Fixed-Income Style Analysis and Optimal Manager Structure

FREDERICK E. DOPFEL

FREDERICK E. DOPFEL is a managing director and senior strategist with the Client Advisory Group at Barclays Global Investors in San Francisco.
fred.dopfel@barclaysglobal.com

The institutional investor's goal for fixed-income is to assemble a portfolio of managers that provides a consistent alpha (realized return in excess of the strategic benchmark) with an acceptable level of active risk, or tracking error. The ability of individual managers to produce consistent returns and the way managers' investment styles are blended both have a significant impact on portfolio performance. Despite the importance of this problem, most institutional investors do not have a scientific way to assemble a portfolio of fixed-income managers, and as a result experience disappointing performance.

Exhibit 1 shows the rolling annual realized alpha of a sample of the largest active fixed-income managers described as *core* or *core-plus*. This time series simulates the performance of an average diversified portfolio of managers benchmarked to the U.S. Lehman Aggregate Index. Over the entire period, realized alpha has been close to zero, although in some periods the managers as a group have materially outperformed or underperformed. Why?

Plotted on the same graph (different axis) is the performance of the Lehman High Yield Credit Index, which appears linked to the performance of the managers. In fact, there is a 79% correlation in the monthly series over a 15-year period and a 90% correlation over the most recent 5 years. In other words, the aggregate performance of managers is driven largely by the average manager's persistent style biases to credit.

This is *not* what investors should be paying for. Investors have asked and paid for alpha (pure active risk exposure), but they have received beta (systematic exposure). Absent an improved system for portfolio construction, investors would be better off reproducing this performance (at lower cost) by holding a combination of index funds rather than active managers.

With or without systematic biases, the dismal performance of fixed-income managers as a whole is not surprising. Active management in any asset class is a zero-sum game, as explained by Sharpe [1991]. But is there a scientific way to improve your chances by picking the right managers and then assembling an optimal portfolio? This important question is the subject of this article, and I do believe that skillful active management, while difficult, is not impossible.

First, I define the various categories of fixed-income active return and risk and the prospects for skillful active management from each source. Then, we will see how style analysis helps to quantify the exposures managers are taking and helps to develop meaningful forward-looking assumptions about manager performance. A case study demonstrates how to best blend managers with diverse styles, to control risk, and to structure an optimal portfolio of fixed-income managers.

EXHIBIT 1

Fixed-Income Managers' Annual Realized Alpha and High Yield Index Performance (1989-2003)

Source: eVestment Alliance and Barclays Global Investors.

I. SOURCES OF ACTIVE RISK AND ACTIVE RETURN

A typical institutional investor allocates 30%-40% of total assets to fixed-income, and in the United States these assets are usually benchmarked to the Lehman Aggregate Bond Index or some other common fixed-income index. (See Siegel [2003] for a discussion of benchmarks.) Beyond the government, investment-grade credit, and MBS/securitized components of the Lehman Aggregate, an investor may define a customized benchmark to include components of high-yield, non-U.S. bonds, and Treasury Inflation Protected Securities. The investor may also set a target interest rate duration that could be the same as or different from the natural duration of the benchmark. The target duration is set in order to partially match the duration of the assets and the duration of the liabilities, with the goal of reducing surplus risk.[1]

Investors implement the fixed-income strategy by hiring some mixture of active and indexed fixed-income managers.[2] Implicitly, investors assume that managers remain on their stated benchmarks, and that the resulting overall portfolio is thus consistent with the investor's benchmark. These assumptions are typically imperfect, but there are tools for correcting this problem (discussed later).

Near-perfect consistency with the benchmark can be attained by investing solely in an index fund or a combination of index funds representing the investor's benchmark. In fact, this rarely happens because most investors seek to do better than the benchmark by hiring active managers. Fixed-income portfolios are predominantly active; an average institutional fixed-income portfolio has approximately 85% of the assets with active managers and only 15% indexed.

In seeking active return, investors are necessarily accepting active risk. Active return is measured by the total return less the benchmark return for the portfolio. Active risk is measured by the standard deviation of the active returns. How much active risk do you have—is it too much, too little, or just enough? For most, the question of how much risk is unasked and unanswered, but the amount is assumed to be small.[3]

The typical range for active risk in large institutional portfolios is between 0.40% and 1.20%. The higher end of the range is typical of portfolios incorporating a high-yield component, and the lower end of the range is typical of portfolios composed primarily of core managers.

Style Bias

At the highest level, active risk and return are derived from two major components: style bias, and the residual. Style bias refers to the persistent difference between the sector and duration exposures of a portfolio in comparison to the exposures of the investor's fixed-income benchmark, sometimes called *misfit*. Style bias may be broken into two major components:

- *Sector bias (style)* – The persistent overweight or underweight to fixed-income sectors relative to the weighting of those sectors in the investor's benchmark.
- *Duration bias (style)* – The persistent higher or lower duration (yield curve position) relative to the duration of the investor's benchmark.

Sustained sector and duration biases in a manager's portfolio with respect to the manager's stated benchmark exposures are usually best viewed as evidence that a manager has a normal style, or effective benchmark, reflecting these sustained positions. The manager's normal style (and stated benchmark) may diverge from the investor's stated benchmark. The difference between the normal style of the manager and the investor's benchmark, if not offset by other managers with complementary styles, represents a net active beta exposure and thereby adds active risk. The active style positions reflect how the normal portfolio of the manager (or the investor) diverges from the investor's benchmark.

The expected active return associated with misfit is a matter of financial market equilibrium pricing, not truly about active value-added. As I noted in the introduction, it is beta; it is *not* alpha.[4]

Residual Risk

The residual component of active risk refers to the remaining net exposures associated with both individual security selection and market timing changes after consideration of the style of the portfolio. The residual may be broken into two major components:

- *Selection bets (residual)*—Investment in individual securities (credit risk selection), in weights different from their weights in the normal portfolio, after taking account of the sector and duration biases described above.
- *Tactical bets (residual)*—Timed fluctuations in sector and duration allocations with respect to the normal portfolio, after taking account of the sector and duration biases described above.

