997, pp. 117-147.

Barr, D.G., and J.Y. Campbell. "Inflation, Real Interest Rates, and the Bond Market: A Study of UK Nominal and Index-Linked Government Bond Prices." *Journal of Monetary Economics*, 39 (3) (August 1997), pp. 361-383.

Brown, R.H., and S.M. Schaefer. "The Term Structure of Real Interest Rates and the Cox, Ingersoll, and Ross Model." *Journal of Financial Economics*, 35 (1993), pp. 3-42.

Campbell, J.Y., A.W. Lo, and A.C. MacKinlay. *The Econometrics of Financial Markets*. Princeton: Princeton University Press, 1997.

Campbell, J.Y., and R.J. Shiller. "A Scorecard for Indexed Government Debt." NBER Working Paper No. 5587, 1996.

Cooper, N., and M. Hodgson. "Investing in Inflation-Protected Bonds." Credit-Suisse First Boston, 2001.

Cox, J.C., J.E. Ingersoll, Jr., and S.A. Ross. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica*, 53 (2) (1985a), pp. 363-384.

——. "A Theory of the Term Structure of Interest Rates." *Econometrica*, 53 (2) (March 1985b), pp. 385-407.

Cross, H. "OATei: EMU-Inflation Indexed OAT." BNP Paribas, 2001.

"Daily Time-Series of Various Maturities." Federal Reserve Board, 2003. Available as follows: one-year constant-maturity yield curve: http://www.federalreserve.gov/releases/h15/data/b/tcm1y.txt; two-year constant-maturity yield curve: http://www.federalreserve.gov/releases/h15/data/b/tcm2y.txt; three-year constant-maturity yield curve: http://www.federalreserve.gov/releases/h15/data/b/tcm3y.txt; and five-year constant-maturity yield curve: http://www.federalreserve.gov/releases/h15/data/b/tcm5y.txt.

"European Interest Rate Products." Goldman Sachs Research, 2003.

Foresi, S., A. Penati, and G. Pennacchi. "Reducing the Cost of Government Debt: The Role of Index-linked bonds." In *Managing Public Debt*. Cheltenham: Edward Elgar, 1997, pp. 93-116.

Fuller, W.A. *Introduction to Statistical Time-Series,* 2nd ed. New York: John Wiley & Sons, 1996.

Gorton, G., and G. Pennacchi. "Security Baskets and Index-Linked Securities." NBER Working Paper No. 3711, May 1991.

Hunter, D.M., and D.P. Simon. "Are TIPS the "Real" Deal? A Conditional Assessment of their Role in a Nominal Portfolio." Working paper, University of South Florida and Bentley College, July 1, 2002.

"Inflation-Indexed Bonds—US Inflation-Indexed Bonds: Q & A." Bridgewater Associates, September 1996.

"Inflation-Linked Bonds for €-Based Investors." Bridgewater Associates, December 2001.

Ingersoll, J.E., Jr. *Theory of Financial Decision Making*. Savage, MD: Rowman and Littlefield, 1987.

Kandel, S., A.R. Ofer, and O. Sarig. "Real Interest Rates and Inflation: An Ex-Ante Empirical Analysis." In *Managing Public Debt*. Cheltenham: Edward Elgar, 1997, pp. 148-169.

Kothari, S.P., and J. Shanken. "Asset Allocation with Conventional and Indexed Bonds." Working paper, MIT, March 26, 2002.

McCulloch, J.H., and J.A. Stec. "Generating Serially Uncorrelated Forecasts of Inflation by Estimating the Order of Integration Directly." Working paper, Ohio State University, June 2000.

"Le Meilleur de l'Euro—*OATei*." Agence France Trésor. Available at ftp://ftp.francetresor.gouv.fr/pub/oatei/docs/slides_oatei_01_fr.pdf

Mulvey, J.M., and S.A. Zenios. "Capturing the Correlations of Fixed-Income Instruments." *Management Science*, 40 (10) (October 1994), pp. 1329-1342.

Ross, S.A. "The Arbitrage Theory of Capital Asset Pricing." *Journal of Economic Theory*, 13 (December 1976), pp. 341-360.

Spiegel, M.M. "Central Bank Independence and Inflation Expectations: Evidence from British Index-linked Gilts." *FRBSF Review*, 1 (1998), pp. 3-14.

"TIPS Indexation Coefficients." U.S. Department of the Treasury, 2003. Available at: http://www.publicdebt.treas.gov/of/ofhiscpi.htm.

Valkanov, R. "The Term Structure with Highly Persistent Interest Rates." Working paper, University of California at Los Angeles, October 1998.

Vasicek, O. "An Equilibrium Characterization of the Term Structure." *Journal of Financial Economics*, 5 (1977), pp. 177-188.

Woodward, G.T. "The Real Thing: A Dynamic Profile of the Term Structure of Real Interest Rates and Inflation Expectations in the United Kingdom, 1982-89." *Journal of Business*, 63 (3) (1990), pp. 373-398.