Tactical sector bets are based on forecasting changes in relative credit spreads and are distinguished from persistent style biases. Tactical duration bets are based on forecasting changes in the level, slope, and shape of the yield curve, and are distinguished from persistent duration biases. Expected returns associated with residual bets represent true alpha, but they are conditional on our belief in the investment skill of the managers.

Without skill, there should be *no expectation* that residual returns generated by tactical sector and duration bets or by selection bets will yield a positive alpha. As noted earlier, active management is a zero-sum game.

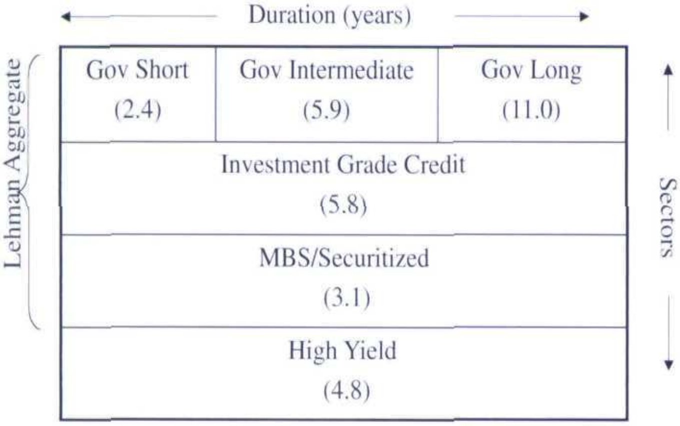
Fundamental Law of Active Management

What information would be useful to us to identify the types of managers who might have better chances than others? One tool to help assess the relative possibilities for successful generation of positive alpha is the *fundamental law of active management*, which says that the expected information ratio (ratio of expected alpha to volatility of alpha) should be related to: 1) the square root of the breadth of decisions within each category; 2) our ability to forecast changes in interest rates and spreads (measured by the information coefficient); and 3) portfolio constraints (measured by the transfer coefficient). See Grinold and Kahn [2000].

For example, tactical duration bets diverging from the normal duration style have low breadth because they amount to a forecast of the yield curve that might occur once per quarter, or a breadth of only four times per year (less if the forecasts are not truly independent). Residual tactical sector bets likewise have low breadth because only a handful of sectors are typically rotated, and fresh information to drive new decisions does not come in very frequently.

Selection bets (usually on credit), on the other hand, have high breadth because of the large number of individual securities available from which to decide active positions. Therefore, assuming equal information coefficients (skill), selection bets would have a higher expected information ratio than tactical duration bets and sector bets. The assessment may vary from investor to investor

EXHIBIT 2
Style Factors for Fixed-Income Managers



Estimated duration as of 12/31/2003.
Source: Barclays Global Investors and LehmanLive.com.

and from manager to manager, but it is generally true that a higher information coefficient would be needed for tactical bets than for selection bets to attain the same expected information ratio because of the greater breadth of opportunities for selection bets.

The transfer coefficient measures how efficiently insights can be translated into portfolio performance, given the impact of a long-only constraint or other portfolio constraints. See Clarke, de Silva, and Thorley [2002]. Most core fixed-income portfolios are long-only, but because they are run at fairly low risk levels, the loss in efficiency is moderate. High-yield portfolios, however, are run at higher risk levels, causing a material loss of efficiency and, all things equal, a lower expected information ratio. Long-short strategies in fixed-income, though typically run at higher risk, would not experience the same impact because the manager's signals can be translated into portfolio performance more efficiently.

Thus, the biases and types of bets a manager is making influence the expected information ratio, a crucial measure of portfolio performance. But how can we get a better understanding of the biases and bets an individual manager is making?

II. STYLE ANALYSIS

Style analysis amounts to measuring a manager's historical average beta exposures, and thus provides insight on the manager's forward-looking or normal exposures to sector and duration risk. Style analysis may be performed by a detailed analysis of actual holdings. In many cases, holdings data are not available or are incomplete, and the holdings-based style analysis may be impractical. It is often more convenient and in some cases more accurate to use return-based style analysis to determine historical fixed-income style, as developed by Sharpe [1988, 1992].

Return-based style analysis compares actual manager returns with (style) factor returns to determine *effective style*. Since its introduction, this robust technique has been popular with consultants and investors to help understand equity exposures in terms such as value versus growth or large-cap versus small-cap. It has not been applied in practice as broadly to fixed-income as to equity managers.

In some studies fixed-income style analysis is used to test hypotheses on the persistence of manager performance and to understand risk allocation over time. See Kahn and Rudd [1995, 1997], Kao [2000], and Brown and Marshall [2001]. Here my goal is to use style analysis to understand managers' exposures relative to the investor's benchmark, hoping to learn enough to provide better forward-looking style characteristic inputs into fixed-income manager structure optimization.

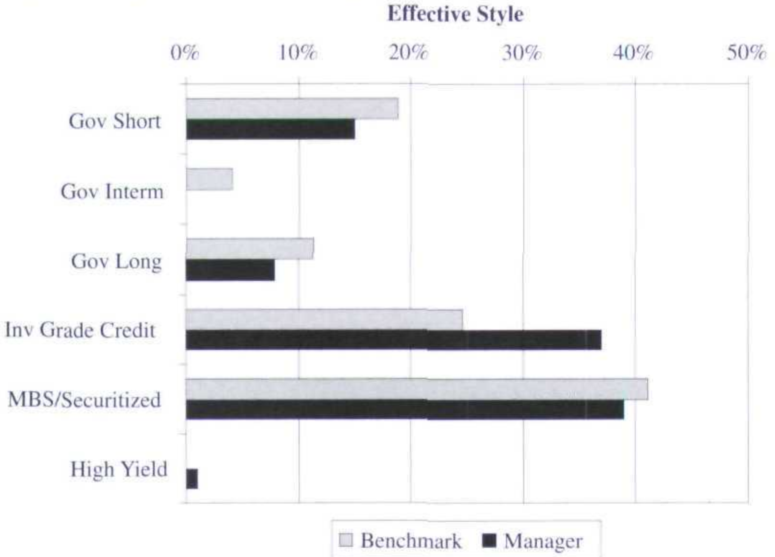
Style analysis applies a regression-like optimization process to a manager's historical returns in comparison to several style factors; see Appendix A. The results are often interpreted as the average historical portfolio weights for various style buckets. But what factors, or styles, should be used in the fixed-income context? In our case study, we shall suppose an investor's portfolio has a custom benchmark weighted at 85% Lehman Aggregate and 15% Lehman High Yield. Then, it is logical to use the Lehman Aggregate components plus a high-yield component as the factors when we perform style analysis of the investor's managers.

The structure of the factors for this example is shown in Exhibit 2. These factors cover the three macro sectors of the Lehman Aggregate plus high-yield. In addition, the government sector is broken into three maturity buckets that may further explain duration exposures that are not sufficiently explained by sectors.

This set of style factors is just one of a large set of possibilities for fixed-income style analysis. It is very suitable for most cases that involve core and core-plus managers combined with high-yield managers. In practice, the style factors will depend on the managers' stated benchmarks, the managers' normal portfolios, and the investor's benchmark.

In our case study, historical style factor weights are determined by a constrained regression of the manager's returns against the style factors in Exhibit 2. In essence, we are creating a factor model to explain the historical performance of an individual manager and, by consolida-

EXHIBIT 3
Style Analysis of Core Manager



tion, an entire portfolio of managers. This model may, if we are confident that it reveals a persistent pattern of style exposures, enable us to better predict the future returns of the managers from the factor model and better understand how the managers are blended to produce overall portfolio expected return and risk.

Example of a Core Manager

Exhibit 3 shows the results of style analysis for an individual core fixed-income manager. The results demonstrate a *material overweight to investment-grade credit* (style weight of 37% compared to a stated benchmark weight of 25%) and an underweight to governments. Further, the allocation across government maturities shows a barbell structure. There is an insignificant weight to high-yield, consistent with the fact that this manager is not invested in high-yield securities and that high-yield is not part of the Lehman Aggregate benchmark.[5]

The R^2 fit of the constrained regression is 98%, indicating that the simple factor model explains all but 2% of the variance in returns. The remaining 2% of unexplained variance may be classified as selection and tactical components of pure active risk. It amounts to approximately 0.6% pure (style-adjusted) active risk.

Example of a Core-Plus Manager

Exhibit 4 shows the style analysis of an individual core-plus manager. This term is often used to describe managers officially benchmarked to the Lehman Aggregate, but allowed and expected to invest outside that benchmark, typically in high-yield bonds and foreign government debt. As expected, there is a significant style weight to high-yield of 5%, consistent with the manager's core-plus scope, and an overweight to investment-grade credit (style weight of 36% compared to a stated benchmark weight of 25%). This particular core-plus manager also shows *a shift to longer maturities* within the government portfolio, contributing to a longer duration than the manager's stated benchmark.

The factor model explains 93% of the manager's variance in returns. The remaining residual risk amounts to approximately 1.2% pure active risk, about double the residual risk of the core manager.

Example of a High-Yield Manager

Exhibit 5 shows the style analysis of a high-yield manager. As expected, the style of this high-yield manager is most highly correlated with a high-yield index, but the style analysis also shows a significant weighting to credit of 10%. Perhaps the result is explained by the high-yield manager's tilt toward higher-quality credits compared to the average quality of the high-yield index. In any case, an allocation to this manager provides an exposure to high-yield that may be less than originally intended, and that may contribute to misfit risk at the investor level if it had been assumed that the manager is pure high-yield.

The style analysis of the high-yield manager explains 85% of the manager's variance in returns, lower than in the other examples. It is typical for a factor model of a high-yield manager to explain somewhat less of the total variance in returns because the model is based primarily on Lehman Aggregate components with only a single high-yield factor. In addition, high-yield managers typically make more selection bets and have larger risk budgets than core managers. These differences explain why a greater portion of the variance in returns of high-yield managers will be unexplained by the factor model.

The pure active risk for this manager is approximately 3.0%, significantly higher than the residual risk of the core and core-plus managers.[6]

EXHIBIT 4
Style Analysis of Core-Plus Manager

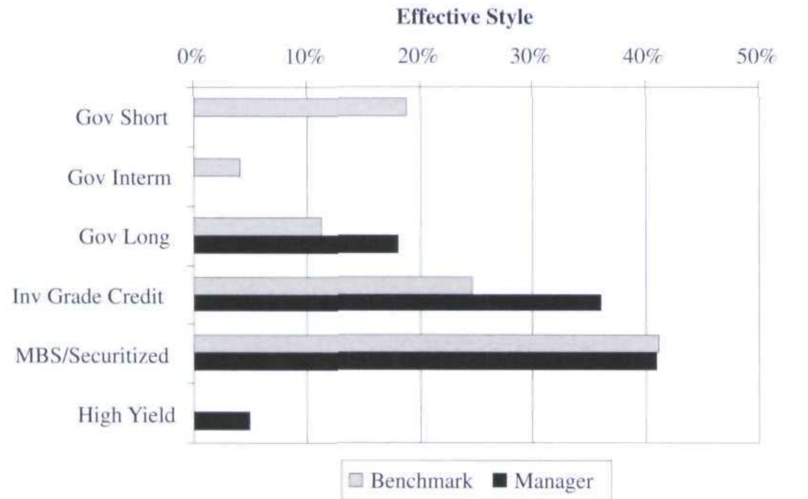


EXHIBIT 5
Style Analysis of High-Yield Manager

III. MANAGER STRUCTURE OPTIMIZATION

The analysis of manager style has been helpful in explaining how the type of biases and quality of bets each one is taking determine a large part of a manager's overall performance. This information could be helpful in developing a forecast of the normal portfolio for each manager. A meaningful assumption for the manager's normal style should be based on a *forward-looking* view of the manager's intentions, though, rather than automatically assuming that the manager's historical average style weights will be persistent.[7]

Forward-Looking Assumptions

In the absence of any additional information about the managers, we will use the historical style weights to forecast a manager's normal portfolio. The historical residual risk for the manager, calculated from the style analysis, is a useful indicator to forecast pure active risk. Assuming that the manager's investment process is stable, the amount of active risk generated by selection and tactical bets should be fairly steady, and in most cases the analyst will feel comfortable using the historical risk as a good estimate of future risk.

One should also look at the historical correlations of alphas across managers. Statistically significant correlations between bond managers are often observed that may be caused by similarity in investment processes. This correlation is detrimental to portfolio performance because, relative to a portfolio of managers with uncorrelated alphas, a correlated portfolio has higher risk.

Forecasting expected pure alpha for a manager is the most subjective and challenging part of the analysis. A common approach is simply to use historical alphas as a forecast of future alphas. This is a mistake, because in most cases historical performance is insufficient to demonstrate skill by the manager, and there is no guarantee that performance will be sustainable.[8]

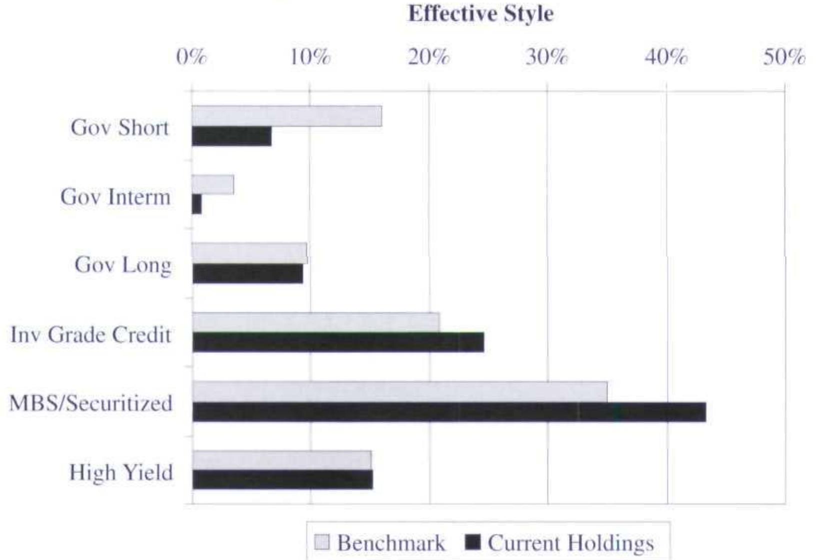
Financial analysts should be guided by their fundamental analysis of the manager's strategy and by the fundamental law of active management. To take best advantage of these insights, it may be helpful to first forecast an information ratio for each manager. Then, the forecasted information ratio is multiplied by the expected residual active risk to obtain expected alpha. Other analysts prefer to forecast an expected alpha directly. In either case, the expected alpha should be calculated after fees and costs.

Exhibit 6 summarizes the forecasted alpha assumptions for the case study and the assumed allocation to managers in the beginning portfolio (current holdings). This forecast, combined with the style analysis assumptions for each manager, provides the basic information we need to analyze the performance of the current hold-

EXHIBIT 6
Expected Alpha and Active Risk Assumptions for Managers

Manager	Type	Current Holdings	Expected Information Ratio	Residual Active Risk	Exp Alpha
Mgr A	Lehman Index	20%	0.00	0.10%	0.00%
Mgr B	Core	20%	0.67	0.60%	0.40%
Mgr C	Specialist	15%	0.40	1.20%	0.48%
Mgr D	Core-Plus	30%	0.30	1.50%	0.45%
Mgr E	High-Yield	15%	0.00	4.00%	0.00%
Mgr F	High-Yield	0%	0.20	5.00%	1.00%

EXHIBIT 7
Style of Current Holdings Portfolio



ings portfolio and to find alternative portfolios with improved performance.

Performance of Current Holdings Portfolio

Using the investor's current manager allocations, one can calculate portfolio alpha, portfolio active risk, and the effective style of the portfolio by weighting the managers' forward-looking assumptions by the investor's percentage allocation to each manager. The difference between the effective style of the total portfolio and the investor's benchmark is the total portfolio misfit.

Exhibit 7 shows the misfit of the current holdings portfolio, measured by the difference between the effective style of the portfolio and the 85% Lehman Aggregate/15% Lehman High Yield benchmark. There is a material overweight to credit and MBS/Securitized and underweight to the government sector. The net impact of this misfit is to produce a misfit risk of 0.18% and an expected return due to misfit of 0.05%.[9]

The misfit of the current portfolio is attributable mostly to differences in sector weights, with a negligible impact from differences in duration. In this instance, the duration of the current portfolio is 4.70 compared to benchmark duration of 4.64. The presence of misfit and misfit risk portrayed in this example is typical of fixed- income portfolios.

The expected alpha of the portfolio is 0.29% with a selection risk of 0.78%, which combine for a total active return of 0.34% and total active risk of 0.80%. The positive expected alpha is simply a result of our beliefs that the managers we have selected, as a group, have the skill that is needed to generate a consistent positive alpha. But is the portfolio optimal in any sense?

Optimization

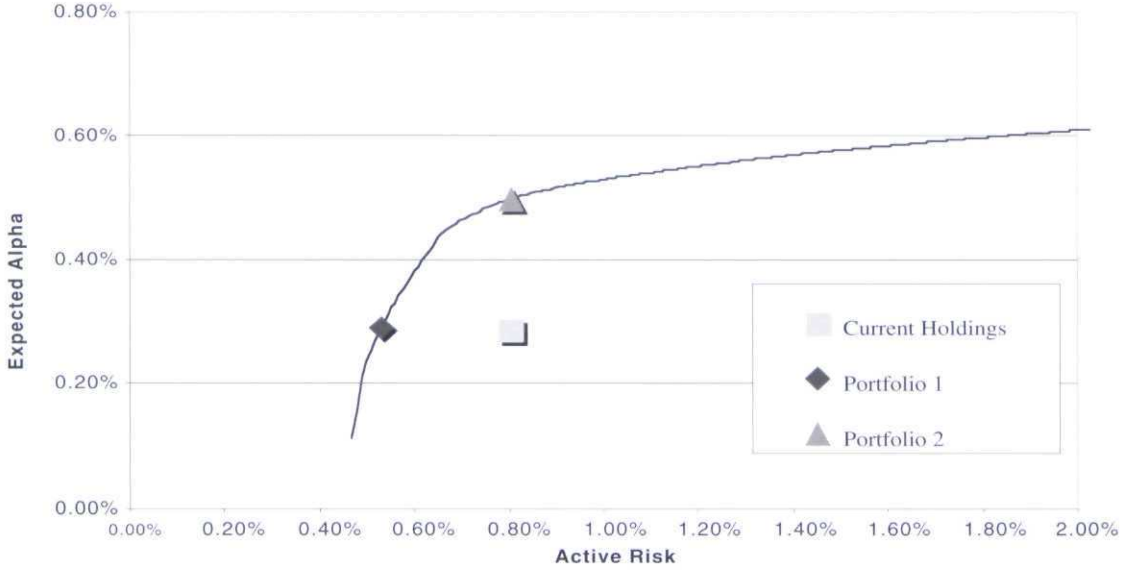
Building a portfolio of managers is an optimization problem. The objective is to maximize expected alpha subject to a penalty, or alternatively a constraint, for expected risk; see Appendix B. The decision variables are the percentage allocations to the various candidate managers. The mathematics of this problem are very similar to the calculation of an efficient frontier for the strategic asset allocation problem, and the solution is a set of optimal portfolios at various active risk levels.

Exhibit 8 shows the solution to the optimization problem as an efficient frontier plotted in active space—expected alpha and expected active risk.[10] It represents the set of portfolios with maximum alpha at a chosen level of active risk. The current holdings portfolio is also plotted, and apparently falls below the efficient frontier, signifying that there are alternative portfolios that would have better expected performance.

Two portfolios in Exhibit 8, among many, have better performance than the current holdings. Portfolio 1 has the same expected alpha, but active risk is reduced by 0.27 of a percentage point. Portfolio 2 has the same active risk budget, but provides an increase of 0.21 of a percentage point in expected alpha.

The comparison of current holdings with Portfolios

EXHIBIT 8
Active Efficient Frontier



1 and 2 is tabulated in Exhibit 9. The holdings of both Portfolios 1 and 2, as compared to the current holdings, reflect a shift toward higher information ratio managers. This is not a coincidence. The optimal solution for active management implies that the holdings of a manager should be approximately proportional to the expected information ratio of the manager and inversely proportional to the manager's contribution to active risk. See Grinold and Kahn [2000].

Note that Portfolio 2 is 100% active. An analysis of the efficient frontier indicates that at active risk levels above 0.66%, the index manager is not part of the optimal portfolio. Below that level, some portion of the portfolio must be held in the index fund in order to achieve a lower risk level. Therefore, the long debate on the question of active versus passive management of fixed-income portfolios is solved by optimization.

The optimal mix of active and passive investment depends on the investor's forecast of the performance of the active managers. It also depends on the active risk budget, or, equivalently, the investor's risk-return preferences. Any portfolio that is other than 100% passive is implicitly assuming that managers have skill in generating alpha.

The latter point emphasizes the importance of facing the challenge of forecasting expected alpha. There is always a temptation to use historical alphas for managers, or to take the easy way out. But expected alpha is very unlikely to be the same as historical alpha. Instead, the investor must carefully estimate both forward-looking alpha and forward-looking manager style to gain the benefits of manager structure optimization.

IV. SUMMARY

The dismal average performance of fixed-income portfolios may be explained by their implicit biases or exposures at variance with the benchmarks and other bets that are compensated only in the hands of the most skillful. Fortunately, there is a better way to control risk and structure a portfolio of managers, which starts with measuring the actual exposures of managers in reference to the investor's benchmark.

The process of understanding managers' exposures can be framed by fixed-income style analysis and supplemented by fundamental due diligence on managers. The manager's investment process, the historical performance, and the types of bets the manager is making can be helpful to inform our judgment of expected alpha. Successful implementation requires meaningful forward-looking inputs because the optimization analysis adds no value if the inputs to it are not skillfully formed.

Optimization at the fixed-income portfolio level implies finding alternative portfolios that provide the highest expected alpha at a given level of active risk. Managers of all different types—index, core, core-plus, high-yield, and specialist—can be considered. The key to better performance is to find managers who can produce con-

EXHIBIT 9
Alternative Portfolios

Manager	Type	Current Holdings	Portfolio 1	Portfolio 2
Mgr A	Lehman Index	20%	32%	0%
Mgr B	Core	20%	38%	50%
Mgr C	Specialist	15%	7%	18%
Mgr D	Core-Plus	30%	10%	20%
Mgr E	High-Yield	15%	7%	0%
Mgr F	High-Yield	0%	6%	12%
Total		100%	100%	100%
Expected Alpha		0.29%	0.29%	0.50%
Selection Risk		0.78%	0.50%	0.77%
Misfit Risk		0.18%	0.19%	0.25%
Active Risk		0.80%	0.53%	0.80%
Duration		4.70	4.63	4.54
Misfit Return		0.05%	0.02%	0.03%

sistent pure, or style-adjusted, alphas, i.e., alphas relative to their own normal portfolios. Then, a portfolio can be constructed by manager structure optimization techniques to blend the variety of styles and exposures contributed by each manager.

APPENDIX A

Style Analysis

Style weights for the n-th manager are found by solving the optimization problem: minimize $\sigma^2_{e_n}$ subject to: $r_{nt} = \boldsymbol{\beta}^T_n \mathbf{f}_t + e_{nt}$, $\boldsymbol{\beta}^T_n \mathbf{1} = 1$ and $\boldsymbol{\beta}_n \geq 0$, where $\sigma^2_{e_n}$ is the sample variance of the residual variable; r_{nt} is the manager's realized return across months t; $\boldsymbol{\beta}_n = \{\beta_{nk}\}$ is the vector of fitted values (style weights) for the n-th manager across the k styles over the time period studied; $\boldsymbol{f}_t = \{f_{kt}\}$ is the vector of realized style factors (realized benchmark component returns) for month t; and residual e_{nt} is the portion of the manager's return for month t that is not explained by the style factors. It is usually convenient to base the choice of style factors on benchmark return components that span the investor's benchmark.

The style weights represent the best-fit coefficients of a constrained regression problem, and may be interpreted as model holdings of the indexes that would most closely simulate the observed returns of the managers. The constraints, requiring that the style weights are non-negative and sum to one, make the optimization problem different from an ordinary regression. Alternatively, without the constraints, the style weights may be interpreted as factor betas, measuring the sensitivity of changes in the manager's returns to changes in the respective factors. The average value of the residual variable e_n is interpreted as the realized monthly pure or style-adjusted alpha, and the standard deviation of the residual variable is interpreted as the realized monthly style-adjusted active risk.

Style analysis, combined with fundamental analysis, is useful in estimating a manager's forward-looking exposures to components of the investor's benchmark. Once estimated, the forward-looking normal styles of the managers are represented by the matrix $\mathbf{B} = \{\beta_{nk}\}$ where is the n-th manager's exposure to the k-th benchmark factor. The financial analyst also needs to estimate the forward-looking expected alpha for each manager $\boldsymbol{\alpha} = \{\alpha_n\}$. Then, the expected (excess) return of the portfolio of managers may be expressed as:

$$r = \beta r_b + \alpha \tag{A-1}$$

where r is the expected return of the portfolio; $\beta = \frac{\mathbf{h}^T\mathbf{B}\mathbf{r}_b}{\mathbf{h}_b^T\mathbf{r}_b}$ is the portfolio's beta relative to the benchmark; $r_b = \mathbf{h}^T_b\mathbf{r}_b$ is the expected return of the benchmark; $\alpha = \mathbf{h}^T\boldsymbol{\alpha}$ is the expected residual return; $\mathbf{h} = \{h_n\}$ is the vector of portfolio holdings of managers; $\mathbf{h_b} = \{h_{bk}\}$is the vector of benchmark weights; and $\mathbf{r}_b = \{r_{bk}\}$ is the vector of expected benchmark component returns.

By subtracting the expected benchmark return from the total portfolio return, the expected active return may be written:

$$r_a = \beta_a r_b + \alpha \tag{A-2}$$

where r_a is the expected active return, and

$$\beta_a = \beta - 1 = \frac{(\mathbf{h}^T\mathbf{B} - \mathbf{h}_b^T)\mathbf{r}_b}{\mathbf{h}_b^T\mathbf{r}_b} \equiv \frac{\mathbf{b}_a^T\mathbf{r}_b}{\mathbf{h}_b^T\mathbf{r}_b} \tag{A-3}$$

is the (scalar) active beta, where $\mathbf{b}_a$ is the vector of net active positions (benchmark components) for the portfolio that have been determined by the managers' individual exposures and the investor's proportionate holdings of the managers.

Then, the volatility of active returns can be calculated:

$$\sigma^2_a = \beta^2_a\sigma^2_b + \omega^2 \tag{A-4}$$

where σ^2_a is the variance of active returns; σ^2_b is the variance of benchmark returns; and ω^2 is the variance of residual returns. In this model, the first terms of Equation (A-2) and Equation (A-4) are the expected active return and expected active risk components, respectively, for style (misfit) associated with the investor's normal portfolio. The second terms of Equation (A-2) and Equation (A-4) are the expected active return and expected active risk components, respectively, associated with the combined effects of residual selection and residual tactical bets of all the managers relative to the investor's normal portfolio.

We now separate the vector of active beta positions $\mathbf{b}_a$ into two additive components – one associated with active sector biases, $\mathbf{b}_s$, and another associated with active duration biases, $\mathbf{b}_d$, so that $\mathbf{b}_a = \mathbf{b}_s + \mathbf{b}_d$. Vector $\mathbf{b}_s$ has non-zero elements for the dimensions primarily associated with sector weights, and vector

$\mathbf{b}_d$ has non-zero elements for the dimension primarily associated with different maturities or duration weights.

This additive constraint also yields additivity of the active beta components: $\beta_a = \beta_s + \beta_d$, where the component betas are defined analogously to Equation (A-3). Then, the expected active return on the portfolio may be written:

$$r_a = \beta_s r_b + \beta_d r_b + \alpha \tag{A-5}$$

Equation (A-5) shows that the active style (misfit) return component of total active return can be separated into its active sector and duration bias components. The active risk calculated from this decomposition is:

$$\sigma_a^2 = \beta_s^2\sigma_b^2 + \beta_d^2\sigma_b^2 + 2\beta_s\beta_d\sigma_b^2 + \omega^2 \tag{A-6}$$

The first two terms of Equation (A-6) represent the active risk contributed by sector and duration biases, respectively, implicit in the normal portfolio; the third term shows an additional active risk component caused by the interaction of the two effects.

It is possible to allocate the cross-term of Equation (A-6) to sector and duration components so that the variances due to sector bias, duration bias, and selection can be summed up as the total active variance. This can be accomplished by a regression first on the sector factors and then regressing the residuals on the duration factors. A more advanced technique employs a one-step generalized moment method.

It is also theoretically possible to partition the residual terms of Equations (A-5) and (A-6), α and ω^2, into the components associated with tactical bets (deviations from the investor's normal sector and duration biases) and selection bets (representing the pure active return). Tactical bets may be observed by examining the 36-month rolling style analysis of a manager over a number of years. A frequent change in style weights from one 36-month period to the next is evidence to support the presence of tactical residual bets.

APPENDIX B

Objective Function for Manager Structure Optimization

The objective function for manager structure optimization follows the development in Grinold and Kahn [2000] and Waring et al. [2000]. Following the notation in Appendix A, the expected return of the portfolio in excess of the risk-free rate is:

$$r = r_a + r_b = \beta_a r_b + \alpha + r_b \tag{B-1}$$

and the variance of return is:

$$\sigma^2 = \sigma_a^2 + \sigma_b^2 + 2\mathrm{cov}(r_a, r_b) = \beta_a^2\sigma_b^2 + \omega^2 + \sigma_b^2 + 2\beta_a\sigma_b^2 \tag{B-2}$$

The proposed objective function trades off expected return against risk, but here we allow for different risk penalties associated with different types of risk.

We assume that λ_a is the risk penalty associated with active risk, λ_b is the risk penalty associated with market risk and the covariance term. Then, the overall objective function is to maximize:

$$U = \underbrace{r_b - \lambda_b\sigma_b^2}_{SAA} + \underbrace{\beta_a r_b + \alpha - \lambda_a(\omega^2 + \beta_a^2\sigma_b^2) - 2\lambda_b\beta_a\sigma_b^2}_{MSO} \tag{B-3}$$

The terms relating to the strategic asset allocation (SAA) objective are separated from the terms for the manager structure optimization (MSO) objective. Since the MSO problem optimizes the manager holdings separate from the SAA problem, for our purposes we may focus solely on the MSO components from the objective function. Further, by using an equilibrium condition for λ_b, namely, $r_b = 2\lambda_b\sigma_b^2$, the MSO objective function simplifies to:

$$U_a = \alpha - \lambda_a(\omega^2 + \beta_a^2\sigma_b^2) \tag{B-4}$$

The expected misfit return and the covariance term are dropped. In words, the MSO objective (in the absence of exceptional forecasts) is to maximize expected pure alpha subject to total active variance. Active variance includes both residual risk associated with alpha and misfit risk associated with the active beta.

It may be argued that the risk penalty associated with the covariance term could be a value λ_c intermediate between λ_a and λ_b, reflecting a compromise position between aversion to active risk and aversion to market risk. Under this assumption, the MSO objective function in Equation (B-3) would instead be:

$$U_a = \alpha + \beta_a r_b - \lambda_a(\omega^2 + \beta_a^2\sigma_b^2) - 2\lambda_c\beta_a\sigma_b^2$$

Using the equilibrium condition for λ_b, it simplifies to:

$$U_a = \alpha - \lambda_a(\omega^2 + \beta_a^2\sigma_b^2) - 2(\lambda_c - \lambda_b)\beta_a\sigma_b^2$$

The case examples in this article use the objective function of Equation (B-4). The assumptions behind (B-4) imply that, even though the investor experiences misfit return as part of active return and experiences covariance risk as part of total return risk, there is no net impact of these terms on the investor's expected utility.

ENDNOTES

The author thanks Barton Waring and colleagues in the Client Advisory Group and the Fixed Income Group of Barclays Global Investors for their insights and encouragement.

[1]See Arnott and Bernstein [1988]. Setting target duration is a decision in the realm of the strategic asset allocation process, rather than the manager structure problem that is the subject of this article. Derivative instruments can also be used to extend duration while enabling the investor to separate the manager structure decision from the duration-matching objective. Once the target duration is set, duration-matching of a fixed-income portfolio is just part of the larger problem of managing misfit risk. In the case study presented here, the duration of the investor's custom Lehman Aggregate/High Yield benchmark is approximately 4.6 years while the individual managers in the example have durations in the range of 4.6 to 5.0, except for the MBS specialist manager who has a much lower duration of 3.1. The durations of current holdings, optimized Portfolio 1, and Portfolio 2 are within $\pm$ 0.1 year of the benchmark duration. The manager structure optimization process automatically corrects for duration misfit as long as there are candidate managers with an expected duration range wide enough to encompass the target duration.

[2]See Waring and Siegel [2003] for a comprehensive review of the principles related to holding active managers in a portfolio. Many large institutional investors manage a significant portion of their fixed-income investments in-house. In this case, the investor should evaluate the internal manager's performance and role in the portfolio in the same manner as proposed here for external managers. The fully allocated costs of the internal manager can be used to simulate management fees.

[3]Modern portfolio theory does not prescribe what level of active risk an investor should bear even in the presence of skill—only that the decisions on investment allocation ought to be mean-variance-efficient. The level of active risk taken in the fixed-income portfolio is usually significantly less than the active risk taken in the equity portfolio. At low levels of risk, assuming a constant information ratio, alpha is consumed by fees; see Kahn [1998]. Therefore, it can be argued that investors are not taking enough active risk in their fixed-income portfolios.

How should the investor go about spending more active risk while seeking higher alpha? Some investors are addressing this issue by reducing the index allocation and replacing it with risk-controlled core managers. Risk-controlled managers usually do not deviate from benchmark style and, assuming skill, can add expected alpha efficiently (without misfit risk). At the other end of the spectrum, some investors are allocating more funds to credit long-short strategies and high-alpha, high-risk, market-neutral strategies that can be "bondized." These strategies, in the presence of skill, show great promise when compared to traditional active strategies.

[4]For example, an overweight exposure to the Lehman Credit Index compared to the Lehman Aggregate Index will result in an unconditional (but not guaranteed) added expected return on the order of 0.4%, based on historical averages and capital asset pricing relationships. It will also add a predictable amount of additional risk. In equilibrium, the net impact on investor utility associated with the return component of this overweight to credit would be canceled out by a component of the associated risk. The investor's desired exposures to style are more effectively incorporated in the strategic asset allocation policy, rather than appearing as a bias in any manager's active style.

[5]Although it is not evident in this instance, it is often possible to see more significant weights to high-yield for core managers if their investment-grade credit investments have a slant to lower-quality credit. The style (regression) analysis will interpret higher spreads associated with a lower-quality credit as a blended portfolio of average quality investment-grade credit and high-yield credit.

[6]The example style results for all the managers in the case study are tabulated below, including an estimate of duration for each manager based on the manager's historical style and the durations of the component factors. There is also an index fund, a mortgage specialist, and a second high-yield manager.

Manager	Type	Gov Short
Mgr A	Lehman Index	18.9%
Mgr B	Core	15.0%
Mgr C	Specialist	0.0%
Mgr D	Core Plus	0.0%
Mgr E	High Yield	0.0%
Mgr F	High Yield	0.0%

Manager	Gov Interm	Gov Long	Inv Gr Credit
Mgr A	4.1%	11.4%	24.6%
Mgr B	0.0%	8.0%	37.0%
Mgr C	0.0%	0.0%	0.0%
Mgr D	0.0%	18.0%	36.0%
Mgr E	0.0%	0.0%	10.0%
Mgr F	0.0%	0.0%	0.0%

Manager	MBS/Securitized	High-Yield	R Squared	Duration
Mgr A	41.1%	0.0%	100.0%	4.60
Mgr B	39.0%	1.0%	98.1%	4.59
Mgr C	100.0%	0.0%	86.1%	3.10
Mgr D	41.0%	5.0%	93.0%	5.51
Mgr E	0.0%	90.0%	85.1%	4.98
Mgr F	0.0%	100.0%	93.7%	4.90

[7]The historical style weights, reflecting the sector and duration biases of the manager, can be evaluated for consistency over time by a rolling-window analysis. It is common practice to examine rolling 36-month periods. This is a compromise between a period long enough for statistical purposes while short enough to show possible changes in the manager's style over time. In most cases, a manager's style will be found to be consistent over time, and if so it is reasonable to forecast that the manager's style will be similar in the future. Managers may exhibit a gradual style drift, and in some cases will show aggressive style rotation. These latter cases challenge the analyst's judgment in forming the estimate of the forward-looking normal portfolio. In practice, a fundamental analysis of the

manager should be combined with the history to form a judgment-based forecast of the manager's normal portfolio.

[8]The analyst can perform a statistical test of the hypothesis that the historical performance was produced by luck rather than skill. See Grinold and Kahn [2000, pp. 325-327].

[9]The expected return due to misfit is based on market expectations of the spreads of the components of the benchmark. For this example, the estimates are as follows:

	Benchmark Weights	Duration	Expected Market Return Relative to Benchmark
Gov Short	16.0%	2.4	-0.65%
Gov Interm	3.5%	5.9	-0.40%
Gov Long	9.7%	10.8	-0.15%
Inv Gr Credit	20.9%	5.7	0.35%
MBS/Securitized	34.9%	3.1	-0.40%
High Yield	15.0%	4.9	1.35%
Total (Benchmark)	100.0%	4.6	-

The duration estimates for components of the benchmark are also presented for reference.

[10]The efficient frontier of Exhibit 8 and the summary presented in Exhibit 9 are the result of optimizing expected alpha against an active risk penalty associated with selection and misfit variance. The investor will also experience expected misfit return and additional covariance risk proportional to the active beta exposure of the portfolio. In our case example, the estimated impact is as follows:

	Current Holdings	Portfolio 1	Portfolio 2
Expected Misfit Return	0.05%	0.02%	0.01%
Additional Covariance associated with Active Beta	0.0024%	0.0007%	0.0006%

As noted in Appendix B, there is a canceling effect, after considering capital market equilibrium conditions, ensuring that the expected utility of each portfolio is unaffected by these impacts. Therefore, the investor's preferences for Portfolio 1 and Portfolio 2 over current holdings are not changed.

REFERENCES

Arnott, Robert D., and Peter L. Bernstein. "The Right Way to Manage Your Pension Fund." *Harvard Business Review*, January-February 1988.

Brown, David T., and William J. Marshall. "Assessing Fixed-Income Manager Style and Performance from Historical Returns." *The Journal of Fixed Income*, March 2001, pp. 15-25.

Clarke, Roger, Harindra de Silva, and Steven Thorley. "Portfolio Constraints and the Fundamental Law of Active Management." *Financial Analysts Journal*, September/October 2002.

Grinold, Richard C., and Ronald N. Kahn. *Active Portfolio Management*, 2nd ed. New York: McGraw-Hill, 2000.

Kahn, Ronald N. "Bond Managers Need to Take More Risk." *The Journal of Portfolio Management*, Spring 1998, pp. 70-76.

Kahn, Ronald N., and Andrew Rudd. "Does Historical Performance Predict Future Performance?" *Financial Analysts Journal*, November/December 1995, pp. 43-52.

——. "The Persistence of Fixed Income Style Performance: Evidence from Mutual Fund Data." Chapter 18 in F.J. Fabozzi, ed., *Managing Fixed Income Portfolios*. New York: John Wiley & Sons, 1997, pp. 299-307.

Kao, Duen-Li. "Analyzing Investment Styles of Fixed Income Managers." *The Journal of Investment Consulting*, vol. 3, no. 1 (December 2000).

Sharpe, William F. "The Arithmetic of Active Management." *Financial Analysts Journal*, January/February 1991.

——. "Asset Allocation: Management Style and Performance Measurement." *The Journal of Portfolio Management*, Winter 1992, pp. 7-19.

——. "Determining a Fund's Effective Asset Mix." *Investment Management Review*, November/December 1988, pp. 59-69.

Siegel, Laurence B. "Benchmarks and Investment Management." The Research Foundation of the Association for Investment Management and Research, 2003.

Waring, M. Barton, and Laurence B. Siegel. "The Dimensions of Active Management." *The Journal of Portfolio Management*, Spring 2003.

Waring, M. Barton, Duane Whitney, John Pirone, and Charles Castille. "Optimizing Manager Structure and Budgeting Manager Risk." *The Journal of Portfolio Management*, Spring 2000.

To order reprints of this article, please contact Ajani Malik at amalik@iijournals.com or 212-224-3205.